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MARCO FANNO'S TAX INCIDENCE THEORY: A FORMAL EXPOSITION

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Abstract. Marco Fanno's contributions to the theory of supply at joint cost and the theory of demand for substitute goods contain some original analyses of tax incidence, based on a "less partial" application of the Marshallian supply and demand paradigm. Fanno's overall theory, however, soon fell into oblivion, partially due to the enormous success of the emerging Hicks-Allen approach, at the end of the 1930s; and so did his more practical results. In this paper, we present a modern formalisation of Fanno's tax incidence theory, which tries to do justice to a series of results which have still today some normative validity.

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1. Introduction.

In the early 1910s, Marco Fanno conceived of a very ambitious project concerning the theory of value of his time, that of extending the Marshallian partial equilibrium analysis to a variety of empirically relevant cases, in which equilibrium in one market must be studied in connection with *a group* of other markets. Market equilibrium for joint or rival products, produced means of production, substitute or complement commodities, could not be studied thoroughly in isolation; yet general equilibrium analysis was considered too abstract to yield definite, relevant results. Hence, the need for an intermediate framework of analysis. That project had been fulfilled by Fanno only partially, albeit remarkably, in two monographs on supply at joint costs (Fanno 1999 [1914]) and on the demand for substitute goods (Fanno 1926), and in a series of articles on the general aspects of connected prices (Fanno 1998 [1929], Fanno 1933, Fanno 1934).

The search for practical applications, for new positive and normative results was, therefore, at the basis of Fanno's project. It is not coincidental that most of his success has been among some American economists of his time, who were aimed at "a recasting of the old problems [of value] into new forms amenable to statistical attack" (Mitchell 1925: 3). [1] Fanno, however, did not produce empirical investigations in this field: he was content with showing that some new *qualitative*, yet practical, conclusions can be drawn from his new framework of

[1] In particular Moore 1929 and Schultz 1928 based their statistical curves of demand on the premise that we can hardly observe any change in demand for a good due to its own price change alone and the presence of substitute and complement commodities featured prominently in this respect. Schultz acknowledged explicitly the importance of Fanno's pioneering work on substitute commodities (See, in particular, Schultz 1928, p. 581, n. 17).

analysis. From this perspective, his results on tax incidence theory are certainly of no secondary importance: it was in view of such practical, positive results that his theoretical programme was motivated.

This short paper reviews and discusses Fanno's contributions to incidence theory and, when necessary, translates them into more modern forms. The comparatively negligible impact that Fanno's results has had on public finance[2] and on the Italian school of finance in particular, is in fact in sharp contrast with the undisputed relevance of his overall contributions to the theory of value, and a modern exposition may perhaps favour a better understanding and do some justice to Fanno's originality in this field.

Sections 2 and 3 offer a brief overview of the state of the art in Fanno's times with respect to connected prices and tax incidence theory, respectively. Then, in sections 4 and 5, we consider Fanno's specific contributions to tax incidence theory in a connected prices framework; his main results will be reproduced synthetically by pushing his formalisation further. We consider, in fact, the effect of a *marginal* variation in an excise tax, rather than the effect of a *new* tax, as Fanno did: by a more extensive use of differential calculus, we can master the variety of possible cases much more easily, presenting them in terms of parameters –a procedure which is now much more familiar than it was at Fanno's times. Section 6 concludes.

2. Partial equilibrium and connected prices

Alfred Marshall's treatment of "Joint and composite demand. Joint and composite supply" (Book V, Ch. VI of Marshall, 1920) offered at the time of Fanno's apprenticeship the main reference concerning interrelated groups of commodities. Along with Marshall, a series of other authors, like Fisher (1925)[1892] and Edgeworth (1915), stressed their empirical importance and provided some broad partial equilibrium analyses. A more direct stimulus, however, had been provided to Fanno by Maffeo Pantaleoni, who convinced him of the

[2] A notable exception is A. Da Empoli's book on "oblique shifting" (Da Empoli, 1926).

analytical and empirical importance of "connected prices" in general. In the Preface to his 1926 book, Fanno informs us that

> The present study belongs to a collection of essays aimed at establishing some correlations amongst prices. This theme had been suggested to me 12 years ago [1913] by Maffeo Pantaleoni. It was one of his favourite topics. Following his advise, I prepared my first work on joint costs and then, with some delay due to the war, I attended to the preparation of this study on substitute goods. (Fanno 1926, pp. 11-12; my translation)

In fact, according to Pantaleoni,

> Thus then, strictly speaking, there are only connected prices and the theory of connected prices, which usually looks like an unusual and complicated case, is in fact the only complete theory of prices. (Pantaleoni, 1925 [1909])

This emphasis on connected prices may be superficially interpreted as an implicit plea in favour of general equilibrium analysis. This was certainly *not* the case for Pantaleoni and Fanno, however. In his obituary of Pareto, Pantaleoni wrote:

> I think Pareto's generalisations of economic equilibrium (…) have reached a limit beyond which I do not see much benefit for science to go: it is a chapter of science relatively or for some time closed. I would like to write 'finis' in this direction (…). Let us pay attention to the points of application of forces. Particularly the systems which constitute connected prices and the exact nature of these functions is not only an immense field of research, but now a necessary one to go over very carefully. (Pantaleoni 1923, pp. 585-6; emphases added)

In the words of Fanno:

> To know these laws [the laws of prices] thoroughly it is not enough to state that all prices are interrelated nor is it enough to provide the complete system of equations simultaneously determining the prices of all goods and services. We also need to show the nature of the relations between the different prices, the direction of their respective movements, the relative strength of these movements, and the way in which the various groups of prices are linked. (Fanno 1999 [1914], p. 3)

“Connected prices” was therefore an independent research programme (see also Bellanca, 1994 and Opocher, 2003), which Umberto Ricci considered as a ‘theory of equilibrium which I would call intermediate’, a sort of ‘third theory of equilibrium’ (Ricci 1939, p. 81).

3. Tax incidence theory

The distribution of the burden of a tax, say an excise tax, between producers and consumers (tax incidence) was at the turn of the last century (and to some extent still is) a typical problem of partial equilibrium and comparative statics, and was “an integral part of the general theory of value” (A. Marshall, quoted in Edgeworth 1897, p. 46). An equilibrium in a certain market was first described by means of the Marshallian curves of supply and demand, and then assumed to be disturbed by the introduction of a tax. A new equilibrium would be established. By comparison, one may assess how the distribution of the tax burden is affected by the specific market circumstances. The works of Carver (1896 and 1924), Edgeworth (1897) and Seligman (1927) [1899] argued that, in competitive conditions,[3] such a distribution crucially depends on the comparative elasticity of the supply and demand curves in the neighbourhood of equilibrium. They based their argument on the geometrical evidence offered by the Marshallian curves: the more elastic the demand curve (at a given elasticity of supply), the lower the increase in the price paid by the consumer and the higher the burden borne by the producers. Conversely, the more reactive is supply (at a given elasticity of demand), the higher the price rise and the burden on consumers.[4] Synthetically, the tax mainly strikes the comparatively less reactive side of the market. This result has been shown to be quite independent of whom has the legal obligation to pay the tax: incidence turned out to be the same irrespective of whether the demand curve shifts downwards (the consumers are obliged to pay) or the supply curve shifts upwards (the producers

[3] Tax incidence in the case of monopoly had been widely studied by these authors as well as by Fanno. For brevity, however, this paper is concerned only with the case of competition.

[4] See, e.g., Edgeworth (1897), p. 48 and Carver (1924), p. 578.

are obliged to pay) by the same amount. [5] This latter property gave a perfect symmetry to the question of shifting: in given market circumstances, shifting is backward or forward according to whether the consumers or the producers must pay the tax, but the final net positions are the same. A separate question was the determination of the tax revenue. Here it was the "absolute" rather than the "comparative" elasticity of supply and demand which mattered: clearly, the higher the elasticity, the most reactive the equilibrium size of the market and the less the tax revenue.[6]

The crucial importance of elasticity led the early authors to inquire into the *reasons why* a supply or demand curve may be more or less elastic, and these reasons were found to be closely related to other commodities: the presence of joint or "rival" outputs producible with the same inputs was relevant for the elasticity of supply; the presence of substitutes or complements in consumption was relevant for the elasticity of demand for a certain commodity. Thus, Carver argued that "in attempting to decide in which of the four classes, as represented by our diagrams [with combinations of high or low elasticities of demand and supply], a given commodity falls, we must find out what other alternatives are available to the consumers on the one hand, and the producers on the other" (Carver, 1924, p. 583).

With this in mind, it was quite natural to extend the problem of incidence of a tax on a certain commodity to consumers and producers of *other commodities*. Edgeworth, for instance, argued that "a tax upon one of two rival products will rise the price of both. A tax on one of two complementary products will rise the tax of the taxed one, and lower the price of the untaxed one" (Edgeworth, 1897, p. 54). Similar propositions are derived for commodities which are "complementary" or "rival" in consumption (on this, see also Seligman, 1927 [1899], p. 231).

All of these conclusions were based on very informal deductive arguments, rather than on rigorous proofs. Moreover, the idea that a tax on one commodity may ultimately be borne, in part, by producers and consumers of *other* commodities had not been fully developed. Both a

[5] This theorem is referred back by Edgeworth to Carver (1896). See Edgeworth (1897), p. 5, n. 2.

[6] See Carver (1924), p. 580.

mathematical model of incidence for the cases of joint outputs and of substitute commodities, and a very explicit theory of "oblique shifting" (*traslazione obliqua*), to use A. Da Empoli's terminology (Da Empoli, 1926), were to be introduced by Marco Fanno in his "twin" monographs of 1914 and 1926, to which we now turn.

4. Joint products and incidence theory

Let us first consider the case of joint products. There are two commodities, *A* and *B*, which are always produced jointly in the fixed proportion of *k* units of *B* for each unit of *A*.[7] We may clearly express global production (that is, the production of *A* and *B* taken together) with an index, Q, of a composite commodity whose metric unit is formed by one unit of *A* and *k* units of *B*.[8] Denoting the two outputs by X_A and X_B, we have clearly $X_A = Q$, and $X_B = kQ$. Let $C(Q)$ be the industry cost, and $C'(Q)$ be what Fanno called the "global marginal cost", which is "the analytical expression of the curve of supply for *A* and *B* expressed as a function of the quantity of global production",[9] under competitive conditions of the industry. The demand price[10] of commodity *A* is expressed by $p_A(X_A) = p_A(Q)$ and the demand price of commodity *B* is $p_B(X_B) = p_B(kQ)$. Let now t_A be a tax per unit levied on commodity *A*. The equilibrium index of production requires that the "global price" be equal to the "global marginal cost" plus the tax per unit, that is

[7] See Fanno (1999), p. 18 and Appendix I. The implications of a *variable k* are considered by Fanno at a later stage of the argument, but rather informally.

[8] Fanno represents such an index by the unfortunate notation (*x* + *y*) (See Fanno, 1999, p. 18 and p. 99), where *x* and *y* are the two outputs; of course, he alerted the reader to the fact that, in so doing, he was *not* meaning a sum proper, but an index of such a sum: see n. 3, p. 18. Hereon our notation departs slightly from Fanno's.

[9] Fanno (1999), p. 18. Fanno's theory of supply was that of a faithful "Marshallian". Here marginal cost may be indifferently interpreted as the marginal cost of the least efficient productive unit, or as the marginal cost of the representative firm.

[10] Fanno refers to the demand price as the "law of demand" (*Ibid.*). He assumes in this context that the demand price for one commodity is independent of the consumption of other commodities.

$$p_A(Q)+kp_B(kQ)=C'(Q)+t_A \qquad (1)$$

At a *Q* satisfying (1), we can determine the equilibrium outputs and the equilibrium prices, as well as the tax revenue.

We may now pause to note that (1) confirms at once an interesting, albeit seemingly "strange" (Fanno, 1999, p. 26), equivalence theorem proved by Fanno in Appendix I: if a tax on *A* were to be replaced by a tax on *B of equal revenue*, then equilibrium would be unaffected. Let in fact $kt_B = t_A$. Replacing t_A in (1) by kt_B obviously leaves the equilibrium unaffected and since $X_B = kX_A$ we have $t_A X_A = t_B X_B$. The same would be true of a tax on "global" production. Fanno drew the important practical conclusion that "in the case of joint costs, if there is parity of fiscal pressure, the manner of application of the tax has no bearing on the resulting disturbance" (Fanno, 1999, p. 27).

Let us now consider a marginal variation in the tax on *A*. Incidence can thus be studied by simply differentiating (1) totally. Let η_A and η_B be the two elasticities of demand (with $\eta_A, \eta_B < 0$) and ε be the elasticity of marginal cost (with $\varepsilon \geq 0$) –all calculated *in the neighbourhood of equilibrium*. It can easily be proved that

$$dQ = -Q\frac{|\eta_A|}{p_A + k(\eta_A/\eta_B)p_B + \varepsilon|\eta_A|C'}dt_A \qquad (2)$$

From the definition of *Q*, we have $dX_A = dQ$ and $dX_B = k(dQ)$. Thence $dp_A = (dp_A/dX_A)dQ$ and $dp_B = (dp_B/dX_B)k(dQ)$. By (2), then, we have

$$dp_A = \frac{p_A}{p_A + k(\eta_A/\eta_B)p_B + \varepsilon|\eta_A|C'}dt_A \qquad (3)$$

$$dp_B = \frac{p_B}{p_B k + (\eta_B/\eta_A)p_A + \varepsilon|\eta_B|C'}dt_A \qquad (4)$$

Equations (2), (4) and (5) contain all of Fanno's results on the incidence of a (marginal) tax levied on commodity *A*. We immediately see that, in general, output falls and the consumers of *both* commodities are affected by the tax. Ceteris paribus, they bear a lower burden the higher ε is. In the words of Fanno:

> If the global cost falls rapidly with the fall in production, i.e. if the curve of costs shows a steep rise, the global price after tax will only have to go up by a small amount (…). Hence equilibrium will be re-established by a small decrease in production and if all other conditions remain the same by a small increase in the two prices after the tax (…). Disturbances of prices resulting from the tax are greatest in the case of constant [$\varepsilon = 0$] or slowly rising costs. (Fanno, 1999, p. 25)

Also demand elasticities, taken together, affect the burden on consumers: the higher they are, the lower the burden.

Turning now to the *distribution* of the burden between the consumers of the two commodities, we see at once that that depends on the *relative* elasticities of demand. The burden on the consumption of one commodity is inversely related to its *relative* elasticity. It may even happen that, if $|\eta_A|$ is very high with respect to $|\eta_B|$, the burden on the consumption of the non-taxed commodity *B* is higher than that of the taxed commodity *A*. Fanno noted this possibility as a “singular anomaly” (Fanno, 1999, p. 25):

> Hence in this case there will be the apparently paradoxical result that a tax on just one of the two goods produced jointly raises the prices of goods exempt from it much more than it raises the price of the good on which it is actually levied. (…) Thus we have the very weird result that where two goods are produced jointly but taxed differently, the good carrying the heavier tax will rise in price less than the one carrying the lighter tax. (Fanno, 1999, p. 26).

In the essay on joint costs, Fanno did not pay particular attention to the determinants of the tax *revenue*, because they were not specific to jointness in production and nothing new was to be added to what was already known. For completeness, let us very briefly note, however, that the change in revenue (dT_A) due to a marginal tax on commodity *A* is

$$dT_A = dt_A Q + t_A dQ$$

Substituting dQ , we get

$$dT_A = Q\left(1 - \frac{t_A|\eta_A|}{p_A + k(\eta_A/\eta_B)p_B + \varepsilon|\eta_A|C'}\right)dt_A.$$

Not surprisingly, the value and the sign of dT_A/dt_A depend on *both* demand elasticities, because they affect the measure of dp_A and dQ. A rigid demand for both commodities makes for a 'high' (marginal) revenue. Singling out the role of the non-taxed commodity *B*, we see at once that the higher the fraction of commodity *B* in production (in value terms), the higher the (marginal) revenue of a tax on *A*; this effect is reinforced if the demand for *B* is less elastic than the demand for *A*.

5. Substitute commodities and incidence theory

Fanno's second monograph is devoted to supply and demand for a group of substitutes, assuming independence between the commodities belonging to that group and all the others (Fanno, 1926, p. 40). Equilibrium conditions for the whole group are first analysed and then, following the same procedure of the 1914 monograph, some comparative statics results are drawn. Among them, particular attention is devoted once again to those concerning the effect of a tax.

The 1926 monograph could rightly be claimed to anticipate some aspects of the theory of demand, which Hicks and Allen were to make popular some 10 years later.[11] As with many precursors, it contains some analytical aspects which may seem now unusual if not odd. They still await a thorough comparative study. For the purpose of our exposition, it will suffice to "translate", when necessary, Fanno's model of market equilibrium into a more usual formalisation.

[11] See Dominedò (1933), pp. 805-6; Di Nardi (1951), p. 217; Müller (1952), p. 90, all mentioned in Fanno (1954), pp. 146-7, n. 38.

The original model of Fanno for the case of only *two* substitute commodities consists of four main equations. [12] The first is the equality between the marginal rate of substitution and relative prices – in modern terms, the "income-consumption line"

$$\theta(X_A, X_B) = \frac{p_A}{p_B} \qquad [I]$$

The second equation is a sort of a budget constraint, limited to the pair of commodities under consideration, which reads

$$p_A X_A + p_B X_B = p_A f_A(p_A) \qquad [II]$$

where $f(p_A)$ is the amount of commodity *A* which would be demanded if commodity *B* was unavailable. This hypothetical demand function is taken by Fanno as given. Clearly, it is assumed for consistency that $p_A f(p_A) = p_B f_B(p_B)$, where $f_B(p_B)$ has a similar meaning. [13] In modern terms, a *sufficient* (but by no means necessary) condition for (II) to hold is that a constant fraction of total income be always spent on commodities *A* and *B* taken together.

In any interpretation, (I) and (II) define two demand functions

$$X_A = X_A(p_A, p_B) \qquad (5)$$

$$X_B = X_B(p_A, p_B) \qquad (6)$$

We may now revert to the *modern* interpretation of (5) and (6) and define the elasticities $\eta_A, \eta_B, \eta_{AB}, \eta_{BA}$ with the familiar meaning (of course, the two cross price elasticities need *not* be equal, because *gross* substitutability is involved).

Turning to supply, marginal cost is expressed as a function of the *industry* output. In competitive conditions, we have

$$p_A = C'_A(X_A) \qquad [III] \quad (7)$$

$$p_B = C'_B(X_B) \qquad [IV] \quad (8)$$

[12] Fanno (1926), p. 58. In the interest of simplicity, we changed slightly some notation and eliminated one unknown and one equation.

[13] See Fanno (1926), pp. 42-43.

The complications arising from diminishing costs are carefully avoided by Fanno: “we shall normally consider only curves with increasing or possibly constant costs” (Fanno, 1926, p. 57; my tanslation).

We are now in a position to formalise Fanno’s results concerning incidence.

Let *A* be the taxed commodity and let (7) be accordingly replaced by

$$p_A = C'_A\left(X_A\right) + t_A \qquad (7')$$

Equations (5), (6), (7’), and (8) determine two pairs of equilibrium prices and outputs (assuming equilibrium exists and is unique). Differentiating totally at the equilibrium point we have

$$dX_A = \eta_A \frac{X_A}{p_A} dp_A + \eta_{AB} \frac{X_A}{p_B} dp_B$$

$$dX_B = \eta_{BA} \frac{X_B}{p_A} dp_A + \eta_B \frac{X_B}{p_B} dp_B$$

and

$$dp_A = \varepsilon_A \frac{p_A}{X_A} dX_A + dt_A$$

$$dp_B = \varepsilon_B \frac{p_B}{X_B} dX_B$$

Making the necessary substitutions, we obtain

$$\begin{aligned} dp_A &= \frac{1+\varepsilon_B|\eta_B|}{(1+\varepsilon_B|\eta_B|)(1+\varepsilon_A|\eta_A|)-\varepsilon_A\varepsilon_B\eta_{AB}\eta_{BA}} dt_A \\ dp_B &= \frac{\varepsilon_B\eta_{B,A}}{(1+\varepsilon_B|\eta_B|)(1+\varepsilon_A|\eta_A|)-\varepsilon_A\varepsilon_B\eta_{AB}\eta_{BA}} \frac{p_B}{p_A} dt_A \end{aligned} \qquad (9)$$

which, of course, involve

$$\frac{dp_B}{p_B} = \frac{\varepsilon_B\eta_{BA}}{1+\varepsilon_B|\eta_B|}\frac{dp_A}{p_A} \qquad (10)$$

If both industries have constant costs ($\varepsilon_A = 0 = \varepsilon_B$), then

$$dp_A = dt_A$$
$$dp_B = 0$$

All the burden of the tax is borne by the consumers of the taxed commodity *A*. If $\varepsilon_A > 0, \varepsilon_B = 0$, then

$$dp_A < dt_A$$
$$dp_B = 0$$

and the burden is distributed between the producers and the consumers of *A*. The latter bear less of the burden the higher their elasticity of demand (and the higher the elasticity of marginal cost).

The case with $\varepsilon_A, \varepsilon_B > 0$ is more complex. Fanno found that in this case

$$0 < dp_B \frac{p_A}{p_B} \leq dp_A < dt_A$$

In his own words:

> We know from the laws of tax shifting that, in the case of production at increasing costs, the price of the taxed commodity rises by less than the tax and, coeteris paribus, it rises less the higher is the elasticity of the demand for the taxed commodity. (…) [The tax] is borne by the consumers of *A* in a lower measure than in the case without substitutes. But, as a counterbalance, it is shifted (if we can speak of a proper shifting) to the consumers of *B*. (…) The rise in the price of *B* relative to the price of *A* depends on the system of indifference curves in the market. (Fanno 1926, pp.68-69; my translation)

The ratio between the two proportional price changes (expressed by our eq. (10)) was called by Fanno the "coefficient of correlation" (*coefficiente di correlazione*) (Fanno 1926, p. 60) and was assumed to be equal to unity in the case of linear indifference curves (perfect substitutes, in modern terms) and lower than unity (but always non-negative) in the other cases.

Fanno considered then the revenue of the tax. Let again dT_A be the *change* in revenue due to a marginal tax on *A*. Clearly, we have $dT_A = t_A dX_A + X_A dt_A$. The measure (and even the sign) of dT_A crucially depends on the strength of the overall effect of the tax on the consumption of commodity *A*. Fanno alerted the reader (and the legislator!) to the possibility that a new tax on a commodity that has substitutes may have no revenue at all. This is the case in which the taxed commodity has close substitutes *and* the latter can be produced at constant costs. Solving our differential equations for dX_A, dX_B, setting $\varepsilon_B = 0$, we get

$$dX_A = -\frac{X_A}{p_A}\frac{|\eta_A|}{(1+\varepsilon_A|\eta_A|)}dt_A$$

$$dX_B = \frac{X_B}{p_A}\frac{\eta_{BA}}{(1+\varepsilon_A|\eta_A|)}dt_A$$

The higher are $|\eta_A|$ and η_{BA} (the closer substitutes are *A* and *B*) the less the revenue of the tax. In the words of Fanno

> In the case in which the production of *B* is at constant costs, and the rate of substitution of the two commodities is constant, the introduction of a tax on the production of *A* tends to shift the entire demand towards *B*, with the result that the taxed commodity would no longer be produced and the revenue of the tax would be null. (Fanno 1926, p. 69; my translation)

6. Concluding remarks

Fanno's works on the theory of value attracted the interest of some leading economists of his time, like Edgeworth (1915) and, later, Henry Moore (1929) and Henry Schultz (1928), just to mention some of them, and anticipated some tools of analysis that Hicks and Allen were to make popular more than 10 years later. They also prompted some further contributions in the specific field of tax incidence, like that of Attilio da Empoli (1926). But the emergence of what may be called the Hicks-Allen approach offered a simpler and more refined theoretical framework both for the analysis of joint production and for that of the demand for substitute goods and Fanno's early works soon fell into oblivion. Yet they command the respect of the historian of thought as well as of the theorist. In particular, Fanno's (and Pantaleoni's) idea that economic theory, to be *practically* relevant, should be neither committed to a too simple partial nor to a too abstract general equilibrium analyses is still interesting and certainly can *not* be found in the Hicks-Allen theory. The “interrelated prices” approach led Fanno to some interesting and sometimes counter-intuitive practical conclusions. Among them, the results concerning tax incidence and tax revenue are certainly, still today, of no secondary importance. We have presented in this note a formal exposition which may render Fanno’s results somewhat more palatable to the modern reader.

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