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SOPHISTICATED BIDDERS IN BEAUTY-CONTEST AUCTIONS

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Sophisticated Bidders in Beauty-Contest Auctions*

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Abstract

In this paper, we study bidding behavior by firms in beauty-contest auctions, i.e. auctions in which the winning bid is the one which gets closest to some function (average) of all submitted bids. Using a dataset on public procurement beauty-contest auctions in Italy and exploiting a change in the auction format, we show that firms' observed bidding behavior departs from equilibrium and can be predicted by an index of *sophistication*, which captures the firms' accumulated capacity of bidding well (i.e., close to optimality) in the past. We show that our empirical evidence is consistent with a Cognitive Hierarchy model of bidders' behavior. We also investigate whether and how firms learn to think and bid strategically through experience.

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1 Introduction

The competition among firms in market economies generates winners and losers: some firms survive and grow up, others go bankrupt and exit the market; some firms have large profits, pay dividends to their shareholders and their market values increase, others have poor performances and their market values fall. This outcome can be consistent with the predominant hypothesis used to model firms behavior in competitive environments, that is Nash equilibrium. If firms play Nash equilibrium, their decisions cannot be improved upon; hence, all firms' strategies are equally good. How can optimal decisions lead to negative outcomes? And how can equally good decisions lead to strongly heterogeneous outcomes? First, this can be a consequence of the fact that some firms (the winners) have some structural advantage with respect to others (the losers). Second, and most importantly, the real world is dominated by uncertainty: when firms take decisions, not all relevant information are perfectly available, so they base their decisions on the probability distribution of future payoffs. Thus a decision which is *ex-ante* optimal, may nevertheless generate poor performances *ex-post*, i.e., when uncertainty unfolds. Moreover, the information about uncertain variables is often asymmetrically distributed across firms, thus generating different decisions and heterogeneous performances for otherwise similar firms.

On the other hand, the presence of firms with poor, even negative, performances may simply be due to the fact that they made the *ex-ante* wrong decisions: they simply did not play their Nash equilibrium strategies. To date, there is a large body of experimental evidence showing that deviations from Nash equilibrium are systematic and significant even in relatively simple games. However, it is not clear to what extent the insights from the lab can be transferred to the field, especially when the decision makers are firms: differently from subjects involved in laboratory experiments, firms (i.e., managers) are probably less prone to psychological biases; moreover, the market provides much heavy penalties for errors and, thus, much stronger incentives to take the best decisions. Hence, it is still an open question whether and when the behavior of firms in competitive environments is better modeled by the equilibrium hypothesis or whether a nonequilibrium approach is more appropriate. In the field, addressing this question is often very difficult, if not impossible. The reason is that, most of the times, a sufficiently precise equilibrium prediction that serves as a benchmark cannot be obtained, as it is related to variables that are often impossible to observe, the most relevant being the information available to the firm.

In this paper we study deviations from Nash equilibrium in average bid auctions. These auctions resemble beauty-contest games in that the winning bid is the one which gets closest to some function (average) of all submitted bids. Average bid auctions have very precise Nash equilibrium predictions which are essentially unaffected by variables that are often unobservable: in equilibrium, either all or - possibly - most bids should be equal. This makes average bid auctions an ideal setting to investigate deviations from equilibrium.

Using an original dataset of procurement average bid auctions in the Italian region of Valle d'Aosta, we observe that actual bids significantly depart from equilibrium, being characterized by a systematic heterogeneity. We hypothesize that this heterogeneity could be the result of the interaction of firms with different abilities in performing an iterated process of strategic reasoning, in the spirit of the Cognitive Hierarchy model by Camerer et al. (2004). Applied to our context, this model predicts that more sophisticated firms, being able to formulate more accurate beliefs about how others are going to bid, make "better" bids, i.e., closer to the truly optimal one. We formulate an empirical reduced form model which shows that, in accordance with the main prediction of the Cognitive Hierarchy model, the firm's index of sophistication, measured by the accumulated capacity of bidding well in the past, is strongly and positively correlated to the goodness of that firm's bid, measured by the distance from the auction's reference point, which proxies the unconditionally optimal bid. This result is robust to several specifications of the empirical model; most importantly, it is also confirmed when we focus our analysis on a sample of auctions awarded with a new average bid format, which includes an aleatory element. Interestingly, our evidence shows that repeated participation and better past performance in the same format of auctions help firms become better strategic bidders.

Literature. This paper mainly contributes to two strands of literature. First, we relate to two recent papers which fit structural econometric Cognitive Hierarchy model on real data. In particular, Goldfarb and Yang (2009) study the decision by Internet Service Providers whether or not to adopt the then new 56K modem technology in 1997. Goldfarb and Xiao (2011) investigate the choice by U.S. managers of competitive local exchange carriers (CLECs) to enter local telephone markets after the Telecommunication Act in 1996. Both papers uncover significant heterogeneity of sophistication among managers, with more sophisticated managers less likely to adopt the new technology or to enter markets with more competitors. They also show that the level of sophistication is higher for firms operating in larger cities, with more competitors or in markets with more educated populations (Goldfarb and Yang) and for more experienced, better educated managers (Goldfarb and Xiao). Both these papers assume a Cognitive Hierarchy model, but do not address whether their model fits better than an equilibrium model.[1] In our paper, instead, we do not assume any structural model but show that the capacity of firms of making better decisions has a systematic component which goes in a direction coherent with a Cognitive Hierarchy model.

Second, our paper contributes to a recent empirical and experimental literature on average bid auctions. Decarolis (2014) and Bucciol et al. (2013) empirically compare the performances of average bid and first-price auctions for the procurement of public works in Italy. These papers show that the first-price is in general associated with lower awarding prices but worse performances in terms of cost and time overruns in the completion time. Conley and Decarolis (2013) argue that the average bid auction is weak to collusion as the members of a cartel, by placing coordinated bids, may pilot the average, thus increasing the probability that one of them wins. Using a dataset (different from ours) of Italian average bid procurement auctions, they construct a test to identify suspected cartels and validate it exploiting a subsample which includes cartels that were identified and condemned by the court. Applying then the test to their dataset, they found that a large fraction of auctions (no less than 30%) is likely to be affected by the presence of cartels; thus, they conclude that the observed deviations from Nash equilibrium are mostly due to a cooperative behavior by bidders. Our paper suggests a complementary explanation to the observed bidding behavior in this type of auctions, but based on a non-cooperative argument. Nevertheless, we provide and discuss some arguments supporting the robustness of our findings to the possible presence of collusion. Chang et al. (2013) experimentally investigate whether a simple average bid auction can be an effective alternative to first-price auctions for an auctioneer concerned with reducing winner's curse phenomena in common value settings. Their results suggest a positive answer: in the average

[1] Brown et al. (2012) use a Cognitive Hierarchy model to explain empirical evidence on box-office premiums associated to cold-opened movies, i.e., movies that are not shown to critics prior their release. In their paper, consumers, not firms, have limited capacity of strategic thinking and firms exploit the consumers' naïveté to extract more surplus by not disclosing information on the low quality movies.

bid auction, subjects do not coordinate on high prices as the Nash equilibrium would predict; rather, they follow a bidding strategy which, surprisingly, is almost identical to the one followed by subjects playing the first-price auction. Given that the bids are identical but the pricing rules are different, this leads to prices being higher in the AB than in the first-price auction, thus reducing losses and virtually eliminating default problems. The authors also discuss the observed bidding behavior in the average bid format. They argue that the observed pattern of bids is better captured by an almost-equilibrium behavior, rather than by a level-k model. In reaching this conclusion, they assume that level-0 bidders bid randomly in the full support of possible costs, thus totally ignoring the informative content of their signals. We suspect that a different assumption on level-0 bidders' behavior may reduce the discrepancy between their evidence and the theoretical predictions from the level-k model.

The rest of the paper is organized as follows: in Section 2, we illustrate the formats of auctions considered, describe our dataset and present some preliminary descriptive evidence; in Section 3, we show that our evidence is clearly inconsistent with Nash equilibrium and obtain a testable prediction from a CH model; this prediction leads to the empirical analysis, provided in Section 4; Section 5 offers a discussion of our results, with further supporting evidence; Section 6 briefly concludes.

2 Auction formats, data and descriptive evidence

The large majority of public works in Italy are procured by means of average bid auctions: these are auctions in which the winner is not the firm that offers the best (i.e., lowest) price, but the one whose offer is closest to some endogenous function (average) of all submitted offers. Participating firms submit a (sealed) price consisting of a percentage discount on the reserve price set by the Contracting Authority (CA).[2] Once the CA has verified the firms' legal, fiscal, economic, financial and technical requirements, the winning firm is determined according to the following mechanism (see Figure 1, top panel): discounts are ordered from the lowest to the highest and a first average ($A1$) is computed excluding the 10% highest and lowest bids.[3] Then, a second average ($A2$) is computed by averaging all bids strictly above $A1$ (again excluding the 10% highest bids). The winning bid is the one immediately *below* $A2$. In the event that all bids are equal, the winner is chosen randomly. We call this auction format "Average Bid", or simply AB.

Our dataset collects auctions for public works issued by the Regional Government of Valle d'Aosta in the period 2000-2009 (data are from Moretti and Valbonesi, 2012). It contains all bids submitted in each auction, together with several detailed information at the firm- and auction-level: for each participating firm, we know the identity (i.e., company name) and some characteristics such as size and subcontracting position; for each auction, we have information on the reserve price, the task of the tendered project and the estimated duration of the work.

An interesting feature of our dataset is that it covers a change in the auction format. In fact, while public works before 2006 were awarded through the AB format described before, since 2006, and *only* in the region of Valle d'Aosta, a new average bid awarding mechanism

[2]Hence, a higher discount means a lower price paid by the CA. In the rest of the paper, we will use the terms bids and discounts interchangeably.

[3]For example, if there are 20 bids, the 2 lowest and the 2 highest bids are excluded in the computation of $A1$. When this 10% is not an integer, the number of excluded bids is obtained by rounding up: for example, if there are 25 bids, the 3 lowest and the 3 highest bids are excluded.

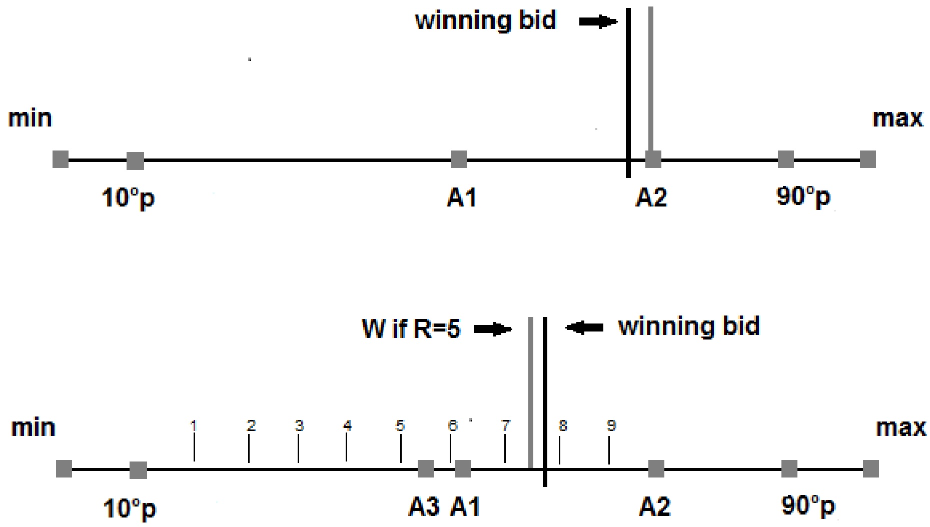

Figure 1 – AB (top panel) and ABL (bottom panel) auction.

has been introduced. The new auction format differs from the previous one as it includes an aleatory element; for this reason, we call it "Average Bid with Lottery" auction, or simply ABL. The ABL auction works as follows (see Figure 1, bottom panel): given the average $A2$ computed as in AB, a random number (R) is extracted from the set of nine equidistant numbers between the lowest bid above the first decile of bids and the bid immediately below $A2$. Averaging R with $A2$, the winning threshold W is obtained and the winning bid is the one closest from *above* to W. Again, if all bids are identical, the winner is chosen randomly. To be precise, if we denote by $d_{10\%}$ the discount immediately above the first decile of the bid distribution and by d_{A2} the discount immediately below $A2$, then the winning threshold is $W = [A2 + R]/2$ where $R = d_{10\%} + (d_{A2} - d_{10\%})i/10$ and i can be any integer between 1 and 9. Hence, the winning threshold will necessarily fall within an interval whose lower and upper bounds are $[A2 + d_{10\%} + (d_{A2} - d_{10\%})/10]/2$ and $[A2 + d_{10\%} + (d_{A2} - d_{10\%})9/10]/2$, respectively. We denote the lower bound of this interval by $A3$.

Figure 2 shows non-parametric kernel density estimation of the bid distributions in the AB and ABL formats (dashed line for AB and straight line for ABL). For each auction, discounts have been re-scaled using a min-max normalization (the lowest discount in an auction takes value 0, while the highest takes value 1).

Figure 2 highlights two relevant features. First, in either formats, bids are clearly neither uniformly, nor normally distributed. Second, the distributions are clearly asymmetric and different across the two formats: in AB, most bids are concentrated in the right end of the support of the distribution of bids; in ABL, most bids are concentrated below the midpoint of the support.

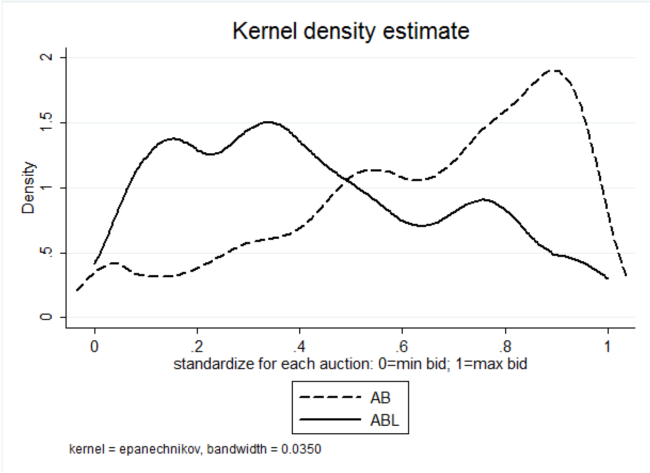

Figure 2 – Discounts in AB and ABL: Kernel density estimation.

3 Theory: Equilibrium vs. Cognitive Hierarchy

The descriptive evidence presented in Figure 2 suggests that the bidding behavior by firms in our dataset is characterized by some regularities. In this section, we first investigate whether this evidence can be consistent with the standard notion adopted to model bidders' behavior in auctions, i.e. Nash equilibrium. To this end, we consider a standard symmetric independent private value model with n firms: a firm's cost of completing the job is private information to that firm, but it is commonly known that all firms' costs are independently and identically distributed according to a cumulative distribution function F.

Under these assumptions, we obtain rather sharp predictions on the (Bayes-) Nash equilibria in the two formats, that we summarize as follows:[4]

(i) *In the AB auction, there is a unique symmetric equilibrium in which firms submit a* 0*-discount irrespective of their costs.*

(ii) *In the ABL auction, it is a symmetric equilibrium for firms to submit a constant discount d irrespective of their costs, provided that d guarantees a non-negative profit even to the highest cost firm.*

(iii) *In the ABL auction, in any equilibrium, all firms' whose costs are below a certain threshold $\hat{c}$ make the same discount $\bar{d}$, while all other firms make a discount which is no greater than $\bar{d}$. The threshold must satisfy $F(\hat{c}) > (n - \tilde{n})/(n + 1)$, where $\tilde{n}$ denotes the smallest integer greater than or equal to $n/10$; hence the ex-ante probability that a given firm bids $\bar{d}$ must be sufficiently large.*[5]

[4] For the proofs, see Appendix A.

[5] If we relax our assumptions (private costs, symmetry across firms, no cost uncertainty, no default risks), the all-zero equilibrium of the AB auction will not change. Similarly, in ABL, we would still have equilibria in which all firms make the same bid, as long as this common bid is sufficiently low. The point is that, unlike a first-price auction where a higher bid always increases the probability of winning and thus stronger bidders - those with lower production or default costs - will bid higher, here to increase the probability of winning a bidder has to make a bid which is neither too high, nor too low; hence, having a lower production or default

Results *(i)* and *(ii)* positively identifies equilibria in which all firms make the same bid, whatever their actual cost is. This is clearly inconsistent with our evidence, where bids are far from being equal (the standard deviation of the distribution of bids is, on average, 4.7% in AB and 4.0% in ABL). For the AB auction, this conclusion is reinforced by the evidence that bids are significantly greater than zero (the average discount is 17.9%), while equilibrium predicts all bids equal to zero. In the ABL auction, given the multiplicity of equilibria, there is a potential problem of coordination, and one could object that our evidence is just the result of a coordination failure. However, this explanation does not seem fully convincing: first, it would apply to the ABL format only, leaving the observed behavior in AB unexplained; second, even restricting this explanation to the ABL case, the observed regular asymmetry in the distribution of bids would raise the following question: why do many firms reach a good coordination on relatively low discounts, whereas other firms seems totally unable to coordinate?

We must also take into account that, in the ABL auction, there might be other equilibria (prediction *(iii)*). In these equilibria (provided they exist), all firms whose cost is below a certain threshold $\hat{c}$, make the same bid $\bar{d}$, and all other firms bid lower. Most importantly, the threshold that separates these two sets of firms, must be sufficiently high: in particular, $F(\hat{c}) > (n - \tilde{n})/(n + 1)$. As a consequence, if firms were indeed playing an equilibrium of this kind, the outcome we should expect to observe is one with a large fraction of firms making the same, highest bid $\bar{d}$, with the remaining (few) firms bidding lower.[6] This is clearly at odds with our descriptive evidence, according to which the typical frequency distribution of bids in an ABL auction has its mode below the midpoint of the range of bids.

We conclude that Nash equilibrium does not seem to be a correct modeling hypothesis for the bidding behavior of firms in our dataset. Although we do reject the equilibrium hypothesis that *all* firms are bidding optimally, our intuition is that *some* of them are doing so, while others are not. One model that supports this intuition is the Cognitive Hierarchy (CH, henceforth) model. This model has been introduced by Stahl and Wilson (1994, 1995) and further developed and applied by, among others, Camerer et al. (2004). Strictly related to the CH model is the level-k model introduced by Nagel (1995) and applied to first- and second-price auctions by Crawford and Iriberri (2007). The CH model has proved to be particularly fruitful in explaining experimental evidence in beauty-contest games. Since average bid auctions are nothing but incomplete information versions of beauty-contest games, the CH model is a natural candidate to explain our evidence.

The CH model holds that individuals (players) involved in strategic situations differ by their level of *sophistication*, i.e., their ability of performing an iterated process of strategic thinking. The proportion of each level in the population is given by a frequency distribution $P(k)$, where $k = 0, 1, 2, \ldots$ is the level of sophistication. Level-0 players are completely unsophisticated and simply play randomly (according to some probability distribution, in general uniform); a level-k player, with $k \geq 1$, believes that her opponents are distributed, according to a normalized version of $P(k)$, from level-0 to level-$(k-1)$ and chooses her optimal strategy given these beliefs. For example, a level-1 player believes that all her opponents are of level-0; a level-2 player believes that her opponents are a mixture of level-0 and level-1 players, where the proportion of level-0 players is $P(0)/(P(0) + P(1))$; and so on. In other

cost is much less of an advantage than in a first-price auction. As a consequence, the cost and the information structures play a less important role in shaping the equilibrium.

[6] For example, if there are 10, 25, 50, 100 participants, the probability that a given firm has a cost lower than the threshold $\hat{c}$ (and thus make the highest bid $\bar{d}$) should be at least 0.81, 0.85, 0.88, 0.89.

words, a level-k player's strategy is optimal *conditional on her beliefs*, but since her beliefs do not contemplate the presence of players of the same or higher level, the resulting strategy will in general be suboptimal. Clearly, a player with higher level of sophistication has in mind a more comprehensive picture of how other players think and play; hence, we expect her strategy to be closer to the unconditionally optimal one.

The logic behind the CH model seems particularly appropriate in our context. In an average bid auction, all bids affect the position of the relevant average and thus the probability that a given bid will be the winning. Therefore, it is crucial to have correct guesses on how all other firms are going to bid. But predicting the behavior of all other firms involves answering to a complicated chain of questions of the kind: what bid b will a firm make if she thinks others are going to bid a? And what bid c will a firm make if she anticipates that others are going to bid b because they think others are going to bid a? And so on. Firms who are able to push this chain of reasoning further will have an advantage over those who perform less steps of such reasoning, in the sense that they will end up with more precise predictions on the actual behavior of others. As a consequence, they are expected to make better (i.e., closer to optimality) bids.

To get a more clear picture of the implications of a CH model in our context, we performed some simulation exercises.[7] The simulations were run assuming that firms' levels of sophistication range from 0 to 2 and that they are distributed according to a truncated Poisson.[8] Level-0 firms are assumed to draw their bids from a uniform distribution over a restricted interval,[9] chosen in a way that ensures that level-0 firms will never play dominated strategies. We computed the *conditionally* optimal bids by level-1 and level-2 firms (i.e., those bids that maximize their expected payoffs under the belief that all other firms are of lower levels). Given the behavior of level-0, level-1 and level-2 firms, we computed the *unconditionally* optimal bid, which is the bid that would maximize the expected payoff of a firm who has fully correct beliefs about the behavior of other firms. Finally, since our objective is to check the consistency of the results of the simulations with real data, we allowed firms to make logistic (i.e., payoff-sensitive) errors. As expected, all simulations performed show that:

(CH1) *In either auction, higher-level firms make "better" bids than lower-level firms, in the sense that the distance of a firm's bid from the unconditionally optimal bid is decreasing in her level of sophistication.*

Moving this prediction to data, however, is problematic, as we do not observe what the unconditionally optimal bid in any auction is. However, we can proxy it. In the AB auction, the intuition suggests that the optimal bid cannot be too far from the expected value of the winning threshold $A2$. By definition, a firm's optimal bid must maximize her expected payoff given the behavior of others. And since the evidence provided by Figure 2 suggests that there are typically many bids around the realized value of $A2$, a bid will have a significant probability of winning the auction only if it is rather close to $A2$ itself.

[7]Given the complexity of the AB and ABL auctions, we cannot obtain precise theoretical predictions. The results of the numerical simulations are reported in Appendix B.

[8]We consider only level-1 and level-2 firms because optimal bids by level-1 and level-2 firms turn out to be rather close, especially when the number of firms is large; thus, considering higher levels will not give additional insights, at least qualitatively. Moreover, experimental evidence has shown that the majority of subjects performs no more than 2 levels of iteration (see, e.g., Crawford et al. 2013).

[9]This is the most common assumption adopted in the CH-literature for level-0 players.

In the ABL auction, taking the winning threshold W as a proxy for the unconditionally optimal bid does not seem fully convincing: even if a firm correctly anticipates the bidding behavior of others, she cannot estimate the exact location of the winning threshold, as this is the result of an unpredictable lottery. However, such firm can estimate the expected lower and upper bounds of the interval where the winning threshold will fall. Now, given good expectations on these bounds, the intuition suggests that it should be better to place a bid close to the lower than to the upper bound of this interval: the probability of winning is similar in both cases, but a bid closer to the lower bound will guarantee a higher payoff in case of winning. This intuition leads us to consider $A3$ (the lower bound of this interval) as possible proxy for the optimal bid in ABL.

To verify the accuracy of $A2$ (for the AB auction) and $A3$ (for the ABL auction) as proxies for optimal bidding, in the simulations we also computed their expected values, which, as a matter of fact, turn out to be very close to the unconditionally optimal bid. Hence, we take $A2$ and $A3$ as proxies for optimal bidding in the two auctions, and we call them the *reference points* for AB and ABL, respectively. Most importantly, the negative relationship between sophistication level and distance from optimal bid translates into an analogous relationship between the former and the distance from the reference point. In other words, prediction (CH1) can be rephrased in terms of the (observable) reference points as follows:

(CH2) *In either auction, the distance of a firm's bid from the auction's reference point is decreasing in the level of sophistication of the firm.*

4 Empirical analysis

The previous section has shown that, in our context, the CH model implies that, if firms have different sophistication levels, this should reflect in different bids by them. An heterogeneity in bidding behavior is indeed apparent in our data (see Figure 2); however, deeper statistical analysis is needed to asses whether such heterogeneity is related to firms' sophistication in the direction prescribed by the CH model, namely that more sophisticated firms bid closer to optimality (prediction (CH1)). The fact that the optimal bid can be well approximated by the (observable) reference point allows us to obtain a simpler testable prediction (CH2). However, to empirically test (CH2), we first need to measure firms' sophistication level.

4.1 A measure of firms' sophistication

In accordance with the fundamental idea of the CH model, a measure of firms' (i.e., managers') sophistication should capture their ability of thinking strategically in interactive situations. Needless to say, measuring this ability is a complicated task. One possibility would be to rely on some instruments, like some measure of ability, education or professional achievements of firms' managers.[10] We refrain from following this strategy for two reasons. First, we lack information on firms' managers or other firms' characteristics that may proxy strategic ability. Second, and most importantly, although innate and/or previously acquired skills certainly

[10]Burnham et al. (2009) showed experimentally that subjects who obtained higher scores in a psychometric test of cognitive ability performed better in a beauty-contest game. Goldfarb and Xiao (2011), who fitted a CH model to the entry decisions by managers in the US local telephone markets, uncovered a significant positive relationship between managers' strategic ability on the one hand, their education and experience as CEOs on the other.

matter, the intuition and the literature suggest that individuals can learn to think strategically in games as they play over and over again. Hence, in a context like ours in which we observe the same firms bidding repeatedly, an out-of-sample static measure of sophistication would miss this learning component. Instead, we need a measure of sophistication that can change dynamically within the sample. To this end, we follow a completely different approach: for each auction in our sample, we measure a firm's sophistication by the relative distance of that firm's bids from the reference point in the preceding auctions of that format to which she participated in. The idea is that, if the CH model is indeed a good model of firms' bidding behavior, then we can "invert" prediction (CH2) and take the distance from the reference point as an outcome-based measure of her capacity of thinking strategically.[11]

Specifically, the index of sophistication of firm i at the moment in which she participates in auction j is computed as:

$$BidderSoph_{ij} = \sum_{k=1}^{j-1} \left(1 - \frac{\Delta_{ik} - \Delta_k^{min}}{\Delta_k^{max} - \Delta_k^{min}}\right) \times \mathbb{1}_{[i \text{ participated to } k]} \times \mathbb{1}_{[k \text{ is the same format as } j]} \tag{1}$$

where Δ_{ik} is the distance of firm i's bid from auction k's reference point and Δ_k^{min} and Δ_k^{max} are the distances from the reference point of the closest and furthest bid submitted in auction k. Notice that each term in the summation in (1) is between 0 and 1 and takes value 0 (1) if firm i's bid was the furthest (closest) to the reference point in that auction.

The index of sophistication (1) is clearly dynamic, as it changes from one auction to the next depending on the outcome of the last auction. Hence, it allows a firm's level of sophistication to increase or decrease relative to the others. The idea is that firms may learn to think strategically as they gain experience in the auction mechanism. Similarly, a firm may lose positions in the sophistication ranking if she does not take into account that other firms may become better strategic thinkers through learning. Notice that our index of sophistication is auction format-specific, in the sense that participations to AB do not contribute to the firm's index of sophistication when she bids in ABL. The idea is that what matters is not experience per se, but experience in that particular strategic situations.[12]

Figure 3 shows the distributions of bids in AB (left panel) and ABL (right panel) for highly and lowly sophisticates firms (i.e., firms with sophistication index above the 90th percentile vs. those with sophistication index below the 10th percentile). These graphs point out an heterogeneity in bidding behavior which goes in the direction suggested by predictions (CH1) and (CH2). In particular: *(i)* bids by highly sophisticated firms are more concentrated than those by lowly sophisticated ones; *(ii)* highly sophisticated firms' bids are concentrated in the right tail of the distribution of bids in AB and in the left tail in ABL.

4.2 Empirical model and results

In this section, we first present the empirical model; we then provide descriptive statistics on our dataset and estimation results. Finally, we focus on the learning process.

[11] In some sense, we are adopting an approach similar to revealed preference: we derive the determinants of behavior by induction from the behavior itself.

[12] The use of a measure of a firm's sophistication that weighs all previous participations not only allows to take into account that a firm may learn to think strategically through experience, but also gives more robustness to the results: in a single auction, a firm may bid close to optimality by chance; in a series of auctions, a firm systematically bids close to optimality only if she is a good strategic thinker. By using a cumulative measure, the impact of lucky bids results downsized.

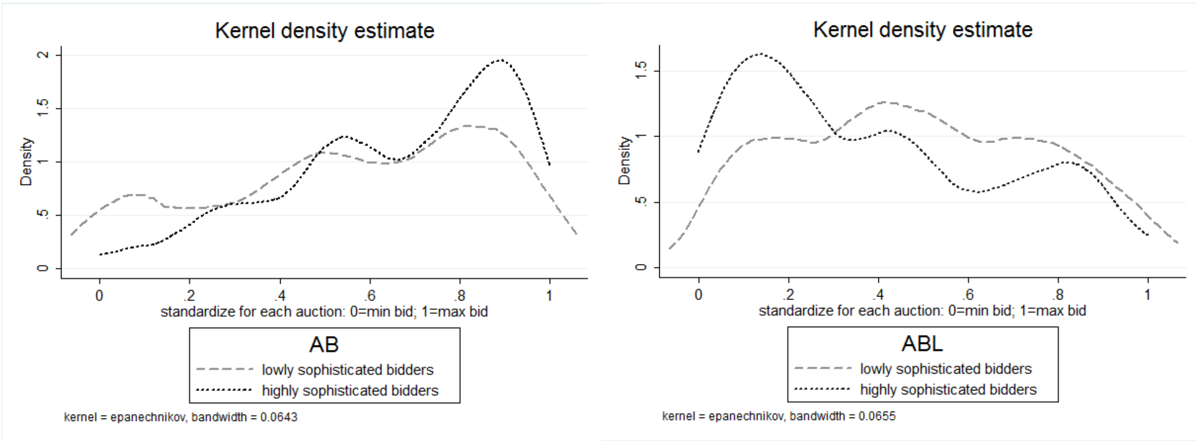

Figure 3 – Discounts in AB (left panel) and ABL (right panel): Kernel density estimation by highly and lowly sophisticated firms.

Model. Given the measure of firms' sophistication described in the previous section, we can now introduce and estimate a reduced form model aimed at testing prediction (CH2). The model specification is the following:

$$\log|Distance_{ij}| = \alpha + \beta \log(BidderSoph_{ij}) + \gamma F_i + \sigma FP_{ij} + \theta P_j + \epsilon_{ij}. \tag{2}$$

In (2), the dependent variable, $\log|Distance_{ij}|$, is the logarithm of the difference (in absolute value) between firm i's bid in auction j and auction j's reference point ($A2$ if auction j is AB, $A3$ if it is ABL). $BidderSoph_{ij}$ is firm i's sophistication index at the moment of participation in auction j, as defined by (1). F_i represents a set of characteristics of firm i which do not vary over time, including proxies for size and location.[13] To reduce the omitted variable problems, in some specifications we also included firms' fixed effects to adjust for firm-specific characteristics; this enables us to focus on the within firm variation in the sophistication status. FP_{ij} is a set of firm's characteristics which vary for each auction. This set includes the firm's backlog of works (i.e., the number of pending projects the firm has at the moment she bids in auction j; it is a proxy for capacity constraints) and the firm's subcontracting position.[14] P_j is a set of variables to control for the characteristics of the auction (number of participants, year dummy variables to adjust for temporal shocks to the firms and the CA) and of the auctioned work (dimension and complexity).[15]

[13]Because we do not have data on firms' employees or total assets, we construct proxies for firms' size based on the type of business entity: *Small* = one-man businesses, limited and ordinary partnerships; *Medium* = limited liability companies; *Large + cooperatives* = public corporations and cooperatives. The use of these proxies is motivated by the evidence of a positive correlation between the type of business entity and the size of Italian firms (see Moretti and Valbonesi, 2012, and Coviello et al., 2013). To proxy firms' location, we take the geographical distance between Aosta (i.e., the seat of the CA) and the chief town of the province in which the firm has her headquarters. We assign a distance of 30 kilometers to firms located in Valle d'Aosta.

[14]According to the Italian regulation on public procurement, fully qualified firms are allowed to freely choose to subcontract the works once they win, while partially qualified firms are required to subcontract the works for which they are not qualified. Moretti and Valbonesi (2012) show that firms' discounts at the bidding stage are affected by their subcontracting positions.

[15]In the procurement literature, the complexity of a project is usually proxied by the project's value or the auction's reserve price, the expected contractual duration of works, dummies for the categories of works included in the project. We use all these proxies in our estimation.

Table 1 – Estimated sample.

	AB			ABL		
	Obs.	Mean	SD	Obs.	Mean	SD
Firm-auction level:						
(log) $\|Distance\|$	8927	-0.601	1.648	1501	-0.464	1.421
(log) $BidderSoph$	8927	2.585	1.302	1501	0.959	1.074
(log) Backlog	8927	0.932	0.769	1501	0.739	0.708
Optional Subcontracting	8927	0.871	0.336	1501	0.817	0.387
Auction level:						
Reserve price (euro)	232	1,120,365	895,493.5	28	1,109,662	681,532.5
Expected duration (days)	232	301.431	166.172	28	402.857	177.353
No. Bidders	232	53.216	28.613	28	82.857	41.662
Building construction	232	0.134	0.341	28	0.107	0.315
Road works	232	0.388	0.488	28	0.286	0.460
Hydraulic works	232	0.306	0.462	28	0.321	0.476
Firm level:						
Small size	514	0.158	0.365	319	0.160	0.367
Medium size	514	0.589	0.492	319	0.624	0.485
Large size	514	0.253	0.435	319	0.216	0.412
Distance firm-CA (km)	514	449.463	448.476	319	344.765	391.891

Descriptive statistics. Table 1 shows summary statistics of the sample we used in our estimations, broken down by auction format.[16] The sample of 232 AB auctions includes 8,927 bids offered by 514 different firms; the sample of 28 ABL auctions includes 1,501 bids offered by 319 different firms. The average auction's reserve price is around 1.1 million euros in both types of auctions and the average number of participating firms per auction is about 53 in AB and 83 in ABL.[17] Most of the auctions concern road works (38.8% of the AB auctions; 28.6% of the ABL auctions), hydraulic works (30.6% of the AB auctions; 32.1% of the ABL auctions) and building construction (13.4% of the AB auctions; 10.7% of the ABL auctions). Also firms' other characteristics, such as size, backlog and subcontracting position, are similar in the two formats.

Main results. In Table 2, columns (1) and (2), we present our estimation results for the sample of AB auctions. The negative and statistically significant coefficient of $BidderSoph$ shows that firms with a higher index of sophistication tend to bid closer to the reference point ($A2$ in this case), thus supporting prediction (CH2). This result is robust to the inclusion of covariates at auction-, firm- and firm-auction-level (column (1)), or firms' fixed effects (column (2)). The inclusion of firms' fixed effects allows us to explore the within firm variability and to reduce selection-bias and omitted variable problems. Moreover, at least for firms whose management did not change along the sample period, the fixed effect captures the role of the innate component of sophistication peculiar to that firm/manager.

[16] These descriptive statistics refer to the sample used for the empirical analysis proposed in this section. The original sample was slightly larger (267 auctions). The sophistication index is computed on this larger sample to avoid being influenced by partial observations. However, due to missing values in some control variables, our regression analyses are based on the restricted sample. Note that we focus on firms that bid at least twice because, by definition, for the first participation, $BidderSoph$ is equal to 0.

[17] Here the number of participating firms is used as a proxy for the level of competition in the auction, thus it was computed on the larger original sample.

Table 2 – Empirical results.

Dependent variable	$\log \lvert Distance \rvert$			
Auction format	AB	AB	ABL	ABL
	(1)	(2)	(3)	(4)
$\log(BidderSoph)$	-0.181***	-0.171***	-0.371***	-0.388***
	(0.022)	(0.038)	(0.043)	(0.063)
Auction/project controls	YES	YES	YES	YES
Firm controls	YES	NO	YES	NO
Firm fixed effects	NO	YES	NO	YES
Firm-auction controls	YES	YES	YES	YES
Observations	8,927	8,838	1,501	1,410
R-squared	0.198	0.270	0.296	0.470

OLS estimations. Robust standard errors clustered at firm-level in parentheses.
Inference: (***) = $p < 0.01$, (**) = $p < 0.05$, (*) = $p < 0.1$.
Auction/project controls include: the auction's reserve price, the expected duration of the work, the number of bidders, dummy variables for the type of work, dummy variables for the year of the auction. *Firm controls* include: dummy variables for the size of the firm, and the distance between the firm and the CA. *Firm-auction controls* include: a dummy variable for the firm's subcontracting position (mandatory or optional), and a measure of the firm's backlog.

Table 2 also reports the results of the regressions for the ABL auction sample. Looking at the ABL sample is illuminating because it allows us not only to test the role of firms' sophistication in a different average bid format, but also to address potential measurement error problems. In fact, while the AB format has long been and is widely used in Italy to award public works, the ABL format was introduced in 2006 and only in Valle d'Aosta. As a consequence, the sophistication index for the AB sample does not take into account that firms may have gained experience (and thus sophistication) by participating in AB auctions issued by other Italian CAs and/or in the past;[18] in the ABL case, instead, the sophistication index is immune from these measurement error problems. Now, also for the ABL sample, the relationship between sophistication index and the distance from the reference point is highly significant and negative, as predicted by the CH model. This is true in both specifications, without (column (3)) and with (column (4)) firms' fixed effects.

Learning dynamics. The previous analysis showed that, in line with the prediction obtained from a CH model, there is a stable relationship between firms' index of sophistication and the distance of their bids from the auction's reference point. However, that analysis does not say much about the dynamics behind this relationship. In particular, do firms learn to think and bid strategically as they participate in more and more auctions? And, if so, what are the determinants and the characteristics of this learning process? Our starting point is the evidence suggested from the kernel density distribution of bids in AB auctions issued during the first (2000) and last (2005) year covered by our dataset. Figure 4 shows that, compared to year 2000, bids in 2005 are generally much more concentrated on the right side of the distribution, thus suggesting that a learning process is most likely taking place.

[18] Anyhow, we believe that the impact of the experience gained outside Valle d'Aosta should be limited because the knowledge of the specificity of each market (first of all, its players) is extremely important. Moreover, the sophistication accumulated in the past (i.e., before year 2000) should be captured by the fixed effects.

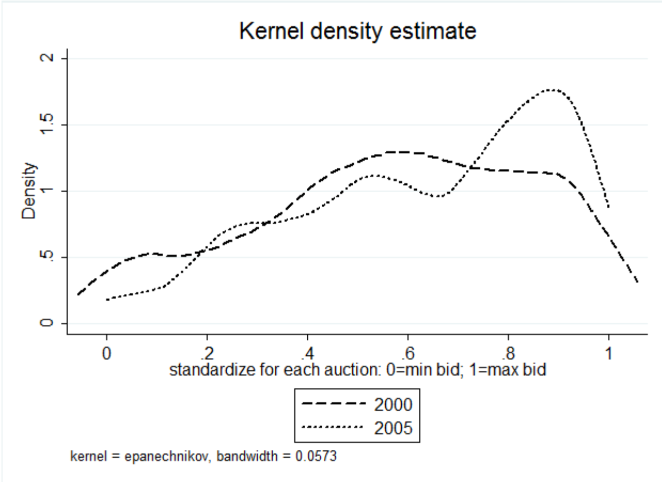

Figure 4 – Discounts in year 2000 and 2005: Kernel density estimation.

To investigate such process more in depth, we decompose firm i's sophistication index at auction j into two components. The first component is simply the number of past participations by firm i in auctions of the same format as j and is meant to capture the pure role of experience; we denote this variable by $PastPart$. The second component is the average performance of firm i in all previous auctions until j, measured as (the absolute value of) the distance of her bid from the auction's reference point. This variable, denoted by $PastPerf$, is intended to proxy the degree at which the firm learns to think and bid strategically from her past performance. Furthermore, we take into account also a third component, given by the innate (i.e., at time 0) strategic skills of the firm; this component is captured by the firms' fixed-effects. If a firm learns from her past experience (everything else being equal, including her past performance and innate ability), we expect $PastPart$ to negatively affect the distance between her bid and the reference point in future auctions. Similarly, if a firm learns from her past performance (everything else being equal, including her past experience and innate ability), we expect $PastPerf$ be negatively associated with future performance.

Table 3, columns (1)-(2) shows the results obtained by estimating a regression model like (2) with firms' fixed effects, with $BidderSoph$ replaced by the variables $PastPart$ and $PastPerf$ as regressors. Column (1) shows that, in the AB auctions, $PastPart$ has a negative and statistically significant coefficient, while the sign of the estimated coefficient of $PastPerf$ is positive but only sightly significant. When we focus on the sample of ABL auctions, instead, both coefficients are negative but the coefficient of $PastPerf$ is not statistically significant (column (2)).

A natural way to get deeper evidence on this issue is to investigate whether the learning process is actually characterized by non-linearities. Estimation results of equations including the quadratic terms for both $PastPart$ and $PastPerf$ show that the learning process is indeed non-linear (columns (3) and (4)). In particular, for both the AB and the ABL samples, we obtain negative and significant coefficients of the linear terms of $PastPart$ and $PastPerf$, but positive and statistically significant coefficients of their quadratic terms. These results underline that a higher number of participations and a better past performance are significantly associated with future bids closer to the reference points, but these marginal effects

Table 3 – Learning dynamics.

Dependent variable	log $|Distance|$							
Auction format	AB	ABL	AB	ABL	ABL	ABL	ABL	ABL
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$PastPart$	-0.004**	-0.031**	-0.021***	-0.142***				
	(0.002)	(0.014)	(0.003)	(0.034)				
$PastPart$, sq.			0.000***	0.004***				
			(0.000)	(0.001)				
$PastPerf$	0.616*	-0.653	-4.900***	-4.264***				
	(0.366)	(0.457)	(1.023)	(1.216)				
$PastPerf$, sq.			4.717***	3.503***				
			(0.794)	(1.128)				
log($BidderSophAB$)					-0.094**	-0.018		
					(0.038)	(0.040)		
log($BidderSoph$)						-0.387***		-0.385***
						(0.047)		(0.048)
$PastPartAB$							-0.002*	-0.000
							(0.001)	(0.001)
$PastPerfAB$							-0.953*	-0.672
							(0.508)	(0.476)
Auction/project controls	YES	YES	YES	YES	YES	YES	YES	YES
Firm controls	NO	NO	NO	NO	YES	YES	YES	YES
Firm fixed effects	YES	YES	YES	YES	NO	NO	NO	NO
Firm-auction controls	YES	YES	YES	YES	YES	YES	YES	YES
Observations	8,838	1,410	8,838	1,410	1,356	1,356	1,356	1,356
R-squared	0.269	0.459	0.276	0.471	0.255	0.301	0.256	0.303

OLS estimations. Robust standard errors clustered at firm-level in parentheses.
Inference: (***) = $p < 0.01$, (**) = $p < 0.05$, (*) = $p < 0.1$.
Auction/project controls include: the auction's reserve price, the expected duration of the work, the number of bidders, dummy variables for the type of work, dummy variables for the year of the auction. *Firm controls* include: dummy variables for the size of the firm, and the distance between the firm and the CA. *Firm-auction controls* include: a dummy variable for the firm's subcontracting position (mandatory or optional), and a measure of the firm's backlog.

are decreasing.[19] These effects show that the learning process has a convergence path: an additional participation and an improvement in the past performance have larger impacts for firms with few past participations and poor past performance, respectively.

Given the peculiarity of our dataset characterized by a change in the auction format, and given the results about the learning dynamics just illustrated, it is interesting to understand whether firms in ABL auctions drew lessons from what they learned in the AB auctions (in our sample, 240 firms participated both in AB and ABL auctions). Recall that our sophistication index was constructed assuming that sophistication is auction format-specific, in the sense that participations to AB do not contribute to the firm's index of sophistication when she bids in ABL. Hence, answering this question is an indirect way to test how restrictive this assumption is. To this end, we focus on the sample of ABL auctions and introduce in our model (2) an additional variable, $BidderSophAB$, representing, for each firm, the highest level of the sophistication index achieved in the period of AB auctions. Table 3, column (5), shows that a higher sophistication index achieved in the AB period is associated with a lower distance from the reference points in ABL auctions. However, when we re-introduce (in column (6))

[19] Note that, for both variables, the turning points - i.e., the value above which the marginal effect becomes positive - are outside the range of observation. Note also that, in columns (1) and (2), signs and statistical significance do not change when we replace $PastPart$ and $PastPerf$ with their logs.

the firm's sophistication index associated to the ABL auctions ($BidderSoph$), the coefficient of the former indicator is not statistically different from zero, while the auction-specific index of firm's sophistication is still negative and statistically significant. This result indicates that sophisticated firms in AB auctions would tend to offer bids closer to the reference point also in ABL auctions, but the strategic ability they acquired in AB auctions does not have any effect once we control for the ability acquired within ABL auctions; in fact, it is the latter that significantly contributes to explain the distance from the reference point in ABL auctions. Similar results are obtained when we introduce in the model specification the number of participations ($PastPartAB$) and the average past performance ($PastPerfAB$) in AB auctions (columns (7) and (8)). The estimated coefficients of these two variables are not statistically different from zero, once we control for the ability acquired by the firm during ABL auctions.

5 Discussion

The analysis presented in the previous section provides evidence that supports, at least qualitatively, a non-equilibrium model of bidding behavior by firms in average bid auctions: observed deviations from the optimal bid are related to a measure of firms' capacity of bidding strategically, the sophistication index; this relation goes in the direction predicted by a CH model. Therefore, our (continuous) sophistication index proxies the (discrete) CH-level of sophistication by firms. The analysis showed that the relation between sophistication index and bidding behavior is robust to a number of determinants, including auction's, firm's and firm-auction's specific characteristics. Most importantly, the relation holds also when we analyze the ABL format, which is new to the firms and characterized by an aleatory element that makes it more complicated for firms to formulate their bidding strategies.[20]

One might wonder whether our findings are robust to the consideration of other factors as well as to a deeper investigation. Below, we discuss some of these issues.

A very interesting aspect that is worth addressing here is related to possible collusive behaviors by firms. In a recent paper, Conley and Decarolis (2013), using a different dataset of AB auctions, argue that this format can be characterized by the presence of colluding firms which drive the winning threshold to let one member of the cartel win. The empirical evidence on AB auctions would thus be the result of a cooperative behavior by groups of firms. Instead, our approach is totally different: we cannot exclude the presence of colluding firms, but we provide some evidence that also a fully non-cooperative non-equilibrium behavior might be at work. In this sense, our work suggests a complementary explanation of the observed behavior by firms in average bid auctions. Nevertheless, we can provide some arguments supporting the robustness of our findings to the presence of collusion. First, it seems reasonable to assert that, if collusion is at work, it is less likely to be present in ABL than in AB auctions: given the inherent uncertainty in the determination of the winning firm, in ABL a successful collusive strategy is much more complicated to be implemented. Interestingly, as shown in Table 2, not only we find a significant correlation between firms' sophistication and distance from the auction's reference point in both AB and ABL, but also the estimated coefficient is larger in the latter. Second, without any intention to provide evidence of the presence of cartels in the auctions issued by the Regional Government of Valle d'Aosta (note that,

[20]For a full set of robustness checks, including controls for the influence of outliers, definition of firms' sophistication as a discrete variable and selection bias issues, see Appendix C, Tables 10-12.

unlike in Conley and Decarolis, in our sample of auctions no cartels have been detected and sanctioned by the court; this makes more difficult to study possible collusive behavior in our setting), we tried to isolate the influence of *potential* collusive groups. To this end, we identified potential collusive groups following Conley and Decarolis (2013). In particular, using information on objective links between firms (e.g., firms sharing the same managers, the owners, the location, subcontracting relationship, joint bidding, etc.), the Conley and Decarolis' algorithm indicates that, in our sample, 172 potential groups of firms are present. Once detected these groups, we proceeded in two ways. Firstly, we included in our baseline model specification two variables measuring, for each firm and each auction: *(i)* the number of (potentially) associated firms bidding in that auction; *(ii)* this number over the total number of firms belonging to that group. Secondly, and more effectively, we estimated our baseline model on a restricted sample including only firms that did not have any links with any other firm bidding to that auction. In both cases, our main result continues to hold, thus supporting the idea that our explanation captures bidding behavior by firms, at least for those that act non-cooperatively.[21]

The interpretation of our data in terms of a CH model of bidding behavior was validated by testing the main prediction that more sophisticated firms' bids will be closer to the unconditional optimal one, which can be approximated by the observable auction's reference point. Further support to the validity of our explanation can be offered by investigating more deeply the correspondence between our empirical evidence and some additional predictions of the CH model that can be drawn from the simulation exercise.[22]

A first prediction is that, given the different awarding rules in the two formats, firms should make relatively lower bids in ABL than in AB. This is clear from our data: Figure 2 shows strong evidence in this direction; furthermore, if we run a regression on a sample of (min-max rescaled) bids offered both in AB and ABL auctions (taking all the covariates included in equation (2)), the coefficient for the ABL auction dummy is negative and statistically significant.

A second prediction that can be derived from the numerical simulations of the CH model is that, when the number of participants increases, the unconditional optimal bid and the auction's reference point tend to increase in AB and to decrease in ABL. The intuition is straightforward: in the viewpoint of a sophisticated firm, who determines her bid on the basis of her own estimates of the distribution of the winning threshold, a lower number of participants increases the variance of this distribution. Since the winning bid is the one that gets closer to the winning threshold from *below* in AB and from *above* in ABL, a sophisticated firm will find it optimal to bid cautiously: in AB, a little below the expected value of the winning threshold, in ABL a little above. As the number of participants increases, the variance of the winning threshold will reduce, and sophisticated firms can be more confident in bidding very close to the expected value of the reference point. This prediction is confirmed both looking at descriptive statistics (the simple correlation between the number of participants and the reference point in AB is positive, while it is negative in ABL) and estimating a regression with the auction's reference point as the dependent variable and the number of participants and other auction-level controls as regressors.

Finally, a third prediction is that, not only the *average* distance from the reference point

[21]Estimation results for this analysis are available in Appendix C, Table 13. We really thank Francesco Decarolis for providing us with his codes and data.

[22]See predictions (CH3), (CH4) and (CH5) in Appendix B. The results of the regressions run to test these predictions are available in Appendix C, Table 14.

is decreasing in the firm's sophistication level, but also the *variance* of this distance follows the same pattern. Again, the intuition is simple: to compute her (conditionally) optimal bid, a level-k firm estimates the distribution of the winning threshold on the basis of the behaviors of level-0 to level-$k-1$ firms. For higher level firms, this distribution has lower variance, being less affected by the random behavior of level-0 firms. As a consequence, their bidding behavior will be more precise (remember that, in the simulations, we allowed for payoff-sensitive errors). This implication is confirmed in the data: in a regression in which the dependent variable is the standard deviation of the distance from the auction's reference point, the coefficient of the sophistication index is negative and significant in both types of auctions.

6 Conclusion

This paper studies bidding behavior by firms in two versions of average bid auctions adopted by a regional contracting authority in Italy for the procurement of public works. Our empirical evidence is inconsistent with Nash equilibrium behavior, i.e. a situation in which all firms are playing their best response to other firms' bids. We proposed an interpretation based on a non-equilibrium CH model of bidding behavior: more sophisticated firms, being better strategic thinkers, are able to get more accurate predictions on the behavior of other firms and bid closer to the unconditionally optimal bid.

Introducing a dynamic measure of sophistication which takes into account the goodness of a firm's bids in all past auctions of the same format in our sample, we showed that, in line with the prediction of the CH model, more sophisticated firms bid closer to the auction's reference point, which proxies the unconditionally optimal bid. We also investigated whether and how firms learn to think and bid strategically through experience, showing that both the number of participations and the average past performance explain firms' performance in future auctions and that this learning process has a convergence path. We finally discussed some issues that give robustness to our interpretation.

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A Equilibria of the AB and the ABL auctions

The model. A single contract is auctioned off. There are n firms participating in the auction. Each firm i has a cost c_i of completing the job. This cost is private information to the firm, but it is commonly known that costs are independent and identically distributed according to a strictly increasing cumulative distribution function $F(\cdot)$ over the interval $[\underline{c}, \bar{c}]$. Firms submit sealed bids formulated in terms of percentage discounts over the reserve price R. Let $d_i \in [0,1]$ denote firm i's bid (discount). The firm submitting the winning bid d^* obtains the contract and it is paid $(1-d^*)R$. Firm i's expected payoff is thus:

$$\pi_i(d_i; c_i, \delta_{-i}) = [(1-d_i)R - c_i]\,\mathrm{PW}(d_i; \delta_{-i}),$$

where $\mathrm{PW}(d_i; \delta_{-i})$ is the probability that firm i wins when she bids d_i and the other firms bid according to δ_{-i}.

In the AB auction, the winning bid is the bid closest *from below* to $A2$. If all firms submit the same bid, the contract is assigned randomly.

In the ABL auction, the winning bid is the bid closest *from above* to W, provided that this bid does not exceed $A2$. If no bid satisfies this requirement, the winning bid will be the one equal, if there is one, or closest from below to W.

We now characterize the properties of symmetric equilibria of these auctions.

Lemma 1. *Let $\delta_K(c)$, $K = AB, ABL$, denote a symmetric (Bayes-) Nash equilibrium of either auction formats and assume it is continuous at $\bar{c}$. The following three properties hold for both auction formats.*

(i) In equilibrium, the probability of winning the auction is strictly positive for all types $c \in [\underline{c}, \bar{c})$.

(ii) Equilibrium bids are weakly decreasing.

(iii) Equilibrium bids are flat at the bottom: there exists $\underline{c} < \hat{c} \leq \bar{c}$ such that $\delta_K(c) = \bar{d}$, for all $c \in [\underline{c}, \hat{c}]$. Notice that, because of (ii), $\bar{d}$ is the highest discount offered.

Proof.

(i) Let $\mathrm{PW}(c)$ denote the probability of winning in equilibrium of a firm with cost c. Notice first that, if $\mathrm{PW}(\hat{c}) > 0$, then $\mathrm{PW}(c) > 0$ for all $c < \hat{c}$ (this follows from incentive compatibility). Hence, to prove the statement, we just need to show that $\mathrm{PW}(\hat{c}) > 0$, for $\hat{c}$ arbitrarily close to $\bar{c}$. Now, since $\delta_K(\cdot)$ is continuous at $\bar{c}$, it is always possible to find a sufficiently small $\Delta > 0$ such that $\delta_K(\cdot)$ is everywhere continuous on $(\bar{c} - \Delta, \bar{c})$. If there exists $\Delta > 0$, such that $\delta_K(\cdot)$ is constant on $(\bar{c} - \Delta, \bar{c})$, then clearly $\mathrm{PW}(\hat{c}) > 0$, for all $\hat{c} \in (\bar{c} - \Delta, \bar{c})$. In this case, in fact, if all firms have costs in the interval $(\bar{c} - \Delta, \bar{c})$ - and this event has a strictly positive probability - any firm has a $1/n$ probability of winning the auction. Consider, instead, the case in which there is no $\Delta > 0$ such that $\delta_K(\cdot)$ is constant on $(\bar{c} - \Delta, \bar{c})$. Take any $\hat{c} \in (\bar{c} - \Delta, \bar{c})$ and let $\hat{d} = \delta_K(\hat{c})$. There are two possible cases:

a. $m \equiv \inf_{x \in (\bar{c}-\Delta, \bar{c})} \delta_K(x) < \hat{d} < \sup_{x \in (\bar{c}-\Delta, \bar{c})} \delta_K(x) \equiv M$. Now, because $\delta_K(\cdot)$ is continuous on $(\bar{c} - \Delta, \bar{c})$, for all $\varepsilon_1 > 0$ it is always possible to find a subinterval

$I_1(\varepsilon_1)$ whose image is $(m, m+\varepsilon_1)$, and for all $\varepsilon_2 > 0$ it is always possible to find a subinterval $I_2(\varepsilon_2)$ whose image is $(M-\varepsilon_2, M)$. We now show that, it is always possible to find $\varepsilon_1 > 0$, $\varepsilon_2 > 0$, n_1 and n_2 (with $n_1 + n_2 = n-1$) such that, if n_1 firms have costs drawn from $I_1(\varepsilon_1)$ and n_2 firms have costs drawn from $I_2(\varepsilon_2)$, then $\hat{d}$ will be the winning bid. We only treat the case of the AB auction (the ABL case is similar). Now, if $\hat{d} - m - (M-\hat{d})/(n-2\tilde{n}-2) > 0$, it is sufficient to take $0 < \varepsilon_1 < \hat{d} - m - (M-\hat{d})/(n-2\tilde{n}-2)$, $0 < \varepsilon_2 < M-d$, $(n-2\tilde{n})\varepsilon_1 + \varepsilon_2 < M+d-2m$, $n_1 = n-\tilde{n}-2$ and $n_2 = \tilde{n}+1$. In this case, A1 will necessarily lie between the highest bid of those firms whose costs are in the interval $I_1(\varepsilon_1)$ and $\hat{d}$ itself; hence, A2 will lie between $\hat{d}$ and the lowest bid of those firms whose costs are in the interval $I_2(\varepsilon_2)$; the winning bid will thus be $\hat{d}$. If instead $\hat{d} - m - (M-\hat{d})/(n-2\tilde{n}-2) \leq 0$, it is sufficient to take $\varepsilon_1 < \hat{d} - m$, $\varepsilon_2 < M - \hat{d}$, $n_1 = n-\tilde{n}-2$ and $n_2 = \tilde{n}+1$. In this case, A1 will necessarily lie between $\hat{d}$ and the lowest bid of those firms whose costs are in the interval $I_2(\varepsilon_2)$; A2 will thus coincide with the lowest bid of those firms whose costs are in the interval $I_2(\varepsilon_2)$ and the winning bid will again be $\hat{d}$.

b. $\hat{d} = m$ or $\hat{d} = M$. In this case, there must be $\tilde{c} > \hat{c}$ such that $m < \delta_K(\tilde{c}) < M$, hence $\text{PW}(\tilde{c}) > 0$. But this would imply that also $\text{PW}(\hat{c}) > 0$.

(ii) Suppose, by contradiction, that $\delta_K(\cdot)$ is not weakly decreasing. Then, there must exist types c_1 and c_2, with $c_1 > c_2$ such that $\delta_K(c_1) > \delta_K(c_2)$. Now, since $\delta_K(\cdot)$ is an equilibrium, it must hold that $[(1-\delta_K(c_1))R - c_1]\text{PW}(c_1) \geq [(1-\delta_K(c_2))R - c_1]\text{PW}(c_2)$. Since $\delta_K(c_1) > \delta_K(c_2)$ and since $\text{PW}(c_2) > 0$ (see point (i)), the previous inequality can be satisfied only if $\text{PW}(c_1) > \text{PW}(c_2)$. But this contradicts the fact that, in equilibrium, the probability of winning must be weakly decreasing in c.

(iii) Suppose, to the contrary, that $\delta_K(\cdot)$ is strictly decreasing at the bottom, i.e. that $\delta_K(\underline{c}) > \delta_K(c)$, for all $c \in (\underline{c}, \bar{c}]$. This implies that $\text{PW}(\underline{c}) = 0$, which contradicts point (i).

From the properties above, we can now derive more precise predictions on the (Bayes-) Nash equilibria in the two formats.

Lemma 2.

(i) In the AB auction, there is a unique symmetric equilibrium in which firms submit a 0-discount irrespective of their costs. The contract is assigned randomly.

(ii) In the ABL auction, it is a symmetric equilibrium for firms to submit a constant discount d irrespective of their costs, provided that $d \in [0, 1-\bar{c}/R]$.

(iii) Consider the ABL auction. Let $\Phi(w|m, n-m-1, \bar{d}-\varepsilon)$ denote the conditional distribution of W when m bidders have costs greater than or equal to $\hat{c}$ and bid according to their equilibrium bidding function, $n-m-1$ bidders have costs smaller than $\hat{c}$ and bid according to their equilibrium bidding function (i.e., they bid $\bar{d}$) and one bidder bids $\bar{d}-\varepsilon$. If, for all $\tilde{n}+2 \leq m \leq n-\tilde{n}-1$, $\Phi(w|m, n-m-1, \bar{d}-\varepsilon)$ is right continuous at $\varepsilon = 0$, then, in equilibrium, the set of firms' types who make the highest discount $\bar{d}$ must be sufficiently large (i.e., $\hat{c}$ must be sufficiently high). In particular, $F(\hat{c}) > (n-\tilde{n})/(n+1)$.

Proof.

(i) By Lemma 1, point *(iii)*, we know that, in any equilibrium of the AB auction, types $c \leq \hat{c}$ make the same bid $\bar{d}$. Now, consider a firm i of this type: she will win the AB auction if and only if all other firms bid $\bar{d}$, in which case every firm will have a $1/n$ chance of winning. If, instead, firm i decreases her bid below $\bar{d}$, in case all other firms bid $\bar{d}$ she will be the sole winner (moreover, with a smaller discount), which is clearly profitable. The only situation in which no such profitable deviation exists is when it is not possible to decrease bids, i.e. when $\bar{d} = 0$. In this case, an upward deviation is not profitable either, as in this case $A2$ will necessarily be equal to 0 and firm i will have a null probability of winning as her bid exceeds $A2$.

(ii) In the ABL auction, if all firms' make the same bid $d \in [0, 1 - \bar{c}/R]$, whatever their actual cost is, every firm will have a $1/n$ chance of winning, with an expected payoff equal to $[(1-d)R - c_i]/n \geq [(1 - 1 + \bar{c}/R)R - c_i]/n = [\bar{c} - c_i]/n \geq 0$. If firm i (of any type) increases her bid, then $A2$ will necessarily be equal to d and firm i will have a null probability of winning as her bid exceeds $A2$. If instead firm i (of any type) decreases her bid below d, then W will necessarily be equal to d and the winner will be one of the other firms. Again, the probability of winning of firm i will fall to zero. Clearly, all firms' types making the same bid $d > \bar{c}/R$ cannot be an equilibrium, as in this case the expected payoff of a type $\bar{c}$ firm will be strictly negative.

(iii) Suppose there exists a symmetric equilibrium of the ABL auction in which firms with cost $c \leq \hat{c}$ bid $\bar{d}$ and firms with cost $c > \hat{c}$ make strictly lower bids (where $\hat{c} \in (\underline{c}, \bar{c})$). Denote by m the number of firms with cost $c > \hat{c}$ (these firms, in equilibrium, make bids strictly below $\bar{d}$), by $n - m - 1$ the number of firms with cost $c \leq \hat{c}$ (these firms, in equilibrium, bid exactly $\bar{d}$) and by $\tilde{n}$ the lowest integer greater than or equal to $n/10$. Also, denote by $p = 1 - F(\hat{c})$ the probability that a firm has cost above $\hat{c}$ and by $B_k(j)$ the probability that j firms out of k have cost above $\hat{c}$. Consider firm i and suppose this firm has cost $c_i \leq \hat{c}$. In equilibrium, this firm should bid $\bar{d}$; hence, her equilibrium payoff is:

$$\begin{aligned}
\pi_i(\bar{d}; c_i) &= \left[(1-\bar{d})R - c_i\right] \left[\sum_{m=0}^{\tilde{n}+1} \frac{B_{n-1}(m)}{n-m} + \sum_{m=\tilde{n}+2}^{n-(\tilde{n}+1)} \frac{B_{n-1}(m)}{n-m} \Pr(d^{(m)} \leq W | m, n-m) \right] \\
&= \left[(1-\bar{d})R - c_i\right] \frac{1}{n(1-p)} \left[\sum_{m=0}^{\tilde{n}+1} B_n(m) + \sum_{m=\tilde{n}+2}^{n-(\tilde{n}+1)} B_n(m) \Pr(d^{(m)} \leq W | m, n-m) \right].
\end{aligned}$$

In the expression above, $\Pr(W > d^{(m)} | m, n-m)$ is the probability that the winning threshold W is greater than the highest bid of the m firms who bid below $\bar{d}$, conditional on the fact that m firms bid less than $\bar{d}$ and $n - m$ firms bid exactly $\bar{d}$. The expression above can be read as follows. Firm i can win the auction in either of these two situations: *(i)* if $m \leq \tilde{n} + 1$, in which case W will coincide with $\bar{d}$; *(ii)* if $\tilde{n} + 1 < m \leq n - (\tilde{n} + 1)$ and W is above the highest bid of those firms who bid less than $\bar{d}$. In both cases, firm i will win with probability $n - m$ (the winning firm will be extracted from those firms who bid $\bar{d}$).

Now, suppose firm i deviates from her equilibrium bid and bids $\bar{d} - \varepsilon$, with $\varepsilon > 0$. In this case, her payoff would at least be:

$$\pi_i(\bar{d}-\varepsilon;c_i) \geq \left[(1-\bar{d}+\varepsilon)R-c_i\right]\Big[B_{n-1}(\tilde{n}+1)\Pr(d^{(\tilde{n}+1)}<\bar{d}-\varepsilon|\tilde{n}+1,n-(\tilde{n}+1)-1,\bar{d}-\varepsilon) + \sum_{m=\tilde{n}+2}^{n-(\tilde{n}+1)} B_{n-1}(m)\Pr(d^{(m)}\leq W<\bar{d}-\varepsilon|m,n-m-1,\bar{d}-\varepsilon)\Big].$$

In the expression above, $\Pr(d^{(m)} \leq W < \bar{d}-\varepsilon|m,n-m-1,\bar{d}-\varepsilon)$ is the probability that the winning threshold W is greater than the highest bid of the m firms who bid below $\bar{d}$ but lower than $\bar{d}-\varepsilon$, conditional on the fact that m firms bid less than $\bar{d}$, $n-m-1$ firms bid exactly $\bar{d}$ and one firm (firm i) bids $\bar{d}-\varepsilon$ (the term $\Pr(d^{(\tilde{n}+1)} < \bar{d}-\varepsilon|\tilde{n}+1,n-(\tilde{n}+1)-1,\bar{d}-\varepsilon)$ has a similar interpretation). The expression above can be read as follows. Firm i wins in at least two situations: *(i)* if $m=\tilde{n}+1$ and $d^{(\tilde{n}+1)} < \bar{d}-\varepsilon$, in which case W will necessarily lie between $d^{(\tilde{n}+1)}$ and $\bar{d}-\varepsilon$; *(ii)* if $\tilde{n}+1 < m \leq n-(\tilde{n}+1)$ and W is above the highest bid of those firms who bid less than $\bar{d}$ but less than $\bar{d}-\varepsilon$.[23]

In the supposed equilibrium, firm i should bid $\bar{d}$. Hence, it must necessarily hold that $\pi_i(\bar{d};c_i) \geq \pi_i(\bar{d}-\varepsilon;c_i)$, which implies that

$$\sum_{m=0}^{\tilde{n}+1} B_n(m) + \sum_{m=\tilde{n}+2}^{n-(\tilde{n}+1)} B_n(m)\Pr(d^{(m)}\leq W|m,n-m) > n(1-p)\Big[B_{n-1}(\tilde{n}+1)\Pr(d^{(\tilde{n}+1)}<\bar{d}-\varepsilon|\tilde{n}+1,n-(\tilde{n}+1)-1,\bar{d}-\varepsilon) + \sum_{m=\tilde{n}+2}^{n-(\tilde{n}+1)} B_{n-1}(m)\Pr(d^{(m)}\leq W<\bar{d}-\varepsilon|m,n-m-1,\bar{d}-\varepsilon)\Big].$$

Now, since, for all $\tilde{n}+2 \leq m \leq n-\tilde{n}-1$, $\Phi(w|m,n-m-1,\bar{d}-\varepsilon)$ is right continuous at $\varepsilon=0$, the right hand side of the expression above is right continuous as well. Hence, the inequality above must be preserved in the limit, i.e. when $\varepsilon \to 0$. This translates into

$$\sum_{m=0}^{\tilde{n}+1} B_n(m) + \sum_{m=\tilde{n}+2}^{n-(\tilde{n}+1)} B_n(m)\Pr(d^{(m)}\leq W|m,n-m) \geq n(1-p)\left[B_{n-1}(\tilde{n}+1) + \sum_{m=\tilde{n}+2}^{n-(\tilde{n}+1)} B_{n-1}(m)\Pr(d^{(m)}\leq W|m,n-m)\right],$$

which can be simplified into

$$\sum_{m=0}^{\tilde{n}} B_n(m) + (2+\tilde{n}-n)B_n(\tilde{n}+1) \geq \sum_{m=\tilde{n}+2}^{n-\tilde{n}-1}\Big[(n-m-1)B_n(m)\Pr(d^{(m)}\leq W|m,n-m)\Big].$$

[23]Firm i can win also in situations in which $d^{(m)} \geq \bar{d}$. When ε is small, this event has a very small probability.

Since the right hand side of the above inequality is positive, we must necessarily have that

$$\sum_{m=0}^{\tilde{n}} B_n(m) + (2 + \tilde{n} - n)B_n(\tilde{n} + 1) \geq 0,$$

or that

$$\frac{\sum_{m=0}^{\tilde{n}} B_n(m)}{B_n(\tilde{n} + 1)} \geq n - \tilde{n} - 2.$$

A necessary condition for the last inequality to be satisfied is that $B_n(\tilde{n}) \geq B_n(\tilde{n} + 1)$. Suppose not, i.e. suppose that $B_n(\tilde{n}) < B_n(\tilde{n} + 1)$. But this implies that $B_n(j) < B_n(\tilde{n}+1)$, for all $j \leq \tilde{n}$, which, in turn, implies that $() \sum_{m=0}^{\tilde{n}} B_n(m))(B_n(\tilde{n}+1)) < \tilde{n}+1$. But $\tilde{n} + 1$ is always lower than $n - \tilde{n} - 2$ and the inequality above will not be satisfied. Hence, we conclude that a necessary condition to have a symmetric equilibrium of the ABL auction in which firms with cost $c \leq \hat{c}$ bid $\bar{d}$ and firms with cost $c > \hat{c}$ make strictly lower bids is that $B_n(\tilde{n}) \geq B_n(\tilde{n} + 1)$, i.e. that $F(\hat{c}) > (n - \tilde{n})/(n + 1)$.

B Numerical simulations

In this section, we present the results of some simulation exercises from a CH model of bidding behavior in AB and ABL. We fix the reserve price to 100 and assume that firms' costs are uniformly distributed on the interval $[\underline{c} = 50, \bar{c} = 70]$, with increments of 0.2. We assume that firms' levels of sophistication range from 0 to 2 and that they are distributed according to a truncated Poisson with parameter λ.[24] Level-0 firms are assumed to draw their bids from a uniform distribution over the interval $[0, 0.3]$. This assumption is roughly consistent with our evidence (the minimum and maximum discounts observed in our sample are 0 and 0.421 in AB and 0.016 and 0.317 in ABL) and ensures that level-0 firms will never play dominated strategies.[25]. Level-1 firms choose their bids to maximize their expected payoffs under the belief that all other firms are level-0, while level-2 firms choose their bids to maximize their expected payoffs under the belief that other firms are a mixture of level-0 and level-1. Given the behavior of level-0, level-1 and level-2 firms, we compute the expected value of the reference point and, for each level, the expected value and the variance (in square brackets) of the distance between their bids and the reference point. Since our objective is to check the consistency of the results of the simulations with real data, we must allow for errors. Hence, the distance from the reference point is computed supposing that level-1 and level-2 firms' bids are subject to logistic errors: every bid is played with positive probability but the probability that a level-l firm ($l = 1, 2$) with cost c bids $\hat{d}$ is $\exp(\eta\Pi_l(\hat{d}; c))/\sum_d \exp(\eta\Pi_l(d; c))$,

[24]Hence, the probability that a firm's level of sophistication is l ($l = 0, 1, 2$) is equal to $\frac{e^{\lambda}\lambda^l/l}{\sum_{i=0}^{2} e^{\lambda}\lambda^i/i} = \frac{\lambda^l/l}{1+\lambda+\lambda^2/2}$. Hence, a higher λ means that firms are, on average, more sophisticated.

[25]In this sense, level-0 firms have at least a minimum degree of rationality. Their random behavior could be interpreted as the consequence of a total absence of any precise beliefs about the behavior of others.

where $\Pi_l(d;c)$ is the expected payoff of a level-l firm when her cost is c and she bids d, and where η denotes the error parameter (with $\eta = 0$ meaning random behavior and $\eta \to \infty$ meaning no errors). We also computed the truly optimal bid, i.e., the bid that would maximize the expected payoff of a firm who has fully correct beliefs about the behavior of other firms. The results of the simulations are reported in Tables 4-9, for different values of the parameter of the distribution of levels ($\lambda = 0.5, 1, 2$), of the number of firms ($n = 25, 50, 100$) and of the parameter of the error distribution ($\eta = 0.5, 1, 2$).

Table 4 – Simulation results for the AB auction with $\eta = 0.5$.

n	λ	*ref. point*	*distance from ref. point*			*opt. bid*	*distance from opt. bid*		
			level 0	*level 1*	*level 2*		*level 0*	*level 1*	*level 2*
25	0.5	20.3	8.5 [2.4]	5.2 [0.9]	4.2 [0.6]	20.4	8.5 [2.4]	5.3 [0.9]	4.3 [0.6]
	1	20.1	8.4 [2.4]	5.2 [0.9]	4.0 [0.5]	19.5	8.2 [2.3]	5.1 [0.9]	3.6 [0.4]
	2	19.8	8.3 [2.3]	5.1 [0.9]	1.6 [0.1]	19.5	8.2 [2.3]	5.1 [0.9]	1.3 [0.1]
50	0.5	21.0	8.8 [2.6]	6.7 [1.5]	5.9 [1.2]	20.1-21.3	8.5 [2.4]	6.6 [1.4]	5.8 [1.1]
	1	20.7	8.6 [2.5]	6.6 [1.5]	5.9 [1.2]	20.4	8.5 [2.4]	6.6 [1.4]	5.8 [1.1]
	2	20.6	8.6 [2.5]	6.6 [1.5]	2.6 [0.2]	20.4	8.5 [2.4]	6.6 [1.4]	2.4 [0.2]
100	0.5	21.0	8.8 [2.6]	7.7 [2.0]	7.1 [1.7]	20.4	8.5 [2.4]	7.5 [1.9]	7.0 [1.6]
	1	20.7	8.6 [2.5]	7.6 [1.93]	7.5 [1.88]	20.4	8.5 [2.4]	7.5 [1.9]	7.4 [1.8]
	2	20.6	8.6 [2.5]	7.6 [1.9]	4.2 [0.6]	20.4	8.5 [2.4]	7.5 [1.9]	4.1 [0.5]

Table 5 – Simulation results for the AB auction with $\eta = 1$.

n	λ	*ref. point*	*distance from ref. point*			*opt. bid*	*distance from opt. bid*		
			level 0	*level 1*	*level 2*		*level 0*	*level 1*	*level 2*
25	0.5	20.3	8.5 [2.4]	2.7 [0.2]	1.1 [0.0]	20.4	8.5 [2.4]	2.7 [0.2]	1.2 [0.0]
	1	20.1	8.4 [2.4]	2.6 [0.2]	1.5 [0.1]	19.5	8.2 [2.3]	2.6 [0.2]	0.9 [0.0]
	2	19.8	8.3 [2.3]	2.6 [0.2]	0.6 [0.0]	19.5	8.2 [2.3]	2.6 [0.2]	0.3 [0.0]
50	0.5	21.0	8.8 [2.6]	4.2 [0.6]	2.1 [0.1]	20.1-21.3	8.5 [2.4]	4.1 [0.6]	2.4 [0.2]
	1	20.7	8.6 [2.5]	4.1 [0.6]	2.4 [0.2]	20.4	8.5 [2.4]	4.0 [0.5]	2.2 [0.2]
	2	20.6	8.6 [2.5]	4.1 [0.6]	0.5 [0.0]	20.4	8.5 [2.4]	4.0 [0.5]	0.3 [0.0]
100	0.5	21.0	8.8 [2.6]	6.0 [1.2]	3.3 [0.4]	20.4	8.5 [2.4]	5.9 [1.2]	3.6 [0.4]
	1	20.7	8.6 [2.5]	6.0 [1.2]	4.7 [0.7]	20.4	8.5 [2.4]	5.9 [1.2]	4.5 [0.7]
	2	20.6	8.6 [2.5]	6.0 [1.2]	0.9 [0.0]	20.4	8.5 [2.4]	5.9 [1.2]	0.7 [0.0]

Table 6 – Simulation results for the AB auction with $\eta = 2$.

n	λ	*ref. point*	*distance from ref. point*			*opt. bid*	*distance from opt. bid*		
			level 0	*level 1*	*level 2*		*level 0*	*level 1*	*level 2*
25	0.5	20.3	8.5 [2.4]	1.1 [0.03]	0.2 [0.00]	20.4	8.5 [2.4]	1.1 [0.04]	0.3 [0.00]
	1	20.1	8.4 [2.4]	1.0 [0.03]	0.8 [0.02]	19.5	8.2 [2.3]	1.0 [0.03]	0.3 [0.00]
	2	19.8	8.3 [2.3]	1.0 [0.03]	0.4 [0.00]	19.5	8.2 [2.3]	1.0 [0.03]	0.1 [0.00]
50	0.5	21.0	8.8 [2.6]	1.5 [0.08]	0.2 [0.00]	20.1-21.3	8.5 [2.4]	1.5 [0.07]	0.9 [0.03]
	1	20.7	8.6 [2.5]	1.4 [0.06]	0.5 [0.01]	20.4	8.5 [2.4]	1.4 [0.06]	0.3 [0.00]
	2	20.6	8.6 [2.5]	1.4 [0.06]	0.3 [0.00]	20.4	8.5 [2.4]	1.4 [0.06]	0.0 [0.00]
100	0.5	21.0	8.8 [2.6]	2.8 [0.26]	0.5 [0.01]	20.4	8.5 [2.4]	2.8 [0.26]	1.0 [0.03]
	1	20.7	8.6 [2.5]	2.7 [0.25]	1.2 [0.05]	20.4	8.5 [2.4]	2.8 [0.26]	1.0 [0.03]
	2	20.6	8.6 [2.5]	2.7 [0.25]	0.2 [0.00]	20.4	8.5 [2.4]	2.8 [0.26]	0.0 [0.00]

Table 7 – Simulation results for the ABL auction with $\eta = 0.5$.

n	λ	*ref. point*	*distance from ref. point*			*opt. bid*	*distance from opt. bid*		
			level 0	*level 1*	*level 2*		*level 0*	*level 1*	*level 2*
25	0.5	13.5	7.7 [2.0]	5.6 [1.1]	4.5 [0.7]	15.0	7.6 [1.9]	5.0 [0.8]	4.1 [0.5]
	1	14.5	7.6 [1.9]	5.2 [0.9]	3.2 [0.4]	15.3	7.6 [1.9]	4.9 [0.8]	3.1 [0.3]
	2	15.6	7.6 [1.9]	4.9 [0.8]	2.1 [0.1]	15.3	7.6 [1.9]	4.9 [0.8]	2.0 [0.1]
50	0.5	12.7	7.8 [2.0]	6.8 [1.6]	6.0 [1.2]	13.5	7.6 [1.9]	6.6 [1.4]	5.7 [1.1]
	1	13.5	7.7 [2.0]	6.6 [1.4]	5.0 [0.8]	15.0	7.6 [1.9]	6.3 [1.3]	4.7 [0.7]
	2	15.3	7.6 [1.9]	6.2 [1.3]	3.3 [0.4]	15.3	7.6 [1.9]	6.2 [1.3]	3.3 [0.4]
100	0.5	12.7	7.8 [2.0]	7.3 [1.8]	6.9 [1.6]	15.6	7.6 [1.9]	6.9 [1.6]	6.6 [1.5]
	1	13.1	7.7 [2.0]	7.2 [1.7]	6.5 [1.4]	14.1	7.6 [1.9]	7.0 [1.6]	6.4 [1.4]
	2	14.3	7.6 [1.9]	7.0 [1.6]	5.4 [1.0]	14.1	7.6 [1.9]	7.0 [1.6]	5.4 [1.0]

Table 8 – Simulation results for the ABL auction with $\eta = 1$.

n	λ	*ref. point*	*distance from ref. point*			*opt. bid*	*distance from opt. bid*		
			level 0	*level 1*	*level 2*		*level 0*	*level 1*	*level 2*
25	0.5	13.5	7.7 [2.0]	4.2 [0.6]	2.3 [0.2]	15.0	7.6 [1.9]	3.1 [0.3]	1.5 [0.1]
	1	14.5	7.6 [1.9]	3.4 [0.4]	1.2 [0.0]	15.3	7.6 [1.9]	3.0 [0.3]	0.9 [0.0]
	2	15.6	7.6 [1.9]	2.8 [0.3]	0.5 [0.0]	15.3	7.6 [1.9]	3.0 [0.3]	0.5 [0.0]
50	0.5	12.7	7.8 [2.0]	5.8 [1.1]	4.1 [0.5]	13.5	7.6 [1.9]	5.4 [1.0]	3.6 [0.4]
	1	13.5	7.7 [2.0]	5.4 [1.0]	2.6 [0.2]	15.0	7.6 [1.9]	4.8 [0.8]	2.0 [0.1]
	2	15.3	7.6 [1.9]	4.8 [0.8]	0.9 [0.0]	15.3	7.6 [1.9]	4.8 [0.8]	0.9 [0.0]
100	0.5	12.7	7.8 [2.0]	6.8 [1.5]	6.0 [1.2]	15.6	7.6 [1.9]	6.2 [1.3]	5.5 [1.0]
	1	13.1	7.7 [2.0]	6.6 [1.4]	5.2 [0.9]	14.1	7.6 [1.9]	6.3 [1.3]	4.9 [0.8]
	2	14.3	7.6 [1.9]	6.3 [1.3]	3.0 [0.3]	14.1	7.6 [1.9]	6.3 [1.3]	3.1 [0.3]

Looking at the results of these numerical simulations, we detect some regularities, that we summarize in the following predictions:

(CH1) *In either auction, for all values of n, λ, and η, the distance of a firm's bid from the unconditionally optimal bid is decreasing in her level of sophistication.*

(CH2) *In either auction, for all values of n, λ, and η, the distance of a firm's bid from the*

Table 9 – Simulation results for the ABL auction with $\eta = 2$.

n	λ	*ref. point*	*distance from ref. point* *level 0*	*level 1*	*level 2*	*opt. bid*	*distance from opt. bid* *level 0*	*level 1*	*level 2*
25	0.5	13.5	7.7 [2.0]	3.3 [0.4]	1.7 [0.1]	15.0	7.6 [1.9]	1.9 [0.1]	0.5 [0.0]
	1	14.5	7.6 [1.9]	2.4 [0.2]	0.9 [0.0]	15.3	7.6 [1.9]	1.7 [0.1]	0.4 [0.0]
	2	15.6	7.6 [1.9]	1.5 [0.1]	0.2 [0.0]	15.3	7.6 [1.9]	1.7 [0.1]	0.3 [0.0]
50	0.5	12.7	7.8 [2.0]	4.5 [0.7]	2.3 [0.2]	13.5	7.6 [1.9]	3.9 [0.5]	1.7 [0.1]
	1	13.5	7.7 [2.0]	3.9 [0.5]	1.6 [0.1]	15.0	7.6 [1.9]	2.9 [0.3]	0.6 [0.0]
	2	15.3	7.6 [1.9]	2.8 [0.3]	0.3 [0.0]	15.3	7.6 [1.9]	2.7 [0.3]	0.3 [0.0]
100	0.5	12.7	7.8 [2.0]	5.7 [1.1]	4.2 [0.6]	15.6	7.6 [1.9]	4.6 [0.7]	3.4 [0.4]
	1	13.1	7.7 [2.0]	5.4 [1.0]	3.0 [0.3]	14.1	7.6 [1.9]	5.0 [0.8]	2.6 [0.2]
	2	14.3	7.6 [1.9]	4.9 [0.8]	0.9 [0.0]	14.1	7.6 [1.9]	5.0 [0.8]	1.0 [0.0]

auction's reference point is decreasing in the level of sophistication of the firm.

(CH3) *For given n, λ, and η, level-1 and level-2 firms' bids are, on average, lower in ABL than in AB.*

(CH4) *For given λ and η, the unconditional optimal bid and the auction's reference point are increasing in n in AB, decreasing in n in ABL.*

(CH5) *In either auction, for all values of n, λ, and η, the variance of the distance from the reference point is decreasing in the sophistication level of the firm.*

C Empirical evidence: robustness checks, collusion, further predictions

The main result of our empirical analysis (presented in Section 4.2) is robust to a full set of checks, namely: *(i)* controlling for the influence of the outliers through robust regression (Table 10); *(ii)* estimating model (2) employing discrete variables for the firms' sophistication levels (Table 11); *(iii)* controlling for selection bias problems through a two-step Heckman model (Table 12).

In Table 13, we report the results of the regressions we performed to control for the influence of potential collusive groups. These results were discussed in Section 5.

Finally, in Table 14, we report the results of the regressions we performed to test the additional predictions (CH3), (CH4), and (CH5) obtained by looking at the results of the numerical simulations (see Section B of this Appendix). These results were discussed in Section 5.

Table 10 – Controlling for outliers (robust regressions).

Dependent variable:	log $\|Distance\|$			
Auction format	AB	AB	ABL	ABL
	(1)	(2)	(3)	(4)
log($BidderSoph$)	-0.194***	-0.190***	-0.373***	-0.409***
	(0.015)	(0.030)	(0.034)	(0.063)
Auction/project controls	YES	YES	YES	YES
Firm controls	YES	NO	YES	NO
Firm fixed effects	NO	YES	NO	YES
Firm-auction controls	YES	YES	YES	YES
Observations	8,927	8,838	1,501	1,410
R-squared	0.221	0.294	0.333	0.493

Robust regression is an iteratively re-weighted least squares procedures (IRLS), which downweighs observations with large residuals using the Huber weight function. Standard errors in parentheses.
Inference: (***) = $p < 0.01$, (**) = $p < 0.05$, (*) = $p < 0.1$.
Auction/project controls include: the auction's reserve price, the expected duration of the work, the number of bidders, dummy variables for the type of work, dummy variables for the year of the auction. *Firm controls* include: dummy variables for the size of the firm, and the distance between the firm and the CA. *Firm-auction controls* include: a dummy variable for the firm's subcontracting position (mandatory or optional), and a measure of the firm's backlog.

Table 11 – Controlling for categorical definition of firms' sophistication levels.

Dependent variable:	log $\|Distance\|$			
Auction format	AB	AB	ABL	ABL
	(1)	(2)	(3)	(4)
$MediumBidderSoph$	-0.218***	-0.124*	-0.710***	-0.579***
	(0.055)	(0.066)	(0.102)	(0.110)
$HighBidderSoph$	-0.490***	-0.326***	-0.995***	-0.801***
	(0.068)	(0.098)	(0.128)	(0.153)
Auction/project controls	YES	YES	YES	YES
Firm controls	YES	NO	YES	NO
Firm fixed effects	NO	YES	NO	YES
Firm-auction controls	YES	YES	YES	YES
Observations	8,927	8,838	1,501	1,410
R-squared	0.194	0.269	0.286	0.469

OLS estimations. Robust standard errors clustered at firm-level in parentheses.
Inference: (***) = $p < 0.01$, (**) = $p < 0.05$, (*) = $p < 0.1$.
$MediumBidderSoph$ is a dummy variable which takes the value 1 if the firm has a value of the indicator of bidders' sophistication between the 33' and 66' percentile of the indicator's distribution. $HighBidderSoph$ is a dummy variable which takes the value 1 if the firm has a value of the indicator of the bidders' sophistication above the 66' percentile of the indicator's distribution. *Auction/project controls* include: the auction's reserve price, the expected duration of the work, the number of bidders, dummy variables for the type of work, dummy variables for the year of the auction. *Firm controls* include: dummy variables for the size of the firm, and the distance between the firm and the CA. *Firm-auction controls* include: a dummy variable for the firm's subcontracting position (mandatory or optional), and a measure of the firm's backlog.

Table 12 – Controlling for selection bias problems (two-step Heckman model).

Auction format	AB			
Type of work	Road works			
Estimator	OLS		Heckman selection model	
Dependent variable:	$\log\|Distance\|$	$Prob.participation$	$Selection$	$\log\|Distance\|$
	(1)	(2)	(3)	(4)
$\log(BidderSoph)$	-0.168***	0.070***	0.247***	-0.533***
	(0.026)	(0.007)	(0.022)	(0.038)
$\log(TimeToBid)$		0.031**	0.145***	
		(0.014)	(0.035)	
Auction/project controls	YES	YES	YES	YES
Firm controls	YES	YES	YES	YES
Observations	3,877	13,517	13,517	3,877

Robust standard errors clustered at firm-level in parentheses.
Inference: (***) = $p < 0.01$, (**) = $p < 0.05$, (*) = $p < 0.1$.
Auction/project controls include: the auction's reserve price, the expected duration of the work, the number of bidders, dummy variables for the year of the auction. *Firm controls* include: dummy variables for the size of the firm, and the distance between the firm and the CA.
The analysis focuses on AB auctions for road works because they represent the largest share of projects in our data (87 auctions). OLS regression in column (1) shows the coefficient of $BidderSoph$ estimated on the subsample of road works. The potential market for road works is defined as those firms that, according to our dataset, bid at least once for road works in a given year. As an exogenous instrument that is related to the probability of firms' participation but has an influence only on the cost of participation, we use $TimeToBid$, which is the length of time between the date when the project is advertised and when the bid letting occurs (this instrument is also used by Gil and Marion 2013, and Moretti and Valbonesi 2012). The hypothesis is that the longer is the time between the beginning of project's publicity and the deadline for bid's submission, the longer is the time for firms to evaluate the project and to submit their bids, the lower is the cost associated with entry. Our data show that there is variability in terms of auctions' advertise lead time, with an average of 28.6 days (and a standard deviation of 11.4 days). In columns (3) and (4), the first and second stage of a two-step Heckman selection model are reported.

Table 13 – Controlling for collusive groups.

Dependent variable:	log $\|Distance\|$											
Auction format	AB	AB	ABL	ABL	AB	AB	ABL	ABL	AB	AB	ABL	ABL
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
log($BidderSoph$)	-0.173***	-0.195***	-0.352***	-0.354***	-0.171***	-0.192***	-0.335***	-0.355***	-0.162***	-0.202***	-0.263***	-0.321***
	(0.023)	(0.037)	(0.051)	(0.066)	(0.022)	(0.037)	(0.053)	(0.066)	(0.031)	(0.053)	(0.071)	(0.109)
log($GroupMembers$)	-0.142***	-0.186***	0.050	-0.043	0.037	-0.137	0.282**	0.157				
	(0.031)	(0.048)	(0.055)	(0.122)	(0.068)	(0.109)	(0.119)	(0.432)				
$ShareGroupMembers$					0.186**	1.306**	0.424	0.134				
					(0.092)	(0.555)	(0.257)	(1.351)				
log($GroupMembers\times$ $ShareGroupMembers$					-0.380***	-0.752***	-0.465*	-0.344				
					(0.121)	(0.249)	(0.239)	(0.445)				
Auction/project controls	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES
Firm controls	YES	NO	YES	NO	YES	NO	YES	NO	YES	NO	YES	NO
Firm fixed effects	NO	YES	NO	YES	NO	YES	NO	YES	NO	YES	NO	YES
Firm-auction controls	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES
Observations	8,037	7,993	1,316	1,258	8,037	7,993	1,316	1,258	3,861	3,834	578	543
R-squared	0.206	0.266	0.283	0.450	0.207	0.267	0.288	0.451	0.184	0.301	0.251	0.573

OLS estimations. Robust standard errors clustered at firm-level in parentheses.
Inference: (***) = $p < 0.01$, (**) = $p < 0.05$, (*) = $p < 0.1$.
GroupMembers is, for each firm in each auctions, the (log of 1 plus the) number of firms belonging to the same potential collusive group of that firm and participating in that auction. The potential groups are defined following Conley and Decarolis (2013): in particular, starting from the observation of actual links among firms (i.e., firms sharing: the same managers, the owners, the location, subcontracting relationship, joint bidding, etc.), the Conley and Decarolis's (2013) algorithm suggests that, in our sample of bidders, 172 potential groups of firms are present.[a] Summary statistics indicate that, on average, for each firm in each auction, there are 1.5 firms belonging to the same potential collusive group. *ShareGroupMembers* is equal to *GroupMembers* divided by the total number of firms belonging to the same potential group.[b] In columns (9)-(12), the samples are restricted to bids offered by firms that did not have any links with any other firm participating in that auction. *Auction/project controls* include: the auction's reserve price, the expected duration of the work, the number of bidders, dummy variables for the type of work, dummy variables for the year of the auction. *Firm controls* include: dummy variables for the size of the firm, and the distance between the firm and the CA. *Firm-auction controls* include: a dummy variable for the firm's subcontracting position (mandatory or optional), and a measure of the firm's backlog.

[a] We cannot test which of these potential group actually collude, since there is no judicial evidence of the presence of collusive cartels in auctions for the procurement of public works in Valle d'Aosta, which would serve as counterfactual.

[b] Note that the estimated sample is slightly smaller than the sample used for the main analysis because of missing values in the variables used to detect the links between firms.

Table 14 – Testing predictions (CH3), (CH4) and (CH5).

Dependent variable:	min-max normalized rebates		Reference points		SD	
Auction format	AB+ABL		AB	ABL	AB	ABL
	(1)	(2)	(3)	(4)	(5)	(6)
ABL	-0.339***	-0.425***				
	(0.024)	(0.018)				
$\log(No.Participants)$			0.464**	-0.712**		
			(0.221)	(0.284)		
$\log(MeanBidderSoph)$					-0.366**	-1.128***
					(0.164)	(0.368)
Auction/project controls	YES	YES	YES	YES	YES	YES
Firm controls	YES	NO	NO	NO	NO	NO
Firm fixed effects	NO	YES	NO	NO	NO	NO
Firm-auction controls	YES	YES	NO	YES	NO	NO
Observations	10,428	10,248	232	28	229	28
R-squared	0.189	0.244	0.848	0.993	0.262	0.681

In columns (1) and (2): OLS estimations and robust standard errors clustered at firm-level in parentheses.
In columns (3)-(6), an IRLS estimator is used to account for the influence of outliers (given the small samples).
Inference: (***) = $p < 0.01$, (**) = $p < 0.05$, (*) = $p < 0.1$.
In columns (1) and (2), the dependent variable is the (min-max rescaled) discount offered by firms. ABL is a dummy variable which takes value 1 (0) if the auction is ABL (AB). Though not reported, the index of sophistication is included among the covariates.
In columns (3) and (4) the dependent variable is the (auction-specific) reference point.
In columns (5) and (6) the dependent variable SD is the standard deviation of the (absolute value of the standardized) distance of bids from the reference point. $MeanBidderSoph$ is the average of the sophistication index across firms in the auction.
Auction/project controls include: the auction's reserve price, the expected duration of the work, the number of bidders, dummy variables for the type of work, dummy variables for the year of the auction. *Firm controls* include: dummy variables for the size of the firm, and the distance between the firm and the CA. *Firm-auction controls* include: a dummy variable for the firm's subcontracting position (mandatory or optional), and a measure of the firm's backlog.