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PRICE CONVERGENCE WITHIN AND BETWEEN THE ITALIAN ELECTRICITY DAY-AHEAD AND DISPATCHING SERVICES MARKETS

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Price convergence within and between the Italian electricity day-ahead and dispatching services markets

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Abstract

In the paper we study the convergence of prices in the electricity markets, both at the day-ahead level and for the dispatching services (such as balancing and reserves). We introduce two concepts of price convergence, the convergence of zonal prices within each market (within convergence), and the converge of prices in a given zone between the two markets (between convergence). We provide an extensive analysis based on Italian data of within and between convergence. The zonal time-series of the prices are evaluated, seasonally adjusted and tested to assess their long-run properties. This evaluation induces us to focus on the behavior of the three largest and most interconnected continental zones of Italy (North, Center-North and Center-South). The fractional cointegration methodology used in the analysis shows the existence of long-run relationships among the series used in our study. This signals the existence of price convergence within markets, even though for the dispatching services market the evidence is less robust. The analysis also shows the existence of price convergence between markets in each zone, even though the evidence is more clearly affirmed for the North (the largest Italian zone), less so for the other two zones. Results are interpreted on the basis of the characteristics of the markets and the zones.

Keywords: zonal prices; convergence between zones; convergence within zones; fractional cointegration; long-run equilibrium.

J.E.L. codes: Q40, Q41, C32, C51
1 Introduction

In the electricity markets, prices should converge over time and across zones by the law of one price. Indeed, the price differential should induce investments for energy generation to move towards areas where they generate more profits, and the load (in particular the industrial one) moves where energy costs less. Thus, in the electricity market, like in any other commodity market, the single price convergence, due to arbitrage, is expected to take place. However, price convergence in power markets can be hampered by elements that are specific to the structure and characteristics of the electricity systems. In the short run, the availability of interconnection capacity can limit the possibility to import and export electricity, thus hindering the determination of a no-arbitrage price. In the long run, the different availability and cost of primary energy sources across areas, the different regulatory and market settings, and the distinct features of load and generation can also lower or impede the convergence over time.

The law of one price should also apply to different power markets in a given area, which are linked by the specific time features or characteristics of the products exchanged. Notable examples of different types of electricity products are the day-ahead and the dispatching services contracts. In the former, energy forwards are exchanged close to real time delivery; in the latter, the system operators acquire ex-ante balancing services and reserves to be used in real time. Day-ahead energy and dispatching services products are linked, since there are common elements that can influence the behavior of the agents or the costs of the power plants or the load that participate to both markets. An example is given by the technological characteristics of the power plants: a flexible technology, for instance, is able to adjust quickly to changes in load at the day-ahead level and thus providing balancing and reserves for the dispatching services. Another example of a common feature is given by the level of competitiveness of the generators that participate to both markets: if a generator has a market power can exert it and thus influences prices in both markets. Also, the existence of incentives or priority rules can influence the costs and the merit order at both the wholesale and dispatching services level. Therefore, we can expect that prices at the day-ahead level and for dispatching services are jointly related and converge over time. However, there can be other elements that might impact the wholesale markets and the dispatching services ones in the opposite direction, hampering their price price convergence. As an example, consider the impact in both markets of the renewable energy sources (RES): the low or null marginal cost to produce renewable energy, coupled with the random nature of their availability, can reduce the costs of energy generation at the wholesale level but can increase the
costs of dispatching services.

Therefore, it is relevant to evaluate to what extent the day-ahead and the dispatching services markets are converging over time. This is the research question of this article. In particular, we distinguish two concepts of convergence: the converge within a single market, across zones, which we shall call *within-converge*, and the converge between markets, namely, the day-ahead and the dispatching services ones, in a given zone, that we shall call *between-convergence*. For the former, the convergence can be influenced, as mentioned, by the physical structure of the electricity systems and the institutional regulations of the markets. Rules and structures of markets (in particular price making rules, price caps, type of bids, market timing and gate closures) can be an obstacle to price convergence. Therefore, it is important to find markets that are as homogeneous as possible in order to test for possible price convergence. For the convergence between markets, as mentioned, there are arguments that can both support or deny the hypothesis of the price convergence over time between day-ahead and dispatching services prices. On the one hand, it is possible to conjecture that there should be no relationship among the two prices, since the underlying of both markets could be seen as distinct, namely, energy to be consumed in the former and energy needed to deliver power in a secure and reliable way in the latter. Thus, no long run convergence should occur. On the other hand, it is possible to argue that the factors that affect energy prices can also influence dispatching services. In this case, there can be a positive or a negative correlation, depending on which factor (generator competitiveness or RES) dominates.

There exists a literature on convergence of day-ahead electricity prices that focuses mostly on European markets. Bollino et al. (2013) provide evidence supporting a common long run behaviour of wholesale prices in four interconnected European markets. Zachman (2008) finds support of partial binary convergence of European wholesale prices. Robinson (2008) show convergence for most but not all European Union member states in the considered sample. Huisman (2013) consider the impact of interconnection on five European wholesale markets, showing their role in reducing price volatility. Balaguer (2011) and Gebhardt (2013) focus on specific European wholesale markets, paying attention to the role of interconnectors and other determinants in the market integration process. Outside Europe, Apergis et al. (2017) show cluster convergence of regional wholesale markets in Australia. Bunn ad Gianfreda (2010) consider several forward markets, from day-ahead to two-months ahead, and find evidence of market integration. To the best of our knowledge, there is no paper in the literature that has tried and estimated the long-run
converge between day-ahead and dispatching services markets. We aim at overtaking such a gap by evaluating the convergence within and between markets, using data of the Italian day-ahead and dispatching services markets.

There are three advantages of using Italian data: the Italian market is divided in zones, which means that the same regulatory framework applies to the whole market; however, Italian zones are different in terms of size, number of operators, market power, penetration of RES (AESEGi, 2016). Therefore, by considering the different Italian zones, we can use the Italian case as a natural experiment that allows us to control of exogenous factors in a more homogeneous framework about market rules than the case of distinct state-wide markets. Moreover, the Italian market operator provides a large dataset that includes hourly prices for both day-ahead and dispatching services. Consequently, data at the day-ahead and dispatching services level can be compared with the same time and space horizon to test for the convergence between markets. Finally, Italy is a large market (305TWh in 2014, the 11th biggest market in the World) so it can provide a possible interesting case study.

In order to undertake our analysis, assessing the existence and nature of a long run common trend across prices, we need to take into account the seasonal nature of these prices. Electricity prices are subject to a complex seasonal structure, at the daily, weekly and annual level. There is a large stream of literature focusing on the seasonality of wholesale electricity prices (see Cartea and Figueroa (2005), Weron (2006), Koopman et al. (2007), Taylor (2010a, b), Caporin et al. (2012), Taylor and Snyder (2012), Janczura et al. (2013), among many others). The same study has not been undertaken at a closer-than-day-ahead level. We close this gap by evaluating the characteristics of the deterministic patterns of electricity prices at both the wholesale and the dispatching services level. Therefore, we first compare, with a descriptive view, the periodic patterns in the two markets across zones, pointing out similarities and differences. Then, we apply a filtering methodology that allows to remove the periodic components of the data and later focus on the analysis of seasonally adjusted prices. Our purpose is to verify if the market prices within each market across zones or between markets in given zones do share common patterns, i.e. they converge to a common long-run trend.

From an econometric perspective, price convergence calls for the presence of price cointegration. We thus proceed in steps and first discuss the integration properties of the seasonally adjusted zonal prices. Our analyses show that the prices are not integrated, thus excluding a-priori the possible presence of cointegration. However, since all the price series (filtered from the
periodic patterns) show evidence of long range dependence, or long memory, we cannot exclude the possible presence of fractional cointegration (Johansen, 2008). The latter feature allows for the presence of a long-run link among price series that have long memory. Following this research line, we first estimate the memory properties of the prices and then determine if the series are fractionally cointegrated. Our results show that, for the case of convergence within markets, considering the three main interconnected Italian continental zones, there is a single stochastic trend with long memory for both the wholesale and the dispatching services markets; for the latter, however, there is a weak convergence to the long-run equilibrium and a milder integration across zones. This is true taking into account three Italian zones, those that are highly interconnected. For the convergence between markets, we see that the wholesale and the dispatching services markets are linked in the long-run, even though the evidence is more clear for the largest Italian zone (North), less so for the other two zones.

The paper is structured as follows. In section 2, we present the main features of the Italian Day-ahead and dispatching services markets. In Section 3 data is discussed and analysed. Section 4 introduces the methodological approach followed. Results are presented and commented in section 5. Final remarks and references follow. A sample of the tables and figures is reported in the text. Additional tables and figures are reported in the Appendix, that contains also a longer discussion of the methodologies applied.

2 The Italian day-ahead and dispatching services markets

The Italian Power Exchange (IPEX), managed by the Gestore del Mercato Elettrico (GME), is organized in several markets, depending on products delivered and on the time horizon for the delivery. For the purpose of this analysis the relevant markets are the following: a) the Day-Ahead Market (Italian acronym MGP, Mercato del Giorno Prima), where producers, wholesalers, and eligible final customers may sell/purchase electricity for the next day. b) the Dispatching Services Market (Italian acronym MSD, Mercato del Servizio di Dispacciamento), where the Italian TSO (TERNA s.p.a.) provides the dispatching services needed to manage, operate, monitor and control the power system. The MSD consists of the scheduling stage (ex-ante MSD), and of the Balancing Market (BM). In the ex-ante MSD, the TSO accepts energy demand bids and supply offers in order to relieve residual congestion and to create reserve margins. In the BM, the TSO acquires energy to do secondary regulation and maintains the system balanced. At the BM, essentially secondary and tertiary reserves are exchanged.
Both the MGP and the MSD have a zonal configuration. There exists at present 6 market zones: North (NO), Centre-North (CN), Centre-South (CS), South (SO), Sicily (SI) and Sardinia (SA). The level of interconnections differs across zones. Table 1 below summarizes the zonal configurations of the Italian (physical) zones and the amount of interconnection capacity across zones. We can see in Table 1 that there are relevant interconnection constraints, in particular towards the islands of Sardinia and Sicily, and also towards the zone South.

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Table 1: Summary of interconnections capacity of Italian market zones. Source: our calculations from the limits provided by TERNA (2016) for representative winter and summer periods. Legend: X stands for high interconnections (above 2500 MW); x stands for limited interconnections (less than 1000 MW); (a) indicates that the zone has high interconnections with foreign zones; (b) points out that the zones are interconnected through a foreign zone (Corse) with limited interconnections; (c) highlights the presence of a limited interconnection to foreign zones (Greece); (d) reports the presence of limited interconnection through the virtual zone of Rossano.

A relevant difference between the MGP and the MSD refers to the equilibrium pricing rule in the auction. The day-ahead market, MGP, works with uniform auctions, that fix the system marginal price at each hour. The winning bidders receive the system marginal price of the zones in which they are located. The load pays a weighted average, namely, the average of the (possibly) different prices originated at the zonal level weighted by the volume of effective exchanges (net of purchases for pumping and from virtual foreign zones). This is called Single National Price (Italian acronym PUN Prezzo Unico Nazionale). The equilibrium pricing rule of the dispatching services market is a pay-as-you-bid-rule. Firms receive the price they have offered/demanded, if their offer to sale/purchase dispatching services to/from the TSO has been accepted. More precisely, power plants make offers to rise or reduce the energy they had already offered at the MGP. A plant sells energy to the TSO whenever the latter, for instance, forecasts the need of more power than the one bought at the Day-Ahead Market to relieve a congestion or preserve a sufficient reserve margin. These are called sales offers, or offers to "increase" energy. Similarly, power plants sell to TERNA offers to reduce production, called purchase offers or
offer to “decrease” energy, that TERNA might need, for instance, whenever there is a possible imbalance due to an excess supply of energy for a given hour and zone. TERNA cashes in accepted offers to decrease energy, and pays accepted offers to increase energy. The MSD is structured in two markets, in which there are offers refer to reserve energy needed ex ante to reduce forecasted zonal congestions and create reserve margin (called MSD ex ante), or offers of secondary and tertiary reserves (called balancing markets, Italian acronyms MB Mercato del Bilanciamento). The prices at MSD are given for every hour of the day and for every zone of the Italian electricity market. Each sale (or purchase) offer that is accepted in the MSD is then priced at its own price (pay-as-you-bid). Therefore, no proper single price arises at the MSD level. However, the market operator (GSE) provides data of weighted average of accepted offers to increase or decrease energy, in which each price is weighted by the amount of services (going up or down) that has been effectively purchased. In order to calculate the net dispatching service price in a given hour and zone, we compute the difference between the price to increase and the price to decrease energy. This represents the effective cost for the electricity system of the provision of dispatching services dealing with aggregated imbalances in a given hour and zone. This net imbalance price corresponds to the imbalances cost due to the differences between the predicted day-ahead quantities and the quantities needed by the TSO to maintain the system balanced. In other words, it represents the social costs (for the electricity system users) of having the electricity system balanced by the TSO. This is what we refer to as the dispatching services price.

3 Data description and the estimation of periodic patterns in MGP and MSD

We make use of publicly available data provided by GME in its website. The prices are hourly, zonal, ranging from 1st January 2010 to 31st December 2015, for a total of 52584 observations for each zone in each market. The MGP prices are the system marginal price of each zone and hour. For the dispatching service prices, recall that there are two markets, the MSD ex-ante and the MB. In the latter, there is a relevant amount of zeros or null observations, more frequent than the prices at the MSD ex-ante. This is as expected since the MB refers to services that are less frequently needed by the TSO than the ones purchased at the ex-ante MSD. For the same

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\[\text{We remind that 2012 was a leap year.}\]
reasons, if it happens that some offers are accepted, both to increase or decrease energy, these prices can be high but often for a limited amount of energy. We calculate the net imbalance and reserves prices as the difference between the weighted average prices to increase and the weighted average price to decrease energy, per each hour and zone, for both ex-ante MSD and MB. More precisely, we take the weighted averages of accepted offers (to increase or decrease energy) of the ex-ante MSD, as made available by GME, and add to each weighted average the corresponding price of the MB, in that hour and zone, weighted by the respective volume, if present. This provides the weighted averages of both MSD ex-ante and MB prices to increase and decrease energy. Then, we calculate the difference between prices to increase and decrease energy per each zone and hour. This provides the net dispatching services prices.

Table 2 reports descriptive statistics for MGP and MSD prices by zone.\(^2\) There are clear differences between MGP and MSD. Zones are quite different in terms of price values, as well as with respect to the presences of zeros or negative values. For what concerns the MGP prices, they have quite close median values, around 60, with the only exception of Sicily, whose prices are characterized by an extreme dispersion, especially on the right part of the prices distribution. In particular, we note that the maximum price in Sicily reached the value of 3,000 Euros. This can be explained noting that Sicily is a market zone that has a very limited amount of thermal capacity installed, and the largest share of RES across all zones. Moreover, it has a very limited interconnection capacity. Sardinia is second in terms of price dispersion, as measured by the range between minimum and maximum, or by the interquartile ranges. Moreover, Sardinia has a peculiar generation portfolio, characterized by limited amount of thermal capacity, the absence of NG power plants and a large penetration of RES. It has also a limited interconnection capacity with the other zones. The other four zones have a more similar behavior among each other. The MGP prices do not contain relevant fractions of zeros, being them in all cases less than the 1% of the data sample.

Moving to MSD prices, we observe larger differences between zones. First, all zones report negative prices. It is worth recalling that these prices are effectively social costs payed by the TSO (which rebate them to the end consumers through a specific tariff component). Negative figures therefore are effectively negative costs, namely, net gains for the TSO, that arise whenever the willingness of generators to pay to reduce energy outweighs their willingness to be payed to

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\(^2\) See Figures (A.1) to (A.5) in the Appendix for the prices evolution over time and over zones, as well as the scatter plots between zones, within the MGP and the MSD markets, and the scatter plots within each zone, between the MGP and MSD prices.
Table 2: Descriptive analysis of MGP and MSD prices. The table reports, by zone, minimum and maximum values, the 5%, 25%, 50%, 75% and 95% quantiles, the fractions of null and negative prices, the Max-Min range and the interquartile range.
generate. This occurs if the zone is long on energy, and generators cannot adjust quickly their production. Negative prices can be observed in a limited number of cases, about 2% for NO, and between 3% and 5% for SA, SI and SO. Differently, for CN and CS, the percentage of negative prices reaches much larger frequencies, about 24% and about 15%, respectively. This could be explained noting that these two zones are the most interconnected ones, among themselves and with the other zones, have a rather limited load, and some large baseload generators. Therefore, these figures point out to the fact that, in the observed period, these zones went long more frequently than the others, and generators had difficulties to reduce their scheduled programs. The fraction of zero prices is also a relevant quantity, as the distribution of zeros across zones shows in which zones dispatching services were less used in the sample period. Zeros are the largest fraction of the sample for SO (about 84% of the recorded hourly prices), a relevant fraction for CN (about 23%), and a somewhat limited fraction of the sample for CS, SA and SI. Notably, NO is the only zone without zero prices in the sample. Recalling that a zero price signals that the energy for dispatching services is not needed in that hour and zone (and therefore has a null value), if follows that NO needs a continuous balancing of energy. The differences in terms of negative and zero prices among the zones make them heterogeneous when focusing on medians and quantiles. The most peculiar case is, obviously, that of SO, followed by SA and SI, that have sensibly smaller interquartile and min-max ranges.

Table 3 focuses on the correlation among prices, showing a clear evidence of the different nature of the MGP and MSD markets. In fact, within the MGP market, prices are much more (positively) correlated than in the MSD market. Moreover, the correlation between the MSD and the MGP prices in each zone is very weak.

The high values of the (linear) correlations in Table 3 might be the consequence of the presence of time trends (which seems, however, not evident from the time plots of prices) and/or periodic patterns. The latter, due to the different demand/consumption levels during the day and the night, during different days of the week, during different periods of the year, are know as a typical feature of energy price series. These patterns are associated with physical elements (e.i. day/night alternation, seasons) and are thus, ideally, correlated among zones. In order to shed light on this aspect, we first proceed to a graphical analysis of the zonal prices in both MSD and MGP.

Figure 1 reports, as an example, the autocorrelation functions (ACF) for both MGP and MSD
<table>
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<tr>
<td>SA</td>
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Table 3: Linear correlation matrix between MGP and MSD zonal prices. Correlations are computed on 52584 observations. All correlations, even those taking very small values (in absolute term) are statistically significant at the 1% confidence level given the large sample size.
Figure 1: MGP and MSD graphical analyses for NO zone. MGP on the left plots, MSD on the right plots. Upper plots, autocorrelation functions of the zonal hourly prices. Lower plots, hourly box-plots of the zonal prices.
hourly prices for the NO zone. Moreover, the Figure includes the box-plots of the hourly prices on the two markets. The Appendix contains similar plots for all other zones. The correlograms of both the MGP and the MSD prices show a clear periodic pattern. However, the MGP and MSD periodic behaviors are quite different.

While for MGP the periodic behavior is clearly visible also on the box-plot, for MSD prices, we note a large number of outliers that hide, at least in part, the periodic pattern. However, as it emerges from the the ACF, a seasonal pattern is also present in the MSD series. We observe similar findings in all zones.

3.1 Seasonality in the MGP and MSD prices

As previously noted, the electricity prices include a periodic behavior that results as a superposition of several cyclical patterns: the diurnal ones, due to the differences in energy demand between day and night; the weekly pattern, with different energy demands during workdays and week-ends (with holidays usually behaving as Sunday); the yearly one, due to the alternation of seasons and summer breaks in the industrial activities. In order to cope with those elements we follow, among the various methods proposed in the literature, the approach by Bernardi and Petrella (2015) that introduce a flexible exponential smoothing method to capture seasonal cycles in time series. Their model allows to deal with yearly (associated with months), weekly and intra-daily patterns. Note that by adopting the method of Bernardi and Petrella (2015) and given the existence of a yearly cyclical pattern in the series, the filtering procedure leads to a reduction of the series length by one year. Appendix A briefly describes the model of Bernardi and Petrella (2015) and the settings we adopted.

We filter from the zonal prices the periodic patterns, obtaining the series plot in Figures A.13 and A.14 in Appendix A. The filtered series start in January 2011. Figure 2 reports the cyclical pattern observed over two weeks for the NO zone and the ACF of the MGP and MSD price series filtered from the seasonal behavior. Appendix A includes also similar plots for all other zones. In general, we note that the cyclical patterns have more stable behaviors and they are similar across zones in the MGP market, a somewhat expected result given the preliminary descriptive analysis. When focusing on the MSD prices, the graphical evidence suggests that the periodic behavior, despite being present in all zones, is different across zones. A possible justification might be the different RES penetration across zones, as well as the distinct structural aspects of the energy generation or of the load across zones. Indeed, each zone has its own average production mix and
there is an uneven distribution of flexible thermal production plants. Moreover, southern and insular zones are smaller compared to the continental ones and less sensitive to industrial load. Finally, regions in Italy are highly heterogeneous in terms of average temperatures and rainfall levels both in winter and in summer.

If we consider the linear correlation among the seasonal patterns (see Table A.1 in Appendix A), we find a confirmation of the previous findings: the correlations are higher among the MGP series and much smaller among the MSD series. Moreover, the correlations are also very small when considering cross-market linear relations. Moving to the filtered series, the ACF show evidences of two phenomena. First of all, both MSD and MGD filtered prices appear still slightly contaminated by a seasonal behavior, as highlighted by the periodic behavior of the correlograms with an oscillation with a period of 24 observations (one day). This suggests that some residual stochastic periodic component is still present in the filtered series. Secondly, all series display long range dependence, as the ACF slowly decreases toward zero and it is still highly significant after 100 lags in all cases,. This suggests that the adjusted price series might follow a stationary and predictable process with long memory and not, as usually expected for prices in financial markets, a random walk process.

After removing the periodic component, the MGP remains correlated across zones. The correlations range from 0.23 between SA and SI up to 0.96 between CN and CS zones, with 9 out of 15 correlations taking values larger or equal than 0.5. Notably, the zones CN, CS, NO, SO are the most correlated. Within the MSD market, correlations range from 0.01 between CS and SO or SA, up to 0.16 between SI and NO, very small values. Finally, focusing on the correlation between the MSD and MGP seasonally adjusted prices in a given zone, the minimum correlation is observed in SO, equal to -0.01, and the largest one is observed in SA, where there is correlation equal to 0.28.

4 Long-run equilibria within and between MGP and MSD prices

On the basis of the observations of the previous section, we further analyze the price series filtered from the periodic patterns. The existence of common trend in prices among markets or within zones points at the existence of a long-run relationship. In particular, the classic way to determine whether two or more series are linked in the long-run, i.e. there is an equilibrium

\[\text{See Table A.2 in Appendix.}\]
Figure 2: MGP and MSD graphical analyses for NO zone. MGP on the left plots, MSD on the right plots. Upper plots, estimated seasonal patterns from the 16th to the 29th November 2015. Lower plots, ACF of seasonally adjusted zonal prices.
Table 4: Linear correlation matrix between MGP and MSD seasonally adjusted zonal prices. Correlations are computed on 43848 observations. All correlations, even those taking very small values (in absolute term) are statistically significant at the 1% confidence level given the large sample size.
relation between the series with non persistent deviations from it, is by means of the well known concept of cointegration. Unfortunately, the concept of cointegration is typically restricted to $I(1)$ time series, whose dynamic behavior resembles that of a random walk. Thus, we first carry out the augmented Dickey-Fuller and Philips-Perron tests to verify if the filtered zonal MGP and MSD prices are unit root processes. The results of the tests (not reported) are concordant in strongly excluding that the dynamics of the two series are coherent with those of a unit root process. Consequently, the pre-requisite for the classic definition of cointegration is missing, i.e. the series are not $I(1)$. However, such a finding does not completely exclude the possible presence of long-run links among the variables of interest. In fact, all series share a relevant feature; they are all characterized by strong persistence. This suggest that a specific form of long-run relations might exist, the one associated with the concept of fractional cointegration, which arises between series that are not $I(1)$ (or $I(2)$), but are nevertheless characterized by long-range persistence. The latter thus becomes a pre-requisite for fractional cointegration.

As a first step, we proceed to the estimation of the persistence, or memory, of the series following the semiparametric approach of Shimotsu and Phillips (2005) and Shimotsu (2010), that is robust to deterministic terms. Table 5 reports the estimated memory coefficients, $d$. A significantly positive coefficient indicates the presence of long memory (or long-range persistence). In particular, if $d < 0.5$, the series is long memory but stationary. The semiparametric estimator of Shimotsu and Phillips (2005) and Shimotsu (2010) is defined in the frequency domain so that its asymptotic properties (bias and variance) depend on the number of frequencies used in the estimation, namely the bandwidth ($m_d$). Table 5 reports the estimates for two different bandwidth: in all cases the memory coefficient is positive, and in most of them, the memory coefficient is lower than 0.5. Consequently, we state that all the zonal prices, filtered from the periodic patterns, display significant long memory and are stationary.

Given that the memory levels are very close, we proceed to the estimation of a dynamic model coherent with both the presence of long-memory and the possible converge within or between zones. However, the combined presence of negative and zero prices coupled with the strong overdispersion of prices in the South region and in the regions of the two islands (Sardinia and Sicily) induces us to restrict the analysis to the most regular zones (NO, CN and CS), which are the largest zones in Italy in terms of capacity installed and also the most interconnected ones.

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4We chose the test specifications in terms of lags and deterministic terms (constant, trend and time dummies) by means of information criteria.
Table 5: Estimates of the memory parameters on the seasonally adjusted series following the approach of Shimotsu and Phillips (2005) and Shimotsu (2010). $m_d$ denotes the bandwidth chosen for the estimation of the long memory (or fractional) parameter. $m_d$ is set proportional to $T$ (the sample size); see Shimotsu and Phillips (2005).

4.1 A model for the converges of MSD and MGP prices

On the basis of the preliminary evidence outlined above, we proceed with the estimation of a fully parametric model coherent with fractional cointegration, to shed further light on the long-run dependence between MGP and MSD within and between markets and zones. We adopt the FCVAR$_{d,b}$ model of Johansen (2008) and Johansen and Nielsen (2012) to study if the series of de-seasonalized hourly MSD and MGP are characterized by common trends across the different zones of the Italian electricity market. Among others, the FCVAR$_{d,b}$ model has been adopted by Dolatabi et al. (2014) in the context of commodity prices and by Caporin et al. (2013) in the context of high-frequency financial data. An alternative VAR specification subject to long memory, fractional cointegration and regime switches has been adopted by Haldrup et al. (2010) to study the occasional congestion periods in the Nordic power market. Compared to the model of Haldrup et al. (2010) which is able to accommodate the congestion periods that are rather frequent on the Nordic power market due to geographical conditions, the FCVAR$_{d,b}$ is more flexible in the characterization of the fractional cointegration relation and offers the possibility to test for the number of cointegrating relations between series. Moreover, the asymptotic theory of the ML estimator for the FCVAR$_{d,b}$ has been fully derived in Johansen and Nielsen (2012). The FCVAR$_{d,b}$ model is

$$
\Delta^d X_t = \alpha \beta' \Delta^{d-b} L_b X_t + \sum_{i=1}^{k} \Gamma_i \Delta^d L_i X_t + \varepsilon_t \quad \varepsilon_t \sim iid(0, \Omega),
$$

where $X_t$ is a $p$-dimensional vector, $\alpha$ and $\beta$ are $p \times r$ matrices, where $r$ defines the cointegration rank. $\Omega$ is the positive definite covariance matrix of the errors, and $\Gamma_j$, $j = 1, \ldots, k$, are $p \times p$ matrices loading the short-run dynamics. $\varepsilon_t$ is the i.i.d. error term with finite eighth moment, see
Johansen and Nielsen (2012). The operator $L_b := 1 - \Delta^b$ is the so called generalized lag operator, which, as noted by Johansen (2008), is necessary for characterizing the solutions of the system. The model in (1) has $k$ lags and $\theta = vec(d, b, \xi, \alpha, \beta, \Gamma_1, ..., \Gamma_k, \Omega)$ is the parameter vector. The parameter space of model is

$$\Theta = \{\alpha \in \mathbb{R}^{rx}, \beta \in \mathbb{R}^{px}, \xi \in \mathbb{R}^p, \Gamma_j \in \mathbb{R}^{px}, j = 1, ..., k, d \in \mathbb{R}^+, b \in \mathbb{R}^+, d \geq b > 0, \Omega > 0\}.$$ 

where $r$ is the cointegration rank, such that $p - r$ determines the number of common stochastic trends between the series. For the reasons explained above, we first apply the model in (1) focusing only on the three continental highly interconnected zones (NO, CN, CS). We consider then several model specifications designed to verify convergence between markets at the single zone level, or convergence within markets considering several zones and a single market. In both cases, the existence of price convergence is associated with the existence of a unique common trend.

Therefore, in the case of convergence between markets, where we estimate the FCVAR$_{d,b}$ over the MSD and MGP price series for a single zone, the convergence requires the existence of one cointegrating relation and of one common trend. Differently, when we focus on the convergence within markets, we estimate the FCVAR$_{d,b}$ model on three variables, i.e. the three zonal prices for a single market. In that case, the convergence is associated with the presence of two cointegrating relations and of one common trend.

We start with the analysis of the convergence within markets, in the three largest Italian zones. For both the MGP and MSD markets, the test of Johansen and Nielsen (2012) indicates a cointegration rank of 2, meaning that there is a single stochastic trend with long memory across the three zones. The existence of a single stochastic trend thus suggests that the three zones are strictly related and there is evidence in favor of within markets convergence for both MGP and MSD.

Table 6 reports the estimates of the FCVAR fitted on the three zonal prices, separately for MSD and MGP. We stress that in both cases, the estimated FCVAR$_{d,b}$ model has the following specification:

$$\begin{bmatrix}
\Delta^d X_{t}^{CS} \\
\Delta^d X_{t}^{NO} \\
\Delta^d X_{t}^{CN}
\end{bmatrix} =
\begin{bmatrix}
\alpha_{11} & \alpha_{12} \\
\alpha_{21} & \alpha_{22} \\
\alpha_{31} & \alpha_{32}
\end{bmatrix}
\begin{bmatrix}
ECM_{t}^{CS, CN} \\
ECM_{t}^{NO, CN}
\end{bmatrix} +
\sum_{i=1}^{k^*} \Gamma_i \Delta^d L^i X_t +
\begin{bmatrix}
\varepsilon_{t}^{CS} \\
\varepsilon_{t}^{NO} \\
\varepsilon_{t}^{CN}
\end{bmatrix}$$

(2)
Table 6: FCVAR estimates for the MGP and MSD series of the three main zones (NO,CN,CS). In parenthesis the standard errors. The optimal lag length ($k^*$) has been chosen according to the BIC criterion under the identification constraint discussed in Carlini and Santucci de Magistris (2017). The optimal cointegration rank ($r^*$) has been selected following the asymptotic distribution of Johansen and Nielsen (2012) and tabulated by McKinnon and Nielsen (2014). The estimation has been carried out with the MATLAB codes of Nielsen and Morin (2016). The parameters of the short-run matrices, $\Gamma_i$, are not reported due to space constraints.

where the two error correction terms are

$$ECM_t^{CS,CN} = \Delta^{d-b}X_t^{CS} - \beta_1\Delta^{d-b}X_t^{CN}$$

$$ECM_t^{NO,CN} = \Delta^{d-b}X_t^{NO} - \beta_2\Delta^{d-b}X_t^{CN}$$

and $X_t^{(i)}$ denotes either the MGP or the MSD series for each zone, with $i = CS, CN, NO$. We point out that the ordering of zones in the model is needed for identification purposes in order to obtain two long-run relationships, i.e., the two error correction terms. We have ordered the zones according to their physical interconnections, i.e., North to Center-North and Center-North to Center-South. Further, we stress that the error correction terms represents deviations from the common trends and are thus expected to be stationary and with a persistence level lower than the one characterizing the original series. The persistence level is monitored by the memory parameters $d$ and $b$. The parameter $d$ is associated with the original series and the parameter $b$ is the reduction in persistence due to the presence of fractional cointegration. Therefore, we expect both $d$ and $b$ to be positive, with $d \geq b$, and with a persistence level for the error correction terms
equal to $d - b$.

For what concerns MGP, the long memory parameter estimate is $d = 0.76$ signaling a non-stationary process and the cointegration gap coefficient, $b$, is also equal to 0.76. This means that the residuals of the equilibrium relation, i.e. the ECM terms, are short memory, such that the three series quickly adapt to restore the equilibrium as long as a shock hits the system. Looking at the cointegration parameters, $\beta_1$ and $\beta_2$, they are both very close to $-1$, signaling that the three series equally contribute to the long-run equilibrium.\footnote{The identification strategy discussed among others in Johansen (2010) is adopted here such that $\beta_{1,1,1,1} = I$, where $I$ is a $r \times r$ identity matrix. Unfortunately, the asymptotic theory of Johansen and Nielsen (2012) shows that the distribution of the estimates of the matrix $\beta$ is non-standard (mixed Gaussian). To test the null hypothesis, $\beta = \beta_0$, one could adopt the LR statistic as discussed in Johansen and Nielsen (2012, p.2700).} We read this as a further confirmation of the within market convergence for MGP. The strength of the cointegration relation also emerges by looking at the adjustment parameters $\alpha$, where $\alpha_{11}$ and $\alpha_{22}$ have the expected signs (negative) and are strongly significant.

An evidence similar to that of MGP arises for the MSD series, although the parameters $\beta_1 = -0.678$ and $\beta_2 = -0.739$ suggest that the MSD series of CS and NO have to move less than proportionally to restore the long-run equilibrium with CN. We therefore conclude that the strength of the common dynamics is more pronounced for MGP than for MSD. In economic terms, this means that the MGP markets are more integrated than the MSD ones. Nevertheless, our results show evidence of within market convergence for both MGP and MSD, despite the fact that the evidence is stronger for the former.

Figure 3 displays the ECM terms of equations (3) and (4). Both $ECM_{t}^{CS,CN}$ and $ECM_{t}^{NO,CN}$ display a degree of autocorrelation that is strongly reduced compared to that of the de-seasonalised MGP prices reported in Figure 2. The persistence in the ECM terms could be attributes, at least in part, to the residual seasonality which is not completely removed by the de-seasonalization procedure adopted in the first step. We link that to the periodic patterns that are still clearly visible in the lower panels of Figure 3. It should be stressed that the model residuals $\hat{\varepsilon}_{t}$ do not display significant autocorrelation.\footnote{The FCVAR$_{d,b}$ model could be possibly extended to account for long memory seasonal patterns by including a seasonal term in the generalized lag operator $L_{b}$. This extension is beyond the scope of the present paper and it is left to future research.}

We now focus on the converge between markets. We first consider the long-run relationship between MGP and MSD in each zone separately. In all three zones, the test of Johansen and Nielsen (2012) signals that the cointegration rank is one, meaning that MGP and MSD are linked in the long-run. This suggest that in all the three zones we do have evidences of between markets...
Figure 3: ECM term of MGP and MSD across zones. The figure reports the time series and associated ACF for the ECM of the MGP and MGP prices for the zones CS-CN (left) and NO-CN (right).
convergence. In other words, there is a long-run equilibrium (attractor) towards which the two series converge to. Therefore, we estimate the following FCVAR\(d,b\) model for each pair of MSD and MGP in each of the three zones,

\[
\begin{bmatrix}
\Delta^d MGP_t \\
\Delta^d MSD_t
\end{bmatrix} = \begin{bmatrix}
\alpha_{11} \\
\alpha_{21}
\end{bmatrix} L_b ECM_t + \sum_{j=1}^{k^*} \Gamma_j \Delta^d L_b^j Y_t + \begin{bmatrix}
\varepsilon^i_{MGP,t} \\
\varepsilon^i_{MSD,t}
\end{bmatrix}
\]

(5)

where \(Y_t = [MGP_t^i, MSD_t^i]^T\) and the error correction term is \(ECM_t = \Delta^{d-b} MGP_t^i - \beta_1 \Delta^{d-b} MSD_t^i\), and \(i = NO, CN, CS\).

Table 7 reports the estimation results for the fractional cointegration between MGP and MSD, in each of the three zones.

<table>
<thead>
<tr>
<th></th>
<th>NO</th>
<th></th>
<th>CN</th>
<th></th>
<th>CS</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Est.</td>
<td>S.E.</td>
<td>Est.</td>
<td>S.E.</td>
<td>Est.</td>
<td>S.E.</td>
</tr>
<tr>
<td>(k^*)</td>
<td>1</td>
<td>-</td>
<td>1</td>
<td>-</td>
<td>4</td>
<td>-</td>
</tr>
<tr>
<td>(r^*)</td>
<td>1</td>
<td>-</td>
<td>1</td>
<td>-</td>
<td>1</td>
<td>-</td>
</tr>
<tr>
<td>(d)</td>
<td>0.517 (0.059)</td>
<td>0.536 (0.020)</td>
<td>0.156 (0.060)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(b)</td>
<td>0.517 (0.101)</td>
<td>0.536 (0.021)</td>
<td>0.156 (0.026)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\beta_1)</td>
<td>-1.081</td>
<td>-0.598</td>
<td>-0.844</td>
<td>-</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\alpha_{11})</td>
<td>-0.002 (0.002)</td>
<td>-0.008 (0.001)</td>
<td>-0.056 (0.063)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\alpha_{21})</td>
<td>0.094 (0.119)</td>
<td>0.288 (0.035)</td>
<td>1.202 (1.416)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 7: FCVAR\(d,b\) estimates for the pairs of MGP and MSD of the three main regions (NO,CN,CS). In parenthesis the standard errors. The optimal lag length \(k^*\) has been chosen according to the BIC criterion under the identification constraint discussed in Carlini and Santucci de Magistris (2017). The optimal cointegration rank \(r^*\) has been selected following the asymptotic distribution of Johansen and Nielsen (2012) and tabulated by McKinnon and Nielsen (2014). The estimation has been carried out with the MATLAB codes of Nielsen and Morin (2016). The parameters of the short-run matrices, \(\Gamma_i\), are not reported due to space constraints.

The estimates of the FCVAR\(d,b\) signal that the strength of the cointegration relation in terms of memory gap is maximal, as \(d = b\) in all cases. This means that the error correction term is short memory, as in the case of the within-markets convergence discussed above. The parameter \(\beta_1\) is close to -1 only for the zone NO, while for CN and CS it is larger than -1 (i.e. closer to zero). This means that, for CN and CS, MSD has to adjust less than proportionally after a unit move of MGP to restore the equilibrium. Looking at the estimates of the speed-of-adjustment parameters, \(\alpha\), it emerges that only for CN the two series move significantly to restore the equilibrium, while for NO and CS the adjustment is much weaker. Overall, the evidence suggests that MGP and MSD have common dynamics within each zone. This result is in favor of the between markets...
convergence, although for CN and CS the evidence is weaker than for NO.

Similarly to the case of within convergence, we report in Figure 4 the error correction terms $ECM_i^t$ of equation (5) for $i = NO, CN, CS$. We find again evidences of a reduction in the persistence over the error correction terms compared to what observed among the seasonally adjusted series. This is coherent with the model feature, the presence of fractional cointegration and the associated between convergence. In all cases we note some periodic behavior even if it appears less pronounced than in the within convergence case. There are limited differences across zones in the persistence level of the error correction terms serial dependence, while the level of the $ECM_i^t$ series is more heterogeneous. The latter is not surprising as there are zonal structural features that also play a role in the deviation from the zonal common trends.

5 Concluding remarks

In this paper we have studied the price convergence across electricity zones within the Day Ahead market and the Dispatching Services (balancing and reserves) market (convergence within), as well as the convergence between day ahead and dispatching services markets in each zone (convergence between). In order to do so, we have first constructed a price index for dispatching services which measures the net social cost of those services for the TSO (and to final customers to which the TSO rebates them). Then, in order to assess the possible long-run correlation hypotheses we have investigated the statistical properties of the time-series of those markets for the Italian physical pricing zones, and deseasonalized them capturing the statistical properties of the residuals. Focusing on the three largest and most interconnected continental zones of Italy (North, Center-North and Center-South) we have tested the existence of common long-memory of prices between markets and across zones. For the convergence within markets, results show that there is convergence in the day-ahed market among the three most interconnected zones (i.e. North, Center-North and Center-South). The same holds true for the dispatching services markets, even though the strength of the common dynamics is more pronounced for MGP than for MSD. Moving to convergence between markets, we show that MGP and MSD have common dynamics within each zone even though for the zones Center-North and Center-South the evidence is weaker than for the zone NO. The existence of common long-run trends across zones in the day-ahead market was as expected. After all, it is in the strategic plan of the TSO to reduce transmission congestions across zones, favoring price coupling across zones (as well as with the
Figure 4: ECM term of MGP and MSD. The figure reports the time series and associated ACF for the ECM of the MGP and MGP prices for the three zones.
It is of interest and a novel result of this paper the fact that a similar tendency, yet less pronounced, occurs at the level of market for balancing and reserves. This can be interpreted taking into account that the zonal interconnections have the same positive impact on convergence for both the day-ahead and the balancing and reserves markets. However, there exist also specific characteristics of the dispatching services markets which induce a peculiar behavior of the time series, which are examined and discussed in the paper. Finally, the results of between markets convergence analysis seems to confirm the hypotheses of common features of the day ahead and the dispatching services markets. We cannot assess however whether this is due to the behavior of some common underlying factor, such as the price of fuel, for instance, or a common strategic behavior of agents in both markets. Future research along these lines aimed at evaluating the determinants of the convergence between day-ahead and dispatching services markets is mostly welcomed.

References


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Nielsen, M.O., Shimotsu, K., 2007. Determining the cointegration rank in nonstationary fractional system by the exact local whittle approach. Journal of Econometrics 141, 574-596.

Letters, 14, 473 -476.


TERNA (2016), Valori dei limiti di transito fra le zone di mercato, Rev. 16, (values of transit limits between market zones, in Italian). Available at https://www.terna.it/it-it/sistemaelettrico/mercatoelettrico/proceduradivalutazionelimitielimitiiditransitoareteintegra.aspx (last access June 2017).

A  Additional Tables and Figures
Table A.1: Linear correlation matrix between MGP and MSD seasonal components of zonal prices. Correlations are computed on 43848 observations. All correlations, even those taking very small values (in absolute term) are statistically significant at the 1% confidence level given the large sample size.
Figure A.1: MSD prices by zone
Figure A.2: MGP prices by zone
Figure A.4: MGP scatterplot by zone
Figure A.5: Scatterplot by zone of MGP (x-axis) versus MSD (y-axis)
Figure A.6: Correlograms of MGP prices
Figure A.7: Correlograms of MSD prices
Figure A.8: Hourly Box-Plots of MGP prices
Figure A.9: Hourly Box-Plots of MSD prices
Figure A.10: Seasonal patterns of MGP by zone for the two weeks starting on Monday 16th of November 2015 and ending on Sunday 29th of November 2015.
Figure A.11: Hourly boxplot for the seasonal patterns of MGP by zone.
Figure A.12: Seasonal patterns of MSD by zone for the two weeks starting on Monday 16th of November 2015 and ending on Sunday 29th of November 2015.
Figure A.13: Filtered MSD prices by zone
Figure A.14: Filtered MGP prices by zone
Figure A.15: Correlograms of filtered MGP prices
Figure A.16: Correlograms of filtered MSD prices
B Estimating the seasonal pattern

We follow Bernardi and Petrella (2015) and estimate the following model on the zonal prices. Let $y_t$ be the series of interest, observed from $t = 1, 2, \ldots T$ at a hourly frequency, then

$$ y_t = \mu_{t-1} + \sum_{j=1}^{J} \lambda_j d_{j,t} + \sum_{i=1}^{I} x_{i,t}s_{i,t-24} + \varepsilon_t $$

(6)

$$ \mu_t = \mu_{t-1} + \alpha \varepsilon_t $$

(7)

$$ s_{i,t} = s_{i,t-24} + \left( \sum_{j=1}^{J} \gamma_{i,j} x_{j,t} \right) \varepsilon_t, \quad i = 1, 2, \ldots I $$

(8)

$$ \varepsilon_t = \sum_{i=1}^{p} \phi_i \varepsilon_{t-i} + \sum_{i=1}^{q} \theta_i \zeta_{t-i} + \zeta_t. $$

(9)

The model includes several components. First, $\mu_t$ is the long-run evolution of the series, the trend component, following a random walk plus noise specification. The variables $d_{j,t}$ with $j = 1, 2, \ldots J$ are monthly dummies taking value 1 if a given day belongs to month $j$ (but note that we might set the monthly dummies such that we have $J \leq 12$ dummies, thus $J$ different monthly effects. The collection of $s_{i,t}$, $i = 1, 2, \ldots I$ represents the cyclical component of the model. It captures the differences in the daily patterns across days of the week, with $1 \leq J \leq 7$ different patterns. Note that each $s_{i,t}$ follows a daily seasonally integrated process with a multiplicative error term. In the latter, the variables $x_{i,t}$ are dummies taking value 1 if the observation at time $t$ falls within one of the $I$ intra-weekly seasonal cycles. The error term $\varepsilon_t$ follows an ARMA process whose innovations are assumed to be Normally distributed with mean zero and unit variance.

For details on the implementation and estimation of the model we refer to Bernardi and Petrella (2015). In our analyses, we set $I = 5$ different day types, setting Tuesday, Wednesday and Thursday to share the common intra-daily seasonal cycle. In terms of monthly dummies, we borrow them from the analyses of Bernardi and Petrella (2015) that consider the energy demand in Italy from 2004 to 2014, and consider five monthly patterns, $I = 5$, where the first group of months include January, March, June, September and October, the second group comprises November and December, April and May constitutes the third group while February and July the fourth. Finally, August is separately considered given its peculiar behavior. Similarly to Bernardi and Petrella (2015), we also separately consider irregular days (holidays). For the innovation term, we specify a simple autoregressive process of order 1.

Once we estimate the model, we compute the seasonally adjusted series as
\[ y_{t,SA} = y_t - \sum_{j=1}^{J} \hat{\lambda}_j d_{j,t} - \sum_{i=1}^{I} x_{i,t} s_{i,t-24}. \] (10)

Note that we remove only the cyclical behaviors and maintain the long-term component and the irregular component.

C  Semiparametric estimation of fractional cointegration

Nielsen and Shimotsu (2007) have proposed a test for the equality of integration orders that is robust to the presence of fractional cointegration. Their approach builds upon the results of Robinson and Yajima (2002) with an unknown presence or absence of cointegration, when the fractional integration orders are estimated. Thus, Nielsen and Shimotsu (2007) propose a test statistic for the equality of integration orders that is informative, independently of the existence of fractional cointegration. In the bivariate case, the test statistic is

\[ \hat{T}_0 = m_d \left( S \hat{d} \right)^t \left( S \frac{1}{4} \hat{D}^{-1} (\hat{G} \otimes \hat{G}) \hat{D}^{-1} S' + h(T)^2 \right)^{-1} \left( S \hat{d} \right) \] (11)

where \( \otimes \) denotes the Hadamard product, \( S = [1, -1]' \), \( h(T) = \log(T)^{-k} \) for \( k > 0 \), \( D = \text{diag}(G_{11}, G_{22}) \) and \( G \) is the coherence matrix at the frequency zero (origin). If the variables are not cointegrated, i.e., the cointegration rank \( r \) is zero, \( \hat{T}_0 \to \chi^2_1 \), while if \( r \geq 1 \), the variables are cointegrated and \( \hat{T}_0 \to 0 \). A significantly large value of \( \hat{T}_0 \), with respect to the null density \( \chi^2_1 \), can be taken as an evidence against the equality of the integration orders. The estimation of the cointegration rank \( r \) is obtained by calculating the eigenvalues of the matrix \( \hat{G} \). Because \( G \) does not have full rank when the series are cointegrated, then \( G \) is estimated following the procedure outlined in Nielsen and Shimotsu (2007) which involves a new bandwidth parameter \( m_L \). In particular, \( \hat{d} \) are first obtained with \( m_d \) as bandwidth. Given \( \hat{d} \), the matrix \( G \) is then estimated, using \( m_L \) periodogram ordinates, such that \( m_L/m_d \to 0 \). Let \( \hat{\delta}_i \) be the \( i \)th eigenvalue of \( \hat{G} \), it is then possible to apply a model selection procedure to determine \( r \). In particular,

\[ \hat{r} = \arg \min_{u=0,1} L(u) \] (12)

where

\[ L(u) = v(T)(2 - u) - \sum_{i=1}^{2-u} \delta_i \] (13)
for some \( v(T) > 0 \) such that

\[
v(T) + \frac{1}{m_L^{1/2} v(T)} \to 0.
\]  

(14)

In the bivariate case, equation (13) is minimized if \( v(T) > \min(\delta_i) \), where \( \min(\delta_i) \) is the smallest eigenvalue of \( \hat{G} \) (or alternatively of the estimated correlation matrix \( \hat{P} = \hat{D}^{-1/2} \hat{G} \hat{D}^{-1/2} \)).