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FAVOURITISM IN SCORING RULE AUCTIONS

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Favouritism in scoring rule auctions*

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Abstract

Scoring rule auctions (*SRAs*) can be a powerful mechanism to procure complex works or services, when quality matters. However, given the buyer's discretion in the design of *SRAs*, favouritism can arise as a drawback. In this paper we empirically document potential favouritism in an original dataset of 196 *SRAs* for the procurement of canteen services in Italy over the period 2009-2013. We then sketch a simple model highlighting how an *SRA* with multidimensional quality can be distorted to favour the incumbent bidder winning the competition. Finally, we design and run a new empirical test to verify our theoretical result. We find that *SRAs* can be distorted to favour the incumbent bidder, and that the victory of the incumbent is associated with less competition and higher prices.

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1 Introduction

In the procurement of complex works or services, when suppliers have to meet "quality" specifications (technical characteristics, relevant delivery date and delivery conditions, etc.), the mechanism for awarding the contract plays a crucial role. In such a setting, a competitive bidding mechanism in the form of a first price auction (*FPA*) is usually not adopted, as the quality delivered might be less than optimal (Asker and Cantillon, 2010). A direct negotiation with a potential seller over all the relevant quality dimensions may perform better, but this process could lead to a higher price, in absence of competition.[1] Scoring rule auctions (*SRAs*) are often suggested as a possible solution: these are multidimensional auctions where bids are competitively evaluated using a linear function that weights both the price and (level of) quality dimension. The winner is the bidder that, according to this function, obtains the highest score.

SRAs have been used, for example, to award highway construction projects in California. In this setting, *SRAs* weight the price and time to completion of the work and, as highlighted by Lewis and Bajari (2011), have succeeded in increasing the provision of quality as compared to *FPA* there adopted to award similar projects.

SRAs differ significantly from conventional auctions because the buyer has discretion in - and tools for - defining the quality to be procured. Such discretion operates *ex ante* in the selection of the weight of quality (and price) included in the linear function used to evaluate bids: the buyer can strategically choose which element is assigned the greater weight in the score. And it also operates *ex post* in the assessment of the quality component of each offered bid: the buyer can adopt a subjective valuation and bidders cannot be certain about score they will achieve given the level of quality supplied (Prabal Goswami and Wettstein, 2016; Burguet, 2015; Huang, 2016).

In this paper we empirically and theoretically investigate the buyer's *ex ante* discretion in *SRAs* and how this discretion can be used to distort the *SRA* with the aim of favouring a specific bidder. Buyers could have different reasons to adopt such malfeasance. For instance, favouritism toward an incumbent supplier, *i.e.* the supplier that has previously provided the service, can simply be motivated by the buyer's risk-averse attitude. Moreover, the buyer's intent to continue ongoing efficient outsourcing could lead him/her to favour the current supplier in the competitive selection. And, in public procurement, the prospect of "exchanges" with a predetermined supplier that increase the public buyer's utility would also provide an incentive to manipulate the awarding mechanism (see Wolfstetter and Lengwiler, 2006, for a

[1] In public procurement, according to Spiller (2008) and Moszoro and Spiller (2012), public buyers can have large incentives to adopt auctions: as potentially competitive mechanisms, auctions can protect the buyer from the political cost of opaque negotiations with one bidder. Chong, Staropoli and Yvrande-Billon (2016), running their analysis on a dataset of French public procurement, find that the higher the degree of political scrutiny, the more the public buyer will choose an auction procedure instead of a negotiatated one.

survey of corruption in procurement auctions).

Whatever the motivation leading to favouritism, this paper investigates how such favouritism could be implemented through *SRAs*. With this aim, we exploit an original dataset of 196 scoring rule auctions for canteen services in Italy awarded by public buyers[2] (elected bodies, administrative and semi-administrative bodies) in the period between 2009-2013. We first provide some descriptive empirical evidences on potential favouritism. To interpret such evidences, we then sketch a simple model of an *SRA* with multidimensional quality showing that the public buyer, by strategically choosing the weight of quality in the score, can increase the probability of the incumbent's winning; and that will be done at the cost of a final higher price to be paid. Finally, we test the model's predictions on our original database finding that - on average - the higher the weight of quality in the *SRA*, the higher the price paid in the auction. This positive correlation between quality and price is not always confirmed when the auction is won by the incumbent supplier. In particular, in such case the price results - on average - higher, regardless of the weight of quality in the *SRA*. This gives us room to develop and run an empirical test for favouritism.

To the best of our knowledge, we are the first to develop and test a simple model of favouritism in the form of an *ex ante* distortion of an *SRA* with multidimensional quality. The seminal paper by Che (1993) theoretically investigates a *SRA* where both the quality and the bidder's types are single-dimensional, quality is enforceable by court, and the scoring rule is quasilinear. In such a setting, the most efficient firm will always win, regardless of the weight assigned to quality in the scoring function. But when more than one quality is included in the *SRA* and private information becomes multi-dimensional, it is no longer possible to rank firms according to their overall efficiency. With the aim filling this gap, we investigate *SRAs* where both the scoring rule and private information are multi-dimensional. This is the necessary setting to study the buyer's *ex ante* distortion - *i.e.* favouritism - in the choice of the weights adopted for the *SRA* components.

We contribute to two main strands of literature. The first is on the design of (optimal) *SRAs* in public procurement. For *SRA*s with one quality, Che (1993) theoretically highlights that the higher the quality component in the scoring rule, the lower the competition will be in the awarding phase. Koning and Van de Meerendonk (2014) empirically show that the higher the quality component in the scoring rule, the higher the price paid. Moreover, the higher the quality component in the scoring rule, the larger the buyer's cost is in specifying it; and the more verifiable such quality is, the better the enforcement of the contract (Bajari and Tadelis, 2001). Branco (1997) investigates the properties of optimal mechanisms when bidder types are single-dimensional and correlated; Asker and Cantillon (2008) show that multi-

[2] Auctions for awarding canteen and meal services to schools in Chile have been investigated by Epstein, Olivares, Weintraub and Yung (2012). These authors studied the design of large-scale combinatorial auctions with the aim of defining the optimal packages bidders should be allowed to bid on, and to diversify the supplier base to promote competition.

dimensionality of suppliers' private information can be reduced to a single dimension (*i.e.* their "pseudotype"). We adopt the Asker and Cantillon's pseudotype and investigate a multi-dimensional *SRA* where price and more than one quality are included. This setting is very close to the one used in real life public procurement and allows us to study how a public bidder can manipulate the weights of the *SRA*'s components to favour a predetermined bidder.[3]

There could be a very fine line between favouritism and corruption in a buyer's act to distort an *SRA*. Burguet and Che (2004) consider an *SRA* with two bidders and assume that both bidders are dishonest, *i.e.* along with quality and price they offer a bribe. The buyer manipulates - *ex post* - the evaluation of the bid's quality in favour of the bidder submitting the larger bribe. Similarly, in our model, the public buyer manipulates - *ex ante* - the technical bid (*i.e.* the weights of the components) directly to favour a preferred bidder.

The second strand of literature we contribute to refers to the design of tests to detect competitiveness and collusion in auctions. Conley and Decarolis (2015) present two statistical tests to detect coordinated entry and bidding choice. They run these tests on a dataset of average bid auctions[4] adopted for awarding public works in Turin, a town in the North-West of Italy: in their setting, collusion was detected by the judge of the local court of law. Differently, we do not have any external assessment about which auction, if any, was not competitive. Hence, we develop a mechanism to find non-competitive behavior by investigating the auction's features and outcomes. We compare *SRA*s managed by different buyers and exploit the information on the incumbent to perform the analysis. Our approach is in line with Bajari and Ye (2003), and Aryal and Gabrielli (2013) who designed a test to disentangle collusion and competition when collusion is not directly observed. Both used non-parametric techniques based on the *FPA* estimation of Guerre, Perrigne and Vuong (2000) to construct a statistical test to detect collusion. Notice that in our study, differently from this literature, collusion would be between the buyer and one specific bidder (and not among bidders); as such, we define this event as *favouritism*.

Finally, in considering favouritism as malfeasance, we are close to Garicano, Palacios-Huerta and Prendergast (2005). These authors offer empirical evidence of how professional soccer referees favour home teams in order to satisfy the crowds in the stadium. In their setting, referees have discretion over the addition of extra time

[3]Considering firms' behaviour in *SRAs* and mechanisms that could induce optimal performance from the award of the *SRA*s, a recent paper by Decarolis, Pacini and Spagnolo (2016) - investigating a firm-level field experiment - highlights that the introduction of a reputational index based on objectively measured past performance in terms of quality provided can substantially limit moral hazard.

[4]In the Italian framweork, an average bid auction works as follows: the first 10% of the highest and lowest discounts over the reserve price is eliminated. Then the average among all remaining discounts is computed (A1). A second average (A2) is calculated among all bids strictly above A1. The winning discount is the highest discount strictly lower than A2. See Albano, Bianchi and Spagnolo (2006) and Decarolis (2009) for further details.

at the end of a soccer game to compensate for lost time due to unusual stoppages. They find that referees systematically favour home teams by shortening close games when the home team is ahead, and lengthening close games when the home team is behind. In our setting, public buyers have discretion over the weights of the *SRA's* components: our model shows that public buyers could favour incumbent suppliers by manipulating the *SRA's* design and ending with higher prices. We find empirical support for this theoretical result.

The rest of the paper is organized as follows. Section 2 illustrates descriptive statistics of our dataset and some preliminary results. In Section 3.1 we present a simple model of favouritism toward a predetermined bidder in a scoring rule auction; and in Section 3.2 we empirically test the model's predictions on our dataset. Section 4 sums up and discusses our empirical results. Finally, Section 5 draws conclusions and policy implications.

2 Institutional setting and descriptive analysis

We built an original database of 196 public procurement auctions awarded between 2009 and 2013 for canteen services contracts in Italy. These auctions have a scoring rule format (*SRA*, henceforth) and are adopted to award contracts lasting from 3 to 5 years and with a reserve price - i.e. the maximum price the public buyer is willing to pay - higher than €150,000.[5] Our cross-sectional dataset includes three different types of public buyers managing such auctions: *elected bodies* (i.e. municipalities, 78% of auctions in our dataset); *semi-autonomous bodies* (i.e. hospitals, 7%); and *administrative bodies*, (i.e. firefighters or local branchs of the Italian Tax Agency, 15%). The first group are locally elected every 4 or 5 years, and the canteen services they outsource - canteens for elementary schools in the municipal area, for instance - are politically sensitive. The third group are run by civil servants and their canteens are usually for internal staff only. Finally, in our dataset semi-autonomous bodies consist of public hospitals; their governance is in-between elected and administrative bodies, as their senior management consists of career managers, but managerial/executive positions are appointed by regions, which are locally elected bodies. Hospital canteens are typically for internal staff and patients.

These public buyers have discretion in the awarding process specifically on two dimensions. First, public buyers are free to choose the weights of price and quality in the *SRA*. Our database includes, for each auction, the weight chosen for quality and price: on average, the weight of quality summed up to 60 points over 100. Second, public buyers can decide about firms' entry in the auction, i.e. whether to allow free entry, whether to submit firms to a preliminary screening, or even whether to

[5] Using EU-TED data on Italian public procurement auctions for canteen services in 2015 (the most recent year available), we can observe that the year total sector value has been about 100 million euro; 42 services were awarded to 30 different winners; the average participation in auctions has been 3.2 bidders; and half of these auctioned services went not to local firms.

restrict entry to a small set of firms. In our database, auctions with free entry, with a preliminary screening and with restricted entry represent respectively, 82%, 15% and 3% of the sample. *Table 1* shows the auction distributions by public buyer type, entry restriction, average quality weight and mean reserve price.

Table 1 - Descriptive statistics: auction's mechanisms:

PUBLIC BUYER (type)	ENTRY RESTRICTIONS Free Entry	Restricted proc.	Invited only	TOTAL	RESERVE PRICE (mean, €)	QUALITY'S WEIGHT (average, over 100 points)
Elected (78%)	147	2	3	152	2,100,806	61.9
Semi-autonomous (7%)	9	5	0	14	4,658,779	60.5
Administrative (15%)	6	23	1	30	3,431,993	49.5
TOTAL	162	30	4	196	2,487,271	59.9

(1)

In our database, for each auction, we observe the identity of the winner and if he/she was the canteen's service provider in the period immediately before the recorded auction takes place (i.e. if she/he is the *incumbent*); the winning bid to reserve price ratio (i.e. the winning rebate); the ratios of the maximum and the minimum bid to reserve price; the average number of participants; the number of excluded bidders (if any); the name/type of the public buyer; and geographic characteristics such as location. In the case of an elected public buyer, we also observe population. Finally, we observe the time between the year in which the service was awarded and the next electoral year. We define this variable as *year to elections* and we include it in the empirical analisys to test for the electoral cycle's relevance. *Table 2* shows the auction outcomes in term of the average number of bidders and mean winning rebate, by disentangling when the winner is the incumbent or not.

Table 2 - Descriptive statistics: auction's outcome and participation

PUBLIC BUYER (type)	WINNER (n° auctions) I. wins	I. does not win	BIDDERS *(average number)* I. wins	I. does not win	TOTAL	WINNING REBATE *(mean, % on reserve price)* I. wins	I. does not win	TOTAL
Elected	93	59	1.9	3.0	2.3	2.14%	5.87%	3.59%
Semi-autonomous	8	6	2.0	2.5	2.2	0.89%	3.22%	1.89%
Administrative	8	22	4.7	4.8	4.8	7.09%	9.85%	9.11%
TOTAL	109	87	2.1	3.4	2.7	2.41%	6.69%	4.31%

(2)

Our descriptive statistics in *Table 2* show that the incumbent wins 55% of the auctions in our database; in such auctions, competition and winning rebate are lower - i.e. there is a lower number of bidders and a higher awarding price paid by the public buyer - than in auctions where the incumbent does not win. In particular, the mean number of bidders in auctions where the incumbent has won is 2.1, and where she/he has not it is 3.4; and the mean winning rebate is respectively 2.41%. and 6.69%. These differences in means are statistically significant.

Finally, to examine the public buyer's choice of the weight of quality (and the complementary weight of price) in the *SRA*, we estimate the following:

$$q_i = \alpha' + \beta_1' EB_i + \beta_2' HOSP_i + \gamma' X_i' + \epsilon_i' \qquad (3)$$

where q is the weight of quality in the SRA, EB and $HOSP$ are dummy variables for the public buyer type, (*i.e.* elected body and hospitals, respectively), and X_i' is a vector of the characteristics of the awarded service (year, region, reserve price and whether or not the service was urgent[6]). In *Table 3* we present results from (3). We find that the public buyer being an elected body increases the weight given to quality in the SRA; moreover, the weight of quality q increases with the value of the contract for the awarded service. This result is robust to using different model specifications.

Table 3 - Preliminary analysis: the buyers' choice of the weight of quality in the SRAs

DEP. VAR:	weight for quality in SRA		
	(1)	(2)	(3)
VARIABLES	OLS	OLS	OLS
reserve price	2.79e-07***	3.17e-07***	
	(7.73e-08)	(8.58e-08)	
ln reserve price			1.340**
			(0.606)
eb	10.94***	11.89***	12.06***
	(1.907)	(2.113)	(2.091)
hosp	9.055***	10.28***	10.11***
	(2.967)	(3.230)	(3.242)
p_acc	-3.180	-2.995	-3.317
	(3.685)	(3.730)	(3.846)
t_red	-10.12**	-9.250*	-9.594*
	(4.921)	(5.429)	(5.444)
prein	-0.639	-1.851	-1.341
	(2.828)	(2.888)	(2.914)
n_west		-2.898	-2.794
		(1.825)	(1.821)
n_east		-5.540**	-5.242**
		(2.329)	(2.296)
south		-0.683	-0.455
		(2.175)	(2.145)
islands		0.870	1.549
		(2.917)	(2.841)
y_09		-1.764	-2.121
		(1.544)	(1.583)
y_10		-0.329	-0.656
		(1.920)	(2.017)
y_12		-1.594	-1.840
		(1.924)	(1.946)
y_13		-6.137	-5.890
		(4.965)	(4.878)
Constant	50.83***	53.68***	35.77***
	(1.765)	(2.299)	(8.041)
Observations	196	196	196
R-squared	0.235	0.288	0.284

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

(4)

Summing up, the descriptive statistics (*Table 1* and *2*) and first estimation results (*Table 3*) on our dataset on *SRAs* adopted to award Italian canteen services show

[6] In the case of urgent awarding procedures, the deadline for bidders to submit their offers is shorter than with non-urgent awarding procedures.

that: i) the incumbent supplier is the winner in 55% of auctions; ii) the competition is lower and the price paid by the public buyer is higher when the winner is the incumbent supplier; and iii) the weight of quality in the *SRA* increases with the awarded contract's value and if the buyer is an elected body.

3 Favouritism in SRAs: theory and empirics

Based on the evidence highlighted in the previous section, in what follows we investigate the public buyer's favouritism toward an incumbent supplier by the adoption of a scoring rule auction. We first present a simple theoretical model in which a public buyer aims to increase the incumbent's probability of victory by distorting the weights of quality and price in the *SRA*. We then return to our database and empirically look for such distortion by testing for the public buyer's choice on the weight of quality in the *SRA*, the victory of an incumbent, and the effects on winning price.

3.1 A simple model

We consider a setting in which a public buyer[7], or simply a "buyer" in what follows, has to award a service by choosing between two bidders, $j = (I, E)$, where I is an incumbent firm (*i.e.* a firm that has previously provided the buyer with such service) and E is a new entry firm. According to the previous experience with the incumbent firm I, the buyer could be willing to distort the awarding mechanism in order to let I win, *i.e.* to favour I. Such behaviour (*i.e.* the buyer's favouritism towards the incumbent) is adopted if it increases the buyer's utility, and this could simply occur when the buyer is risk averse, or - in a public procurement setting - as a result of the buyer's aim to continue a positive ongoing outsourcing, or finally as a reward for private exchanges between the buyer and the supplier.

The awarded service is described by two non-monetary characteristics, *i.e.* two qualities, and its price. The buyer gets the following utility function:

$$U(Q, p) = q_1 + q_2 - \alpha p + f| \, [t_I > t_E,] \tag{5}$$

where p is the price he has to pay for the provision of the service of quality $Q = \{q_1, q_2\}$; and α is the relative weight of the price with respect to overall quality Q in the buyer's utility function. If the incumbent firm I wins the award of the service, the buyer will receive an additional utility $f \in [0, +\infty[$. It follows that if $f = 0$ there is no favouritism by the buyer toward I, while if $f > 0$ there is a bias in favour of I.

The buyer awards the service using a scoring rule mechanism t, defined as:

$$t = a_1 q_1 + a_2 q_2 - p \quad . \tag{6}$$

[7]In what follows, as a convention, we will refer to the public buyer using "he" and to the bidder using "she".

Such scoring rule (6) weights each bid $B_j = \{q_{1j}, q_{2j}, p_j\}$, adopting a linear combination with coefficients (a_1, a_2). The bidder j with the highest score t_j wins the auction.

Each bidder j has a type $\theta_j \in [0,1] \subset \mathbb{R}^2$; she has private information on her multidimensional quality, $(\theta_{1j}, \theta_{2j})$, i.e. θ_{1j} and θ_{2j} are *i.i.d.* according to a uniform distribution between 0 and 1.

We assume each bidder's cost function to be quadratic and separable, that is:

$$C_j(Q, \theta_j) = \frac{1}{\theta_{1j}} q_1^2 + \frac{1}{\theta_{2j}} q_2^2. \tag{7}$$

We also assume that the buyer knows the type of the incumbent firm, $\overline{\theta}_I = (1, 0)$, but he does not observe the type of the other bidder E, denoted with $\theta_E = (\theta_{1E}, \theta_{2E})$.

The timing of the game and the agents' choices at each stage are the described in *Figure 1.*

	Choice	*Choice*
	a_1, a_2	$p,\ q_1, q_2$
TIME	$-\ \ ---1---\ \ -\ \ ---2-- \longrightarrow$	
PLAYER	Buyer	Bidders

(Figure 1)

In *stage 2*, equilibrium bids can be derived following Asker and Cantillon (2008). Accordingly, an *SRA* is equivalent to a first price auction where bidders' private values are given by their *pseudotype,* i.e. *the maximum level of social surplus that* a *supplier can generate, given her cost function and the scoring rule chosen*[8], which is:

$$k(\theta_j) = \arg\max_{q_1, q_2} \{a_1 q_1 + a_2 q_2 - C_j(q_1, q_2, \theta_j)\}$$

where $k(\theta_j)$ is the pseudotype for bidder j. Q^* is defined as the level of quality that maximizes the pseudotype: it is a weakly dominant strategy for each bidder to offer it as quality component of her bid (Asker and Cantillon, 2008). In our setting, having assumed a quadratic cost function, the pseudotype becomes a linear combination of $(\theta_{1j}, \theta_{2j})$. Hence, it is possible to derive both its distribution (by convolution) and the equilibrium scores. Then - as the residual in the scoring rule function - we can also obtain the price component of the bids.

The optimal design of an *SRA* with multidimensional private information is a difficult problem to solve, and we are not aware of any general result such as the

[8] Asker and Cantillon (2008), p. 73.

one for the unidimensional case by Che (1993).[9] With the aim of contributing to filling the gap, in a setting with multidimensional private information and using a quadratic cost function, we characterize the optimal *SRA* for two cases: the case where the buyer distorts the *SRA* to favour the incumbent ($f > 0$), and the case with no distortion ($f = 0$).

In *stage 1*, the buyer chooses a_1 and a_2 of the scoring rule t with the aim of maximizing:

$$\max_{a_1,a_2} \; \Pr(I\ win) \cdot U(\overline{\theta}_I|a_1, a_2) + f \cdot \Pr(I\ win) + [1 - \Pr(I\ win)] \cdot E[U(\theta_E|a_1, a_2)] \quad (8)$$

where $U(\theta_j|a_1, a_2)$ is the buyer's utility provided by bidder j, with $j \in \{I, E\}$, conditioned to the scoring rule chosen by the buyer; and $\Pr(I\ win)$ is the probability that I wins the auction. The solution of the buyer's problem in (8) yields the following result:

Proposition 1 *In the case of no favouritism, $f = 0$, the optimal weights of the scoring rule t are $a_1 = a_2 = \frac{3}{4\alpha}$ and bidders' quality provision will be below what could have been achieved with full information.*

In the case of favouritism, $f \geq 0$, the buyer will distort the scoring rule t such that $a_1 \geq a_2$. A finite solution will always exists for a_1, a_2. Define with (a_1^*, a_2^*) the optimal weights with $f \geq 0$. Then, there exist two levels of favouritism $\overline{f}$ and $\overline{\overline{f}} > \overline{f}$ such that if:

(i) $f \in \left[0, \overline{f}\right]$ then $q_{1I}(a_1^*, a_2^*) \geq q_{1I}\left(\frac{3}{4\alpha}, \frac{3}{4\alpha}\right)$ and $p_I(a_1^*, a_2^*) \geq p_I\left(\frac{3}{4\alpha}, \frac{3}{4\alpha}\right)$,

(ii) $f \in \left[\overline{f}, \overline{\overline{f}}\right]$ then $q_{1I}(a_1^*, a_2^*) \leq q_{1I}\left(\frac{3}{4\alpha}, \frac{3}{4\alpha}\right)$ but $p_I(a_1^*, a_2^*) \geq p_I\left(\frac{3}{4\alpha}, \frac{3}{4\alpha}\right)$ and

(iii) $f > \overline{\overline{f}}$ then $(a_1^*, a_2^*) = \left(\frac{2}{3\alpha}, 0\right)$, $q_{1I}\left(\frac{2}{3\alpha}, 0\right) < q_{1I}\left(\frac{3}{4\alpha}, \frac{3}{4\alpha}\right)$ and $p_I\left(\frac{2}{3\alpha}, 0\right) < p_I\left(\frac{3}{4\alpha}, \frac{3}{4\alpha}\right)$.

Finally, $q_{2I}(a_1^*, a_2^*) = 0 \; \forall f$.

Proof. See Appendix B. ■

In the case of no favouritsm, $f = 0$, our result in *Proposition 1* is in line with Che (1993): the optimal scoring rule produces a level of quality below what could have been obtained under full information. The optimal mechanism under informational asymmetry reduces the supplier's quality provision and internalizes the informational cost of the buyer.

In the case of favouritism, $f > 0$, a distortion is introduced in the *SRA*: indeed, the buyer assigns more weight to a_1, that I is more endorsed with, and less weight to the other quality, a_2,the I is less endorsed with. As a result, it is more likely that I wins the auction. At the same time, the higher a_1, the higher will be $I's$ market power and so the price component of her bid. Thus, if the buyer is interested both

[9] Che (1993) shows that it is optimal for the buyer to undervalue quality in order to reduce market power of the most efficient firm.

in the victory of the incumbent and in the quality of the service, then the gain in quality and the increase in the probability of victory of the incumbent will balance the higher price paid, so the weight of a_1 with favouritism will be higher than without. In contrast, if the only concern for the buyer is to award the contract to I, then the increase in the probability of victory of I will be equally obtained by a reduction in both weights (a_1 and a_2), but the reduction will be far greater for a_2.

3.2 Empirical analysis

The descriptive statistics in Section 2 have shown that in auctions where the incumbent has won, the price paid by the public buyer is higher and competition is lower than in auctions where the incumbent has not won. According to our model in Section 3.1, if a buyer that is interested in the quality of the service distorts the *SRA* to favour the incumbent, the price paid for the service would be higher and this could be a possible explanation for our empirical evidence. Unfortunately, we do not observe favouritism *per se*, *i.e.* which auctions (if any) were distorted to increase the probability that the contract is awarded to the incumbent supplier.

Thus, to address our empirical inspection of favouritism, we design the following strategy. We begin by separating our dataset into two samples: one including auctions that were likely competitive, and the other including all the remaining auctions where we do not have any element to conjecture whether there was any non-competitive behavior at play. We then run an econometric model on the competitive subsample and we construct a test as a result. Finally - also referring to the predictions of the simple model in Section 2 - we apply the test to the entire sample. The observations which fail to be predicted by the model are thus referred as being affected by favouritism.

The competitive test

To disentangle competitive from non-competitive auctions in our sample, we assume that an auction is competitive if both of the following conditions were contextually met: (i) the incumbent has not won; (ii) there was no entry restriction. We end up with 84 auctions that satisfy conditions (i) and (ii), and we define these as our prior-competitive subsample. As shown in *Table 4,* this subsample strongly differs in the auctions outcomes from the remaining part of the dataset. In particular, the prior-competitive subsample exhibits a winning rebate which is, on average, 2.5 times higher and a bidders' participation which is 75% greater than in the other auctions.

Table 4 - Summary statistics of the prior-competitive subsample and of the remaining auctions in the dataset

SUBSAMPLE	Winning rebate				N° bidders				Reserve price			
	Mean	*Std*	*Min*	*Max*	*Mean*	*Std*	*Min*	*Max*	*Mean*	*Std*	*Min*	*Max*
Competitive	6.70%	7.35	0	30%	3.46	2.64	1	13	2,265,744	3,640,990	174,000	18,470,079
Other auctions	2.52%	4.69	0	24%	2.07	1.92	1	11	2,653,417	6,291,476	152,000	59,995,590

(9)

A two-sample Kolmogorov–Smirnov (K-S, henceforth) test of the equality of distributions confirms that both the winnng rebate and bidders' participation values are distributed differently into the two subsamples. Surprisingly, the K-S test does not find any difference between the two subsamples in the distribution of the reserve price, in the weight of quality in the scoring function, in the year the contract was awarded, in the electoral cycle, or - using NUTS groups of regions code from Eurostat - in the geographical location of the public buyer. Using the K-S test we find only a very weak difference (*p-value* = 0.095) in the distribution of the public buyer's type. Thus, across the prior-competitive subsample and the other remaing auctions of our dataset, the characteristics of the service awarded and of the *SRA* adopted are identically distributed, but the auction outcomes differs strongly.

On the prior-competitive subsample, we run a parametric estimate of the winning rebate p_{wi} for each auction i as follows:

$$p_{wi} = \alpha_1 + \beta_{11} N_i + \beta_{12} q_i + \beta_{13} X_i \tag{10}$$

and we estimate the difference between the maximum and the minimum price offered by bidders, Δ_{pi}, according to the following model:

$$\Delta_{pi} = \alpha_2 + \beta_{21} N_i + \beta_{22} q_i + \beta_{23} \overline{X}_i \tag{11}$$

where N is the number of bidders, $q \in [0, 100]$ is the weight of quality within the *SRA*, X and $\overline{X}$ are vectors of auction characteristics that include the NUTS groups of regions code, the population of the municipality if the public buyer is an elected body, the buyer's type, the reserve price, the number of years until the next election and whether or not a restricted procedure was used.

In the procurement literature, the ratio of the winning price to the reserve price is used as a measure of competitiveness (see, for example, Coviello and Gagliarducci, 2010). This is usually done in first price auctions and average bid auctions where competition is only on the price component. However, considering an *SRA*, the higher the weight of quality, the less important the price component becomes in the bid. Unfortunately, in our dataset of *SRA*s we do not observe the quality component of the bid and we only observe the price component of the winning bid, p_{wi}. If we included the latter in the analysis, we could end up with an incomplete measure of competition. To reduce this possible incompleteness in measurement, we add to the estimation the difference between the minimum and the maximum price submitted by all bidders, Δ_{pi}. To make it clear how we interpret Δ_{pi}, consider the following example: in an *SRA* where $q_i = 0$ (thus corresponding to a first price auction), competition will only be on the price side and, depending on the bidders' heterogeneity, Δ_{pi} will be at its maximum value. Differently, in a *SRA* where $q_i = 100$, the price component of all bids will be equal to the reserve price, and the difference between the highest and the lowest price discount will be zero. Hence, in the prior-competitive subsample, we expect q to be negatively correlated with Δ_{pi}.

Estimation results of the regression (10) on p_{wi} and of the regression (11) on Δ_{pi} are presented in the first two columns of *Table* 5 and *Table 6*, respectively.

In the first column of *Table 5*, we use a standard OLS model. In the second column, we consider that the awarding mechanism may be endogeneous with respect to the buyer's type and the dimension of the contract, as shown in regression (3). For this reason, we use a two-stage least square (2sls) where q is instrumented using the buyer's type, the reserve price, and whether or not there was a requirement for urgency in the awarding of the service.

Note that in running the regression on Δ_{pi} we have considered only auctions with at least two participants.

Table 5: p_{wi}, prior-competitive subsample

DEP VAR:	*Winning Rebate*			
	(1)	(2)	(3)	(4)
VARIABLES	**OLS**	**IV**	**IV Lewbel**	**3sls**
q	-0.143**	-0.351***	-0.143**	
	(0.0661)	(0.133)	(0.0604)	
q_sra				-0.270**
				(0.110)
N° bidders	0.914***	0.894***	0.851***	1.024***
	(0.247)	(0.238)	(0.243)	(0.260)
year_to_elect	-1.399***	-1.577***	-1.395***	-1.445***
	(0.481)	(0.475)	(0.439)	(0.438)
ln reserve price	-0.264		-0.256	
	(0.580)		(0.530)	
Restricted proc.	4.157	-0.191	4.150	1.827
	(3.314)	(2.630)	(3.027)	(2.033)
population	0.00715***	0.00938***	0.00710***	0.00795***
	(0.00235)	(0.00226)	(0.00215)	(0.00191)
eb	0.270		0.346	
	(2.522)		(2.306)	
bur	-0.877		-0.720	
	(3.723)		(3.408)	
south	7.068***	7.282***	6.945***	8.519***
	(2.293)	(2.164)	(2.102)	(2.071)
islands	-0.461	0.822	-0.571	0.688
	(2.331)	(2.278)	(2.135)	(2.132)
n_east	-1.004	-1.403	-1.076	0.668
	(2.092)	(2.010)	(1.913)	(1.895)
n_west	-4.190**	-4.144**	-4.230**	-2.683
	(2.015)	(1.886)	(1.841)	(1.817)
subcontracting	4.726**	3.658*	4.650**	5.037***
	(2.008)	(1.929)	(1.837)	(1.764)
Constant	17.50*	27.26***	17.60**	20.13***
	(9.451)	(8.810)	(8.633)	(6.986)
Observations	84	84	84	84
R-squared	0.548	0.482	0.547	0.542

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

(12)

Table 6: Δ_{pi}, *prior-competitive subsample,* N° bidders greater than 1

DEP VAR:		*Spread_p*		
	(1)	(2)	(3)	(4)
VARIABLES	OLS	IV	IV Lewbel	3sls
q	-0.121**	-0.197***	-0.130**	
	(0.0573)	(0.0700)	(0.0535)	
q_sra				-0.240***
				(0.0819)
N° bidders	1.050***	1.126***	0.844***	0.831***
	(0.243)	(0.214)	(0.306)	(0.285)
year_to_elect	-0.983**	-0.797*	-0.893**	-0.703
	(0.490)	(0.442)	(0.455)	(0.440)
ln reserve price	-0.690		-0.505	
	(0.560)		(0.517)	
eb	2.489		2.337	
	(2.704)		(2.540)	
bur	6.087*		5.646*	
	(3.102)		(2.895)	
excluded	-0.537	-0.335	-0.557	-0.724
	(1.373)	(1.177)	(1.298)	(1.231)
subcontracting	3.028	1.634		2.403
	(2.100)	(1.816)		(1.884)
south_i	3.511*		2.761	
	(2.016)		(1.899)	
north	-0.448		-0.673	
	(1.786)		(1.685)	
south		6.835***		7.240***
		(1.997)		(2.127)
islands		1.041		0.377
		(2.023)		(2.076)
n_east		-0.357		0.625
		(1.701)		(1.843)
n_west		-1.039		-0.382
		(1.642)		(1.760)
Constant	16.57*	14.14***	15.98**	17.27***
	(8.614)	(4.849)	(8.085)	(5.437)
Observations	71	71	71	71
R-squared	0.458	0.474	0.432	0.450

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

(13)

Our results on the prior-competitive subsample show that the weight assigned to quality in the *SRAs* has a very strong impact on the winning rebate; the higher this weight, the lower the competition on the price component. This is also confirmed by looking at the negative and significant effect of quality on Δ_{pi}, i.e. on the difference between the minimum and maximum rebate over the reserve price in each auction.

As we could expect, the number of bidders is significant and has a positive effect on the winning rebate: greater competition increases the winning rebate, i.e. it reduces price. A higher number of bidders also produces more heterogeneity across bids, *i.e.* Δ_{pi} increases.

Considering public buyers that are electoral bodies, we can observe that the electoral cycle also has an important influence on both p_{wi} and Δ_{pi}: the lower the years remaining to the next election, the lower the price paid and the higher the distance between the minimum and the maximum rebate. This is consistent with the idea of competitive auctions, i.e. where no favouritism is at work. Consider, for example, a municipality which has to manage an *SRA* to award the canteen service for local schools. In this setting, a mayor close to election time will be as efficient as possible in managing such procurement process; in so doing, he/she would appear a capable administrator and save money to be spent on gaining consensus with the aim of being re-elected or on increasing support for a candidate from the same political party.

The results presented in *Table 5* and *Table 6* remain significant using different errors (standard, robust, corrected for small sample and bootstrapped). We further run other tests on the IV model as follows. First, we run an F-test of the joint significance of the additional instruments used for q on q: we find that instruments are sufficently correlated with the endogeneous regressor[10]. Second, we run a Sargan test and verify that the instruments are uncorrelated with the error term. Finally, we use the Durbin-Watson test to verify that q is really endogeneous and, as such, needs to be treated with instrumental variables. We find that q is actually endogeneous for the regression (10) on winning rebate, but not for the regression (11) on Δ_{pi}. Since an exogenous regressor estimated with the IV model is consistent, but less efficient, and since we find in regression (3) that q correlates with the buyer's type and the reserve price, we also use the IV model for the second regression. The results do not change significanty if OLS is instead used. Finally, when we estimate regression (10) on the auctions outside the competitive subsample, we find that the awarding mechanism is no longer significant. Moreover, in this case, we got a much lower R^2 (specifically, 0.09 against 0.48). We obtain similar results on the regression (11) on Δ_{pi}. This confirms that our initial assumptions chosen to extract competitive auctions were well founded. Indeed, our results highlight an unobserved difference between the two subsamples: auction outcomes for observations outside the prior-competitive subsample are not well explained by the auction mechanism and service characteristics.

Robustness check: endogeneous participation As a robustness check, we consider that the number of bidders in the *SRA*, N, can simultaneously be determined with the price decision because participation in the auction has a cost for each bidder. We first estimate a regression where q is assumed to be exogenous and N is simultaneously determined with p_{wi}. We find that N correlates with all the other regressors. Since it is difficult to select an instrument that correlates only with N and not with p_{wi}, and instrumental variables/other solutions are not available, we resort to the model proposed by Lewbel (2012). This approach exploits heteroskedasticity in data to construct an instrument for models with such issues. The results are presented in the third columns of *Table 5* and *Table 6*; they do not differ significantly from the standard OLS estimates.

As a further robustness check, we estimates a three-stage least squares (3sls) model which considers the awarding mechanism to be endogeneous with respect to the size of the awarded contract and the type of the buyer, as suggested in regression (3). Differently from N, the weight of quality q within the awarding procedure can be instrumented using this information. This is why we should treat the endogeneity that arises from q differently with respect to the simultaneity problem that comes from N.

As the first stage of the model, we estimate q using the reserve price, dummies

[10] The significance of the F statistics may not be enough and the value of the test should also be considered. In the case of one endogenous regressor, Stock, Wright, and Yogo (2002) suggest that the F statistic should exceed 10 for inference based on the 2SLS estimator to be reliable. Regressions (10) and (11) both satisfy this additional condition.

for the buyer's type and dummies whether or not the service was urgent. Then, the predicted values of q are used in the second and third stages to estimate the model proposed by Lewbel (2012). In the second stage, we construct an instrument to estimate the number of bidders, N, and finally, in the third stage we estimate p_{wi} and Δ_{pi}, having corrected for the endogeneity of q and for the simultaneity problem of N. The stages are designed as follows:

$$\begin{array}{llll} 1 & q_i \sim res\_price,\, eb,\, hosp & \textit{Decision } q \\ 2.\ \text{(Lewbel)} & N_i \sim \widehat{q}_i, X_i', p_{wi} & \textit{Entry decision} \\ 3. & p_{wi} \sim \widehat{N}_i, \widehat{q}_i, X_i & \textit{Auction outcome} \\ & \Delta_{pi} \sim \widehat{N}_i, \widehat{q}_i, X_i & \textit{Auction outcome} \end{array} \tag{14}$$

The estimation results of (14) are presented in *Table 7* for the first stage and in the last column of *Table 5* and *Table 6* for the third and final stage. Results are consistent with previous estimates.

Table 7: Estimate of q - first stage 3sls model, prior-competitive subsample

DEP. VAR:	weight for quality in SRA
	(1)
VARIABLES	OLS
reserve price	8.11e-07**
	(3.39e-07)
eb	10.53***
	(3.269)
hosp	7.9095*
	(4.667)
p_acc	-12.27**
	(4.858)
t_red	-9.716*
	(5.479)
prein	0.887
	(4.540)
Constant	50.68***
	(3.253)
Observations	84
R-squared	0.444

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

(15)

4 Results on the whole sample and discussion

With the aim of developing an econometric analysis to test for competitiveness in auctions, we now estimate predictions from our empirical model in Section 3 on the the entire sample. If the predictions are within a given confidence interval, the test is passed. We estimate confidence intervals (both above and below the predicted value) for the difference between the maximum and the minimum price bid submitted. We also estimate confidence intervals for the prediction of the winning rebate, but in this case the confidence interval is only calculated below that prediction.

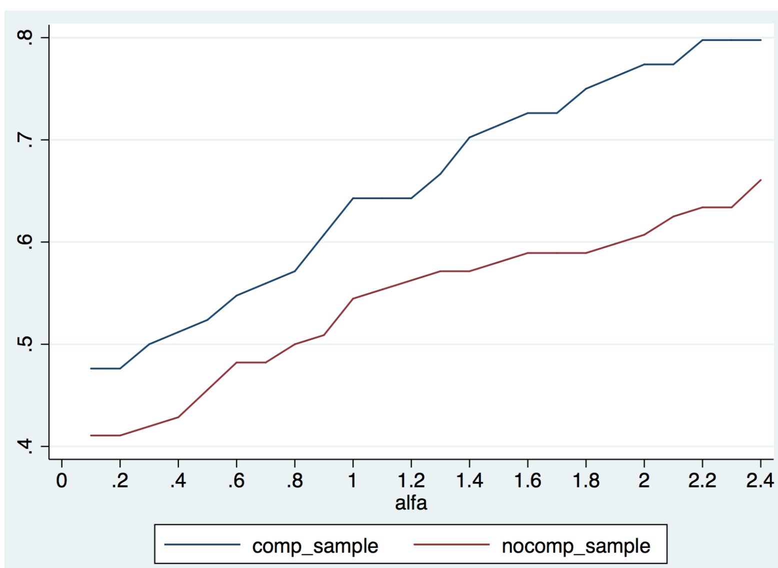
Confidence intervals are calculated as follows:

$$CI = Xb \pm \alpha SE \tag{16}$$

where Xb is the predicted value, SE is the standard error, and α is the t-value parameter that defines the width of the confidence interval. The bigger α is, the wider the confidence interval. We use both standard errors of the prediction (stdp) and standard errors of the forecast (stdf), which are equal to standard errors of the predictions plus the error variance of the regression. By construction, these latter standard errors are larger than the standard errors of the prediction. As a result, they produce larger confidence intervals, so that a lower number of observations will fail to be predicted by the model.

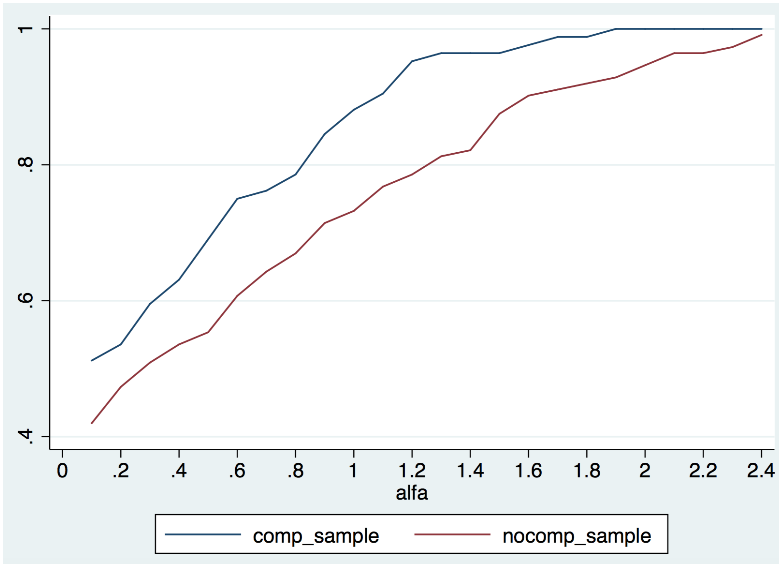
In Figures 1.1 to 2.2, we plot α (the t-value parameter that defines a confidence interval), on the horizontal axis, and the proportion of correctly predicted values – within our prior-competitive subsample in red and on the remaining part of the sample in blue – on the vertical axis. We do this for the winning rebate, p_{wi} (*Figure 1.1* using stdp and *Figure 1.2* using stdf), and for the difference between the minimum and maximum rebate over the reserve price in each auction, Δ_{pi} (*Figure 2.1* using stdp and *Figure 2.2* using stdf).

Figure 1.1: proportion of correctly predicted values, p_{wi}, stdp (%)



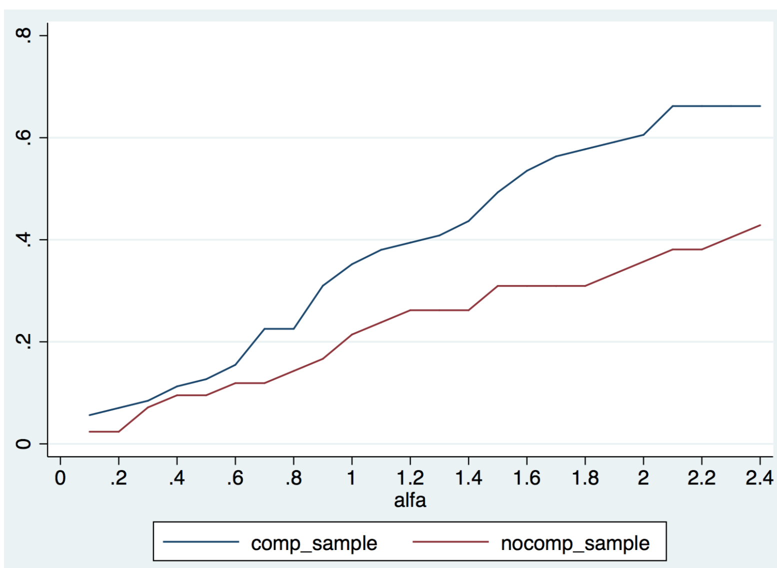
(17)

Figure 1.2: proportion of correctly predicted values, p_{wi}, stdf (%)



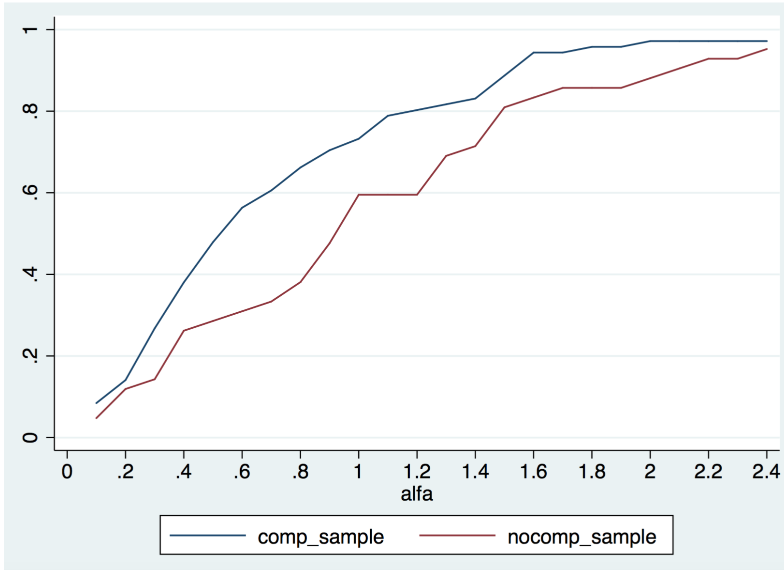
(18)

Figure 2.1: proportion of correctly predicted values, Δ_{pi}, stdp (%)



(19)

Figure 2.2: proportion of correctly predicted values, Δ_{pi}, stdf (%)



(20)

Depending on the subsample used - the prior-competitive subsample or the other one - we find a significant difference in the precision of the model estimating the winning rebate p_{wi} and Δ_{pi}. Whatever the confidence interval and the standard errors used, the prior-competitive subsample is systematically better predicted with respect to the remaining auctions. Note that this is no longer true if the model is estimated on a randomly chosen subsample.

Finally, on the basis of these results, we move from the analysis on our prior-competitive subsample to the estimate of auctions that may have lacked competition. We use predicted values of our model as a test: if the observation is within a given confidence interval of predicted values, then the competitive test is passed. Given the two estimated values (*Test 1* for p_{wi} and *Test 2* for Δ_{pi}) we end up with two tests; if both tests fail, then we define the auction as not competitive.

Table 8: number of incorrectly predicted values with CI = 90%, stdp

PUBLIC BUYER (Type)	Test Failed, *90% CI stdp*					Test OK
	Incumbent effect		Electoral effect		TOTAL	TOTAL
	I. wins	I. does not win	Electoral year	of which, I. wins		
Elected	37	13	13	12	50	102
Semi-autonomous	2	2	3	1	4	10
Administrative	4	2	1	1	6	24
TOTAL	43	17	17	14	60	136

(21)

Table 9: number of incorrectly predicted values with CI = 95% , stdp

PUBLIC BUYER (Type)	Test Failed, *95% CI stdp*					Test OK
	Incumbent effect		Electoral effect		TOTAL	TOTAL
	I. wins	I. does not win	Electoral year	of which, I. wins		
Elected	36	11	13	12	47	105
Semi-autonomous	2	2	3	1	4	10
Administrative	3	2	1	1	5	25
TOTAL	41	15	17	14	56	140

(22)

Table 10: number of incorrectly predicted values with CI = 98%, stdp

PUBLIC BUYER (Type)	Test Failed, *98% CI stdp*					Test OK
	Incumbent effect		Electoral effect		TOTAL	TOTAL
	I. wins	I. does not win	Electoral year	of which, I. wins		
Elected	31	8	13	12	39	143
Semi-autonomous	2	2	3	1	4	10
Administrative	3	2	1	1	5	25
TOTAL	36	12	17	14	48	148

(23)

With a 90%, confidence interval (CI), we find that 60 *SRAs* out of 196 (full sample) fail to pass both *Test 1* for p_{wi} and *Test 2* for Δ_{pi}. Obviously, the larger the confidence interval, the smaller the number of auctions that do not enter within that interval. Accordingly, with a 95% and a 98% confidence interval, 56 and 48 auctions, respectively, fail to pass both the tests. However, the proportion of auctions which lack competition and where the incumbent has won remains constant at between 72% and 75% for all the three confidence intervals. In the full sample, incumbents won 55% of the auctions; thus, it is much more likely for a *SRA* where the incumbent has won to fail our test.

We also find that 17 *SRAs* awarded during an electoral year fail the test under all three CIs. Under the 95% CI, these represent 30% of the non-competitive subsample of 56, a proportion which is as twice as highs as the 31 *SRAs* (representing 15% of the sample) awarded in an electoral year on the entire dataset. Interestingly, 12 of the 13 *SRAs* awarded by an elected body in the electoral year that failed the test were won by incumbent suppliers.

Note that a potentially distorted *SRA* - i.e. one failing our test - is more likely to have been awarded by an elected buyer. Under a 95% confidence interval, we find that 31% of *SRAs* awarded by elected buyers fail both competitive tests, while this proportion drops to 17% for *SRAs* managed by non-elected administrative bodies, with hospitals somewhere in-between but closer to elected bodies (28%).

Finally, the buyer's decision to use preliminary screening, or even to restrict entry, seems not to play a role. Preliminary screening is observed in 15% of auctions in the entire sample, entry restriction in the 3%. Those proportions remain constant, or are even reduced, in the subset of observations that fail the test under all three CIs.

Table 11: number of incorrectly predicted values with CI = 80%, stdf

PUBLIC BUYER (Type)	Test Failed, *80% CI stdf*					Test OK
	Incumbent effect		Electoral effect		TOTAL	TOTAL
	I. wins	I. does not win	Electoral year	of which, I. wins		
Elected	18	2	8	8	20	132
Semi-autonomous	1	0	1	1	1	13
Administrative	1	1	1	1	2	28
TOTAL	20	3	10	10	23	173

(24)

Table 12: number of incorrectly predicted values with CI = 95%, stdf

PUBLIC BUYER (Type)	Test Failed, *95% CI stdf*					Test OK
	Incumbent effect		Electoral effect		TOTAL	TOTAL
	I. wins	I. does not win	Electoral year	of which, I. wins		
Elected	6	0	3	3	6	146
Semi-autonomous	0	0	0	0	0	14
Administrative	0	0	0	0	0	30
TOTAL	6	0	3	3	6	196

(25)

These results are confirmed and are even stronger using standard errors of the forecast. Using an 80% confidence interval, the winning rebate p_{wi} and Δ_{pi} of 23 *SRAs* cannot be predicted using our competitive model. Among these observations, incumbents won in 87% of *SRAs*. Using instead a 95% confidence interval, 6 *SRAs* were not predicted by our model; in all of these, the incumbent won.

With a 80% (95%) confidence interval, 43% (50%) of all *SRAs* that were found to be non-competitive were awarded during an electoral year. Finally, 87% (100%) of the 23 *(6) SRAs* which failed the test were managed by an elected body.

To sum up, in our datatset on Italian canteen services, an *SRA* which has failed the test proposed in this paper is more likely to have been awarded to the incumbent supplier, to have been managed by an elected body and to have been awarded during an electoral year.

5 Conclusions

In this paper we investigate an original database of 196 public procurement auctions for canteen services contracts in Italy awarded between 2009 and 2013. These are scoring rule (*SRAs*), *i.e.* they contain price and quality components, and are awarded by public buyers (elected bodies, semi-autonomous bodies and administrative bodies) endorsed with discretion in chosing the rule for firms' entry in the *SRA* and the weights of the price and quality components in the *SRA*.

Our analysis proceeds in three steps. First, we document with some descriptive statistics and preliminary investigations of our database the potential existence of favouritism towards incumbent suppliers. Second, we study with a simple theoretical setting which mechanism inherent to the *SRA* may underlie the public buyer's favouritism. Third, we return to our database to check predictions we gained from the theoretical setting.

Our simple descriptive analysis highlights that:

i) in 55% of our sample the winner is the incumbent supplier, *i.e.* the firm that has been providing the canteen's service in the period immediately before the recorded auction takes place;

ii) the winning discount is lower (*i.e.* the price paid by the public buyer is higher) when the winner is the incumbent supplier, and the buyer is an elected body;

iii) the value of the awarded contract increases the relevance of quality in the scoring function, *i.e.* the quality weight in the score is higher when the contract value is larger.

We then sketch a simple model to guide the interpretation of this empirical evidence, *i.e.* to highlight how a public buyer can favour an incumbent bidder in an *SRA* with multidimensional quality. *Proposition 1* shows that in presence of favouritism towards the incumbent - but with the quality of the service remaining an important component in the public buyer's utility function - the public buyer will pay a higher price than in the absence of favouritism. Indeed, to increase the probability that the incumbent will win the *SRA*, the public buyer will design the *SRA* with a higher weight for the quality component the incumbent is endorsed with, and in so doing will give the incumbent market power and prompt her to offer a lower winning rebate (*i.e.* to offer the service for a higher price).

Finally, we return to our dataset and empirically test our model's predictions. In particular, exploiting the public buyers' heterogeneity and their choices on firms' entry in the auction and on weights in the *SRA*, we provide an empirical analysis to test for a public buyer's favouritism in an *SRA* toward the incumbent. Defining a prior-competitive subsample, we develop and run an econometric test for competitiveness; we then construct, from the result of this test, a posterior variable that highlights the presence of favouritism.

We find that the weight of the quality dimension in the *SRA* has a very strong impact on the winning rebate; the higher this weight, the less competition on the price side. The weight of quality also increases heterogeneity across bids, *i.e.* it increases the difference between the minimum and maximum rebate over the reserve price in each auction. While the number of observations that fail to be predicted by our model depends on the width of the confidence intervals and the type of standard errors used, we always find that auctions where the incumbent supplier has been awarded the contract are significantly more likely to lack competition. Similarly, those auctions are more likely to have been managed by an elected buyer. All these results are confirmed by different robustness checks.

Taken together, our results suggest that muldimensional *SRAs* with more than one quality component could be easily distorted by public buyers. This is a relevant point, since *SRAs* are increasingly being adopted as public procurement awarding format in many countries, and a buyer's bias toward predetermined suppliers could annihilate competition and its potential positive effects.[11] Finally, note that the methodology developed in our analysis could be adopted for periodical screening by a regulatory authority in charge of monitoring and regulating performance of public auctions. It could represent a starting point for a more specific investigation of favouritism in public procurement, in particular when the public buyer has some discretion in choosing the rules for firms' entry in the auctions, and the relative weights for price and quality in the *SRA*.

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Appendix A: Variables and definitions

PUBLIC BUYER'S CHOICE ON FIRM ENTRY IN THE AUCTION

free entry auctions where any entry restriction has applied;
Restricted proc. auctions where a restricted procedure was used
invited only auctions for invited only bidders;
P_accellerated and t_red auctions with shorter deadline to submit bids
Prein auctions in which time for bidders to decide to participate is higher.

VARIABLE NAMES FOR THE AUCTION'S CHARACTERISTICS AND OUTCOMES

quality, q the weight of quality in a scoring rule auction;
q_sra predicted values of q in 3sls model (Table 7);
Winning rebate the winning rebate to reserve price ratio;
reserve price the auction's starting value (in euros) decided by the public buyer. All auctions considered in this analysis had a reserve price higher than 150,000 euros;
ln reserve price the same variable *reserve price,* but in log;
spread_p the difference between the maximum and the minimum winning rebate;
N° bidders the number of bidders;
excluded dummy equal to one in case any bidders were excluded after having submitted a bid;
subcontracting dummy equal to 1 if part of the service was subcontracted.

VARIABLE NAMES FOR BUYER'S TYPE

Eb dummy for elected bodies;
hosp dummy for hospitals;
bur dummy for bureaucracies, police and firefighters.

OTHER VARIABLE NAMES

Population the population in case the contracting authority is an elected body;
n_west north west of Italy;
n_east north east of Italy;
center central regions of Italy;
south south of Italy;
island the two major islands.

Geographical division follows the NUTS group of regions subdivision by Eurostat: *North* aggregates n_east and n_west, and *south_i* aggregates south and islands.

y_09 to y_13 are dummies equal to 1 if the auction was awarded in a given year (2009 to 2013).

year to elections the distance between the year in which the service was awarded and the next electoral year.

Appendix B: proof of Proposition 1

The model of Section 2 is solved via backward induction starting from stage 2. Algebra and mathematical details are given in the *Online Appendix*.[12]

5.1 Stage 2

In stage 2, we define the equilibrium bid $B_j = \{q_{1j}, q_{2j}, p_j\}$ for bidders $j \in \{I, E\}$. As a convention, we refer to the buyer using "he" and to each bidder using "she".

Following Asker and Cantillon (2008), consider bidder j who has won the contract with a score to fulfill t_j^W. She chooses q_{1j}, q_{2j}, p_j, given the score submitted t_j^W, to maximize her profit:

$$\begin{aligned} \max_Q \ \pi_j &= p_j - \sum_{i=1}^{2} \frac{1}{\theta_{ij}} q_i^2 \\ s.t. \ t_j^W &= \sum_{i=1}^{2} a_i q_{ij} - p_j \end{aligned} \tag{26}$$

Replace p_j in the objective function to obtain:

$$\max_Q \ \sum_{i=1}^{2} \left(a_i q_{ij} - \frac{1}{\theta_{ij}} q_i^2 \right) - t_j^W \tag{27}$$

An important feature here is that, in equilibrium, the optimal provision of quality q_{ij} for bidder j is independent from t_j^W. Define

$$k(\theta_j) = \max_Q \ \sum_{i=1}^{2} \left(a_i q_{ij} - \frac{1}{\theta_{ij}} q_i^2 \right) \tag{28}$$

as the bidder j pseudotype. Solving the pseudotype maximization problem, we obtain that once the scoring rule is fixed, in equilibrium the quality decision of bidder j depends only on the bidder's ability in that quality. The optimal decision of bidder j for quality i is:

$$q_{ij}^* = \frac{1}{2} a_i \theta_{ij} \tag{29}$$

The set of pseudotypes is an interval in R, and the density inherits the smooth property of θ_j (that is distributed according to a continuous joint density function). The maximized pseudotypes becomes:

$$k(\theta_j) = \sum_{i=1}^{2} \frac{1}{4} a_i^2 \theta_{ij} \tag{30}$$

[12] The Online Appendix is available at: https://sites.google.com/site/riccardomarchiadani/home/working-papers

The use of a quadratic cost function results in a pseudotype linear in the random variables θ_1 and θ_2. Denote $\frac{1}{4}a_i^2 = c_i$ to ease notation. By convolution, the cumulative distribution function (CDF) of $k(\theta)$ is given by the following piecewise function:

$$F(k) = \begin{cases} \frac{1}{2}\frac{k^2}{c_1 c_2} & \text{if } 0 \leq k \leq c_2 \text{ and } c_2 \leq c_1 \quad (1) \\ \frac{1}{2}\frac{2k-c_2}{c_1} & \text{if } c_2 < k \leq c_1 \text{ and } c_2 \leq c_1 \quad (2) \\ 1-\frac{1}{2}\frac{(c_1+c_2-z)^2}{c_1 c_2} & \text{if } c_1 < k \leq c_1+c_2 \text{ and } c_2 \leq c_1 \quad (3) \\ \frac{1}{2}\frac{k^2}{c_1 c_2} & \text{if } 0 \leq k \leq c_1 \text{ and } c_1 \leq c_2 \quad (4) \\ \frac{1}{2}\frac{2k-c_1}{c_2} & \text{if } c_1 < k \leq c_2 \text{ and } c_1 \leq c_2 \quad (5) \\ 1-\frac{1}{2}\frac{(c_1+c_2-z)^2}{c_1 c_2} & \text{if } c_2 < k \leq c_1+c_2 \text{ and } c_1 \leq c_2 \quad (6) \end{cases} \tag{31}$$

Consider that $k \in [0, (c_1 + c_2)]$. We then apply Asker and Cantillon's (2008) Theorem 1 and Corollary 1: the equilibrium bid (Q, p) in the scoring rule is equivalent to the equilibrium bid in an equivalent first price auction (FPA) where the bidder' private valuations are given by their pseudotypes and where bidders' scores are replaced by bidders' bids. The equilibrium bid in an FPA is given by:

$$t(k) = k - \frac{1}{F^{N-1}(k)} \int_0^k F^{N-1}(z)dz \tag{32}$$

$t(k)$ always exists, can be analytically estimated, and it is a finite number[13]. To conclude the characterization of equilibrium bids of a scoring rule auction in this setting, we need to define the price component p_j of the bid $B_j = (q_{1j}, q_{2j},\ p_j)$ submitted by player j. It is obtained as the residual component of the scoring function, where both scores and quality have been replaced with equilibrium values derived above:

$$p_j = 2k(\theta_j) - t(k(\theta_j)) \tag{33}$$

In equilibrium a 1:1 relation exists between pseudotypes, scores and prices. That is, each pseudotype $k(\theta_j)$ bids a unique score t_j and a unique price p_j. However, quality provision depends only on the bidder's specific ability to provide that quality. Hence the same pseudotype may produce different level of qualities q_{1j} and q_{2j} because different configurations of $(\theta_{1j}, \theta_{2j})$ may end up having the same pseudotype, depending on the scoring rule chosen by the buyer.

5.2 Stage 1

In stage 1, the buyer observes only bidder's I type and has to decide the optimal mechanism a_1 and a_2 to award the contract. Depending on the scoring rule chosen, the utility he will receive is equal to the utility provided by the incument plus the

[13]Equilibrium bids are derived in Section 1, online appendix.

expected utility provided by the entrant, with each utility weighted for the probability of victory of the related bidder. Finally, we introduce favouritism as an additive utility that the buyer receives if the incumbent wins. We denote it by f. f is a measure of how much the buyer is willing to distort the scoring rule in order to let the incumbent win. If $f = 0$ there is no favouritism, if $f > 0$ there is favouritism. Hence the maximization problem becomes:

$$U = \Pr(a\ win) \cdot U(\overline{\theta}_I | a_1, a_2) + f \cdot \Pr(a\ win) + [1 - \Pr(a\ win)] \cdot E[U(\theta_E | a_1, a_2)] \quad (34)$$

where $U(\theta_j | a_1, a_2)$ is the utility provided by bidder j, with $j \in \{I, E\}$, depending on the scoring rule chosen by the buyer, while $\Pr(a\ win)$ is the probability that the incumbent win.

We use the following solution strategy:

1. We derive the optimal scoring rule without favouritism.
   (a) In doing so, we have to derive $\Pr(a\ win)$ - the probability that the incumbent win - then $U(\overline{\theta}_I | a_1, a_2)$ - the utility of the incumbent -, and finally $E[U(\theta_E | a_1, a_2)]$, which is the expected utility of the entrant.
   (b) Then we solve the maximization problem and we prove that there exists only one couple a_1, a_2 which are a global maximum for the function U because there is no other local maximum, the function is continuous, and because the boundary solutions for $a_i = 0$ and $a_i = +\infty$ provide a lower utility.
2. Then, we introduce favouritism. We prove that:
   (a) if $f \geq 0$ then the optimal weights (a_1^*, a_2^*) have to be constructed such that $a_1^* \geq a_2^*$: the buyer has to assign, in the scoring rule, more importance to the quality in which the incumbent is more efficient. Moreover, the optimal weight in the scoring rule for the quality in which the buyer is less efficient is below its level without favouritism, while the optimal weight in the scoring rule for the quality in which the buyer is more efficient can be above or below.
   (b) With infinite favouritism, due to excessive market power given to the incumbent, the provision of both quality and price is lower than under the case without favouritism. Infinite favouritism can be interpreted as the case where the buyer is no longer concerned by the quality of the service, but in having the preferred bidder win the auction.
   (c) With a finite level of favouritism, the price paid by CA in case of victory of the incumbent may be above (with low values of f) or below (with high

level of f) that in the case without favouritism. In particular, we prove that there exists a threshold $\overline{f}$ such that if $f < \overline{f}$, then - in case of victory of the incumbent - quality 1 provision is higher and price is also higher than in the case without favouritism. There exists also a treshold $\overline{\overline{f}}$ such that if $f \in \left[\overline{f}, \overline{\overline{f}}\right]$ then quality provision will be below the case without favouritism but the price remains above.

5.2.1 Derivation of $\Pr(a\ win)$

The probability that bidder $\overline{\theta}_I$ wins the auction is equivalent to the probability that the unobserved pseudotype $k(\theta_E)$ is lower than the observed pseudotype $k\left(\overline{\theta}_I\right)$, given the scoring rule chosen:

$$\begin{aligned} \Pr(a\ win) &= \\ \Pr\left[k(\theta_E) < k\left(\overline{\theta}_I\right)\right] &= \\ \Pr(a_1^2\theta_{1E} + a_2^2\theta_{2E} &< a_1^2\overline{\theta}_{1I} + a_2^2\overline{\theta}_{2I}) = \\ \Pr(Z &< 4k\left(\overline{\theta}_I\right)) = \end{aligned} \tag{35}$$

where $Z = a_1^2\theta_{1E} + a_2^2\theta_{2E}$ is a convolution of the two random variables θ_{1E} and θ_{2E}. The cumulative density function of Z, evaluated in $4k\left(\overline{\theta}_I\right)$, depends on the optimal values of a_1 and a_2. Six cases are possible, given the relative values of a_1 with respect to a_2 and of a_1,a_2 with respect to $4k\left(\overline{\theta}_I\right)$, as described in Section 2 of the online appendix. However, given the pseudotype $k\left(\overline{\theta}_I\right) = \frac{1}{4}a_1^2$ of the incumbent, three of the cases mentioned above are impossible. In the remaining three cases we have that $\Pr(a\ win)$ is equal to:

$$\begin{array}{lll} \textit{Case} & \Pr\left(Z<4k\left(\overline{\theta}_I\right)\right) & \\ 1. & \frac{1}{2}\frac{a_1^2}{a_2^2} & \text{Only if } a_1 = a_2 \\ 2. & \frac{1}{2}\frac{2a_1^2-a_2^2}{a_1^2} & \text{Only if } a_1 > a_2 \\ 4. & \frac{1}{2}\frac{a_1^2}{a_2^2} & \text{Only if } a_1 < a_2 \end{array} \tag{36}$$

5.2.2 Derivation of $U(\overline{\theta}_I|a_1, a_2)$

The utility of the incumbent $U(\overline{\theta}_I|a_1, a_2)$ depends on its bid $B_I = (q_{1I}, q_{2I}, p_I)$. Quality provision depends only on its ability in the given quality and is equal to $q_{1I} = \frac{1}{2}a_1$ for the quality in which the incumbent is more efficient and to $q_{2I} = 0$ for the quality in which the incumbent is less efficient. As for the price, we replace the incumbent pseudotype - which is equal to $k_I = \frac{1}{4}a_1^2$ - and equation (32) - the equilibrium bid in an FPA with two participants - in equation (33) to obtain:

$$p_I = \frac{1}{4}a_1^2 + \frac{1}{F(k_I)}\int_0^{k_I} F(z)dz \tag{37}$$

While k_I is known by the buyer, the CDF is a piecewise function, the exact piece to be used depends on the optimal scores a_1, a_2 which can be obtained only at the end of the buyer's optimization process. As before, we have to consider six cases that comes from the piecewise distribution of $F(k)$. But only three of these are possible given the incumbent's pseudotype, which are:

$$\begin{array}{lll} \textit{Case} & p_I & \\ 1. & p_I = \frac{1}{3}a_1^2 & \text{Only if } a_1 = a_2 \\ 2. & p_I = \frac{1}{4}a_1^2 + \frac{1}{12}\left(\frac{3a_1^4 - 3a_1^2a_2^2 + a_2^4}{2a_1^2 - a_2^2}\right) & \text{Only if } a_1 > a_2 \\ 4. & p_I = \frac{1}{3}a_1^2 & \text{Only if } a_1 < a_2 \end{array} \tag{38}$$

And finally the utility provided by the incumbent is:

$$U(\overline{\theta}_I | a_1, a_2) = \frac{1}{2}a_1 - \alpha p_I \tag{39}$$

5.2.3 Derivation of $E[U(\theta_E | a_1, a_2)]$

Consider the utility of the expected entrant, in case she wins.

$$\begin{aligned} E[U(\theta_E|a_1,a_2)] &= E[q_{1E} + q_{2E} - \alpha p_E] \\ &= E\left[\frac{1}{2}a_1\theta_{1E} + \frac{1}{2}a_2\theta_{2E} - \alpha p_E\right] \\ &= \frac{1}{2}a_1 E[\theta_{1E}] + \frac{1}{2}a_2 E[\theta_{2E}] - \alpha E[p_E] \end{aligned} \tag{40}$$

Note that, even if the CDF of k is a piecewise function, its expected value is simply equal to:

$$E[k] = \frac{1}{8}\left(a_1^2 + a_2^2\right) \tag{41}$$

To derive $E[\theta_{iE}]$ consider that, in order to be part of $E[k]$, a generic pair $(\theta_{1E}, \theta_{2E})$ has to satisfy three conditions:

$$\begin{aligned} \theta_{1E} &\in [0,1] \\ \theta_{2E} &\in [0,1] \\ \frac{1}{4}\left(a_1^2\theta_{1E} + a_2^2\theta_{2E}\right) &= \frac{1}{8}\left(a_1^2 + a_2^2\right) \end{aligned} \tag{42}$$

The distribution of each θ_{iE} remains uniform because there is a one-to-one relation between θ_{1E} and θ_{2E}, that is, once a value θ_{1E} is fixed, then only one value of θ_{2E} (at most) will be such that the pair satisfy the three conditions above, and this interval is continuous. Hence the problem of finding the expected values of $E\left[\theta_i^W\right]$ given the expected pseudotype $E[k]$ reduces in finding the extreme values of this interval.

Those are given in the table below:

$$
\begin{array}{llllllll}
 & \underline{\theta}_1^w & \overline{\theta}_1^w & \underline{\theta}_2^w & \overline{\theta}_2^w & E\left[\theta_1^w\right] & E\left[\theta_2^w\right] & C1 \\
1 & \frac{1}{2}\left(1-\frac{a_2^2}{a_1^2}\right) & 1 & \frac{1}{2}\left(1-\frac{a_1^2}{a_2^2}\right) & 1 & \left(\frac{3}{4}-\frac{1}{4}\frac{a_2^2}{a_1^2}\right) & \left(\frac{3}{4}-\frac{1}{4}\frac{a_1^2}{a_2^2}\right) & a_1=a_2 \\
2 & \frac{1}{2}\left(1-\frac{a_2^2}{a_1^2}\right) & \frac{1}{2}\left(1+\frac{a_2^2}{a_1^2}\right) & 0 & 1 & \frac{1}{2} & \frac{1}{2} & a_1>a_2 \\
3 & 0 & 1 & \frac{1}{2}\left(1-\frac{a_1^2}{a_2^2}\right) & \frac{1}{2}\left(1+\frac{a_1^2}{a_2^2}\right) & \frac{1}{2} & \frac{1}{2} & a_1<a_2 \\
4 & 0 & \frac{1}{2}\left(1+\frac{a_2^2}{a_1^2}\right) & 0 & \frac{1}{2}\left(1+\frac{a_1^2}{a_2^2}\right) & \frac{1}{4}\left(1+\frac{a_1^2}{a_2^2}\right) & \frac{1}{4}\left(1+\frac{a_2^2}{a_1^2}\right) & (a_1=a_2)
\end{array}
\tag{43}
$$

Case 4 is equivalent to case 1, so we can disregard it.

We have finally to derive the expected value of the price of the potential entrant. The expected price is given by:

$$
E\left[p_E\right]=E\left[k\right]+\frac{1}{F(E\left[k\right])}\int_0^{E[k]}F(z)dz \tag{44}
$$

Since $F(k)$ is a piecewise function, we still have to consider all six cases. But only four are possible:

$$
\begin{array}{lll}
(1) & \text{if} & a_1=a_2 \\
(2) & \text{if} & a_1>a_2 \\
(4) & \text{if} & a_1=a_2 \\
(5) & \text{if} & a_1<a_2
\end{array}
\tag{45}
$$

Cases (1) and (4) are equivalent. The expected price becomes[14]:

$$
\begin{array}{lll}
E\left[p_b\right]=E\left[k\right]+\frac{1}{3}E\left[k\right] & \text{if} & a_1=a_2 \\
E\left[p_b\right]=E\left[k\right]+\frac{1}{16}a_1^2+\frac{1}{48}a_2^2 & \text{if} & a_1>a_2 \\
E\left[p_b\right]=E\left[k\right]+\frac{1}{16}a_2^2+\frac{1}{48}a_1^2 & \text{if} & a_1<a_2
\end{array}
\tag{46}
$$

5.2.4 Solution of the maximization problem without favouritism[15]

To solve the maximization problem, we have to consider three cases:

$$
\begin{array}{rcl}
a_1 & = & a_2 \\
a_1 & \geq & a_2 \\
a_1 & \leq & a_2
\end{array}
\tag{47}
$$

We solve each of these three cases, and the solution must satisfy the above condition, to be accepted.

[14] Calculated with mathematica

[15] All computations are in Section 3 of the online appendix.

We solve the first case, $a_1 = a_2$, and we obtain what is, at least, a local maximum (we consider also the determinant of the Hessian matrix):

$$a_1 = a_2 = \frac{3}{4\alpha} \tag{48}$$

The utility in this case assumes value $U\left(\frac{3}{4\alpha}, \frac{3}{4\alpha}\right) = \frac{3}{16\alpha}$. We then prove that if $a_1 > a_2$, only one solution exists where both the first order conditions are equal to 0 (a necessary condition for a local maximum to exists). However, looking at the Hessian matrix, reveals that this point is a saddle point. Finally, if $a_1 < a_2$, no point exists such that both FOCs are equal to zero.

To prove that the unique local maximum we found is a global maximum, we have to prove that the function is continuous and we have to look at what happens when $a_i = 0$ and when $a_i \to +\infty$ (the boundary values).

- To prove that the function is continuous. It is for any values of a_1 and a_2, because in all the three cases, if $a_1 = a_2$, then $U = \frac{1}{2}a - \frac{1}{3}a^2\alpha$
- Check the utility if $a_i = 0$. If $a_1 = a_2 = 0$ then $U(0,0) = 0$. If $a_1 = 0$ then the maximum value U can assume is equal to $U\left(\frac{2}{3\alpha}, 0\right) = \frac{1}{6\alpha} < \frac{3}{16\alpha}$. Instead, if $a_2 = 0$ then the maximum value U can assume is given by $U\left(0, \frac{2}{3\alpha}\right) = \frac{1}{12\alpha} < \frac{3}{16\alpha}$
- Check the utility if $a_i \to +\infty$. If $a_1 = a_2 \to +\infty$ then $U \to -\infty$. If $a_1 \to +\infty$ then $U \to -\infty$ for any value of a_2. If $a_2 \to +\infty$ then $U \to -\infty$ for any value of a_1.

Hence we can conclude that, without favouritism, there exists a unique couple $a_1 = a_2 = \frac{3}{4\alpha}$ such that the function is maximized, and this is a global maximum.

Comparison with quality provision under first best (full information) case
With full information, the buyer can offer to bidders a contract which maximizes her utility subject to a zero profit condition for bidders. Replacing p in the buyer's utility function and solving the maximization problem, we obtain:

$$q_i^{FB} = \frac{1}{2\alpha}\theta_j \tag{49}$$

which is lower than quality provision under the optimal scoring rule $\left(\frac{3}{4\alpha}, \frac{3}{4\alpha}\right)$, obtainted from equation (29):

$$q_i^* = \frac{3}{8\alpha}\theta_j \tag{50}$$

5.3 Favouritism

We now introduce favouritism. The function the buyer maximizes becomes:

$$\max U = \; H(a_1, a_2) + f \cdot \Pr_{(a\ win)}(a_1, a_2) \tag{51}$$

where $H(a_1, a_2)$ is the continuous function studied in the previous section. It has a single maximum point, where $a_1 = a_2 = \frac{3}{4\alpha}$. We define this global maximum as $(\overline{a}_1, \overline{a}_2)$. Consider also that:

$$\begin{aligned} \lim_{\substack{a_1 \to \infty \\ a_2 \longrightarrow \infty}} H(a_1, a_2) &= -\infty \\ \lim_{a_1 \to \infty} H(a_1, a_2) &= -\infty \\ \lim_{a_2 \to \infty} H(a_1, a_2) &= -\infty \end{aligned} \tag{52}$$

And,

$$\begin{aligned} \nexists \widetilde{a}_1 &: \; H(\widetilde{a}_1, 0) > H(\overline{a}_1, \overline{a}_2) \\ \nexists \widetilde{a}_2 &: \; H(0, \widetilde{a}_2) > H(\overline{a}_1, \overline{a}_2) \\ H(0,0) &< \; H(\overline{a}_1, \overline{a}_2) \end{aligned} \tag{53}$$

$H(a_1, a_2)$ is twice differentiable.

$\Pr(a\ win)$ is a continuous function globally increasing in a_1 and globally decreasing in a_2, as can be seen from first order conditions: $\frac{\partial \Pr(awin)}{\partial a_1} > 0$ and $\frac{\partial \Pr(awin)}{\partial a_2} < 0$. Moreover $\Pr(a\ win) \in [0, 1]$ and f is a positive number.

5.3.1 A maximum exists

Consider that, at least for finite values of f, a maximum exists. In fact, $U(a_1, a_2)$ has an upper bound because:

$$U(a_1, a_2) \leq H(\overline{a}_1, \overline{a}_2) + f \tag{54}$$

since $\max H(a_1, a_2) = H(\overline{a}_1, \overline{a}_2)$ and $\max \Pr(a_1, a_2) = 1$. Moreover consider that:

$$\begin{aligned} \lim_{a_1 \to \infty} H(a_1, a_2) + f \cdot \Pr(a_1, a_2) &= -\infty \\ \lim_{a_2 \to \infty} H(a_1, a_2) + f \cdot \Pr(a_1, a_2) &= -\infty \end{aligned} \tag{55}$$

Hence the function has at least one maximum and this maximum is finite.

5.3.2 Domain of the optimal weights of the scoring function

Define by (a_1^*, a_2^*) the weights of the scoring function that maximizes the buyer's utility given a level of favouritism $f \geq 0$. Then $a_1^* = a_2^* = \frac{3}{4\alpha}$ if $f = 0$. We now prove that $a_1^* > a_2^*$[16] if $f > 0$. Moreover, we prove that $a_1^* \in \left[\frac{2}{3\alpha}, \widehat{a}_1\right]$ where $\widehat{a}_1$ is a finite value such that $\widehat{a}_1 > \frac{3}{4\alpha}$ and $a_2^* \in \left[0, \frac{3}{4\alpha}\right]$. We split the proof into two parts.

[16] Or, at most, $(a_1^*, a_2^*) = (\overline{a}_1, \overline{a}_2)$

First:

$$\text{if } f>0 \rightarrow \nexists\left(\widetilde{a}_1,\widetilde{a}_2\right):\left(\widetilde{a}_1<\widetilde{a}_2\cup U\left(\widetilde{a}_1,\widetilde{a}_2\right)>U\left(\overline{a}_1,\overline{a}_2\right)\right) \tag{56}$$

Consider first that $H\left(\widetilde{a}_1,\widetilde{a}_2\right)<H\left(\overline{a}_1,\overline{a}_2\right)$. Then consider that, if $\widetilde{a}_1<\widetilde{a}_2$, then $\Pr_{(a\ win)}\left(a_1,a_2\right)=\frac{1}{2}\frac{a_1^2}{a_2^2}<\frac{1}{2}$ while $\Pr_{(a\ win)}\left(\overline{a}_1,\overline{a}_2\right)=\frac{1}{2}$. It follows that $U\left(\widetilde{a}_1,\widetilde{a}_2\right)<U\left(\overline{a}_1,\overline{a}_2\right)$. This concludes the proof.

Second:

$$\text{if } f>0 \rightarrow \nexists\left(\widetilde{a}_1,\widetilde{a}_2\right):\left(\widetilde{a}_1=\widetilde{a}_2\cup U\left(\widetilde{a}_1,\widetilde{a}_2\right)>U\left(\overline{a}_1,\overline{a}_2\right)\right) \tag{57}$$

Consider first that $H\left(\widetilde{a}_1,\widetilde{a}_2\right)\leq H\left(\overline{a}_1,\overline{a}_2\right)$. Then consider that, if $\widetilde{a}_1=\widetilde{a}_2$, then $f\cdot\Pr_{(a\ win)}\left(\widetilde{a}_1,\widetilde{a}_2\right)=f\cdot\Pr_{(a\ win)}\left(\overline{a}_1,\overline{a}_2\right)=\frac{1}{2}\frac{a_1^2}{a_2^2}p=\frac{p}{2}$. It follows that $U\left(\widetilde{a}_1,\widetilde{a}_2\right)\leq U\left(\overline{a}_1,\overline{a}_2\right)$. This concludes the proof.

Hence The couple $\left(a_1^*,a_2^*\right)$ that maximizes $U\left(a_1,a_2\right)$ in case $f>0$ must be constructed such that $a_1^*>a_2^*$ or, at most, $\left(a_1^*,a_2^*\right)=\left(\overline{a}_1,\overline{a}_2\right)$. Finally, consider that a necessary condition for a maximum (which we know to exists) is that the point that maximizes U will be the one such that:

$$\begin{aligned}\nabla U\left(a_1,a_2\right) &= 0 \\ \begin{bmatrix}\frac{\partial U}{\partial a_1}\\ \frac{\partial U}{\partial a_2}\end{bmatrix} &= \begin{bmatrix}\frac{\partial H}{\partial a_1}\\ \frac{\partial H}{\partial a_2}\end{bmatrix}+f\begin{bmatrix}\frac{\partial\Pr_{(a\ win)}(a_1,a_2)}{\partial a_1}\\ \frac{\partial\Pr_{(a\ win)}(a_1,a_2)}{\partial a_2}\end{bmatrix}=0\end{aligned} \tag{58}$$

$$\begin{aligned}\nabla H\left(a_1,a_2\right) &= -f\cdot\nabla\Pr_{(a\ win)}\left(a_1,a_2\right) \\ \begin{bmatrix}\frac{\partial H}{\partial a_1}\\ \frac{\partial H}{\partial a_2}\end{bmatrix} &= -f\begin{bmatrix}\frac{\partial\Pr_{(a\ win)}(a_1,a_2)}{\partial a_1}\\ \frac{\partial\Pr_{(a\ win)}(a_1,a_2)}{\partial a_2}\end{bmatrix}\end{aligned} \tag{59}$$

This, with some manipulations, can be expressed as:

$$\frac{\partial H}{\partial a_2}\left(\frac{\partial\Pr_{(a\ win)}\left(a_1,a_2\right)}{\partial a_2}\right)^{-1}=\frac{\partial H}{\partial a_1}\left(\frac{\partial\Pr_{(a\ win)}\left(a_1,a_2\right)}{\partial a_1}\right)^{-1} \tag{60}$$

To define the domain of a_1, the above condition can be expressed in terms of a_1 and the ratio $\varepsilon_a=\frac{a_2}{a_1}$ which we know must be such that $\varepsilon_a\in[0,1]$.We show in Section 4 of the online appendix that there exists for a_1 a lower bound equal to $\frac{2}{3\alpha}$ and an upper bound greater than $\frac{3}{4\alpha}$ (the optimal weight for a_1 without favoritism) such that if a_1 does not belong to this interval, then equation (60) has no solution. Similarly, equation (60) can be expressed in terms of a_2 and the ratio $\varepsilon_a=\frac{a_2}{a_1}$; it has solutions only if $a_2^*\in\left[0,\frac{3}{4\alpha}\right]$.

5.3.3 Infinite favouritism

Suppose $f \to \infty$. Consider the buyer's maximization problem:

$$\max U = \; H(a_1, a_2) + f \cdot \Pr_{(a\ win)}(a_1, a_2) \tag{61}$$

It is optimal to choose $a_2 = 0$. In this case in fact $\Pr_{(a\ win)}(a_1, a_2) = 1\ \forall a_1$[17]. The buyer then will choose a_1 to maximize the residual part of her utility function: $H(a_1, a_2)$. The problem becomes:

$$\max \; H(a_1, 0) \tag{62}$$

$$\max \; \frac{1}{2}a_1 - \frac{3\alpha}{8}a_1^2 \tag{63}$$

The solution is: $a_1 = \frac{2}{3\alpha}$. Since $a_1 < \frac{3\alpha}{4}$ then the quality provision will be below the case without favouritism; the price is also lower. In fact, in this case the buyer is concerned only with ensuring the preferred bidder wins the auction. To do so, he must give a very high market power to the incumbent and, as a result, quality 1 becomes very costly and not as important for the buyer.

5.3.4 Proof of the Theorem in Section 2[18]

Defineby (a_1^*, a_2^*) the weights of the scoring function that maximize the buyer's utility given a level of favouritism $f \geq 0$. Using all results derived above, we are now going to prove that there exist two tresholds in the level of favouritism f - which we define as $\overline{f}$ and $\overline{\overline{f}}$ with $\overline{\overline{f}} > \overline{f}$, such that:

- if $f = 0$ then this is the benchmark case with $(a_1^*, a_2^*) = \left(\frac{3}{4\alpha}, \frac{3}{4\alpha}\right)$ already proved before.
- if $f \in \left[0, \overline{f}\right]$ then $p_I(a_1^*, a_2^*) \geq p_I\left(\frac{3}{4\alpha}, \frac{3}{4\alpha}\right)$ and quality provision is above the case without favouritism: $q_{1I}(a_1^*, a_2^*) \geq q_{1I}\left(\frac{3}{4\alpha}, \frac{3}{4\alpha}\right)$. If $f > \overline{f}$ then $q_{1I}(a_1^*, a_2^*) < q_{1I}\left(\frac{3}{4\alpha}, \frac{3}{4\alpha}\right)$.
- if $f \in \left[0, \overline{\overline{f}}\right]$ then $p_I(a_1^*, a_2^*) \geq p_I\left(\frac{3}{4\alpha}, \frac{3}{4\alpha}\right)$. If $f > \overline{\overline{f}}$ then $p_I(a_1^*, a_2^*) < p_I\left(\frac{3}{4\alpha}, \frac{3}{4\alpha}\right)$.

Favoritism and the relative importance of a_1 w.r.t. a_2 Define by (a_1^*, a_2^*) the weights of the scoring function that maximize the buyer's utility given a level of favouritism $f \geq 0$. It is convenient to use the ratio ε_a of the two weights of the

[17]Recall $\Pr_{(a\ win)}(a_1, a_2) = 1 - \frac{1}{2}\frac{a_2^2}{a_1^2}$

[18]All calculations are in Section 5, online appendix.

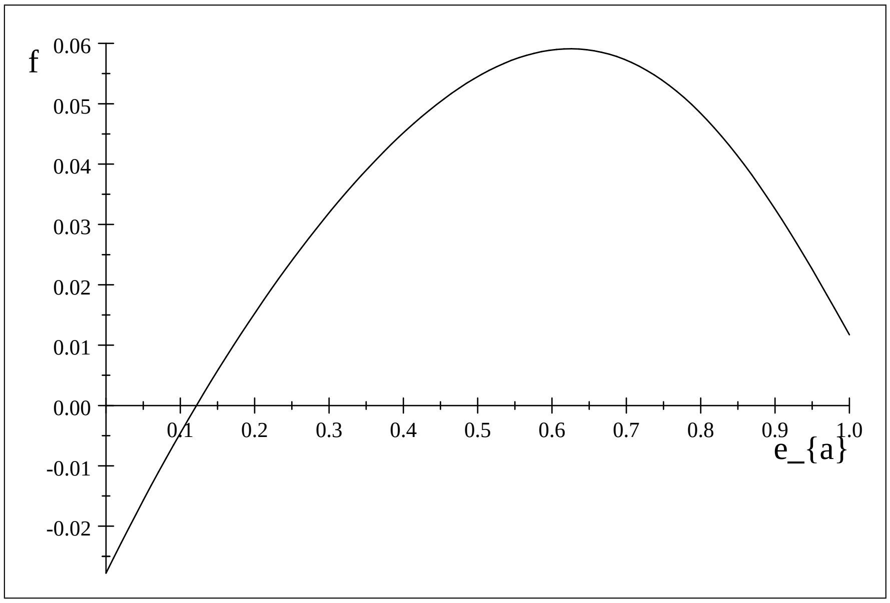
scoring rule to get rid of a_2. To this end, define $\varepsilon_a = \frac{a_2}{a_1}$, such that $\varepsilon_a \in [0,1]$. Then equation (60) can be stated as:

$$a_1^* = \frac{1}{\alpha}\frac{24 + 6\left(\varepsilon_a^3 - \varepsilon_a^2\right)}{11\varepsilon_a^4 - 15\varepsilon_a^2 + 36} \tag{64}$$

Using equation (64) it is possibile to express favoritism as a function of the relative importance of a_1^* and a_2^*:

$$f = \frac{1}{\alpha}\frac{3\left(4 + \varepsilon_a^2\left(\varepsilon_a - 1\right)\right)\left(\varepsilon_a\left(108 + \varepsilon_a\left(11\varepsilon_a^3 - 30\varepsilon_a - 73\right)\right) - 12\right)}{4\left(11\varepsilon_a^4 - 15\varepsilon_a^2 + 36\right)^2} \tag{65}$$

Plotting (65) we obtain that:



(Graph 1)

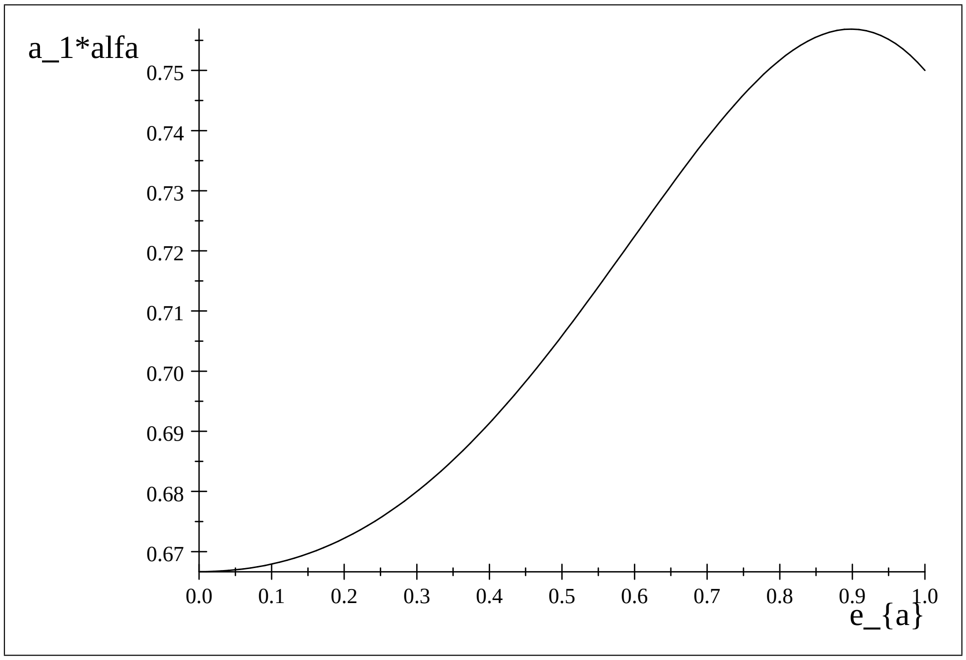
It immediately follows that there exists a level of f, which we denote as $\overline{\overline{\overline{f}}}$, such that if $f > \overline{\overline{\overline{f}}}$ then no internal solution is possible. In this case, the optimal boundary solution is for $(a_1^*, a_2^*) = \left(\frac{2}{3\alpha}, 0\right)$. In fact, the optimal weights have to be within the domain of (a_1^*, a_2^*) derived above and, of all the boundary solutions of that domain, this is the utility-maximizing one. We obtain that $\overline{\overline{\overline{f}}} \approx 0.059$[19].

Consider the case of $\varepsilon_a^* = 1$. In this case, from (65) we obtain $f = \frac{3}{256\alpha}$. For any value of $f \in \left[\frac{3}{256\alpha}, \overline{\overline{\overline{f}}}\right]$, two local maximum (a_1^*, a_2^*) exists. For "normal" values of α, i.e. less than 5, the global maximum is given by the highest of the two solutions for ε_a for any of that level of favouritism[20]. Moreover, the domain of the global-utility-maximizing ratio $\varepsilon_a^* = \frac{a_2^*}{a_1^*}$ can be derived using numerical techniques. It results equal

[19] All numerical solutions are obtained with the software Mathematica.
[20] See Online Appendix.

to $\varepsilon_a^* \in \left[\frac{0.625}{\alpha}, 1\right]$. Hence, it finally follows that the ratio ε_a^* is monotonically decreasing in f.[21]

Quality provision under favouritism Consider that quality 1 provision of the incumbent is proportional to a_1: if $a_1 \geq \frac{3}{4\alpha}$ then the quality provision with favouritism is higher than in the case where $f = 0$; if $a_1 < \frac{3}{4\alpha}$, then the quality provision is lower. Quality 2 provision of the incumbent is always equal to zero. Plotting the solution for a_1 in equation (64), we obtain:



(Graph 2)

From graph (2) it can be seen that there exists a ratio $\overline{\varepsilon}_a$ such that if $\varepsilon_a \in [\overline{\varepsilon}_a, 1]$ then $a_1 \geq \frac{3}{4\alpha}$. Since for each level of favouritism there is an associated optimal ratio ε_a^* which is monotonically decreasing in f, then there exists a level $\overline{f}$ such that if $f \in \left[0, \overline{f}\right]$ then $\varepsilon_a \in [\overline{\varepsilon}_a, 1]$, then $a_1 \geq \frac{3}{4\alpha}$ and hence the quality provision with favouritism is above the quality provision without.

By solving the FOCs of the buyer's utility maximization problem in f and a_2 for $a_1^* = \frac{3}{4\alpha}$ it is possible to derive the maximum level of favouritism such that the quality provision is above than in the case with $f = 0$. Solutions of f are:

$$f = \left\{ \left\{ \frac{3}{256\alpha}, -\frac{1}{2816\alpha}\left(\left(\frac{94}{\sqrt[3]{1314\sqrt{6}+\sqrt{9528\,992}}} + \sqrt[3]{1314\sqrt{6}+\sqrt{9528\,992}} - 7\sqrt{6}\right)^2 - 183\right)\right\} \quad \text{if} \quad \alpha \neq 0$$

The first solution, $f = \frac{3}{256\alpha}$, yields to $a_1 = a_2 = \frac{3}{4\alpha}$ which is not acceptable, given the constraint $a_1 > a_2$. The second one, instead, which can be numerically approximated to $\overline{f} = \frac{0.050367}{\alpha}$, is that threshold.

[21] For $f \in \left[0, \frac{1}{256\alpha}\right]$ any internal solution is impossible. In this case, $(a_1^*, a_2^*) = \left(\frac{3}{4\alpha}, \frac{3}{4\alpha}\right)$ and $\varepsilon_a^* = 1$.

Price under favouritism We now prove that there exists a level $\overline{\overline{f}}$ such that, as long as $f < \overline{\overline{f}}$, then the price paid is higher than in the case without favouritism. Consider first that, for the price with favouritism to be higher with respect to the case where $f = 0$, we need the following condition to be true:

$$p_I\left(a_1^*, a_2^*\right) > p_I\left(\frac{3}{4\alpha}, \frac{3}{4\alpha}\right) \tag{66}$$

where both prices are given by the price equations derived in (37), the first for $a_1^* > a_2^*$ and the second for $a_1 = a_2$. Then the inequality can be simplified setting $a_2 = a_1\varepsilon_a$ to obtain:

$$a_1 > \frac{1}{\alpha}\sqrt{\frac{\frac{3}{4} - \frac{3}{8}\varepsilon_a^2}{\frac{3}{2} - \varepsilon_a^2 + \frac{1}{6}\varepsilon_a^4}} \tag{67}$$

which becomes a numerical problem using the solution for a_1 in (65). For a solution to be acceptable given the constraint $a_1^* > a_2^*$ it must be that $\varepsilon_a^* \in [0, 1]$. However, if $\varepsilon_a^* \precsim \frac{0.625}{\alpha}$ then the solution is no longer a global maximum. Given these constranints, the approximate solution we found for that inequality is $\varepsilon_a \succsim 0.65$, which yields $\overline{\overline{f}} \approx 0.058$.