WHAT DOES A MONETARY POLICY SHOCK DO?
AN INTERNATIONAL ANALYSIS
WITH MULTIPLE FILTERS

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May 2012

“MARCO FANNO” WORKING PAPER N.145
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Abstract  
What does a monetary policy shock do? We answer this question by estimating a new-Keynesian monetary policy DSGE model for a number of economies with a variety of empirical proxies of the business cycle. The effects of two different policy shocks, an unexpected interest rate hike conditional on a constant inflation target and an unpredicted drift in the inflation target, are scrutinized. Filter-specific Bayesian impulse responses are contrasted with those obtained by combining multiple business cycle indicators. Our results document the substantial uncertainty surrounding the estimated effects of these two policy shocks across a number of countries.

Keywords: Multiple filtering, business cycle proxies, new-Keynesian business cycle model, trend inflation, monetary policy shocks.

JEL classification: C32, E32, E37.

*First version: January 2009. We thank Christopher Bowdler (Editor) and two anonymous referees for detailed comments and useful suggestions. We also thank Fabio Canova, Filippo Ferroni, Philip Liu, and Christophe Rault for detailed comments and suggestions on an earlier draft, and Carl Walsh for fruitful discussions on some preliminary results. We are grateful to Carlo Altavilla, Gianluca Cubadda, Huw Dixon, Martin Ellison, Michel Juillard, Juha Kilponen, Fabrizio Mattesini, Giulio Nicoletti, Paolo Paesani, Tommaso Proietti, Antti Ripatti, Vanessa Smith, Jouko Vilmunen, Matti Virén, and participants at presentations held at University of Helsinki, Bank of Finland, Computational Management and Science 2009 (Geneva), AngloFrenchItalian Workshop 2009 (Birkbeck College), University of Rome "Tor Vergata", and ASSET 2009 (Istanbul) for providing me with useful feedbacks. Part of this research was conducted while the corresponding author was visiting the Department of Economics of the University of California at Santa Cruz, whose kind hospitality is gratefully acknowledged. All remaining errors are ours. The opinions expressed in this paper do not necessarily reflect those of the Bank of Finland. Author’s details: Efrem Castelnuovo, University of Padua, Via del Santo 33, I-35123 Padova (PD), phone: +39 049 827 4257, fax: +39 049 827 4211, e-mail account: efrem.castelnuovo@unipd.it.
"There are no innocents ... only different degrees of responsibility."
Lisbeth Salander (in Stieg Larsson, The girl who played with fire)

1 Introduction

What does a monetary policy shock do? Different frameworks, including VARs, FAVARs, and structural models, have been employed to answer this question (for a recent survey, see Boivin, Kiley, and Mishkin (2010)). In this respect, a lot of attention has been devoted to the construction and the estimation of DSGE models. Among other issues, the estimation of these frameworks requires a researcher to map the model-consistent concept of the business cycle, which is a latent factor, to an empirical counterpart. The latter is typically constructed by isolating the frequencies of interest (of, say, the real GDP) with the use of a filter. Some researchers employ statistical filters or information external to the business cycle model they focus on (e.g., Rabanal (2007), Benati and Surico (2008, 2009)). Others, instead, explicitly model low frequency processes such as technology and or/preferences and estimate the DSGE model with growth rates (e.g., Smets and Wouters (2007), Justiniano and Primiceri (2008b), Castelnuovo and Nisticò (2010), Riggi and Tancioni (2010), Ascarei, Castelnuovo, and Rossi (2011)). Both approaches have pros and cons. Statistical filters are robust to model-misspecification, but are somewhat ad hoc - why should one prefer Hodrick-Prescott to linear-detrending? - and may induce spurious evidence in favor of cyclicalities (Cogley and Nason (1995)). Theoretically-consistent detrending is surely appealing, and logically in line with the employment of micro-founded models, but also obviously prone to biases induced by trend misspecification - what if technology is not a log-difference stationary process?\footnote{Of course, an econometrician may bet in favor of a 'reference model' for the trend of a series and undertake a 'robust control' approach by minimizing the largest deviations from such a trend induced by a 'evil' agent who works subject to given 'deviation constraints'. We thank Martin Ellison for proposing this idea, whose elaboration is left to future research.} Unfortunately, different filtering choices may lead to dramatically heterogeneous representations of the business cycle (Canova (1998)). Moreover, the misspecification of the trend component in rational expectations models may drastically alter policy functions and equilibrium laws of motions, so calling for an 'adjustment' by the structural parameters to compensate for such distortions when the model is confronted with the data (Cogley (2001)). Indeed, not much is known on the propagation of cyclical shocks at low frequencies (for an investigation, see Canova, Lopez-Salido, and Michelacci (2010)). Then, it is important to investigate how sensitive the results obtained with estimated
econometric models are to different filtering.\footnote{Throughout this paper, we will use the terms 'detrending' and 'filtering' interchangeably. In fact, as pointed out by Canova (2007, Chapter 3), 'detrending' refers to the process of making economic series (covariance) stationary, while 'filtering' has a much broader applicability, and refers in general to 'manipulations' operated to the frequencies of the spectrum.}

It is worth spending a few more words on 'theoretically-consistent' detrending. Modern business cycle theories have proposed structures where DSGE models feature persistent and permanent dynamics. In such models, agents internalize that the economy displays low frequency movements and their decisions are explicitly based on the degree of persistence of the driving processes. The ability of agents to interpret whether a shocks is temporary or permanent has profound implications on the dynamics of the system. The crucial point is that, if the presence and the nature of non-stationarity were known, then rational expectations modelers would always choose the appropriate model-based transformation and reduced-form representation. However, in absence of such knowledge, transformations implied by the model structures are as problematic as any statistical filtering. Hence, we are back to square one, and statistical filters are not 'per se' inferior to model-based transformation.

This paper asks the question: How relevant is the filtering issue for estimating the effects of monetary policy shocks on a macroeconomic environment? To answer this question, we estimate a new-Keynesian model of the business cycle (NKBC henceforth) with a variety of filtered output measures. We engage in estimations relying either on a single filter or involving multiple filters. As for the latter strategy, we follow Canova and Ferroni (2011), who recently elaborated a novel methodology (along the lines drawn by Boivin and Giannoni (2006a) with their 'data-rich environment') to efficiently combine differently constructed proxies of the business cycle. This approach has got the potential to eliminate, or at least reduce, filter-specific biases. In our empirical exercise we concentrate on two different types of monetary policy shocks: i) a standard policy shock, which is identified with a disturbance affecting the policy rate conditional on a constant inflation target; ii) an inflation target shock, which is identified with a disturbance to an autoregressive inflation target.

In conducting our empirical exercise, we first focus on the U.S. economy, which has been object of an intense scrutiny by empirical researchers. Then, we move our attention to a number of non-U.S. countries to assess to what extent heterogeneous filters may lead to different predictions as for the effects of the monetary policy shocks we are interested into. The set of countries under scrutiny includes the Euro area as a whole, Germany, France, the United Kingdom, Australia, New Zealand, and Canada.
To maintain comparability with the U.S. case, we will stick to the parsimonious small-scale AD/AS structural model often employed to investigate the U.S. economy, a choice also undertaken by Benati (2008).

Our results clearly support filter-heterogeneity as a relevance source of uncertainty surrounding the assessment of the role played by our two monetary policy shocks in driving inflation and the business cycle. Different filtering techniques lead to a (in some cases dramatically) substantial heterogeneity in the posterior densities of the estimated parameters of our DSGE model, which is reflected in our estimated impulse response functions to monetary policy shocks are also clearly filter-specific. This holds true not only for the responses of the ‘output gap’ to policy shocks, \(^3\) but also the reactions of inflation and the policy rate. Importantly, such evidence is extremely robust across countries. Given the relevance of a correct quantification of the effects of a monetary policy shock from the policymakers’ standpoint, our results offer support to the employment of techniques able to operate with a ‘data-rich’ environment, like those recently put forward by Boivin and Giannoni (2006a) and Canova and Ferroni (2011), which are designed to reduce the distortions due to a ‘limited-data’ approach.

While sharing in part the methodology and well as the modeling assumptions with some of the aforementioned contributions, our paper is fundamentally different as for the object of our investigation, i.e., the role of the choice of the business cycle filter for the estimation of the macroeconomic responses to different policy shocks, as well as its scope, which in our case is wider in that it covers eight different economic realities.

Methodologically, we follow Canova and Ferroni (2011) and estimate a structural model with a variety of different contaminated proxies of the business cycle. In terms of focus, however, our paper is different compared to Canova and Ferroni’s. In particular, we aim at quantifying the filter-driven uncertainty surrounding the effects of policy shocks. We do it i) with actual data of eight different areas, as opposed to Canova and Ferroni (2011), who investigate such uncertainty in a MonteCarlo context; ii) by focusing on two different policy shocks, i.e., a standard innovation to the policy rate conditional on a given level of the inflation target, and an innovation to the inflation target, which has recently been stressed as being one of the drivers of the U.S. inflation process (Cogley, Primiceri, and Sargent (2010)); iii) by considering a model in which

\(^3\)In this paper we will interpret the empirical proxies of the business cycle as measures of the ‘output gap’. Justiniano and Primiceri (2008a) work with a medium-scale DSGE model and show that the distance between the theoretically relevant output gap and the statistically constructed one(s) dramatically drops when measurement errors are admitted in the estimation, which is what we do in this paper.
real balances are not allowed to influence the equilibrium values of the macroeconomic variables we focus on (on top of the influence exerted by the nominal interest rate). Our contribution should then be seen as complementary to Canova and Ferroni’s.

The remainder of the paper is structured as follows. The new-Keynesian DSGE model we deal with is proposed in Section 2. Section 3 presents the different measures of the business cycle we work with, and discusses some of their properties. Section 4 discusses some issues on the estimation of the macroeconomic model we focus on, with a particular emphasis on the multiple filters approach. Section 5 presents the results of our empirical exercises as for our reference economy, i.e., the United States. Section 6 extends our analysis to a set of other industrialized countries and areas, including the Euro area, Germany, France, the United Kingdom, Australia, New Zealand, and Canada. Section 7 discusses some related literature. Section 8 concludes.

2 The model with time-varying trend inflation

We work with the following new-Keynesian business cycle framework:

\[
\begin{align*}
\pi_t &= \pi^*_t + \beta E_t(\pi_{t+1} - \pi^*_{t+1}) + \kappa x_t + \varepsilon^\pi_t, \\
x_t &= \gamma E_t x_{t+1} + (1 - \gamma) x_{t-1} - \tau (R_t - E_t \pi_{t+1}) + \varepsilon^x_t, \\
R_t &= (1 - \phi_R)[\phi_a(\pi_t - \pi^*_t) + \phi_x x_t] + \phi_R R_{t-1} + \eta^R_t, \\
\pi^*_t &= \rho_s \pi^*_{t-1} + \eta^*_t, \\
\varepsilon^z_t &= \rho_z \varepsilon^z_{t-1} + \eta^z_t, z \in \{\pi, x\}; \eta^j_t \sim i.i.d. N(0, \sigma_j^2), j \in \{R, *, \pi, x\}.
\end{align*}
\]

Eq. (1) is an expectational new-Keynesian Phillips curve (NKPC). Such a curve dictates the evolution of the inflation rate $\pi_t$ as a function of the contemporaneous inflation target $\pi^*_t$, the expected value of the future realization of the inflation gap (the wedge between raw inflation and its target), whose loading is the discount factor $\beta$, and the output gap $x_t$, whose influence on the inflation rate is regulated by the slope $\kappa$. The presence of the time-varying inflation target in the NKPC may be rationalized by firms’ full indexation to the current inflation target (Woodford (2007)).\footnote{In presence of partial indexation, the inflation schedule displays extra terms and interactions between the steady-state inflation level and some structural parameters entering the NKPC. For some theoretical analysis, see Ascari (2004) and Ascari and Ropele (2007).} Goodfriend and King (2008) employ a very similar model to analyze the U.S. inflation drift observed in the 1970s. The empirical importance of modeling the low-frequency component of inflation...
is supported by, among others, Ireland (2007), Cogley and Sbordone (2008), Cogley, Primiceri, and Sargent (2010), and a variety of other authors (further references and empirical support in favor of of a time-varying trend inflation in the U.S. macroeconomic environment may be found in Castelnuovo (2011)). The presence of such low-frequency process is also likely to reduce the statistical misspecification often detected as far as NKPCs are concerned (Fanelli (2008)). The NKPC at hand is purely forward looking. This choice is motivated by some recent evidence pointing towards a zero-weight assigned to indexation to past inflation in a model (similar to the one employed here) in which the inflation target is allowed to vary over time (Cogley and Sbordone (2008) and Benati (2010)).

The IS eq. (2) describes the evolution of the cyclical component of the real GDP, which is a function of expected and past values - weighted by \( \gamma \) - as well as by the ex-ante real interest rate, the latter loaded by the intertemporal elasticity of substitution \( \tau \). Strictly speaking, \( \gamma \) is a convolution involving the degree of habit formation of the representative agent, and \( \tau \) is a convolution involving the degree of relative risk aversion and that of habit formation. Habit formation offers the rationale for the presence of lagged output in the Euler equation. Following Benati and Surico (2008, 2009), we then prefer to work with the more flexible semi-structural eq. (2).

Eq. (3) is a Taylor rule that postulates a gradual response to movements in inflation and output by the monetary policy authorities. The target (4) is assumed to follow an autoregressive process (whose unconditional mean is normalized to zero), an assumption we share with a variety of previous studies (Cogley and Sargent (2005a), Ireland (2007), Woodford (2007), Goodfriend and King (2008), and Cogley, Primiceri, and Sargent (2010). Notice that we jointly model the standard monetary policy shock and the shock to ‘trend inflation’, i.e., the time-varying inflation target set by the Fed. This is a key-modeling choice. In fact, if the Fed has actually pursued a time-varying inflation target, in assuming a constant target we would force the dynamics of the inflation target to enter the ‘residual’ of the policy rule, and we would label as ‘policy shock’ what, de facto, is a convolution of the true policy innovation and the inflation target dynamics. Bache and Leitemo (2008) show that this misspecification can dramatically bias impulse responses to a monetary policy shock in autoregressive models. Standard assumptions on the stochastic processes (5) close the model.

\footnote{The implicit assumption here is that inflation can be modeled as an AR(p) process. For a comparison between ARMA(p,q) models and models allowing for long memory and nonlinearity in the inflation process, see Caggiano and Castelnuovo (2011).}
3 Different business cycle proxies: A comparison

How to approximate the model-consistent business cycle measure $x_t$, which enters eqs. (1)-(3)? To answer this question, one has to extract the cyclical component from the real-GDP raw time series. As for the U.S. economy, we concentrate on six different trends, very popular among macroeconomists. First, we consider the measure of potential output provided by the Congressional Budget Office, which employs a production-function approach to compute a measure of sustainable output.\(^6\) We employ such a measure to filter low-frequency movements of the real GDP out of the raw series, and we label ‘CBO’ this empirical proxy. The second transformation is obtained by applying the popular Hodrick-Prescott (‘HP’) filter with standard weight 1,600. The third transformation is a classical trend-cycle decomposition obtained by fitting a constant and a linear trend to the raw series without allowing for any break in the sample, and taking the residuals as indicator of the business cycle (‘LIN’). By contrast, the fourth manipulation (‘LBR’) fits a piecewise linear trend with a break in 1980:III in both the constant and the slope parameter. Another proxy we consider is constructed by applying the Baxter and King (1994) band-pass filter (‘BP’) to the log-real GDP so to extract cycles within the [8,32] quarters periodicity (with 12 quarters left as leads/lags). Finally, we take the growth rate of the raw series (‘FD’) as an indicator of the GDP’s cyclical component, a choice that relies upon the random walk with drift as a model for the real GDP trend. We perform all these transformations by considering the sample 1954:III-2008:II, a span longer than the one we employ to estimate our new-Keynesian model. This choice’s aim is that of tackling initial-condition issues concerning some of the filters at hand.

The filters that we consider are very widely employed in the macroeconomic literature.\(^7\) Importantly, they are heterogeneous along different dimensions. Some filters compute the non-cyclical component with quasi-deterministic procedures (LIN, LBR), some assume it is stochastic but very smooth (HP, BK), some very volatile (FD). Some procedures employ univariate information, others a larger set (CBO). Some are one-sided (FD), others two-sided (LIN, LBR, HP, BP). As regards low-frequency distortions, some are likely to overestimate the contribution of the low frequency variability

\(^6\)A detailed explanation on the computation of the CBO potential output may be found at http://www.cbo.gov/ftpdocs/30xx/doc3020/PotentialOutput.pdf.

\(^7\)Of course, the list of filters one may think of is much larger. Canova (1998), Canova (2007) (Chapter 3), Cogley (2008), and Proietti (2009) consider a set of alternative filters and discuss the pros and cons of different filtering strategies at length.
on the business cycle (LIN, LBR), others underestimate it or possibly estimate it fairly precisely (BP).

Figure 1 - left column displays the business cycle empirical proxies obtained with the six filters described above. One may spot similarities and differences across these proxies. Some comments are in order. First, 'eyeball econometrics' suggests a positive correlation across proxies, which is also confirmed by the figures reported in Table 1. However, such correlation varies - in some cases, dramatically - when moving from a pair to another. The highest correlation - 0.94 - regards the pair (HP,BP), while the lowest - 0.10 - involve (LIN,FD). In general, FD is poorly correlated with the rest of the business cycle indicators. This is due to the somewhat erratic behavior displayed by this proxy, which also signals shorter cycles compared to alternatives. The different proxies under investigation display a relevant amount of heterogeneity also in terms of business cycle dating. Taking the NBER recessions (identified by the grey bars in Figure 1) as reference, one may observe that CBO and HP perform reasonably well. By contrast, LIN just misses to capture the 1969:IV-1970:IV, 1973:IV-1975:I, and 1980:I-1980:III recessions, which are considered as simple slowdowns - i.e., realizations of decreasing but positive output gaps, while LBR shows a somewhat better ability in matching such recessions. Still sticking to the dating issue, FD shows the worst performance, with no clear indication of any particular recession, with the exception of the early 1980s one, indeed caught by all the proxies at hand. The magnitude of booms and busts is clearly filter-dependent, with some filters - e.g. LIN - possibly magnifying the deviations compared to 'potential' output and others - e.g. FD - dampening them. Table 1 confirms the high volatility in terms of estimated variance of the cyclical component of output. The highest figure - 11.61 - is associated with the LIN filtered proxy, whose variance is much larger than those of the widely employed CBO and HP- respectively 5.76 and 2.90 - and definitely greater than the one of the real GDP growth rate, with the ratio between the two being close to sixteen! Interestingly, when allowing for a break in the trend coefficients, the variance of the linearly detrended business cycle proxy drops of about 40%, so getting much closer to those of HP and CBO. The FD indicator returns the lowest variance - 0.73, and the BP filter induces the second lowest variance - 1.68.

Such heterogeneity is also reflected by the AutoCorrelation Functions depicted in Figure 1 - middle panel. In terms of autocovariance structure, a very different story is told by filters like HP and BP when contrasted to FD, with the latter showing a very quick drop of persistence after a few lags and a mild oscillatory behavior around zero thereafter, while the former display higher persistence and wide oscillations over
the twenty-five lags considered. Accounting for the break in the linear trend induces a switch of the sign for most of the autocovariances of LBR compared to LIN. Table 1-last row, however, suggests that the estimated persistence of the business cycle is very high, with the exception of the FD manipulation. Figure 1 - right panel depicts the log-Spectra of the proxies at hand. It is easy to spot significant errors in terms of identification of the frequencies of interest. Ideally, business cycle indicators should retain frequencies corresponding to the range 8 to 32 quarters (identified by the vertical black dotted bars in the normalized frequency domain). Notably, these proxies tend to attribute an excessive power to low-frequencies, but the error is clearly heterogeneous across filters, with the BP filter performing (by construction) better than all other filters, the HP filter offering an ‘intermediate’ performance, and others - among which the CBO filter - overemphasizing the relevance of low-frequency fluctuations for the business cycle. In general, problems of leakage (loss of power at the edges of the business cycle frequency band) and compression (increase of power in the middle of the band) are pervasive, a result already pointed out by, among others, Canova (1998), Canova (2007), Chapter 3, Canova (2009a), Proietti (2009), and Canova and Ferroni (2011).

In short, commonly applied filters tend to comove but are very heterogeneous across some dimensions - dating, magnitude, average length, and persistence of the business cycle. These differences are key. One the one hand, they may trigger a ‘filter-induced’ heterogeneity in results, the quantification of which is ultimately what this paper is after. But this heterogeneity is also a source of relevant information. As stressed by Canova and Ferroni (2011), heterogenous business cycle representations in the frequencies of interest enable the econometrician to optimally extract the relevant information embedded by each ‘contaminated proxy’ in the estimation phase. Given that each filter carries its own information in terms of business cycle frequencies, all these filters can constructively be employed (i.e., jointly employed) to estimate our business cycle model. The Kalman-filter will then seek to minimize the prediction errors computed by comparing the latent factors of the model to the observables employed in the estimation. Hence, filters delivering a business cycle representation 'at odds' with the model-consistent one will, in principle, be penalized. The next Sections elaborates on this issue.

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8The log-Spectra is computed with the 'pwelch' Matlab function. A Bartlett window kernel of size 21 was employed to smooth the periodogram and obtain a consistent estimation of the spectra.
4 Model estimation with multiple filters

4.1 Single vs. multiple filters

We estimate the new-Keynesian business cycle model (1)-(5) by using our proxies i) individually, i.e., one at a time, and ii) jointly. Canova and Ferroni (2011) point out that the combined employment of multiple filters has three main advantages. First, it does not require the researcher to take a strong a-priori stand on how to model the trend and the shocks driving it. Given the uncertainty surrounding the evolution of factors like technology and preferences, the fact of being able to remain agnostic on which filter to use is likely to work towards the reduction of biases due to trend misspecification. Second, this methodology allows to employ cyclical data computed with filters having very different features, e.g., one vs. two-sided filters, univariate vs. multivariate, deterministic vs. stochastic, and so on, so making parameter estimates more robust to filter misspecification. Third, errors in the attribution of the business cycle frequencies are proxy-specific. If such errors display a somewhat common pattern across proxies, the joint employment of different empirical indicators of the business cycle should reduce small sample biases in parameters estimates. If such errors are more idiosyncratic, this estimation procedure should wash them out so delivering more precise estimates. Canova and Ferroni’s (2011) Monte Carlo exercises, conducted with a standard DSGE model, confirm that the joint employment of multiple filters reduces the biases of the estimated parameters as well as those associated with the estimation of impulse response functions.

Of course, one would like to use an ’ideal filter’ capable to eliminate the distortions induced by imperfect filtering. The ideal filter should acknowledge the fact that the cyclical component of the DSGE model has the features of an autoregressive process. Canova and Ferroni (2011) show that, if such filter exists and is unique, an iterative approach may be in principle implemented to recover it. Unfortunately, the list of practical issues that render such recovery basically impossible is long. In particular, the estimated weights of the contaminated empirical proxies are likely to be inconsistent. Moreover, the computation of the ideal filter implies the estimation of the model a number of times to converge to the parameter calibration that ensures that correct extrapolation of the model-consistent cyclical component out of the data. Hence, we follow Canova and Ferroni’s ’instructions for practitioners’ and take the ’imperfect filters’ route by combining the different filters with an endogenous set of weights estimated jointly with the parameters of the structural model.
4.2 Measurement equation

To estimate the model (1)-(5), we set up the following encompassing formulation for the set of measurement equations:

$$
\begin{bmatrix}
FFRATE_t \\
INFLGDP_t \\
\vdots \\
\xi_{Nt} \\
\end{bmatrix}
= 
\begin{bmatrix}
\bar{R} \\
\pi_t \\
\vdots \\
\pi_N \\
\end{bmatrix}
\begin{bmatrix}
1 & 0 & \cdots & 0 \\
0 & 1 & \cdots & 0 \\
0 & 0 & \cdots & \lambda_1 \\
0 & 0 & \cdots & \lambda_N \\
\end{bmatrix}
\begin{bmatrix}
\xi_t \\
\xi_{t+1} \\
\vdots \\
\xi_{t+N-1} \\
\end{bmatrix}
+ 
\begin{bmatrix}
0 \\
0 \\
\vdots \\
0 \\
\end{bmatrix}
\begin{bmatrix}
u_{1t} \\
u_{2t} \\
\vdots \\
u_{Nt} \\
\end{bmatrix}
$$

where $FFRATE_t$ is the quarterly federal funds rate at time $t$, $INFLGDP_t$ is the quarterly GDP deflator inflation, $\xi_t = [\xi_{1t}, \ldots, \xi_{Nt}]'$ is the $(N \times 1)$ vector of empirical proxies of the business cycle computed with $N$ different approximations of the trend, $\bar{\xi} = [\bar{\xi}_1, \ldots, \bar{\xi}_N]'$ is a $(N \times 1)$ vector of proxy-idiosyncratic constants to demean the filters, $\lambda$ is a $(N \times N)$ diagonal matrix of 'loadings' relating the model consistent cyclical component $x_t$ to the $N$ empirical proxies $\bar{\xi}_t$, and $u_t = [u_{1t}, \ldots, u_{Nt}]'$.

$\sim i.i.d.(0_{N \times 1}, diag(\sigma^2_{u1}, \ldots, \sigma^2_{uN}))$ is a $(N \times 1)$ vector of serially and mutually uncorrelated filter-specific measurement errors. Then, the slope parameters $\lambda_n, n \in \{1, \ldots, N\}$ captures the weight assigned by the data to the 'signal' associated with a given filter as for the cyclical component of the real GDP. The associated measurement errors indicate the uncertainty surrounding such signals. When implementing the multiple filter strategy, we normalize $\lambda_{CBO} = 1$, and we interpret the remaining $\lambda_s$ as relative loadings compared to the first one. By contrast, when estimating the model with a single proxy, the measurement equation (6) features $N = 1$, $\bar{\xi}_t = [\bar{\xi}_{nt}]'$, $u_t = [u_{nt}]'$, and the restriction $\lambda = [\lambda_n]' = 1$ is imposed. A measurement error to the business cycle equation in (6) is allowed also when a single proxy is employed.\(^9\)

4.3 American dataset

Our observables are the U.S. GDP deflator, the log-real GDP, and the federal funds rate (average of monthly observations), all downloaded from the website of the Federal Reserve System.\(^10\) Our observables are demeaned before estimation. Consequently, we

\(^9\)Given the filters we employ, our modelization is compatible with a deterministic trend in technology in a nonlinear microfounded version of the model.

\(^10\)URL: http://research.stlouisfed.org/fred2/.
set $\mathbf{\Gamma}, \pi$, and the vector $\mathbf{x}$ to zero. Several authors have detected a break in the U.S. monetary policy conduct at the end of the 1970s (Clarida, Gali, and Gertler (2000), Lubik and Schorfheide (2004), Boivin and Giannoni (2006b), Canova (2009b)). Castelnuovo and Surico (2010) show that the switch towards a more aggressive monetary policy in the U.S. is an important ingredient to explain the instabilities detected in the impulse responses to a standard monetary policy shock estimated with small-scale VARs. Castelnuovo (2010) detects a break at the end of the 1970s in the correlation between real interest rates and inflation expectations in the United States. Surico (2006) discusses the perils coming from merging two subsamples characterized by different equilibria when conducting empirical exercises on NKPCs. Castelnuovo (2012) shows that monetary aggregates, not modeled in our framework, have influenced the volatility of U.S. output in the pre-Volcker period. Hence, we focus on the sample 1982:IV-2008:II, the first observation representing the beginning of the post-Volcker experiment’ period. Our end-of-sample choice enables us to avoid dealing with the acceleration of the financial crises began with the bankruptcy of Lehman Brothers in September 2008, which triggered non-standard policy moves by the Federal Reserve (Brunnermeier (2009)).

4.4 Bayesian estimation: Priors

We perform econometric estimations by relying upon Bayesian techniques, widely employed in the applied macroeconomic literature (see Canova and Sala (2009) for a discussion of the pros and cons of this methodology vs. alternatives). Our priors are standard, and are fairly similar to those in Benati and Surico (2008, 2009) and Cogley, Primiceri, and Sargent (2010). In line with this last contribution, we set the autoregressive parameter $\rho_s$ of the inflation target process (4) to 0.995 to capture low-frequency movements in inflation, for which the DSGE model might not provide a satisfactory description (see Justiniano and Primiceri (2008a) and the references cited therein).

Differently, the standard monetary policy shock is assumed to be white-noise. This difference enhances the identification of the two monetary policy shocks. As it is customary in the literature, we calibrate the discount factor $\beta$ to 0.99. The remaining priors are quite standard. In particular, they are uninformative as for the persistence parameters. Finally, we assume the loadings of the empirical proxies to be independently distributed as $\lambda_i \sim N(1, 0.5)$. Measurement errors are also assumed to be independently distributed and follow $u_{it} \sim Inverse\ Gamma(0.25, 2)$.\footnote{The figures reported in brackets refer to the mean and standard deviation of the distributions of interest. Our full list of priors is detailed in our Appendix (Table 1).}
4.5 Two-sided filters: A note

A note on the use of two-sided filters is in order. Admittedly, the estimation of a framework with likelihood-based techniques with two-sided filtered business cycle indicators violates the timing assumption underlying the standard Kalman recursion. However, two considerations are in order. Firstly, this paper documents the heterogeneity of results stemming from the employment of a variety of business cycle indicators widely adopted in the literature. To our knowledge, this is the first paper documenting the sensitivity of the quantification of the effects of monetary policy shocks to the employment of a wide array of different filters. Secondly, this ‘timing misspecification’ is not necessarily more distortionary than the model misspecification that is likely to occur when specifying the technological process with simple autoregressive stochastic processes, which is typically done when estimating business cycle monetary models with growth rates. Overall, while being aware of the timing issues related to likelihood-based techniques, we believe our exercise may convey relevant information on the sensitivity of some macroeconomic estimates to the employment of different business cycle proxies.

5 Empirical results: The U.S. case

5.1 Impulse responses to monetary policy shocks

We move directly to the target of our analysis, which is the sensitivity of impulse responses to the use of differently computed business cycle indicators.\(^{12}\) Figure 2 displays the impulse response functions to two monetary policy shocks, the ‘traditional’ shock to the nominal interest rate in the Taylor rule (3) conditional on a constant inflation target, and the shock to the trend inflation process (4). In all cases, the reactions have the expected sign. A monetary policy tightening induces an increase in the policy rate as well as in the real interest rate, a decrease in the output gap, and a demand-driven deflation. A positive inflation target shock triggers a take-off in inflation and calls for a monetary policy tightening. Given that policymakers react with gradualism, the real interest rate takes negative values in the short run, which leads to a temporary expansion. These reactions are qualitatively in line with those put forward by Ireland (2007).

\(^{12}\) An Appendix available upon request reports information on i) the estimation methodology, ii) our estimated parameters’ posterior densities, iii) a model comparison involving our benchmark model and an alternative model featuring a constant inflation target and price indexation to past inflation, and iv) the forecast error variance decomposition to different horizons.
and Cogley, Primiceri, and Sargent (2010). \footnote{Credible sets ('confidence bands') are intentionally not displayed. The point here is that of assessing the heterogeneity due to the filtering choice, and not the sample uncertainty surrounding objects like impulse responses. The discussion on the statistical differences between pairs of impulse responses is entertained in the following subsection.}

Importantly, while the dynamics of the system are qualitatively clear, Figure 2 also shows that there are significant quantitative differences. In fact, the business cycle reaction to both shocks is extremely heterogeneous across different indicators. To assess this heterogeneity, we compute the percentage deviations of each estimated reaction compared to the MF filter. Table 2 summarizes the information in the figures regarding the 4 and 8-quarter ahead percentage deviations. The estimated differences across filters and between each filter and the MF representation are striking. As regards the standard policy shock, figures related to output at the 4-quarter horizon range from the zero deviation suggested by the CBO filter to the 75% of FD. Interestingly, accounting for a break in the linear trend dampens the percent deviation compared to the ‘Canova-Ferroni’ scenario. Similar pictures emerge when focusing on the two-year ahead horizon.

Filter-uncertainty clearly affects also the estimated reaction of inflation to a standard monetary policy shock. Again, CBO suggests milder deviations from MF when contrasted to HP and LIN, and LBR somewhat dampens the effects induced by LIN. Notably, the widely employed HP filter is associated with a percentage deviation of about 40% (8-quarter ahead), a very large departure indeed. The growth rate, once more, turns out to be the filter deviating the most compared to the MF ‘weighted average’, with figures over 80%.

Also inflation target shocks trigger quantitatively very different business cycle responses. The 4-quarter ahead business cycle reaction to a trend inflation shock inducing a 1% on impact hike in the inflation rate reads about -26%, -7%, and 1% when respectively - HP, LIN, and FD filters are considered. Larger figures are in general recorded at a longer horizon, with the FD’s departure reading 75%. Interestingly, inflation’s deviations compared to MF turn out to be modest, the highest figure being 5.78% (FD, 8-quarter ahead). The ‘filter-induced uncertainty’ affecting our estimated impulse responses is, indeed, remarkably high. \footnote{One may be tempted to rank the different indicators employed in the multiple-filter investigation on the basis of the estimated loadings \( \lambda \). The closer a loading is to one, the argument could go, the closer is the corresponding the indicator to the ideal filter. A simple example, however, shows that this reasoning does not take scaling effects into account. Suppose the ideal indicator \( x_b \) featuring \( \lambda_b = 1 \) and \( \sigma_b \rightarrow 0 \) exists. Define \( \bar{x}_b \equiv x_b/r, r \in R\{1\} \). This implies \( \bar{\lambda}_b = r\lambda_b \neq 1 \). However, the indicator \( \bar{x}_b \) is, by construction, just as good as the indicator \( x_b \) as for the properties of the business cycle. This}

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5.2 Pairwise comparisons

One may wonder if these filter-specific impulse responses are different from a statistical standpoint. Consider two filters \( i \) and \( j \), with \( i \neq j \), and label as \( IRF(h, z, \varepsilon, k, j) \) the value \( h \) sampled from the posterior distribution of the impulse response function of the variable \( z \) to a shock \( \varepsilon \) at a horizon \( k \) computed by estimating our new-Keynesian model with the output gap proxy conditional on the filter \( j \). Then, the density of the difference between the two impulse response functions of the variable \( z \) to a shock \( \varepsilon \) at a horizon \( k \) conditional on the filters \( i \) and \( j \) is computed as

\[
IRF(h, z, \varepsilon, k, j) - IRF(h, z, \varepsilon, k, j), h = [1, \ldots, H],
\]

i.e., we sample \( H \) realizations from the posterior densities of the impulse responses of interest, and we compute the difference of the two \([1 \times H]\) vectors.

To fix ideas on the outcome of our exercise, Figure 3 depicts the histogram constructed by sampling realizations from the density of the CBO filter-based impulse response of inflation eight quarters after a standard monetary policy shock has occurred and subtracting to such realizations those sampled from the HP filter-based impulse response of inflation to the same shock/horizon. The vertical black lines identify the 16th and 84th percentiles, which approximate our 64% credible set. Notably, the zero value, which is identified by the vertical dashed red line, falls outside the so-constructed 64% set. Then, how large has the credible set to be in order to embed the zero value? The very first cell of Table 3 reports the answer to this question, which reads 74% and involves the credible set constructed by involving the 13th and 87th percentiles of the distributions in Figure 3.

In general, when considering two filters, a very large (small) number (credible set width) reported in Table 3 suggests that the difference between the moments induced by such two filters is 'big' ('little'). Hence, substantial differences may be found when contrasting the indications of the FD filter as opposed to all the remaining ones. As for inflation, different reactions to a standard monetary policy shock are computed by the example suggests that looking at the loadings as a strategy to rank the different indicators at hand may be misleading. Differently, the estimated standard deviations characterizing the set of measurement equations are informative, in that the smaller is the standard deviation of the 'measurement error' associated to a given indicator, the higher is the information content of such indicator. Our estimates imply the following ordinal ranking (estimated posterior mean of the standard deviations of the indicators in brackets): BP(0.17) > HP(0.30) > LBR(0.79) > CBO(1.10) > LIN(1.18) > FD(2.99). We stress, however, that this paper is concerned with the uncertainty surrounding the effects of monetary policy shocks, more than the ordering of different indicators on the basis of a given loss function.
CBO filter when compared to the HP and MF filters. The LIN and BP filters return different estimates as for the reaction of output to a monetary policy shock. The CBO and BP filters return quite similar indications as for our objects of interest in both cases under scrutiny. LBR and BP filters return responses which are quite close to those estimated by all filters jointly. Interestingly, the 'ubiquitous' HP filter seems to offer estimates different compared to those arising from the combined filters approach.

Wrapping up, we can at least conclude that, as far as the U.S. economy is concerned, i) a substantial heterogeneity affects macroeconomic responses to monetary policy shocks conditional on different filters for output; ii) the FD filter is clearly an outlier in terms of induced impulse responses. Consequently, one may think of combining different filters à la Canova and Ferroni (2011), perhaps excluding the FD filter that appears to convey more noise than signal.

5.3 Robustness checks

We engaged in some robustness checks to verify the solidity of our results.\textsuperscript{15}

- we excluded either the FD indicator or the BP one from the set employed to estimate the MF model. The FD filter appears to be an outlier when contrasted to the remaining filters. In fact, the information content of the FD business cycle proxy is weighted 'endogenously' via the estimated $\lambda_{FD}$. However, to assess if our results are driven by the presence of the FD filter, we estimated the MF model with the remaining five filters. As for the BP indicator, the exclusion is based on its similarity compared to the HP proxy of the business cycle. Our results, not shown here for the sake of brevity, turn out to be robust to these perturbations of the set of filters employed;

- we also experimented with a combination of the six multiple filters 'MF' with the a different normalization, i.e., $\lambda_{HP} = 1$. The idea of taking the HP filter as a reference is due to its wide popularity in the empirical literature. Our results, shown in Figure 4, turn out to be very similar to the ones proposed in our benchmark analysis, the only difference being a slightly milder reaction of output to our monetary policy shocks. Our main finding i.e., the large heterogeneity implied by the use of different filters in the estimation of DSGE models and the implied impulse responses to monetary policy shocks, is fully confirmed by our robustness checks;

\textsuperscript{15}The results not documented here for the sake of brevity are available upon request.
• we re-estimated the model by considering the cyclical representation also of inflation and the policy rate on top of that of real GDP. E.g., the model ‘HP’ has been estimated with HP filtered log-real GDP, HP filtered GDP deflator inflation, and the HP filtered federal funds rate. The filter-specific transformation was applied to inflation and the policy rate also as for the remaining filters. The main conclusion of this paper, i.e., the pervasive heterogeneity induced by different filtering strategies as regards IRFs, is unaffected;

• in the baseline battery of estimations we employed demeaned data and set \( \bar{R} \), \( \bar{\pi} \), and the vector \( \bar{x} \) to zero. In fact, as pointed out by Canova and Ferroni (2011), estimating the constants of the model may be informative on the ‘level biases’ associated with each filter. As a ‘quality-check’, we re-estimated the MF models by allowing for independently distributed filter-specific constants \( \bar{x}_n \sim N(0, 0.5) \), \( n \in \{1, \ldots, N\} \). A large departure from the zero-value of a given filter-specific constant would cast doubts on that filter’s ability to correctly identify the mean of the business cycle process. However, the vector \( \bar{x} \) is estimated to be very close to zero, and with small standard errors, a result suggesting the absence of level biases.\(^{17}\)

6 International evidence

The results discussed so far are conditional on the U.S. data. Of course, one may wonder to what extent impulse responses to identified policy shocks are heterogeneous not only across filters, but also across countries. Hence, this Section estimates the new-Keynesian model presented in Section 2 for a number of countries and aggregates, i.e., the Euro area, Germany, France, the United Kingdom, Australia, New Zealand, and Canada.

\(^{16}\)The LBR filter was not considered in the MF application due to multicollinearity issues. We did not filter inflation and the policy rate in the CBO case, which is constructed on the basis of the potential output as computed by the Congressional Budget Office.

\(^{17}\)A suggestion we got regards the possibility of estimating our models with the trend inflation estimate obtained by Cogley, Primiceri, and Sargent (2010) with a VAR with drifting parameters. This exercise could be carried out in a two-step fashion. Firstly, we could estimate the trend inflation process with reduced-form VARs. Then, we could undertake an estimation conditional on the level of trend inflation process estimated during the first step. While interesting, we suspect this exercise would deliver very similar results to those presented in this paper. As stressed by Cogley et al (2010, pp. 17-18), which estimate a model similar to ours: "[...] the [DSGE] model-implied evolution of the Central Bank inflation objective [...] resembles quite closely the VAR-based estimate of the permanent component of inflation [...]."
Our main source of information as for the macroeconomic data employed in this Section is the OECD Main Economic Indicators database. We compute our filters by considering the longest series available as provided by such a database. Our estimations focus on samples that are free from institutional breaks. Following Benati (2008), we select our samples by considering as a major institutional break the official adoption of the ‘Inflation Targeting’ scheme by the countries we are focusing on. In line with our analysis of the U.S. case, we also exclude the recent turbulences due to the last financial crises by selecting the second quarter of 2008 as the last observation in our samples. As for the Euro area, we work with synthetic data conditional on the period starting from 1999, i.e., we do not take a ‘counterfactual’ pre-1999 ECB into account. This choice is due to our willingness of analyzing ‘true’ monetary policy shocks without dealing with a fictitious central bank. We estimate models for France and Germany for the very same sample, the idea being that of being able to contrast impulse responses of the Euro area as a whole vs. those of the two largest (in real GDP terms) countries belonging to this area. Hence, the sample employed to estimate our structural models read as follows: Euro area, Germany, and France, 1999:I-2008:II; U.K., 1992:IV-2008:II; Australia, 1990:I-2008:II; New Zealand, 1993:1-2008:II; Canada, 1990:I-2008:II.

As for the selection of the filters, we are forced to implement some variations compared to the analysis of the U.S. economy. First of all, the U.S. Congressional Budget Office does not publish measures of potential output as for non-U.S. countries. Then, we replace the ‘CBO output gap’ measure with the country-specific output gap estimates provided by the World Economic Outlook database, which is computed by appealing to a Production-Function Approach (‘PFA’).18 As for the ‘LBR’ output gap, breaks in linear trends are not as well documented for non-U.S. countries as it is the one affecting the U.S. real GDP’s linear trend. We then replace ‘LBR’ with a one-sided backward-looking HP filter (‘HPB’) à la Stock and Watson (1999), which is flexible enough to capture possible variations in real GDP ‘trends’.19

One may wonder if one should in principle expect different results from our international analysis than those obtained with U.S. data. Just as an example, Figure 5 plots the HP-filtered real GDPs of the eight areas under analysis. Clearly, the pairwise

18Such database contains information taken from the World Economic Outlook (WEO) report, which presents the IMF staff’s analysis and projections of economic developments of a variety of countries. We constructed quarterly output gap series by interpolating the year-by-year output gaps provided by the WEO with the ‘cubic-match last’ spline methodology provided by EVIEWS.
19The computation of the one-sided optimal HP filter was conducted by exploiting the MATLAB codes kindly provided by Alexander Meyer-Gohde (Humboldt University) and freely downloadable from http://ideas.repec.org/c/dge/qmrbcd/181.html .
correlations may be very heterogeneous. This may depend on a large variety of issues, including the economic situation of each given economy per se, the different samples employed to compute each given filter, the different data quality, etcetera. However, the focus of this paper is on the consequences of having different filters at work on the impulse responses to identified monetary policy shocks per each given country of interest.

We therefore turn to analyzing such estimated reactions. In order to have a sense of the differences compared to the U.S. case, we plot each given country/response of a given variable to a given shock across different filters jointly with the min and max reactions recorded for the U.S. economy as for the very same variable/response to the same shock. Again, the idea is to help the reader spotting similarities and differences compared to the reference American economy. Let’s not turn to our results.

**Euro area.** The estimated reactions of the Euro area, depicted in Figure 6, are extremely close each other. Indeed, the dispersion is very low when excluding those computed with the FD filter, which appears to be a clear outlier here, in that it basically tends to dampen all the estimated reaction in the short/medium run with the exception of the reaction of the policy rate to an inflation target shock. We therefore leave the FD-driver aside and comment on the remaining responses. In terms of output reaction to a monetary policy shock, the estimated responses are comparable (as for throughs and peaks) to those of the U.S. case. Differently, the reaction of inflation is signalled to be stronger, ranging from -0.6% to -0.5% on impact depending on the filter one considers. This may be due to the fact that the nominal interest rate, conditional on a standard monetary policy shocks, is higher than what suggested by the U.S. economy, again at least as far as its lower bound is concerned. Moving to the trend inflation shock, we observe a milder reaction of output compared to the largest one associated with the U.S. economy. This is consistent with a milder (indeed, in relative terms) increase in the short-term policy rate after the shock joint with a reaction of inflation quite comparable to that suggested by some of the U.S. filters.

Is the Euro-area less prone to filter uncertainty? This may be a possible conclusion arising from this exercise. In this sense, it is informative to explore the two biggest (in real-GDP terms) areas belonging to the European Monetary Union, i.e., Germany and France. Are they characterized by the same ‘degree of convergence’ across filters?

**Germany.** When looking to the responses estimated with German data, the answer to the last question seems to be negative. Figure 7 plots the responses of interest. The heterogeneity displayed by the responses of output to a monetary policy shock is very
evident. The PFA filter suggests a very deep and prolonged recession, leading to a large deflation and a substantial ‘overshooting’ of the policy rate, which goes well down to zero. As a matter of fact, all filters but FD predict a recession stronger than that associated with the U.S. case. Moreover, the predicted reactions are not only different in terms of magnitude, but also as far as the suggested timing is concerned, with a clear filter-dependence as for the throughs after the shock. More similar in terms of shape are the reactions of inflation, which anyhow display a substantial heterogeneity as for the on impact reactions are concerned, with values ranging from -0.5% to -1.5%. The reactions of the policy rate are also quite heterogeneous but comparable to the American ones with the exception of the ‘PFA’ filter. Similar considerations can be made as for the reactions to an inflation target shock. However, the predicted magnitude is comparable, in terms of the largest realizations, to that associated with the U.S. economy as far as output is concerned. Differently, the reactions of inflation are larger (in absolute value), with the exception of what suggested by the FD filter. Interestingly, this happens in face of an increase of the nominal interest rate after the shock which is predicted to be much milder than the one estimated with U.S. data.

**France.** Somewhat different results are obtained when scrutinizing French data - see Figure 8. The magnitude of output responses to a standard policy shock are much milder than the German ones, and do not exceed the American ones in terms of maximum value in absolute terms. Apart from two filters, i.e., FD and PFA, the reactions of output are quite synchronized. Differently, inflation reactions are somewhat larger than the American ones, but are less heterogenous than the German ones, first and foremost as for the on-impact values. The responses of the policy rates display a fair amount of heterogeneity. All of them (but PFA), however, converge to zero within 25 quarters. Turning to the inflation target shock, output reactions are milder than the American ones, a consideration that can be done also as for the reactions of the policy rates and, to a large extent, of inflation. Apart from the PFA outlier, all the reactions display an appreciable degree of synchronization.

We asked earlier if the Euro-area is less prone to filter uncertainty. The outcome of our exercises conducted with German and French data suggests that this is not necessarily the case, at least as for the leading economic countries in the Area are concerned. Differently, it may very well be that aggregation of different countries’ data leads to canceling out some of the effects of the Euro area countries’ shocks on inflation and output.

**United Kingdom.** A remarkable dispersion is found to affect also the estimated
reactions of the U.K. economy, which are shown in Figure 9. In particular, the linear trend is a clear outlier here, in that it suggests a reaction of output to a standard monetary policy shock about three-to-four times as large as those suggested by other filters. Some reactions are in general barely in line with those of the U.S. economy in terms of largest realization in absolute value (although not in sync with it). The frequency-based filters (HP, HPB, BP) display reactions slightly more aggressive than the largest U.S. ones, while - again - the FD filter tends to suggest a conservative estimate of the output costs triggered by a monetary policy shock. A wide range of estimates are also available as for the on-impact reaction of inflation to a monetary policy tightening, with the extremes being provided by LIN (about -1.8) and FD (close to -0.5). The policy rate also displays a persistence that is clearly filter-specific. As for the shock hitting trend inflation directly, one may again notice how the LIN indicator implies a much stronger reaction of output and inflation, but not necessarily of the policy rate. Heterogeneous responses are offered also when focusing on the remaining filters, which anyhow suggest a larger reaction of inflation in the medium-term (apart from FD) and a milder response of the policy rate compared to our findings as for the U.S. economy.

**Australia.** Similar comments can be made for the Australian case, which also sees the LIN filter as the one implying the largest reaction of output and inflation to a monetary policy shock, as clearly depicted in Figure 10. As for output, the remaining reactions remain all within the ‘American bounds’. The same does not hold true as for inflation, for which some filters signal a stronger deflation after a monetary policy shock (the MF combination of filters included), but not for the policy rate, for which different filters suggest different reactions, but all within the extremes values as identified by the U.S. responses. In terms of reaction to trend inflation shocks, impulse responses suggest heterogeneity in terms of timing and magnitudes quite comparable to those of the U.S. economy. Quantitatively, the largest peak as far as the reaction of output is concerned is comparable to the U.S. one, while inflation takes larger values in absolute terms.

**New Zealand.** It is of interest to notice that the comments on the Australian responses can be replicated, to a large extent, when turning to New Zealand data. Again, LIN is a kind of ’outlier’ suggesting strong reactions of output and inflation to standard monetary policy shocks, while FD works as the filter offering ’conservative’ estimates of such reactions. Somewhat interesting is the difference in terms of correlation between the reactions suggested by the PFA filter and the MF combination of filters, which is very high in the Australian case and somewhat mild as for New Zealand. However, the
overall similarity between responses of these two countries is clearly evident. Figure 11 documents such reactions.

Canada. Figure 12 depicts the dynamics reactions to monetary policy shocks as for the Canadian economy. Again, the LIN filter suggests somewhat larger reactions than those implied by other filters. The remaining filters induce reactions whose maximum values in absolute term are in line with those recorded with U.S. data, with the sole exception of the reaction of inflation to an inflation target shock, which shows to be larger and persistently so. Once again, the FD filter is the one offering the most conservative estimates of the entire panel of indicators.

Wrapping up, our international analysis i) documents the heterogeneity in the various country-specific reactions to our identified monetary policy shocks; ii) confirms the main message of this paper, i.e., the (at times very!) substantial filter-induced uncertainty surrounding each country’s macroeconomic reactions to monetary policy shocks; iii) puts in evidence the tendency of the FD filter to offer very conservative estimates of the effects of the shocks under scrutiny.

7 Contacts with the literature

This paper is closely related to some recent contributions regarding filtering and the estimation of DSGE models. Close to the (already reported) Canova and Ferroni (2011) paper are Ferroni (2011) and Canova (2009a). Ferroni (2011) contrasts the standard ’first filter, then estimate’ two-stage approach with a novel ’jointly filter and estimate’ one-step strategy. The novelty hinges upon the joint estimation of trend and structural parameters. Importantly, this strategy allows a researcher to exploit the cross-equation restrictions of the DSGE model when performing the trend-cycle decomposition, to compare the descriptive ability of different filters, and to employ the resulting information to construct robust estimates via Bayesian averaging. Ferroni’s (2011) ”trend agnostic” methodology turns out to be more consistent than alternatives also in case of model misspecification. He also estimates a standard AD/AS model with U.S. data and show that different filters may indeed induce different estimates of the parameters/moments of interest. Canova (2009a) also proposes a ’single step’ methodology that allows for a flexible link between unfiltered raw data and the theoretical model at hand, and in which cyclical and non-cyclical components are allowed to have power in all the frequencies of the spectrum. Simulations performed by the author show that standard data transformations induce distortions in structural estimates and policy conclusions that
are drastically reduced when applying his methodology. Compared to Ferroni (2011) and Canova (2009a), we undertake a more conventional ‘two-stage strategy’ to highlight the consequences of detrending in the context of a modern monetary policy model of the business cycle that embeds, among others, trend inflation shocks, i.e. possibly one of the main drivers of the great moderation in inflation (Cogley, Primiceri, and Sargent (2010)). Moreover, we jointly consider a variety of differently filtered business cycle representations and let the data speak about their relative weights. In terms of data, we undertake an international analysis involving eight different economic areas, which confirms that relevance of analyzing different filters for the assessment of the macroeconomic effects of two distinct monetary policy shocks.

Cogley (2001) suggests to estimate the model with GMM techniques before solving the Euler equations for rational expectations, so to avoid to specify the driving processes at the estimation stage. Differently, we stick to the ‘first solve, then estimate’ sequence typically called for by likelihood-based estimation techniques, also to overcome the weak-instrument problem often arising when employing GMM techniques. Gorodnichenko and Ng (2010) focus on the problem of estimating a DSGE model under trend misspecification, and elaborate a robust covariance estimator to tackle this issue. They show that, when the data are generated by a RBC model, the distortions due to trend misspecification can be dramatic, but their moment-based estimator can importantly reduce such distortions. Compared to Gorodnichenko and Ng (2010), we concentrate on a monetary policy DSGE model of the business cycle and document the uncertainty surrounding the impulse responses to two different policy shocks, a dimension that they do not scrutinize.

A recent contribution by Delle Chiaie (2009) contrasts the estimates of a medium-scale DSGE model for the Euro area conditional on the employment of two different filters, linear vs. HP filter. Dramatic differences in terms of posterior densities and impulse response functions arise. Compared to Delle Chiaie’s (2009) investigation, our paper employs a larger variety of filters, and it combines them according to the proposal by Canova and Ferroni (2011). Moreover, we consider two different types of monetary policy shocks, one of the two - the trend inflation shock - being possibly very relevant as a driver of the U.S. inflation in the sample at hand (Cogley et al, 2010).

From a more exquisitely economic standpoint, our contribution intersects those concerned with the modeling of the U.S. inflation and output. One of the main features of inflation is its persistence, which has often been modeled via somewhat ad hoc indexation mechanisms. Going against this tendency, Cogley and Sbordone (2008) and Benati
(2010) show that, once trend inflation is embedded in the new-Keynesian Phillips curve, price indexation is statistically not significant. The remarkable evidence supporting the hypothesis of a time-varying inflation target pursued by the Fed (see also Ireland (2007), Stock and Watson (2007), Cogley, Primiceri, and Sargent (2010)) motivates our choice of working with a model in which trend inflation is allowed to play an active role in shaping the inflation process of the U.S. as well as a number of other industrialized economies.

8 Conclusions

This paper has assessed the effects of monetary policy shocks by estimating a new-Keynesian monetary policy DSGE model for the U.S. as well as a set of other industrialized areas with different empirical proxies for the business cycle. These proxies are employed i) one-by-one, and ii) following a recent proposal by Canova and Ferroni (2011), in a combined fashion. The effects of two different policy shocks - an unexpected interest rate hike conditional on a constant inflation target, and an unpredicted drift in the inflation target - have been scrutinized. The sensitivity of impulse responses to two identified monetary policy shocks have been assessed.

Our main findings are the following. Firstly, substantially different indications may arise out of the employment of different filters for the estimation of impulse response functions to identified policy shocks. Secondly, filters that are typically considered as ‘close substitutes’, e.g., Hodrick-Prescott and Band-Pass, can actually imply dramatically different responses. Thirdly, the use of the growth rate of the real GDP results much more moderate estimates of the macroeconomic reactions to monetary policy shocks.

Our findings support studies that elaborate strategies to deal with large datasets (Boivin and Giannoni, 2006a; Canova and Ferroni, 2011). These contributions propose promising tools to perform robust evaluations on the impact of macroeconomic shocks and systematic policies on the macroeconomic dynamic of interest. Indeed, a data-rich approach may unveil structural relationships that would be otherwise overlooked, and whose omission would be misleading when seeking to identify macroeconomic drivers or to design optimal reactions to shocks. Given its constant departure compared to most of the remaining filters and across the eight areas we analyzed, one may want to deal with the growth rate of output (if taken as a proxy of the ’business cycle’ per se) with great care.
Interestingly, Canova and Ferroni (2011) show that a data-rich approach lends support to the role of money in a small-scale DSGE context, a support not provided by standard, ‘one-filter only’ estimations. In light of the recent monetary easing implemented by a variety of central banks to tackle the real effects of the on-going financial turmoil, this finding is clearly very important. We look forward to reading other contributions unveiling key-structural relationships involving macroeconomic aggregates via a data-rich approach.

References


Figure 1: Proxies of the Business Cycle: Multiple Filters. Left column: U.S. real GDP filtered with different proxies of the low-frequency component (‘trend’). List of filters indicated in the text. Grey vertical bars identify recessions (from peak to trough) as dates by the NBER. Middle column: AutoCorrelation Functions of the business cycle proxies. Right column: Log-Spectral Density of the business cycle proxies. Blue vertical bars identify the normalized business cycle frequencies in the range [1/16, 1/4] corresponding to 8-32 quarters.
<table>
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<th>CBO</th>
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<th>LBR</th>
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Figure 2: Impulse Response Functions to Monetary Policy Shocks - United States. MF reactions constructed by normalizing the weight of the CBO filter to one. First row: Responses to a standard monetary policy shock - normalized to induce a 1% on-impact increase in the policy rate. Last row: Responses to a trend inflation shock - normalized to induce a 1% on-impact increase in the inflation rate. Median Bayesian impulse responses based on 500 draws from the posterior densities.
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Table 2: **Impulse Response Functions to a Monetary Policy Shock: Percent Deviations with respect to Multiple Filters Models.** Figures computed by relying on median responses.
Figure 3: Impulse Response Functions to Monetary Policy Shocks - United States, An Alternative Normalization. MF reactions constructed by normalizing the weight of the HP filter to one. First row: Responses to a standard monetary policy shock - normalized to induce a 1% on-impact increase in the policy rate. Last row: Responses to a trend inflation shock - normalized to induce a 1% on-impact increase in the inflation rate. Median Bayesian impulse responses based on 500 draws from the posterior densities.
Figure 4: Density of the Difference between Values Sampled from the Eight-quarter ahead CBO Impulse Response to a Standard Monetary Policy Shock and Values Sampled from the HP IRFs - United States. Vertical black lines: 16th and 84th percentiles. Vertical dashed red line: Zero value.
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Table 3: **Densities of the Pairwise Differences: Minimum Width of the Credible Set to Cover the Zero Value.** Figures constructed on the densities of the pairwise differences involving two different filters and the implied impulse responses after eight quarters to a given shock. Figures in the cells indicate the minimum width required for the credible set of each density of the difference to contain the zero value. E.g., the very first cell 'HP, CBO' contains the value '74'. This indicates that the credible set [(100-74)/2,100-(100-74)/2]=[13,87] contains the zero value, which indicates that the estimated effects on inflation of a monetary policy shock after two years, estimated either with the HP filter or with the CBO filter, are similar if one is willing to focus (at least) on the 74 per cent credible set.
Figure 5: HP-filtered Real GDP - Different Economic Regions. Sample: 1999:I-2008:II.
Figure 6: Impulse Response Functions to Monetary Policy Shocks - Euro Area. Sample: 1999:I-2008:II. Blue dashed horizontal lines identify the highest and lowest estimated reactions as for the benchmark U.S. economy. MF reactions constructed by normalizing the loading of PFA to one. First row: Responses to a standard monetary policy shock - normalized to induce a 1% on-impact increase in the policy rate. Last row: Responses to a trend inflation shock - normalized to induce a 1% on-impact increase in the inflation rate. Median Bayesian impulse responses based on 500 draws from the posterior densities.
Figure 7: **Impulse Response Functions to Monetary Policy Shocks - Germany.** Sample: 1999:I-2008:II. Blue dashed horizontal lines identify the highest and lowest estimated reactions as for the benchmark U.S. economy. MF reactions constructed by normalizing the loading of PFA to one. First row: Responses to a standard monetary policy shock - normalized to induce a 1% on-impact increase in the policy rate. Last row: Responses to a trend inflation shock - normalized to induce a 1% on-impact increase in the inflation rate. Median Bayesian impulse responses based on 500 draws from the posterior densities.
Figure 8: Impulse Response Functions to Monetary Policy Shocks - France. Sample: 1999:1-2008:II. Blue dashed horizontal lines identify the highest and lowest estimated reactions as for the benchmark U.S. economy. MF reactions constructed by normalizing the loading of PFA to one. First row: Responses to a standard monetary policy shock - normalized to induce a 1% on-impact increase in the policy rate. Last row: Responses to a trend inflation shock - normalized to induce a 1% on-impact increase in the inflation rate. Median Bayesian impulse responses based on 500 draws from the posterior densities.
Figure 9: Impulse Response Functions to Monetary Policy Shocks - United Kingdom. Sample: 1992:IIV-2008:II. Blue dashed horizontal lines identify the highest and lowest estimated reactions as for the benchmark U.S. economy. MF reactions constructed by normalizing the loading of PFA to one. First row: Responses to a standard monetary policy shock - normalized to induce a 1% on-impact increase in the policy rate. Last row: Responses to a trend inflation shock - normalized to induce a 1% on-impact increase in the inflation rate. Median Bayesian impulse responses based on 500 draws from the posterior densities.
Figure 10: Impulse Response Functions to Monetary Policy Shocks - Australia. Sample: 1993:1-2008:II. Blue dashed horizontal lines identify the highest and lowest estimated reactions as for the benchmark U.S. economy. MF reactions constructed by normalizing the loading of PFA to one. First row: Responses to a standard monetary policy shock - normalized to induce a 1% on-impact increase in the policy rate. Last row: Responses to a trend inflation shock - normalized to induce a 1% on-impact increase in the inflation rate. Median Bayesian impulse responses based on 500 draws from the posterior densities.
Figure 11: **Impulse Response Functions to Monetary Policy Shocks - New Zealand.** Sample: 1990:I-2008:II. Blue dashed horizontal lines identify the highest and lowest estimated reactions as for the benchmark U.S. economy. MF reactions constructed by normalizing the loading of PFA to one. First row: Responses to a standard monetary policy shock - normalized to induce a 1% on-impact increase in the policy rate. Last row: Responses to a trend inflation shock - normalized to induce a 1% on-impact increase in the inflation rate. Median Bayesian impulse responses based on 500 draws from the posterior densities.
Figure 12: Impulse Response Functions to Monetary Policy Shocks - Canada. Sample: 1991:I-2008:II. Blue dashed horizontal lines identify the highest and lowest estimated reactions as for the benchmark U.S. economy. MF reactions constructed by normalizing the loading of PFA to one. First row: Responses to a standard monetary policy shock - normalized to induce a 1% on-impact increase in the policy rate. Last row: Responses to a trend inflation shock - normalized to induce a 1% on-impact increase in the inflation rate. Median Bayesian impulse responses based on 500 draws from the posterior densities.