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COMPETITION AND IRREVERSIBLE INVESTMENTS UNDER UNCERTAINTY

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Competition and Irreversible Investments under Uncertainty*

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Abstract

We examine the effect of competition on investment decisions in an industry in which each firm has a completely irreversible investment opportunity and the product market has positive externalities for a small market size and negative externalities for a large market size. In the latter case, which corresponds to the traditional competitive industries, firms invest sequentially as market profitability develops. In the former case, which corresponds to industries in which investment is mutually beneficial, firms invest simultaneously after the market's profitability has developed sufficiently to gain all network benefits and to recover the option value of waiting. These extensions of a "real options" analysis may help explain rapid and sudden developments such as recent Internet investment, or explain the late take-off phenomenon of prolonged start-up problems, such as the case of fax machine production.

key words: Irreversible Investments, Real Options, Network Effects.

JEL: C61, D81, G31

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1 Introduction

Investment is defined as the act of incurring an immediate cost in the expectation of future payoff. However, when the immediate cost is sunk (at least partially) and there is uncertainty over future rewards, the timing of the investment decision becomes crucial (Dixit and Pindyck, 1994, p. 3). In particular, irreversibility and uncertainty make firms invest only when the value of the investment is more than the value of the option of waiting before making an irreversible decision.

This paper extends this model, taking strategic interactions into account. Specifically, we analyze the effect on competition on firms' optimal investment strategies in an industry having a large number of identical firms engaged in an investment game to enter a new product market. We consider a sector where each firm has only one completely irreversible investment opportunity and the market has an inverted U-shape relation between profits and industry size. That is, positive externalities tend to dominate for low initial market size, while negative externalities dominate at higher market levels.

Although we do not refer to a particular product, there are many markets that have greater profitability when more than one firm has already invested. In the case of goods with "network externalities", consumer's advantage increases as the total number of consumers purchasing the same or compatible brands increases. An example is the decision by rival firms to set up an interconnected network to satisfy an interdependent demand for telecommunication services by many potential customers (Rohlfs, 2001, p. 34).[1] Another relevant case involves a high degree of complementarity between different goods e.g. for software and hardware (Katz and Shapiro, 1985). Generally, software packages are produced by many firms so that they can be used by the same hardware. Thus, the greater the variety of software supporting a certain hardware, the greater the value of this hardware and the greater the advantage consumers directly gain from the variety of software supporting that hardware. Some authors refer to this as "indirect network externalities" (Shy, 2001, p.52) or "complementary bandwagon effects" (Rohlfs, 2001, p. 47-48).[2] In other cases, the utility of each consumer

[1] Other examples are access to the web via Internet Service Providers, mobile phones using a particular standard (GSM,CDMA), Electronic Messaging System (EMS), videotext system, etc.

[2] Jeffrey H. Rohlfs (2001) coined the term bandwagon effect for the benefit that a person enjoys as a result of others' doing the same thing that he or she does, and specifically he used the term network externalities for the bandwagon effect that applies to the user set

decreases as more consumers buy the good. This occurs because of congestion, as the communication and information-based industries are recently experiencing. Even though the introduction of a new Web site increases the value of the Internet to every existing user, the progressive increase of its use increases congestion measured in term of excessive delay of transmission (longer connection time spent to load a Web page) or loss of service altogether (Odlyzko, 1999). Congestion then reduces consumers' utility of joining the Internet and passes this disadvantage to the firms by reducing the demand of access.[3]

The negative externalities case corresponds to the traditional competitive industry in which the investment of one firm lowers the profitability of the others. In this case the introduction of competition has two opposing effects which offset each other. Firstly, competition reduces the expected profit flow that derives from the investment, which tends to delay investment. Secondly, competition introduces a strategic benefit in favour of the investment as it deters the investments by rivals. Leahy (1993) first discovered this property, showing that the optimal investment strategy of a competitive firm is equal to that of a single firm in isolation. In this case, firms enter sequentially as market profitability increases.

On the contrary, when investments are mutually beneficial, the optimal investment policy is essentially a question of coordination. As the timing of a firm's entry is influenced by the entry decisions of others, Leahy's result cannot be applied. Two equilibria can emerge: either the industry remains locked-in with no entry as long as very pessimistic expectations dominate the market, or a mass of firms simultaneously runs to enter, driven by the expected rents generated by the positive externalities.[4] Excluding the former, we show that this "network run" is triggered when the profitability of the market has developed sufficiently to allow the firms to capture all bandwagon benefits and to recover the option value of waiting due to the irreversibility. This also determines endogenously the optimal start-up size of the industry.

Therefore, our model is an extension of the dynamic equilibrium in a competitive industry presented by Leahy (1993) and Dixit and Pindyck

of a communication network.

[3]See, for example, DaSilva (2000) and Falkner et al. (2000), for a survey on the literature on how to price congestible networks as Internet.

[4]This is what Rohlfs (2001, p. 16-17) defines a "*chicken-egg problem*"; nobody joins the network because the size of the network is zero, but the size of the network is null because no one has joined it.

(1994, ch.5).[5] Furthermore, Nielsen (2002), focusing on a duopoly model with positive externalities, predicts a similar result to ours, namely that firms invest simultaneously at the market profitability given by the duopoly solution.[6] More generally, this result holds in a free entry framework.

The paper is organized as follows. Section 2 states the basic assumptions behind the model. Section 3 gives the main results of the paper, namely the optimal entry strategy in the presence of positive and negative externalities. Section 4 places the paper within the context of the literature on irreversible investment and market structure. Section 5 concludes and the Appendix contains the proofs omitted in the text.

2 Model and assumptions

We have considered the decision to enter a new market subject to uncertain returns by a large number of identical firms. In order to focus exclusively on the competitive timing process, we have abstained from explicitly describing either the product market decisions (price or quantity), or the firm size, and we have assumed that entry costs required to initiate the technology projects are given. These conditions are summarized by the following assumptions:[7]

1. At any time t an idle firm may decide to enter a new market. Firms are risk-neutral and discount the future returns at the risk-less interest rate ρ.[8]

2. All firms are identical. Their size dq is infinitesimally small with respect to the market.[9]

[5] Baldursson (1998) and Grenadier (2002) extends Dixit and Pindyck's model considering Cournot-Nash competition. Their analysis indicates that although qualitatively the investment process is similar in oligopoly and competitive equilibrium, oligopoly quantitatively slows investment.

[6] Huisman (2001, ch.8) extends the Nielsen (2002) model introducing asymmetry into the investment cost of firms. Although cost asymmetry may reduce the positive externality effect, both firms invest simultaneously and early, anticipating that the other will also invest early.

[7] These assumptions rule out market structure and monopoly power, which are beyond the scope of this paper. (see Ericson and Pakes, 1995; Amir and Lambson, 2003).

[8] Introducing risk aversion does not change the results since the analysis can be developed under a risk neutral probability measure (Cox and Ross, 1976; Harrison and Kreps, 1979).

[9] Many infinte time models of industry investment evolution show this assumption (see Dixit, 1989; Dixit and Pindyck, 1994; Jovanovic, 1982; Hopenhayn, 1992; Lambson, 1992; Bartolini, 1993).

3. Each firm can enter by committing forever to a flow cost w or undertaking a single irreversible investment which requires an initial sunk cost $K \equiv w/\rho$.

4. Once the investment is undertaken, it cannot be abandoned.[10]

5. Firms are free to enter. That is, first they decide whether or not to enter (and pay the entry cost K) and then compete for the available rents (generated by positive externalities).

6. After entry, firms sell a continuos flow of one unit of output. Thus, q indicates the number of firms currently active (incumbents) as well as the total demand.[11]

7. Each firm produces a flow of operating profits that we have abbreviated as:
$$\pi(q_t, \theta_t) \equiv D(q_t)\theta_t \tag{1}$$
where θ_t is a multiplicative industry-specific shock at time t.[12] We may consider, in a simpler setting, $D(q_t)$ as the inverse demand function (Dixit and Pindyck, 1994, ch. 9; Bartolini, 1993; Nielsen, 2002), or as a reduced form of a more general profit function (Dixit and Pindyck, 1994, ch. 11; Dixit, 1995; Grenadier, 2002). Time is continuous, $t \in [0, \infty)$, and suppressed if not necessary.

8. The function $D(q)$ is twice continuously differentiable in q, and it is increasing over the interval $[0, \bar{q})$ and decreasing thereafter (see figure 1). That is, there are positive externalities to investment which can be caused by "network externalities" or by complementary products, over $[0, \bar{q})$. After $\bar{q}$ it is better that no other has invested for any individual firm, given that competition and/or congestion may occur (Shy, 2001, ch. 5). We also assume that at zero and at some finite

[10] Besides irreversibility, this assumption avoids the need to consider such operating options such as reducing output or even shutting down, thereby considering reducing variable costs. For further details see e.g., Dixit and Pindyck (1994).

[11] None of the following results depend on this assumption (Grenadier, 2002).

[12] This assumption highly simplifies the analysis of the industry equilibrium without reducing the impact of the results obtained. By considering only industry-wide uncertainty, a firm knows that if θ rises entry becomes, *ceteris paribus,* as attractive for the other firms as for itself. Then the entry of new firms may overaccelerate investment if the industry has positive externalities or it may dampening profits if negative externalities are at work. See Jovanovic, (1982), Hopenhayn, (1992) and Miao (2005) for models of industry equilibrium with only firm-specific shocks, and Caballero and Pindyck (1992) for a more general model where both uncertainties coexist.

number of firms $\mathfrak{Q}$ ($\mathfrak{Q} >> \bar{q}$), profits falls to zero, i.e. $D(0) = 0$, and $D(\mathfrak{Q}) = 0$, whatever the value of θ. As $\mathfrak{Q}$ could be arbitrarily large, this assumption is harmless in our setting.

Figure 1 about here

9. Finally, the industry-specific shock θ follows a geometric diffusion process:

$$d\theta = \alpha\theta dt + \sigma\theta dW \quad \text{with } \theta_0 = \theta \text{ and } \alpha, \sigma > 0. \tag{2}$$

Applying Itô's Lemma to (1) and substituting (2) to eliminate $d\theta$, an expression for the profit process in terms of the shock and the number of firms emerges as:

$$d\pi = \mu(q)\pi dq + \alpha\pi dt + \sigma\pi dW, \quad \text{with } \pi_0 \equiv u(q_0)\theta_0 = \pi \tag{3}$$

where $\mu(q) \equiv D'(q)/D(q)$ indicates the direct effect of entry. From (3), entry influences the level of profits through its effect on the market equilibrium, depending on the initial size of the industry. In particular, given any value of the shock θ, more firms in the market implies a higher or lower equilibrium level of profits depending on the presence of positive $\mu(q) > 0$ or negative $\mu(q) < 0$ externalities, respectively.

3 The main results

This section summarizes the main properties of the entry process, emphasizing the economic reasoning behind it. All proofs are in the Appendix.[13]

3.1 Negative externalities

If the initial size of the industry is $q \geq \bar{q}$, we expect entry to work as follows: for a fixed number of firms, profits move according to the above stochastic process with $\mu(q)\pi dq = 0$. If profits then climb to a level $\pi^* \equiv D(q)\theta^*$, entry will become feasible and profits will drop along the function $D(q)$. In technical terms this means that the threshold π^* becomes an upper *reflecting barrier* on the profit process.[14] Profits will then continue to move

[13] Although the inverted U-shape of (1) implies an entry process that meets positive externalities first, we have solved the investment problem by working backward starting from the negative externalities interval.

[14] The profit function follows a regulated Brownian motion in the sense of Harrison (1985).

stochastically without the term $\mu(q)\pi dq$ until another entry episode occurs. In addition, since the industry-wide shock θ makes all firms symmetric, some random mechanism must be used to select which idle firm will enter first.

A competitive equilibrium can be defined as a symmetric Nash equilibrium in entry strategies which bound the profit process of the firms. It can be built simply from the entry policy of a single firm in isolation regardless of future entry decisions.[15]

This remarkable property, first discovered by Leahy (1993), has an important operative implication: the optimal competitive equilibrium policy need not take account of the entry effect. The profit level, say $\hat{\pi}$, that triggers entry by an individual firm in isolation is identical to that of a firm that correctly anticipates the other firms' strategies π^*. That is, when a firm decides to enter it can claim to be the last to enter the industry, ignoring future entry by other firms. This behavior can be summarized by the following proposition.

Proposition 1 *The candidate policy for optimal entry in a competitive industry, characterized by an initial mass of firms $q \in [\bar{q}, \mathfrak{Q}]$ is described by the following upper profit threshold:*

$$D(q)\theta^*(q) = \frac{\beta}{\beta - 1}(\rho - \alpha)K \equiv \pi^*(= \hat{\pi}), \quad \textit{with } \frac{\beta}{\beta - 1} > 1 \tag{4}$$

where $\rho > \alpha$ and $\beta > 1$ is the positive root of the auxiliary quadratic equation $\Psi(x) = \frac{1}{2}\sigma^2 x(x-1) + \alpha x - \rho = 0$.

Proof. See Leahy (1993) and Appendix ■

With q incumbents, an idle additional firm will invest if the present value of its profits at entry, $\frac{D(q)\theta^*(q)}{\rho-\alpha}$, exceeds the cost of the investment K augmented by the option to wait $\frac{1}{\beta-1}K$.[16] Over the range $[\bar{q}, \mathfrak{Q}]$, additional entry occurs every time profits reach to the known threshold π^*; if profits stay below this barrier, no new investment is undertaken.

Although at first glance this result seems surprising, it is consistent with the properties of the dynamic programming principle of optimality for a

[15] "...., each firm can make its entry decision by finding the expected present value of its profits *as if it were the last firm that would enter this industry*, and then making the standard option value calculation. While the firm should entertain rational expectations about the stochastic process θ, *it can be totally myopic in the matter of other firm's entry decisions*" (Dixit and Pindyck, 1994, p.291).

[16] In other words, the decision to enter entails the exercise of an option to delay. When the firm enters its loss of flexibility is given by $\frac{1}{\beta_1-1}K$.

symmetric Nash equilibrium in entry strategies. The principle of optimality states that, given the initial conditions and control values, an optimal path has the property that the control over the remaining period must be optimal for the remaining problem, with the state due to the early decisions considered in the initial condition. This principle matches with the definition of subgame perfect Nash equilibrium where a strategy profile is a Nash equilibrium if no firm has the incentive to deviate from its strategy given that other firms do not deviate (Fudenberg and Tirole, 1991, p. 108).

We can understand the competitive equilibrium better by writing this threshold in terms of the shock, θ. Since $\pi^* \equiv D(q)\theta^*(q)$ and $D(q)$ is decreasing in the region $[\bar{q}, \mathfrak{Q}]$, the optimal policy can be restated by the following upward-sloping curve (Figure 2):

$$\theta^*(q) \equiv \frac{\beta}{\beta - 1}(\rho - \alpha)\frac{K}{D(q)}, \quad \text{for} \quad q \in [\bar{q}, \mathfrak{Q}] \tag{5}$$

In the area above the curve, it is optimal to enter. A discrete mass of firms will enter in a lump to move the profit level immediately to the threshold curve. In the region below the curve the optimal policy is inaction; firms wait until the stochastic process θ moves it vertically to $\theta^*(q)$ and then again a mass of firms will jump into the market, just enough to keep the profits from crossing the threshold.

3.2 Positive externalities

Working backward, if the initial size q is less than $\bar{q}$, any potential entrant is subject to positive externalities, so that the timing of a firm's entry is influenced by the entry decisions of others. Intuition suggests that Leahy's result cannot be extended to cover this case; a single firm cannot continue to claim to be the last to enter the industry in constructing its optimal entry policy. The gist of our argument relies on the presence of "network benefits"; so the more firms in the industry, the greater the advantage in terms of profit flow. However, although investing is profitable, it is "more expensive" to do it alone than to enter with others or to follow others' decisions. This mean that the Nash equilibrium is represented by the myopic trigger $\hat{\pi}$ and the sequential investment dynamics are no longer subgame-perfect. By the disadvantage of moving first and the strategic nature of the timing decision, each firm can do better by delaying entry.[17]

[17] Potentially conflicting preferences over appropriation of the positive "network benefits" make them face a choice between no entry and agreement.

However, as all firms are subject to the same (industry-wide) uncertainty shock, only two equilibrium patterns are possible: either the industry remains locked-in at the initial size, sustained by self-fulfilling pessimistic expectations, or a mass of firms simultaneously enter, driven by the expected rents.[18] In the latter case, we expect entry to work as follows: for a fixed number of firms, profits move according to (3) with $\mu(q)\pi dq = 0$. If profits reach to $\pi^{**} \equiv D(q)\theta^{**}$, it will trigger an entry of more firms that increases the industry's size instantaneously by a jump. The exact shape of the trigger π^{**} as well as the number of firms that enter it is given in the following proposition.

Proposition 2 *The candidate policy for optimal entry into a competitive industry, characterized by positive externalities and initial mass of firms* $q \in [0, \bar{q})$, *is described by the following upper profit threshold:*

$$\pi^{**} \equiv D(q)\theta^{**}(q) = D(\bar{q})\theta^{*}(\bar{q}), \;\; \textit{for all} \;\; q \in [0, \bar{q}) \tag{6}$$

Proof. See Appendix. ■

Over the range $[0, \bar{q})$, the optimal entry policy is to set the threshold π^{**} equal to the known threshold $D(\bar{q})\theta^{*}(\bar{q})$ where the profit flow is maximum. No firms enter if profits remain below this barrier, but a discrete mass of $(\bar{q} - q)$ new firms simultaneously enters the first time that π^{**} is reached.

To appreciate the intuition behind this result let us consider a possible sequential investment starting at $q < \bar{q}$. As the firms may delay entry until θ reaches the upper level $\theta^{*}(q)$ (i.e., the "optimal" entry trigger for each idle firm in isolation), the first firm (randomly selected) to invest will earn lower profits $D(q)\theta^{*}(q)$ until some new firms decide to invest. Hence, indicating the mass of new firms with dq, as $D(q+dq) > D(q)$, the trigger $\theta^{*}(q)$ must be larger than $\theta^{*}(q + dq)$. But this implies that once $\theta^{*}(q)$ is reached, the new trigger $\theta^{*}(q+dq)$ is already surpassed and all the dq firms invest immediately. As these arguments apply for all $(\bar{q} - q)$, in any equilibrium the firms must invest simultaneously at $\theta^{*}(q)$.[19] Thus, with network externalities, no firm would ever invest at a lower entry trigger than $\theta^{*}(\bar{q})$ since this trigger is based on the most optimistic assessment with respect to other firms, namely they all invest at $\theta^{*}(\bar{q})$. On the other hand no firm finds it convenient to delay its entry when other firms invest; i.e. $\theta^{*}(\bar{q})$, which is also the most optimistic investment trigger of the rivals.

[18] Subgame-perfectness arguments may help to elimiminate the market failure equilibrium where nobody invests (Dixit, 1995; Moretto, 2003).

[19] Huisman (2001), and Nielsen (2002), use the same arguments to confirm simultaneous investment in a duopoly model with positive externalities.

An immediate corollary that follows from propositions 1 and 2 is:

Corollary 1 *The profit threshold that triggers the "network run" of* $(\bar{q} - q)$ *new firms is the same reflecting barrier that triggers the marginal competitive entry under negative externalities at* $\bar{q}$*:*

$$D(\bar{q})\theta^*(\bar{q}) = \frac{\beta}{\beta - 1}(\rho - \alpha)K \equiv \pi^{**}(= \pi^*).$$

Again, we can understand the equilibrium by writing this threshold in terms of the aggregate shock θ. Since $\pi^{**} \equiv D(q)\theta^{**}(q)$ and $D(q)$ is increasing in the region $[0, \bar{q})$, the optimal policy is given by a flat curve starting at $\theta^{**}(0) = \theta^*(\bar{q})$ defined by:

$$\theta^{**}(q) = \theta^*(\bar{q}) \equiv \frac{\beta}{\beta - 1}(\rho - \alpha)\frac{K}{D(\bar{q})}, \quad \text{for all} \quad q \in [0, \bar{q}) \tag{7}$$

Figure 2 summarizes the effect of positive externalities on entry. Starting at q, if the initial shock is below the known trigger at $\bar{q}$, all firms wait until θ rises vertically to this level, and then simultaneously enter, creating the optimal size $\bar{q}$. Once the optimal size is reached, to the right of $\bar{q}$, further decisions to enter proceed as explained in the previous section. Starting at any $q < \bar{q}$, (6) (or (7)) locates the optimal entry threshold so as to maximize the total profits of the incremental number of firms that enter $(\bar{q} - q)$. The shock value $\theta^*(\bar{q})$ that triggers these firms' "network run" is the same threshold that justifies a further marginal entry under negative externalities.

Figure 2 about here

4 Comments on the literature

The previous section showed that for $q < \bar{q}$ the candidate policy $\theta^*(\bar{q})$ is the unique threshold beyond which a mass $(\bar{q} - q)$ of idle firms find it optimal to move simultaneously. It has also been shown that once entry has exhausted positive externalities, new firms will enter following the competitive rule (5), where in equilibrium the option value of waiting drops to zero. Obviously, simultaneous investment may arise under very different circumstances from those considered here. For example in Bartolini (1993), simultaneous investment is driven by a constraint on the total size of the industry. He considers a competitive industry in which the firms initially enter following the optimal policy as in proposition 1, until a "critical" size is reached. At this "critical" size, rent competition generates a "competitive run" that immediately fills the rest of the quota. During this run the firms reduce current profits in an attempt to capture the rent that the industry size is expected to generate. Unlike Bartolini, a run in our model is generated by the maximization of the rent associated with positive externalities. These rents will be dissipated in the future by the competitive entry of firms with negative externalities. In Grenadier (1996), however, simultaneous investment occurs because two firms rush to enter a declining real estate market that will otherwise leave space for only one firm. In Moretto (2000), simultaneity arises because of a bandwagon effect on entry costs. Two firms are engaged in an "attrition" game generated by the presence of incomplete information plus positive externalities on the investment costs; i.e. it is more expensive to go first than to adopt the technology at the same time or later when others have already done so. Although the first-mover disadvantage leads to sequential investment, if the asymmetry between firms is not too high the investment occurs as a cascade.[20] At the opposite end, Huisman and Kort (1999) and Pawlina and Kort (2001) show that under complete information simultaneous investments may arise also in the presence of negative externalities.[21] Finally, Maison and Weeds (2001) show the same result in a similar duopoly model. Although they consider the simultaneous presence of negative and positive externalities, the only case in which both firms enter simultaneously is when they know that if the investment occurred sequentially, the leader would lose out considerably once the follower decided to enter.

5 Conclusion

We have offered a preliminary investigation into the effect of competition on firms' irreversible investment decisions under uncertainty as a generalization of the "real options" approach. We considered a product market that allows simultaneous treatment of positive externalities for a small market size and negative externalities for a large market size. The latter corresponds to a traditional competitive industries where the investment of one firm lowers the profitability of others. In that case, firms invest sequentially as market profitability develops. The former case corresponds to industries in which

[20] Dosi and Moretto (1998, 2007), examine a war of attrition game induced by spillover benefits on the cost of adopting "green" technology. They show that auctioning green investment grants is a better policy to stimulate simultaneous investment than standard subsidies that lower investment costs.

[21] See also Thijssen and Huisman and Kort (2002) for simultaneous entry when firms use mixed strategies.

investment is mutually beneficial, i.e. the investment of one firm increases the profitability of other firms' investments. In this case we find that firms invest simultaneously after the profitability of the market has developed sufficiently. The profit level that triggers an initial investment under negative externalities endogenously determines the optimal start-up size of the industry.

Our theoretical results may help to explain both the rapid and sudden development that has occurred for certain network goods such as the telecommunication services (Williams et al. 1988; Rogers, 1995; Schoder, 2000; Lim et al. 2003), as well as for the recent boom of Internet investment, for example the setting up of dotcoms on the World Wide Web for e-commerce (Odlyzko, 1999), and the many prolonged start-up problems while awaiting market development as in the case of digital fax machines (Rohlfs, 1974, 2001; Economides and Himmelberg, 1995).[22]

Furthermore, our results complement the recent new line of research on adoption and diffusion of new technology. This line of research incorporates the idea that any single decision by a potential user is not between adopting or not adopting, but is a choice between adopting now or deferring the decision later. From this point of view, the adoption of a new technology is similar to any other investment decision under uncertainty about future benefits and irreversibility, which generates an option value of waiting (Stoneman, 2001; Luque, 2002; Hall and Khan, 2003). Then the adopter's decision process can be modelled as suggested by Dixit and Pindyck (1994), and used here, providing another reason why diffusion of new technology may be rather slow (Hall and Khan, 2003, p.3).

Some extensions can easily be incorporated, such as the inclusion of finitely-lived capital projects, stage investments, growth options, and operative options that lead to suspension or definitive abandonment of the investment. The model also permits study of the efficiency of the investment-entry pattern. Is the equilibrium investment-entry time efficient? Does the efficient entry pattern occur in equilibrium? Such a study can be conducted considering the cooperative solution where by the investment decisions are determined by maximizing the sum of the firms' value functions or introducing a true social value function. Finally, a more substantial modification concerns a comparison with the case in which there is a monopolist who possesses all investment opportunities. Although the start-up problem in

[22] These are both examples of interlinked network services competitively supplied. Each consumer enjoys network externalities not only with respect to the consumers of his or her own supplier. The history of the fax machine also illustrates the importance of interlinking in making the demand grow to solve the start-up problem.

that case is much simpler, the analysis of the start-up conditions and the optimal network size is particularly interesting. Specifically, where network externalities are present, it may be profitable for the monopolist to sacrifice profits in the short-run in the hope of raising prices in the future after demand has grown and consumers are enjoying network effects.

A Appendix

The aim of this Appendix is to prove that the candidate policies (4) and (6) are indeed optimal. The analysis is restricted to a single entry trigger strategy, i.e. as if each firm uses a stopping rule (a pure Markovian strategy), that specifies the critical value of the state variable θ beyond which the firms invest. This assumption greatly simplifies analysis as it rules out mixed strategy equilibria. Our choice of pure strategies can be justified for at least for two reasons. First, they are very simple strategies which require firms to have only a low level of rationality. Second, the simultaneous investment scenario with mixed strategies is outcome equivalent to firms employing pure strategies.[23]

For the proof, we have referred to certain dynamic optimization solutions extensively studied in the Operations Research literature where by an Itô process is constrained never to leave an (optimal) region (see Harrison and Taksar, 1983, Karatzas and Shreve 1984; Harrison, 1985), to some well-known applications to a competitive economy (see Leahy, 1993; Bartolini, 1993; Dixit and Pindyck, 1994) and to scale economies (Dixit, 1995).

Let us consider the value of a firm $V(q, \theta)$, that is active in the market, as the expected discounted stream of profits:

$$V(q,\theta) = \max_{\tau} E_0 \left[\int_0^{\infty} e^{-\rho t} D(q_t)\theta_t dt - J_{[t=\tau]} K \mid q_0 = q,\ \theta_0 = \theta \right] \qquad (8)$$

where $J_{[t=\tau]}$ is the indicator function and the expectation is taken considering that the number of active firms may change over time by new entry. The solution to (8) can be obtained starting within a time interval within which no new entry occurs. Over this interval the number of firms is fixed and the firm is an asset which pays a flow of profits $D(q)\theta$ per unit of time, and experiences a "capital" gain $E[dV(q, \theta)]$ as θ evolves stochastically. Assuming $V(q, \theta)$ to be a twice-differentiable function with respect to θ and using Itô's Lemma to expand $dV(q, \theta)$, the solution of (8) is given by the following differential equation (Dixit and Pindyck, 1994, p. 179-180):

$$\frac{1}{2}\sigma^2\theta^2 V_{\theta\theta}(q,\theta) + \alpha^2\theta V_\theta(q,\theta) - \rho V(q,\theta) + D(q)\theta = 0 \qquad (9)$$

[23] Using of a simple discrete-time game, Moretto (2003) showed that, for the problem at hand, by letting the firms play mixed strategies is outcome equivalent to the one in which the firms employ pure strategies. The intuition is that as the market develops closer to a level sufficient to gain all network benefits, the probability of mistakes reduces and the coordination problem among potential entrants becomes less severe.

Provided that $\rho > \alpha$, a family solution of (9) can be written as: $V(q,\theta) = A(q)\theta^{\beta} + B(q)\theta^{\gamma} + \frac{D(q)\theta}{\rho-\alpha}$, where $1 < \beta < \rho/\alpha$, $\gamma < 0$ are, respectively, the positive and the negative roots of the characteristic equation $\Psi(x) = \frac{1}{2}\sigma^2 x(x-1) + \alpha x - \rho = 0$, and A, B are two constants to be determined. To keep $V(q,\theta)$ finite as θ becomes small, i.e. $\lim_{\theta\to 0} V(q,\theta) = 0$, we discard the term in the negative power of θ, setting $B = 0$. Moreover, the boundary conditions also require that $\lim_{\theta\to\infty}\left\{V(q,\theta) - \frac{D(q)\theta}{\rho-\alpha}\right\} = 0$, where the second term in the limit represents the discounted present value of the profit flow over an infinite horizon starting from θ (Harrison 1985, p. 44). Then, the general solution reduces to:

$$V(q,\theta) = A(q)\theta^{\beta} + \frac{D(q)\theta}{\rho-\alpha} \tag{10}$$

Since the last term represents the value of the active firm in the absence of new entry, then $A(q)\theta^{\beta}$ is the correction of the firm's value due to the new entry and $A(q)$ must be negative. To determine this coefficient we need to impose some suitable boundary conditions. First of all, perfect competition (free entry) requires the idle firms to expect zero profits at entry. Indicating by $\theta^*(q)$ the value of the shock, θ, at which the q-th firm is indifferent between entry right away or waiting another instant, the *matching value condition* requires:

$$V(q,\theta^*(q)) \equiv A(q)\theta^*(q)^{\beta} + \frac{D(q)\theta^*(q)}{\rho-\alpha} = K \tag{11}$$

The firm's competitive behavior keeps the value of active firms below the level K, by increasing the number of firms in the market. As we assumed that the firm's size is infinitesimal, then the trigger level, $\theta^*(q)$, is a continuous function in q.

Secondly, as the term θ^{β} in (11) is always positive, any change in q either raises or lowers the whole function $V(q,\theta)$, depending on whether the coefficient $A(q)$ increases or decreases. By totally differentiating (11) with respect to q we get:

$$\begin{aligned}\frac{dV(q,\theta^*(q))}{dq} &= V_q(q,\theta^*(q)) + V_{\theta}(q,\theta^*(q))\frac{d\theta^*(q)}{dq} \\ &= A'(q)\theta^{*\beta} + \frac{D'(q)\theta^*}{\rho-\alpha} + \left[A(q)\beta\theta^*(q)^{\beta-1} + \frac{D(q)}{\rho-\alpha}\right]\frac{d\theta^*(q)}{dq} = 0\end{aligned}$$

where, as long as each firm rationally forecasts the future development of the whole market and new entries by competitors, at the optimal entry threshold we get $V_q(q, \theta^*(q)) = 0$ (Bartolini, 1993; proposition 1; Grenadier, 2002, p. 699). Then:

$$V_\theta(q, \theta^*(q)) \frac{d\theta^*(q)}{dq} \equiv \left[A(q)\beta\theta^*(q)^{\beta-1} + \frac{D(q)}{\rho - \alpha} \right] \frac{d\theta^*(q)}{dq} = 0 \qquad (12)$$

This *smooth pasting condition* states that either each firm exercises its entry option at the level of θ at which its value is tangent to the entry cost, i.e. $V_\theta(q, \theta^*(q)) = 0$, or the optimal trigger, $\theta^*(q)$, does not change with q. While the former means that the value function is smooth at entry and the trigger is a continuous function of q, the latter indicates that a single firm would benefit from marginally anticipating or delaying its entry decision. In particular, if $V_\theta(q, \theta^*(q)) < 0$, it means that the value of a firm is expected to increase if θ drops. On the contrary, if $V_\theta(q, \theta^*(q)) > 0$, it means that a firm would expect to make losses versus a future drop in θ. In both situations (12) is satisfied by imposing $\frac{d\theta^*(q)}{dq} = 0$.

The rest of the proof is devoted to showing that for $q \geq \bar{q}$, the smooth pasting condition reduces to a traditional one, such that $V_\theta(q, \theta^*(q)) = 0$ and $\theta^*(q)$ is increasing in q. For $q < \bar{q}$, we get $V_\theta(q, \theta^*(q)) > 0$ which requires $\frac{d\theta^*(q)}{dq} = 0$.

A.1 Proof of proposition 1

To prove proposition 1 we need to show two results: (1) in the case of $q \geq \bar{q}$, the smooth pasting condition (12) reduces to $V_\theta(q, \theta^*(q)) = 0$; (2) the optimal competitive trigger, $\theta^*(q)$, is equivalent to the trigger of a firm in isolation, that is of a firm claiming to be the last to enter. For (1), let us consider the value of an active firm starting at the point $(q, \theta < \theta^*)$, a firm that would follow the optimal policy hereafter. Indicating by T the first time that θ reaches the trigger θ^*, the optimal policy must then satisfy:

$$\begin{aligned} V(q, \theta) &= \max_{\theta^*} E_0 \left[\int_0^T e^{-\rho t} D(q)\theta_t dt + e^{-\rho T} V(q, \theta^*(q)) \mid q_0 = q,\ \theta_0 = \theta \right] \qquad (13) \\ &= \max_{\theta^*} \left[D(q) E_0[\int_0^T e^{-\rho t}\theta_t dt \mid \theta_0 = \theta] + K E_0[e^{-\rho T} \mid \theta_0 = \theta] \right] \end{aligned}$$

where the second equality follows from the fact that, by (11), $V(q, \theta^*(q)) = K$. Moreover, by using some standard results in the theory of regulated stochastic processes (Dixit and Pindyck, 1994, p. 315-316): $E_0[\int_0^T e^{-\rho t}\theta_t dt \mid \theta_0 = \theta] = \frac{\theta - \theta^\beta (\theta^*)^{1-\beta}}{\rho - \alpha}$ and $E_0[e^{-\rho T} \mid \theta_0 = \theta] = \left(\frac{\theta}{\theta^*}\right)^\beta$, we can rewrite (13) as:

$$V(q,\theta) = \max_{\theta^*} \left[\frac{D(q)\theta}{\rho-\alpha} - \left(\frac{D(q)\theta^*}{\rho-\alpha} - K \right) \left(\frac{\theta}{\theta^*} \right)^{\beta} \right] \tag{14}$$

To choose θ^*, the first order condition is:

$$\frac{\partial V}{\partial \theta^*} = \left[(\beta-1)\frac{D(q)}{\rho-\alpha} - \beta \frac{K}{\theta^*} \right] \left(\frac{\theta}{\theta^*} \right)^{\beta} = 0 \tag{15}$$

which gives:

$$D(q)\theta^*(q) = \frac{\beta}{\beta-1}(\rho-\alpha)K \equiv \pi^*, \quad \text{with } \frac{\beta}{\beta-1} > 1 \tag{16}$$

Since $D(q)$ is decreasing in the interval $[\bar{q}, \mathfrak{Q}]$, $\theta^*(q)$ is increasing. Substituting (16) into (14) we can solve for $A(q)$, which is negative as required by (10):

$$A(q) = -\frac{K\theta^*(q)^{-\beta}}{\beta-1} \equiv -\frac{(\pi^*)^{1-\beta} D(q)^{\beta}}{\beta(\rho-\alpha)} < 0 \tag{17}$$

Finally, substituting (17) into (14) and rearranging we obtain (10):

$$V(q,\theta) = -\frac{(\pi^*)^{1-\beta} D(q)^{\beta}}{\beta(\rho-\alpha)} \theta^{\beta} + \frac{D(q)\theta}{\rho-\alpha} \tag{18}$$

from which it is easy to verify that $V_q(q,\theta) \neq 0$ within the interval $\theta < \theta^*(q)$ and is zero at the boundary.

For (2), let us consider an idle firm pretending to be the last to enter the industry. With q firms already active, if the firm decides to enter when the shock is $\hat{\theta}$, it pays K and receives in return an asset that values $\frac{D(q)\hat{\theta}}{\rho-\alpha}$. Now write $F(q,\theta)$ for the value of the firm's option to enter at time zero. This takes the form:

$$F(q,\theta) = \max_{\hat{\theta}} E_0 \left\{ e^{-\rho T} [\frac{D(q)\hat{\theta}}{\rho-\alpha} - K] \mid q_0 = q,\ \theta_0 = \theta \right\} \tag{19}$$

where T indicates the first time that θ hits the trigger $\hat{\theta}$. Rearranging, we get:

$$\begin{aligned} F(q,\theta) &= \max_{\hat{\theta}} \left\{ [\frac{D(q)\hat{\theta}}{\rho-\alpha} - K] E_0[e^{-\rho T} \mid \theta_0 = \theta] \right\} \\ &= \max_{\hat{\theta}} \left(\frac{D(q)\hat{\theta}}{\rho-\alpha} - K \right) \left(\frac{\theta}{\hat{\theta}} \right)^{\beta} \end{aligned} \tag{20}$$

Taking the derivative of this expression with respect to $\hat{\theta}$ and solving, we can show that the optimal threshold $\hat{\theta}$ is equivalent to (16). By direct inspection of (14) and (20) we immediately to note that the value of an active firm (14) is the difference between the value of an active myopic firm and the value of an inactive myopic firm as expressed by (20). Competition, therefore, not only does not alter the incentive to trade an idle firm for an active firm, but also encourages both to have the same price at entry. Using (16) in equation (18) gives $V(q,\theta^*(q)) - K = 0$, i.e. in equilibrium firms expect zero profit at entry (Dixit and Pindyck, 1994, ch.8).

Since, by (3), the myopic profit process and the competitive profit process are identical until θ^*, the profit flow that the firm is able to obtain following the policy $\hat{\theta}$ is the best that it can do, at least until T. However, by the principle of optimality this choice is also optimal for the rest of the period as (13) shows; if the optimal policy of the single firm calls for it to be active at θ^* tomorrow, it is obvious that the optimal policy today is to enter at $\hat{\theta}$. Finally, as (13) is a continuous function in θ^*, the limit as $\theta^* \to \hat{\theta}$ shows that $\hat{\theta}$ is a Nash equilibrium (Leahy, 1993; proposition 1).

A.2 Proof of proposition 2

In the case of $q < \bar{q}$ we have to show three things: (1) that a single firm can no longer claim to be the last to enter the industry and therefore, the optimal competitive trigger is no longer equivalent to the trigger of a firm in isolation; (2) that the candidate policy described in proposition 2 satisfies the necessary and sufficient conditions of optimality; (3) that it is a subgame perfect equilibrium.

For (1) and (2), let us consider an (idle) firm that follows the optimal policy $\theta^*(q)$. As $\theta^*(q)$ is decreasing in the interval $q < \bar{q}$, the higher the number of firms in the industry, the greater the profit flow at entry. The (idle) firm would then maximize its entry option by claiming to be always the last to enter the market, expecting an inadmissible upward jump in profits. At $\theta = \theta^*(q)$, the firm's value is simply $V(q,\theta^*(q)) \equiv \frac{D(q)\theta^*(q)}{\rho-\alpha}$. Then, we can see that:

$$V(q,\theta^*(q)) - \lim_{\theta \to \theta^*(q)} V(q,\theta) = \frac{K}{\beta - 1} > 0 \tag{21}$$

In (21), the inequality holds since it represents the correction due to the new entry (i.e. $A(q)\theta^\beta$ in (10)). This contradicts the *smooth pasting condition* $V_\theta(q,\theta^*(q)) = 0$, and then the optimality of $\theta^*(q)$. As all (idle) firms are equal, all expect an upward jump in profits at $\theta = \theta^*(q)$ if no other firm enters afterwards. This may induce each to delay entry waiting for the others to enter first. However, as $\theta^*(q)$ is decreasing in the interval $q < \bar{q}$, the upward jump in profits would decrease as more firms have already entered and it disappears at $q = \bar{q}$, where the firm's value function at entry is simply the known function (18). This confirms that: a) the candidate policy for the interval $q < \bar{q}$ is to impose $\frac{d\theta^*(q)}{dq} = 0$; b) the optimal level of shock that triggers entry is $\theta^*(\bar{q})$, where the profit flow is maximum for all the discrete sizes of investment $(\bar{q} - q)$; c) at $\bar{q}$ the necessary condition for optimality, $V_\theta(\bar{q}, \theta^*(\bar{q})) = 0$, turns out to be satisfied again.

To verify whether the necessary conditions are satisfied, we calculate the value of an active firm starting at the point (q, θ), and that follows a policy so defined: wait until T, at which the process θ rises to a level $c > \theta$, which corresponds to an immediate increase of the industry size to $b > q$. Using (13) the expected payoff $V(q, \theta)$ from this policy is equal to:

$$\begin{aligned} V(q,\theta) &= E_0\left[\int_0^T e^{-\rho t} D(q)\theta_t dt + e^{-\rho T} V(b,c) \mid q_0 = q,\ \theta_0 = \theta\right] \quad (22) \\ &= \frac{D(q)\theta}{\rho - \alpha} - \left[\frac{D(q)c}{\rho - \alpha} - V(b,c)\right]\left(\frac{\theta}{c}\right)^\beta \end{aligned}$$

The best moment for the industry size to jump as well as the dimension of the jump, are given by the following first order conditions:

$$\frac{\partial V(q,\theta)}{\partial c} = \left[(\beta - 1)\frac{D(q)}{\rho - \alpha} - \beta\frac{V(b,c)}{c} + \frac{\partial V(b,c)}{\partial c}\right]\left(\frac{\theta}{c}\right)^\beta = 0$$

$$\frac{\partial V(q,\theta)}{\partial b} = \frac{\partial V(b,c)}{\partial b}\left(\frac{\theta}{c}\right)^\beta = 0$$

When b and c are chosen according to the candidate policy so that $b = \bar{q}$ and $c = \theta^*(\bar{q})$, the value function reduces to (10) and the *matching value condition* requires $V(b, c) = K$. These prove that the candidate policy satisfies the first order conditions.

By processing (21) we can say more about the necessary conditions. Let the firm, as in (22), wait until the first time the process θ rises to the myopic trigger level $c \equiv \theta^*(b)$, corresponding to an immediate increase of the industry size to $b > q$. Assume also that the firm expects no more entry after b. Therefore its expected payoff $V(b, \theta)$ from this time onwards equals the discounted stream of profits fixed at $D(b)$, i.e.:

$$V(b,\theta) = \frac{D(b)\theta}{\rho - \alpha} \tag{23}$$

Comparing (23) with (10) gives $A(b) = 0$. Therefore, to obtain the constant $A(q)$, subject to the claim that beyond b no other firm will enter the market, we substitute (10) into the condition $V_q(q, \theta^*(q)) = 0$ to get $A'(q)\theta^*(q)^\beta + \frac{D'(q)\theta^*(q)}{\rho-\alpha} = 0$, resulting in:

$$A'(q) = -\frac{\theta^*(q)^{1-\beta} D'(q)}{\rho - \alpha} \equiv -\frac{(\pi^*)^{1-\beta}}{\rho - \alpha} \frac{D'(q)}{D(q)^{1-\beta}} \tag{24}$$

Integrating (24) between q and b gives:

$$\int_q^b A'(x)dx = -\frac{(\pi^*)^{1-\beta}}{\rho - \alpha} \int_q^b \frac{D'(x)}{D(x)^{1-\beta}} dx$$

Taking account of the fact that $A(b) = 0$, this integral gives the constant $A(q)$ as:

$$A(q) = \frac{(\pi^*)^{1-\beta}}{\beta(\rho - \alpha)} \left[D(b)^\beta - D(q)^\beta \right] \tag{25}$$

Substituting (25) into (10), which we rewrite to make explicit its dependence on the end size b, yields:

$$V(q,\theta;b) = \frac{(\pi^*)^{1-\beta}}{\beta(\rho - \alpha)} \left[D(b)^\beta - D(q)^\beta \right] \theta^\beta + \frac{D(q)\theta}{\rho - \alpha} \tag{26}$$

As long as $D(b) > D(q)$, the first term in (26) is positive and it forecasts the advantage the firm would experience by the entry of $b - q$ firms when θ hits $\theta^*(b)$. That is, if the firm were able to choose the optimal dimension of the jump, it would be $b \to \bar{q}$ which happens the first time that θ reaches $\theta^*(\bar{q})$. Thus, as opposed to before when non-sequential investments are possible, the necessary conditions would coordinate an optimal simultaneous entry by all firms. Finally, if $D''(q) < 0$ the above necessary conditions are also sufficient.

Since each firm foresees the benefit from the entry of others and observes the state variable θ, it instantaneously considers when to enter by maximizing (26). Then, with simultaneous investment the firms' optimal strategies are easy to find; each firm invests as if it were the only to invest but, with the expectation of earning all network benefits; i.e. $\theta^*(\bar{q})$ is a (symmetric) Pareto-dominant Nash equilibrium for all $q < \bar{q}$. In addition, as the reaction lags are literally non-existent, no firm is incentivated to deviate from the entry strategy $\theta \to \theta^*(\bar{q})$ and $b \to \bar{q}$, given that the other firms do not deviate. Finally, since θ is a Markov process in levels (Harrison, 1985, p. 5-6), the conditional expectation (22) depends only from the starting states θ. Therefore, at each date $t > 0$, the firm's values resemble those described in (26), which makes the equilibrium subgame perfect.

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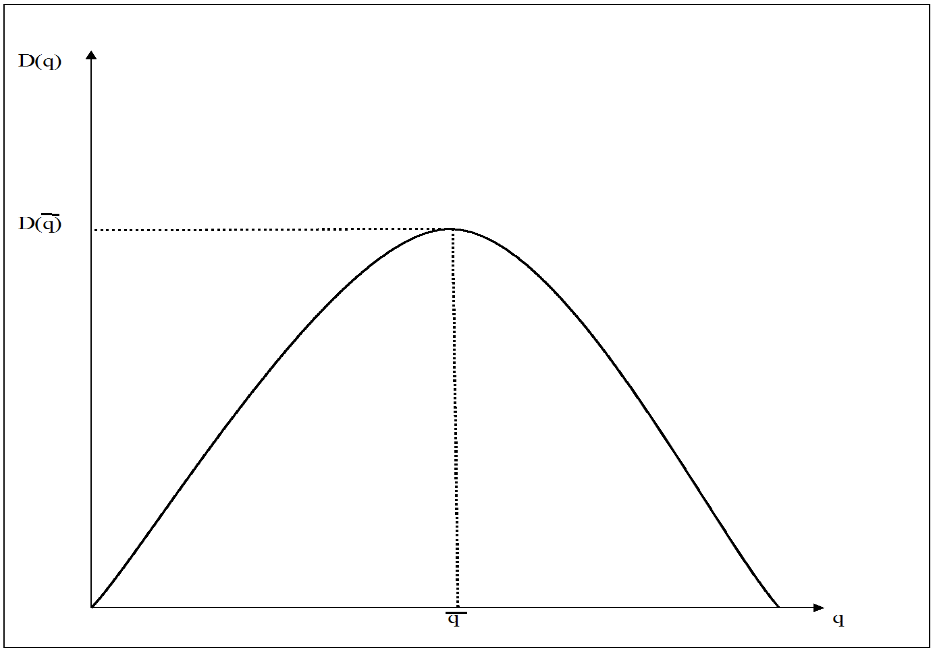

Figure 1

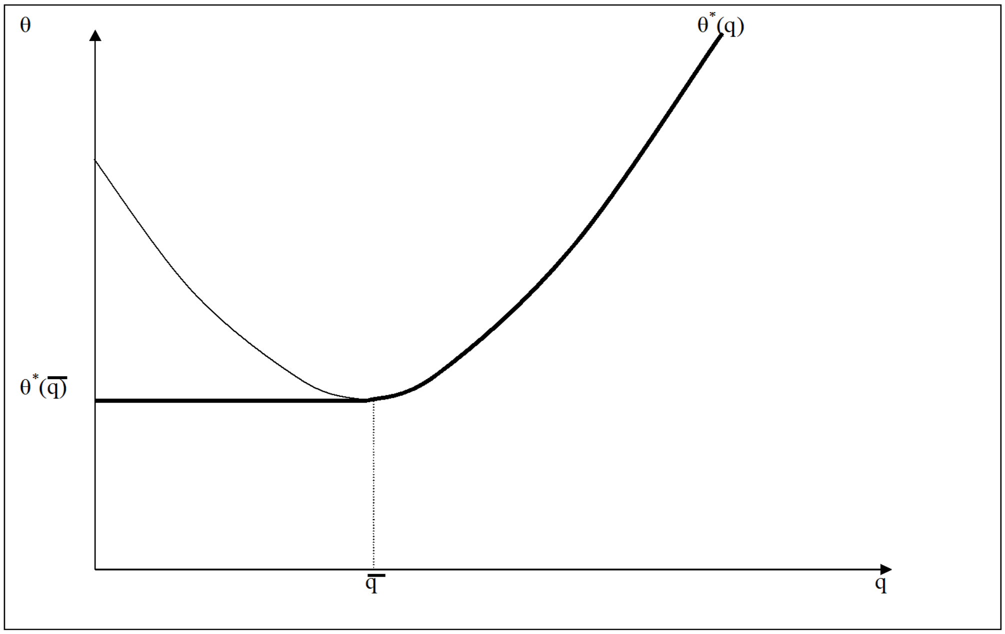

Figure 2