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CREDIT DERIVATIVES: CAPITAL REQUIREMENTS AND STRATEGIC CONTRACTING

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Credit Derivatives: Capital Requirements and Strategic Contracting*

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Abstract

In this paper we investigate the problem of a bank, which, due to the presence of capital requirements, needs to issue credit derivatives. Because of asymmetric information in the loan and credit risk transfer markets, banks face an adverse selection problem, sharpened by the fact that credit derivative contracts are not publicly observable. We show that high-quality banks can use CDO contracts to signal their own type, even when credit derivatives are private contracts. Also a menu of contracts with a first-to-default basket and a credit default swap conditioned to the default of the first asset, can be used as a signalling device. Moreover, this last menu of contracts generates larger profits for high-quality banks than the CDO contract if the cost of capital and the loan interest rates are sufficiently high.

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1. Introduction

Bank loans have usually been considered as illiquid assets. This is mostly explained by the private information banks have about the quality of their loans: since this information is not easily verifiable, potential buyers are reluctant to take risk on such assets.

The recent advent of credit derivatives, however, has provided banks with a whole range of flexible instruments for selling loans and transferring loan risk. For example, pure credit derivatives, such as plain vanilla credit default swaps (CDS) allow banks to buy protection on a single exposure or on a basket of exposures, portfolio products, such as Collateralized Debt Obligations (CDO), enable banks to sell risks from their entire loan portfolio[1]. One main advantage of these new instruments over traditional forms of credit risk transfer is that their flexibility helps to mitigate informational problems.

In this paper we investigate the problem faced by a bank that, because of asymmetric information in the credit risk transfer market, needs to signal its quality, but, at the same time, it has to satisfy minimum capital requirements. This problem is more interesting than it would appear at first glance because protection buyers cannot signal their own type by freely varying the quantity of insurance, which is a standard solution for an insurance contract. This is due to the fact that retaining a large portion of credit risk can be too costly because of the presence of capital constraints. Moreover, in line with current market practice[2], we

[1] BBA (2002), BIS (2003) and BIS (2005) surveys show that the volume of trade in credit derivatives has known a huge increase shortly. To date, credit derivatives are traded almost exclusively on single names CDS: defaults events for corporations that also have publicly traded bonds outstanding; CDO, RMBS, "classical" ABSs experienced some success in recent years.

[2] As reported by BIS (2005) "About two-thirds of the surveyed banks disclose only the aggregate notional size of their positions. [....] In addition, when national accounting rules require

assume that credit derivatives trades are not made pubblic (i.e. credit derivatives are private contracts) so that the protection buyer cannot make a commitment to a specific partial protection level to signal its type. In fact, any protection buyer who purchases partial protection upon its loans with a protection seller can, at the same time, hedge the rest with another protection seller, without the first being informed.

The design problem is thus to choose a contract that balances the issuer's desire to transfer credit risk against capital cost that the bank faces if it retains part of the risk (cost of issuing new capital, or the opportunity cost of capital borne by allocating capital at risk[3]) under the constraint that credit derivatives are private contracts.

Our goal is to study the different menus of contracts presented in the literature and traded in the market to solve the adverse selection problem and to find which one is preferable given: (i) the presence of capital constraints; and (ii) that credit derivative contracts are not publicly observable.

The overall structure of our model is roughly as follows. We assume that banks are of different types, and vary in their ability to screen borrowers. We further assume that there exist "high" type banks that are able to screen their borrowers and choose only "good" loans, and "low" type banks that are unable to do so. In our model there is a one-to-one relation between a bank's ability and the riskiness of its credit portfolio, i.e. banks of diverse type have different loan pools. Protection sellers do not know the true type of the protection buyer (simply "the buyer" from now on), and therefore face an adverse selection problem. The bank

the disclosure of financial guarantees, the total amount guaranteed through credit derivatives is disclosed, even if the bank does not make a more comprehensive disclosure of its position".

[3] Returns on capital generated by allocating capital at risk to other risky activities.

is subject to minimum regulatory capital requirements. As frequently claimed in the literature, we assume that even for banks capital at risk is costly as a result of asymmetric information (see Dewatripont and Tirole (1993) and Froot and Stein (1998)) or because of capital requirements (see Gorton and Winton (1998)). This induces the bank to prefer to hold risk-free rather than risky assets even if it is risk neutral and to attribute a cost to the capital at risk required by loans.

The definition of minimum capital requirements is importantin our analysis. Much of the initial activity in the credit risk transfer market was in response to inconsistencies in the regulatory framework for bank capital allocation (see Jones (2000)). In this paper we want to avoid this aspect and are concerned solely with capital requirements which prevent regulatory arbitrage and help to reduce the probability of bank default. The less intrusive capital adequacy rule suggested by regulators in pursuit of this objective is that banks hold a level of capital at risk at least enough to cover the Value at risk (VaR) of their risky portfolio, where VaR is the maximum unexpected loss of bank asset portfolio given a certain confidence level[4].

In order to solve the adverse selection problem, we consider a sample of contracts that have already appeared in the literature on security design. Namely, we concentrate on two different menus of contracts. The first is the *CDO menu* where the protection buyer transfers a portion of the risk of the portfolio in one or more tranches to the protection seller and retains the other portion. The key aspect of this mechanism is that the risk transferred and the risk retained are of different seniority and, usually, the protection buyer holds the equity tranche (i.e. the most junior tranche). As DeMarzo and Duffie (1999) show, the protection

[4]See Basel Committee on Banking Supervision (1999, 2004)

buyer's retention of the subordinate block reduces the total lemon's premium by creating an incentive to align its interests with those of the protection seller.

The second is a menu of contracts that is quite new in the literature of financial innovation and is based on a basket of loans characterized by different maturities: a first-to-default basket and a plain vanilla credit default swap conditioned on the default of the first asset. The first-to-default basket is a financial contract in which the protection buyer pays a premium to the protection seller in exchange for a contingent payment by the counterpart if any of the underlying assets defaults. A plain vanilla conditioned on the default of the first asset is a commitment to buy, at a fixed price, an insurance contract (a credit default swap) on the rest of the basket after the first default. For sake of simplicity, We call this menu of contracts the *FTD menu.* With an FTD menu, the bank is signaling its type by committing to buy "new" insurance in case of default and therefore showing that its credits have a low probability of default.

Our analysis yields several insights. First, if the cost of capital is relatively low, we show that the CDO contract may be extended to solve the adverse selection problem that arises from opacity and private credit derivative contracts. Second, the FTD menu is able to solve the adverse selection problem. Third, the CDO menu is not always the second best contract when credit derivatives are private contracts (as instead shown by DeMarzo and Duffie (1999) in a framework with public observability of contracts). In fact, one of the main contributions of our paper is to prove that if the cost of capital and interest rate are sufficiently high, then FTD menu Pareto dominates the CDO menu. We believe that this last result is especially interesting, since it proves that theoretical predictions may vary according to whether credit derivative contracts are publicly observable or not.

The paper is organized as follows. The next section describes the related literature. In Section 3 we present the basic model and we analyze the benchmark case with symmetric information. In Section 4 we consider the asymmetric information case. Section 5 concludes.

2. Related literature

The tremendous development in credit derivative markets has received the attention of both regulators and policy makers. Most international and national supervisors have published reports on the topic (e.g. International Monetary Fund (2002), Bank for International Settlement (2003, 2005)). These reports are rather similar in tone. On one hand they emphasize the benefits of credit derivatives in terms of risk sharing and diversification gains. On the other hand, there is common concern that credit derivatives may have implications for financial stability. In creating new markets for credit risk, credit derivative instruments may (i) have an impact on asymmetric information problems existing between borrowers and lenders (see Duffee and Zhou (2001) and Morrison (2005)) and (ii) create new problems in the credit markets (see Kiff, Michaud and Mitchell (2003) for a review of almost all the potential implications of credit risk transfer markets because of the asymmetric information problems in the credit markets). Most of the arguments however, are on a purely informal basis, which is due to the lack of theoretical work on these issues. A recent exception is Morrison (2005) who shows that if credit derivative trades are not pubblished so that the protection buyer cannot make an ex-ante commitment to a specific protection level, banks have a moral hazard incentive to fully hedge their exposition and therefore cease to monitor. This behavior has the negative effect of causing disintermediation and thus reducing welfare. In our paper we show that the extensive flexibility provided by credit derivative products allows a solution to the adverse selection problem created by both the opacity of bank loans and the fact that credit derivatives are not published.

Even ignoring the capital requirement issue and the contract observability problem, the theoretical literature on credit derivatives and asymmetries of information problem is limited and borrows from optimal contract design to solve the adverse selection problem. In their model DeMarzo and Duffie (1999) include general securities whose payoffs may be contingent on arbitrary public information such as CDO contracts. DeMarzo and Duffie focus on liquidity problems with asymmetric information. More precisely, they have shown that, in line with Leland and Pyle (1977) pooling and sharing may be optimal when the protection buyer has superior information. They argue that the sharing process allows the protection buyer to concentrate the "lemon's premium" on the small first-loss or equity tranche and create a relatively large, low-risk senior tranche. Also, the protection buyer's retention of the subordinate tranche reduces the total lemon's premium by creating an incentive fot the buyer to align its interests with those of the protection seller. In our model we also consider this kind of contract design and derive the characteristics of this contract when a buyer cannot credibly commit to retain part of the risk because credit derivatives are private contracts, an aspect not addressed in the previous paper.

Duffee and Zhou (2001) demonstrate that the problem of adverse selection may be overcome by drawing up credit derivatives with a smaller maturity than that of the underlying asset[5]. The key assumption in their model is the hypothesis that the bank's information advantage changes over the time and, in particular, is greater close to the maturity date of the loan. One of the contract we present is similar in spirit to the one proposed by Duffee and Zhou (2001). Nevertheless our approach is different, because we neither assume that the bank's information advantage decreases over time nor that there is perfect observability of credit derivative contracts.

3. The model

3.1. Assumptions

Let us consider a market where there is a bank (buyer) operating in the local loan market which may hedge its expositions in the OTC credit derivative market by selling credit risk to other banks (protection sellers). By definition, the OTC market is characterized by private contracts i.e. details of trades are not made pubblic.

Buyers and sellers are both risk neutral and, for simplicity, the riskless interest rate is zero. The (protection) buyer belongs to one of two different types: high-type (denoted by h) and low-type (denoted by l). Both types vary only with respect to the quality of their loan pools for the credit risk on which the bank seeks protection. The quality of the pools is assumed to depend on borrowers'

[5] Moreover, Duffee and Zhou (2001) show that the mechanism proposed by Gorton and Pennacchi (1995) to reduce the moral hazard problem associated with the loan sales is broadly applicable to any mechanism that transfers loan risk outside of the bank, including credit derivatives.

ability to repay loans. Since the probability of loan default depends on the ability of the bank to discern its borrowers, the buyer's quality can be represented by the probability that its borrowers repay loans. This probability is greater for a high-type buyer than for a low-type. Let p_i for $i = \{h, l\}$ be the probability of success for loans repayment, then $0 \leq p_l < p_h \leq 1$, where p_l and p_h are the probability of loan success held by a low-type and a high-type buyer, respectively.

The model incorporates three dates: 0, 1 and 2. On date 0, the buyer makes two commercial loans with fixed size: I_1 and I_2. I_1 matures on date 1 while I_2 matures on date 2. Both credit lines can default only at the maturity date and are uncorrelated. Making a loan of amount I a buyer $i = \{h, l\}$ obtains an expected profit $\pi_i = p_i (1 + \mu) I - I$, where μ is the interest rate, which is the same for both types. Hence, sellers cannot infer buyers' types from the interest rate.[6] Moreover, we assume that $\mu \leq 1$; this assumption allows us to simplify our analysis greatly and in our opinion is sufficiently mild not to undermine the generality of our results. We assume that $\pi_h > \pi_l \geq 0$ that is both types of loans have non negative net-present-value (NPV)).[7]

The buyer is subject to capital requirements based on a VaR rule i.e. the bank's capital has to be at least equal to the amount of the largest unexpected loss which occurs with probability equal or lower than α. We assume that $p_h < \alpha$. Therefore, in order to issue loans the bank has to hold a buffer of capital called

[6] This assumption is in line with the statement of Duffee and Zhou (2001), according to which there is not a one-to-one relation between the interest rate charged by a bank and the quality of borrowers. Indeed, the interest rate charged on a loan depends on the overall relationship existing between the bank and its borrowers and also on the bank's and borrowers' bargaining power. Moreover, the bank's choice of interest rate is also affected by the presence of informational asymmetries between borrowers and the bank itself. Regarding this topic, we recall the works about credit rationing by Jaffee and Russel (1976) and Stiglitz and Weiss (1981).

[7] Later on we briefly discuss some implications of the case when the low-type's loans have negative NPV.

capital at risk. In line with the literature mentioned above, we assume that there is an opportunity cost of capital $\rho > 0$; this makes the bank's concern with risk management endogenous even if the bank is risk neutral.

Here we focus on the case in which banks use credit derivatives in order to reduce capital requirements and therefore the cost of capital. The credit derivative market we consider is characterized by the presence of different types of contracts. At time 0, the buyer simultaneously offers to purchase credit derivative contracts from the sellers[8]. Since there are many sellers, we assume that the buyer faces a competitive market. At the time of the proposal, the buyer's type is private information. Similarly, we assume that the buyer has full bargaining power and makes a take-or-leave-it offer to a seller.

The buyer may offer a number of different contract menus:

- two plain vanilla credit default swaps (CDS), both of which hedge against a single name, I_1 and I_2 (for sake of brevity, the *CDS basket*);

- a collateralized debt obligation[9] (CDO) on the portfolio I_1 and I_2, and an insurance contract to cover the counterpart's losses up to a certain amount L (the *CDO menu*);

- a first-to-default basket and a plain vanilla CDS contract over I_2 conditioned on the default of the first asset I_1 (the *FTD menu*). The first-to-default basket is a financial contract in which the protection buyer pays a premium to the protection

[8] We assume that protection sellers are not subject to capital requirements because, as in line with empirical evidence, they are largely insurance companies or hedge funds.

[9] Most specifically, the CDO contracts traded in the market are Asset Backed Securities where the bank sells part of its loan portfolio to a special purpose vehicle which refinances itself through the issue of bonds. Payoffs are tranched with claims on the poll separated into different degrees of seniority in bankruptcy and timing of default. The equity (or junior) tranche is the residual claim and has the highest risk. The mezzanine tranche comes next in priority. The senior tranche has the highest priority and is often AAA rated. Usually the bank buys the equity tranche which absorbs all default losses up to its par value, before other tranches have to bear any further losses. For a more detailed description see Das (1998).

seller in exchange for a contingent payment by the counterpart if asset I_1 defaults. In case of a default by I_1 the contract ends. The other contract in the menu is a commitment at time $t = 0$ to buy, at a fixed price, a plain vanilla contract on I_2 at time $t = 1$, conditional on the default of the first asset.

In our model, the credit event is identified with a failure to pay at the maturity date. The credit event payment is defined as the difference between the nominal value plus the accrued interest and the recovery value of the defaulted loan. For simplicity, we assume here that the recovery value is equal to zero, so that the credit event payment will be equal to the nominal value plus the accrued interest of each loan ($I_1(1 + \mu)$ in $t = 1$ and $I_2(1 + \mu)$ in $t = 2$). Moreover, all the cash flows (including payment of the premiums) occur at the maturity of the contracts. Finally, let $0 < q < 1$ be the percentage of high-quality banks among the protection buyers.

3.2. The benchmark case: symmetric information

When the buyer's type is common knowledge, then the lowest premium that a risk neutral seller is willing to accept, in order to hedge the credit risk of an amount $I_j(1 + \mu)$ by means of a plain vanilla contract is:

$$\Phi_i\left(I_j\right) = \left(1 - p_i\right) I_j(1 + \mu) \text{ with } i = h, l \text{ and } j = 1, 2. \tag{3.1}$$

The expected profits of a buyer of quality $i = \{h, l\}$ are:

$$\pi_i\left(I_j\right) = \mu I_j - \Phi_i\left(I_j\right) + \rho(\min\left(0, \mu I_j - \Phi_i\left(I_j\right)\right) \text{ with } i = h, l \text{ and } j = 1, 2. \tag{3.2}$$

Since by assumption the NPV of the loans is positive for both types $i = h, l$, it follows that the capital requirement constraint is never binding. Hence,

$$\pi_i (I_j) = \mu I_j - \Phi_i (I_j) \text{ with } i = h, l \text{ and } j = 1, 2. \tag{3.3}$$

The other possibility is that the protection buyer buys a CDS basket that covers both the loans. Given our assumption about correlation among loans, the premium that a risk neutral seller is willing to accept is simply:

$$\Phi_i (I_1 + I_2) = \Phi_i (I_1) + \Phi_i (I_2) \text{ with } i = h, l. \tag{3.4}$$

It is straightforward to show that, with complete information, the full coverage CDS basket (as well as full coverage plain vanilla contracts) is a first-best contract.

4. Asymmetric information

4.1. Pooling equilibria.

In any pooling equilibrium the minimal premium that a risk neutral seller is willing to accept, in order to sign a plain vanilla contract that hedges the counterpart against the credit risk of the loan I_j is:

$$\Omega (I_j) = q (1 - p_h) I_j(1 + \mu) + (1 - q) (1 - p_l) I_j(1 + \mu) \text{ with } j = 1, 2 \tag{4.1}$$

Signing a full coverage plain vanilla credit derivative, a buyer of type i obtains the following expected profit:

$$\pi_i(I_j) = \mu I_j - \Omega(I_j) \text{ with } i = h, l \text{ and } j = 1, 2. \tag{4.2}$$

As usual, it is easy to find the pooling equilibrium where both types of buyers sign the same contract in equilibrium. In particular, it is straightforward to check that there exists a pooling perfect Bayesian equilibrium such that buyers of both types sign plain vanilla credit derivative contracts. The seller's beliefs are such that, if a full coverage plain vanilla contract (or a full coverage CDS basket) is offered, the buyer is a high-type with probability q; if any contract different than a full coverage plain vanilla (or a full coverage CDS basket) is offered, then the buyer is a low-type with probability 1. It is clear that high-type banks' profits are lower than their profits in a game with complete information and the lower the number of high-type banks in the market, the stronger is the incentive to signal their own type. We devote the next section to analyzing separating equilibria.

4.2. Separating equilibria

In this section we prove the existence of separating equilibria such that, at time zero, a high-type buys one of the two menus of contracts presented above and the low type buys full coverage plain vanilla contracts[10]. First we consider the two menus separately. Then we determine which separating contract is preferred by high-type banks; this depends on how the cost of capital, ρ, and the interest rate, μ, vary.

[10] We do not consider explicitly the plain vanilla credit derivative swap on a basket with partial coverage where the amount against which credit protection is held by the issuer is less than the amount of the credit exposure. This because we already know from DeMarzo and Duffie (1999), and the same holds in our framework, that this type of contracts provide less profit to the protection buyers than the CDO contracts.

In order to overcome the multiplicity of perfect Bayesian equilibria, we only consider separating equilibria which satisfy the intuitive criterion proposed by Cho-Kreps (1987) for a signalling game (denoted "CK perfect Bayesian equilibria"). Given that we employ this refinement concept several times, it is worth giving an informal intuition of how it works. Consider that a buyer makes an out-of-equilibrium proposal and consider any conjecture that it has about how the seller reacts. If it happens that, given the seller's most optimistic conjecture (the seller believes that the proposer is high-type bank with probability one), a high-type bank finds it optimal to deviate while the low-type does not, then the intuitive criterion imposes to assign probability 1 that the proposer of such a contract is a high-type bank.

First, we determine the conditions under which we have a separating equilibrium such that high-type banks choose CDOs to signal their own type. Later we consider the FTD menu.

4.2.1. Separating equilibrium with CDO

With a CDO contract the protection buyer sells its basket portfolio (I_1, I_2) to the protection seller and guarantees the payment, in case of default, of a fraction of the loss suffered by the buyer of the portfolio (the protection seller). We consider a small modification to the CDO contract described above. In our contract the protection buyer pays L to the protection seller when the default of at least one loan occurs and the amount L does not depend on the size of the loss suffered by the protection seller. Given that both parties are risk neutral, a flat refund leads to the minimum size of loss that the high-type has to sustain in order to signal its own type. Therefore, this contract, minimizing the amount of the required capital, cannot be Pareto dominated by any other contract with variable payment. In this section we assume that the buyer can offer to the seller either a plain vanilla, CDS basket credit derivatives or a CDO menu, but not an FTD menu of contracts. Importantly, as pointed out above, we assume that contracts are private, and therefore the protection buyer can hedge the equity tranche without the counterpart of the CDO contract being informed[11].

Proposition 4.1. *If $\rho \leq \frac{(p_h^2 - p_l^2)(1+\mu)}{(p_h+p_l) - p_h p_l (1+\mu)}$, then there exists a unique CK separating perfect Bayesian equilibrium such that*

(i) high type banks sell $(I_1 + I_2)$ loans in exchange of a fixed amount of money equal to $p_h(I_1 + I_2)(1+\mu) + (1 - p_h^2)\hat{L}$, with $\hat{L} = \frac{(I_1+I_2)(1+\mu)}{(p_h+p_l)}$ and commit to pay a refund equal to $\hat{L}$ in case of default of any of the underlying assets;

(ii) low type banks sign full coverage plain vanilla or full coverage CDS basket credit derivative contracts at the fair price.

The counterpart's beliefs are such that if plain vanilla, CDS basket or a CDO with $L < \hat{L}$ are offered then the buyer is a low-type with probability one. If a CDO menu with $L \geq \hat{L}$ is offered, then the buyer is high-type with probability one.

Proof. See the appendix. ■

The intuition behind this result is the following. A bank that sells its loans partially insures the buyer of the loan by committing to pay an amount $\hat{L}$ in case of default of any of the underlying assets. Since the probability of sustaining a default is lower for the high types than for the low-types, then there exists an amount of refund L such that low-types prefer to sign a plain vanilla contract, while high-types prefer to sign a CDO menu. In the appendix we show that the minimum

[11]BIS (2005) indicates that banks issuing CDO have then transferred the equity tranches to hedge funds.

refund $\hat{L}$ which supports a separating equilibrium (and the unique one satisfying the intuitive criterion) induces a positive capital requirement. Therefore, the larger the cost of capital ρ, the smaller the profits of a high-type bank when it signs a CDO menu. In particular, if $\rho > \frac{(p_h^2 - p_l^2)(1+\mu)}{(p_h+p_l)-p_h p_l (1+\mu)}$, then high-type banks also prefer to hedge the equity tranche and, in this case, there is no a separating equilibrium. If $\rho \leq \frac{(p_h^2 - p_l^2)(1+\mu)}{(p_h+p_l)-p_h p_l (1+\mu)}$, then a high-type prefers to incur the cost of capital rather than hedge the equity tranche. Low-types do not over-insure their exposure using both CDS basket and CDO menu (i.e. trying to mimic high-types) because the premium they receive for the refund $\hat{L}$ is lower than the fair value (they receive $(1-p_h^2)\hat{L}$ but the expected value of their payment is $(1-p_l^2)\hat{L}$). Therefore, if a low-type over-insures its exposition, it reduces its expected profits[12]. As a result, our separating equilibrium does not require that a bank has not secretely hedged its portfolio using other contracts.

It is also worth noting that the refund $\hat{L}$ does not depend on the cost of capital ρ (while the existence itself of the separating equilibrium does), because we assume that credit derivatives are private contracts. Since a bank can hedge the refund $\hat{L}$ without the counterpart being informed, the cost of capital does not enter the self selection constraint of the low type. On the contrary, it is easy to show that the penalty will depend on the cost of capital ρ and is decreasing in this parameter if we assume that (i) the protection buyer could credibly commit ex-ante to retain the equity tranche risk and that, ex-post, this is incentive compatible or (ii) credit derivatives are public contracts.

[12] Even the high-type bank is not over-insuring its portfolio using CDS basket because anyone buying that insurance is assumed to be a low-type bank.

Finally, note that under[13] Basel I in some jurisdictions CDOs are considered as a portfolio loan sales and banks face almost no capital requirement for holding the equity tranche (i.e. for this contract ρ is almost equal to zero). It is straightforward to show that if $\rho = 0$ for CDO menu then the high type bank is still able to signal its type by signing a CDO menu with $L = \hat{L}$. In this case the CDO menu is a first best contract for the high-type (given the assumption of risk neutrality). This result may be one reason of why CDO contracts have experienced some success in recent years. Nevertheless, Basel II requires that all first loss positions must be deducted from a bank's capital[14]. Therefore, under Basel II, the cost of capital becomes positive ($\rho > 0$) and CDO contracts are no longer first best contracts.

4.2.2. Separating equilibrium with first to default contracts

In this subsection we show that high-type banks may also use the FTD menu of contracts to signal their own types. By signing an FTD menu, a high-type bank credibly signals its quality in a way that is distinct from signing a CDO menu. In fact, in the last case the high-type bank sells its portfolio of loans and the signalling device is obtained by partially insuring the buyer of the portfolio in case of default. In the former case, the high-type bank buys protection and is therefore the insured party. In this case, a bank signals its own type by accepting a stochastic payment for the insurance. The bank will pay a new premium to insure against a second default, if and only if a default of one of assets has already occurred. In this way, a high-type bank signals its own type with a contract that provides partial coverage, such that it is not the amount of coverage, but the

[13] See Basel Committee on Banking Supervision (1988).
[14] See Basel Committee on Banking Supervision (2004).

amount of the payment (i.e. the premium paid for the insurance), that varies across different states of the world. Again, since the probability of having a first default is higher for low type (since $1-p_l > 1-p_h$), then there exists a premium large enough to deter low-types to sign an FTD menu.[15]

Assuming that a protection buyer can only propose either an FTD menu or a CDS basket credit derivative contract, the following proposition holds:

Proposition 4.2. *If* $\mu \geq \frac{(1-p_l)I_1+(1-p_h p_l)I_2}{p_l(I_1+p_h I_2)}$ *or* $\rho \leq \frac{(p_h-p_l)}{p_l}$, *then there exists a unique* CK *separating perfect Bayesian equilibrium such that:*

(i) high type buyers sign a first-to-default basket contract paying a premium $\hat{\Psi}_{0,h}(I_1,I_2) = (1-p_l)(I_1+p_h I_2)(1+\mu)$, *and a plain vanilla contract on* I_2 *conditioned on the credit* I_1 *default, paying a premium* $(1-p_h)\,I_2(1+\mu)$;

(ii) low type buyers sign full coverage plain vanilla derivative contracts and pay the fair premium.

The seller's beliefs are such that the buyer is a high type bank with probability one if and only if it offers an FTD menu with $\Psi_{0,h}\,(I_1,I_2) \geq \hat{\Psi}_{0,h}(I_1,I_2)$.

Proof. See the appendix. ■

The premium paid at time $t=1$ to hedge loan I_2 in case of I_1 default is equal to the fair premium for the high-types. In fact, we restrict our analysis to renegotiation-proof contracts. In a separating equilibrium only high-type banks sign an FTD menu. Therefore at time $t=1$ in the case of I_1 default, there is complete information on the buyer type and therefore the unique renegotiation-proof premium that the buyer pays to hedge asset I_2 is equal to $(1-p_h)\,I_2(1+\mu)$.[16]

[15] This is a typical signalling device in the literature on asymmetric information. For instance, in Diamond (1993) a borrower can decide to execute an (inefficient) short term contract in order to signal he is not afraid to turn again to the credit market.

[16] We may have different separating contracts if the premium paid at time $t=1$ can be larger than $(1-p_h)\,I_2(1+\mu)$. But consistently with the assumption that the buyer has full bargaining

The capital requirement induced by an FTD menu can be positive or zero, depending on the level of the interest rate. As we show in the appendix, if $\mu \geq \frac{(1-p_l)I_1+(1-p_h p_l)I_2}{p_l(I_1+p_h I_2)}$, then profits are large enough to pay the premium of the plain vanilla credit defauls swap on I_2 when I_1 defaults and therefore there is no capital requirement. If $\mu < \frac{(1-p_l)I_1+(1-p_h p_l)I_2}{p_l(I_1+p_h I_2)}$, a high-type bank which signs the FTD menu faces a positive cost of capital. Therefore, it chooses to not hedge the loss it faces in the worst state of the world (when I_1 defaults and it has to hedge again credit I_2) if and only if $\rho \leq \frac{(p_h-p_l)}{p_l}$. When $\mu < \frac{(1-p_l)I_1+(1-p_h p_l)I_2}{p_l(I_1+p_h I_2)}$ and $\rho > \frac{(p_h-p_l)}{p_l}$, the cost of capital is sufficiently high that high-type banks would prefer to hedge the unexpected loss. The high-type bank has no incentive to over-insure its portfolio issuing more than one FTD menu. To buy insurance is costly for a high type bank and therefore this strategy increases profits only if in this way the bank reduces its capital requirements. For the same reason, the high-type bank has no incentive to over-insure its portfolio using a CDS basket. The effect would be a reduction of expected profits since anyone buying that insurance contract is assumed to be low-quality bank.

Low-type banks do not over-insure their exposition because the overall cost of purchasing any of the contracts (either CDS or FTD menus) is equal to their expected profits. As for the CDO menu, our separating equilibrium does not require that outsiders know that a bank has not secretely hedged its portfolio using other contracts.

When a high-type bank buys an FTD menu, it commits to buy a plain vanilla contract at time $t = 1$ in case of default of the first asset. This commitment is

power, we assume that these contracts would be renegotiated. If we assume that a buyer can commit at time $t = 0$ to pay a premium larger than $(1 - p_h)\, I_2(1 + \mu)$ to the seller in order to insure I_2 in case of I_1 default, then there exists equilibria in which buyer's profits are higher.

ex-post incentive compatible as a result of the continuous application of capital requirements, that is because of the presence of a capital requirement constraint also at date 1. The FTD menu is a costly contract for the high-type banks since they pay an insurance premium that is higher than the fair premium. In particular, the premium paid at time $t = 0$ is $(1 - p_l)(I_1 + p_h I_2)(1 + \mu)$ while the fair premium of a first-to default contract is equal to $(1 - p_h)(I_1 + p_h I_2)(1 + \mu)$. Hence, an FTD menu is not a first best contract when $\rho = 0$. Moreover neither the premium paid at time $t = 0$ nor the premium eventually paid at time $t = 1$, depend on the cost of capital ρ. Again, this occurs since we have assumed that buyers can privately sign credit derivatives and therefore the cost of capital does not influence the self-selection constraint of the low-type banks.

The equilibrium described in Proposition 4.2 is not unique if we enlarge the set of possible contracts that a buyer can offer to the seller. High-type banks, as we have shown above, can signal their own type using either an FTD menu or a CDO menu. Hence, it is worthwhile analyzing the conditions under which a high-type buyer would prefer to signal its own type by means of an FTD rather than a CDO menu.

4.3. Signalling contracts: a comparison

Assuming that a protection buyer can propose an FTD menu, a CDO menu or a CDS basket credit derivative contracts, the following proposition holds:

Proposition 4.3. *If* $\mu \geq \frac{(1-p_l)I_1+(1-p_h p_l)I_2}{p_l(I_1+p_h I_2)}$ *and* $\rho \geq \frac{(p_h^2-p_l^2)(1+\mu)(I_1+p_h I_2)}{(I_1+I_2)(p_h+p_l)-p_h p_l(I_1+I_2)(1+\mu)}$, *then a high-type buyer prefers to signal its own type by drawing an FTD menu than by drawing a CDO menu of contracts.*

Proof. See the appendix. ■

The intuition for this proposition is the following. In proposition 4.2 we prove that if $\mu \geq \frac{(1-p_l)I_1+(1-p_h p_l)I_2}{p_l(I_1+p_h I_2)}$, then the FTD menu does not induce any capital requirement. In contrast, in order to sustain a separating equilibrium, the CDO menu always induces a capital requirement . Therefore, ρ only affects the profits of high-type buyers when they sign a CDO menu and not when they sign an FTD menu. It follows that there exists a cost of capital ρ large enough to make the FTD menu more profitable than the CDO menu for high-type buyers.

DeMarzo and Duffie (1999) show that the CDOs are optimal contracts in presence of asymmetric information when contracts signed in the credit markets are publicly observable. We proved that CDOs still can be used to solve the adverse selection problem when banks have capital requirements and contracts are private. Moreover, as mentioned above, it follows immediately from Proposition 4.1 that the CDO menu is a first best contract when $\rho = 0$. However, Proposition 4.3 shows that if ρ and μ are high enough the FTD contract not only Pareto dominates the CDO contract, but also guarantees higher profits to the buyers.

One potential criticism of our analysis is that our definition of VaR includes also intermediation margin (i.e. μ) while it is not clear under Basel II if this element is included[17]. Nevertheless, as the following proposition shows, it is easy to extend our results to the case where VaR is the maximum total loss (i.e. expected and unexpected loss) of a bank's asset portfolio with a given confidence level α.

Proposition 4.4. *If the capital requirement is based on the total loss, and:*

$\rho \in \left[\frac{(p_h^2-p_l^2)(I_1+p_h I_2)(1-p_l)}{(I_1+I_2)-(1-p_h)(p_h+p_l)I_2}, (p_h - p_l)\right],$

[17] See Suarez and Repullo (2004) and Elizalde and Repullo (2005) for a discussion of this issue.

then the high-type buyers prefer to signal their own type by signing an FTD menu than a CDO menu.

Proof. see the appendix. ■

In this case also the FTD menu induces capital requirements that are non-zero but lower than those for the CDO menu. Indeed, the maximum total loss is only the fair premium for hedging I_2 if I_1 defaults, i.e. the expected loss, $(1 - p_h)(1 + \mu)I_2$; for the CDO menu it is the penalty L and that is strongly related to both the expected and unexpected loss of I_1 and I_2. Therefore, as before, it follows that there exists a cost of capital ρ large enough to make the FTD menu more profitable for high-type buyers.

Another potential limitation is that we consider credit derivatives that apply only to a basket of two loans. The extention to a larger basket will complicate too much our model and we prefer to leave this issue to furthere research. Nevertheless, we expect that our main idea holds even in a more general framework. The key point is that, as soon as the FTD menu has a capital requirement that is lower than the CDO menu, there exists a cost of capital that will make this menu of contracts better than the CDO[18].

One case not explicitly considered in our analysis is when the low-type banks are able to select loans with a negative NPV. It is easy to show that even under this assumption the results will be qualitatively unchanged. More specifically, both FTD and CDO menus allow the high-type to signal. The only significant difference is that in any separating equilibrium the low-type banks do not fund loans with a negative NPV and do not enter the credit derivative markets.

[18]This result is enforced if we consider that, as shown by Franke and Krahnen (2005) the equity tranche bears more than 96-98% of the credit risk of the loan portfolio under the CDO.

A second case related to our analysis but not explicitly considered is the moral hazard problem that arises when the protection buyer signs a full coverage plain vanilla contract and therefore has an incentive to stop monitoring the borrower. In our framework it is easy to show that with an FTD menu in which underling assets have different maturities the same conclusion does not hold necessarily. In fact, it is enough to assume that the probability of default of commercial loans may also depend on the intensity of the monitoring activity exerted by the bank that holds these loans in its portfolio. If a bank signs an FTD menu then, in the event of an asset default, it must sign another contract to cover the exposure of the remaining loans. Hence, if the cost of monitoring is not too high, the bank has an incentive to monitor the portfolio. It is straightforward to extend this result to the CDO menu.

5. Conclusion

A major concern of both policy makers and regulators is the effect of credit derivatives on the performance of credit markets. We show that the existence of a credit derivative market together with capital requirements for credit derivatives induces an adverse selection problem because low-type banks may cover their exposure with credit derivatives. Hence, the introduction of a credit derivative market does not necessarily always benefit the economy and increase social welfare.

The use of classical signaling contracts able to solve the problems that arise from the opacity of the loan portfolios of banks is precluded in such market because (i) the retention of a part of the risk increases banks' capital requirements and (ii) credit derivatives are private contracts and are not explicitly made public by banks. This last point makes the use of contracts based on partial coverage more difficult, because protection buyers are unable to commit to a specific level of protection that is ex-post incentive compatible.

To our knowledge this is the first paper in the academic literature to consider rigorously the implications for the design of credit derivatives contracts of these two characteristics: capital requirements and private credit derivative contracts.

Our main result is that, when the cost of capital is not too high, there may nonetheless exist a separating equilibrium where high-type banks signal their own type by signing derivative contracts, First, we show that a CDO menu is (still) a contract that solves the adverse selection problem, even if the buyer cannot commit itself to not hedge the risk by holding the equity tranche (because credit derivative contracts are private). Nevertheless, in our framework unlike DeMarzo and Duffie (1999) CDOs are not always second best contract in presence of asymmetric information. In fact, our second contribution is to prove that if the cost of capital and the interest rate are sufficiently high, then an FTD menu Pareto dominates the CDO menu.

We believe that this second result is especially interesting for two main reasons. First because it suggests a potential contract design that is able to solve the adverse selection problem when Basel II is implemented. Under Basel I we show that the optimal contract is the CDO menu because it is a first best contract. Second, it shows that theoretical predictions may change whether credit derivative contracts are publicly observable or not. Since the assumption that contracts are not publicly observable seems much more plausible, our result suggests that the analysis of the presence of private contracts in the credit market may deserve further investigation.

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6. Appendix

Proof of Proposition 4.1: Under this framework the low-type could decide to mimic the high-type by issuing a CDO and retaining the equity tranche (this is the one that characterizes the contract if credit derivatives are public contracts) or issuing the CDO and hedging the equity tranche. In the first case it faces capital requirement in the second case the position will be completely hedged and therefore capital requirements will be zero. The first incentive-compatible constraint for the low-type is:

$$\begin{gathered}p_h(I_1+I_2)(1+\mu)+(1-p_h^2)L-(1-p_l^2)L-(I_1+I_2)+\\+\rho\left(\min\left\{0,p_h(I_1+I_2)(1+\mu)+(1-p_h^2)L-L-(I_1+I_2)\right\}\right)\leq\\(I_1+I_2)(1+\mu)-(1-p_l)\left(I_1+I_2\right)(1+\mu)-(I_1+I_2)\end{gathered}\tag{6.1}$$

the second is:

$$\begin{gathered}p_h(I_1+I_2)(1+\mu)+(1-p_h^2)L-(1-p_l^2)L-(I_1+I_2)\leq\\(I_1+I_2)(1+\mu)-(1-p_l)\left(I_1+I_2\right)(1+\mu)-(I_1+I_2)\end{gathered}\tag{6.2}$$

It is straightforward to note that low type bank's profits are equal or higher hedging the equity tranche and therefore the second constraint is more binding. Condition (6.2) implies:

$$L \geq \frac{(I_1 + I_2)(1+\mu)}{p_h + p_l} \equiv \hat{L} \tag{6.3}$$

High-type bank equilibrium profits are:

$$p_h(I_1+I_2)(1+\mu)-(I_1+I_2)+\rho\left(\min\left\{0, p_h(I_1+I_2)(1+\mu)+(1-p_h^2)\hat{L}-\hat{L}-(I_1+I_2)\right\}\right) \tag{6.4}$$

The equity tranche $\hat{L}$ implies a positive capital requirements for the high-type if:

$$p_h(I_1+I_2)(1+\mu) - p_h^2\frac{(I_1+I_2)(1+\mu)}{p_h+p_l} - (I_1+I_2) < 0 \tag{6.5}$$

that is if:

$$(1+\mu) < \frac{(p_h+p_l)}{p_h p_l} \tag{6.6}$$

which holds by assumption that $\mu < 1$.

The high-type bank can also either hedge the equity tranche paying the premium as a low-type, or not hedge the equity trance and faces the cost of capital ρ. High type prefers to not hedge $\hat{L}$ if:

$$\begin{gathered} p_h(I_1+I_2)(1+\mu)-(I_1+I_2)+\rho\left(p_h(I_1+I_2)(1+\mu)-p_h^2\hat{L}-(I_1+I_2)\right) \geq \\ p_h(I_1+I_2)(1+\mu)+(1-p_h^2)\hat{L}-(1-p_l^2)\hat{L}-(I_1+I_2) \end{gathered} \tag{6.7}$$

that is if:

$$\rho \leq \frac{(p_h^2-p_l^2)(1+\mu)}{(p_h+p_l)-p_h p_l(1+\mu)} \equiv \rho'. \tag{6.8}$$

By (6.7) and the incentive-compatible constraint of the low-type, it turns immediately out that the incentive compatible constraint for the high-type is satisfied.

It is easy to check that the equilibrium beliefs are consistent with the equilibrium strategies and the out-of-equilibrium beliefs satisfy the intuitive criterion. Note also that in any separating equilibrium where the equity trance offered by the high-type bank is equal to $\tilde{L} > \hat{L}$, then there exists $\hat{L} \leq L < \tilde{L}$ such that if a high-type deviates offering a CDO contract with this equity tranche, then by the intuitive criterion a seller should assign probability 1 that the proposer is a high-type. ■

Proof of Proposition 4.2: The high-type buyer offers a first-to-default contract which satisfies the following maximization problem:

$$\max_{\{\Psi_{0,i}(I_1,I_2)\}} \mu\left(I_1+I_2\right)-\Psi_h\left(I_1,I_2\right)-(1-p_h)\Phi_h\left(I_2\right)+\rho(\min\left(0,\mu\left(I_1+I_2\right)-\Psi_h\left(I_1,I_2\right)-\Phi_h\left(I_2\right)\right)) \tag{6.9}$$

s.t.:

$$\Psi_h\left(I_1,I_2\right)+\left(1-p_h\right)\Phi_h\left(I_2\right)-\left(1-p_h\right)\left(I_1+I_2\right)\left(1+\mu\right)\geq 0 \tag{6.10}$$

$$\begin{aligned}\mu\left(I_1+I_2\right)-\Psi_h\left(I_1,I_2\right)-(1-p_l)\Phi_h\left(I_1\right)+\rho(\min\left(0,\mu\left(I_1+I_2\right)-\Psi_h\left(I_1,I_2\right)-\Phi_h\left(I_2\right)\right)\\ \leq \mu\left(I_1+I_2\right)-(1-p_l)(I_1+I_2)(1+\mu).\end{aligned}$$

Condition (6.10) is the participation constraint for the seller, and condition (**??**) is the incentive compatibility constraint for the low-type buyer. It is straightforward to note that the buyer wants to minimize the premium. Since the equilibrium is separating and the buyer has full bargaining power, at time $t=1$ the premium paid to hedge I_2 in case of I_1 default is[19]:

$$\Phi_h\left(I_2\right)=\left(1-p_h\right)I_2(1+\mu) \tag{6.11}$$

Case 1:

Let assume that:

$$\mu\left(I_1+I_2\right)\geq\Psi_h\left(I_1,I_2\right)+\Phi_h\left(I_2\right) \tag{6.12}$$

Substituting (6.11) into condition (6.10) and into condition (**??**) it is easy to check that condition (**??**) is more binding. Hence in equilibrium:

$$\Psi_h\left(I_1,I_2\right)\geq(1-p_l)(I_1+p_hI_2)(1+\mu)\equiv\hat{\Psi}_h(I_1,I_2). \tag{6.13}$$

The buyer's expected profits are:

$$\mu\left(I_1+I_2\right)-(1-p_l)(I_1+p_hI_2)(1+\mu)-\left(1-p_h\right)\left(1-p_h\right)I_2(1+\mu), \tag{6.14}$$

and therefore the high-type buyer prefers to sign a first-to-default as a high-type, than signing a plain vanilla contract as a low-type if:

$$\begin{aligned}\mu\left(I_1+I_2\right)-(1-p_l)(I_1+p_hI_2)(1+\mu)-\left(1-p_h\right)\left(1-p_h\right)I_2(1+\mu)\geq\\ \mu\left(I_1+I_2\right)-(1-p_l)(I_1+I_2)(1+\mu)\end{aligned}$$

[19]Equivalently, if the plain vanilla over I_2 conditioned on I_1default, is signed at time $t=0$, the buyer cannot credibly commit to pay a premium higher than $\bar{\Psi}_h\left(I_2\right)=\left(1-p_h\right)I_2(1+\mu)$. In fact, given its bargaining power, it is able to renegotiate the terms of the contract at time $t=1$.

that is:

$$(p_h - p_l)(I_2)(1+\mu)(1-p_h) > 0, \tag{6.15}$$

which holds by assumption. Finally, we have to check under which condition, assumption (6.12) holds true. Substituting $\hat{\Psi}_h(I_1, I_2)$ and $\Phi_h(I_2)$ in (6.12) we obtain the following condition:

$$\mu \geq \frac{(1-p_l)I_1 + (1-p_h p_l)\, I_2}{p_l(I_1 + p_h I_2)} = \widehat{\mu}. \tag{6.16}$$

Equilibrium beliefs are consistent with the equilibrium strategies and out-of-equilibrium beliefs satisfy the intuitive criterion. For any separating equilibrium where the high-type offers an FTD contract with $(\Psi_h(I_1, I_2), \Phi_h(I_2))$ such that $\Psi_h(I_1, I_2) > \hat{\Psi}_h(I_1, I_2)$, then there exists a deviating proposal $(\Psi_h'(I_1, I_2), \Phi_h(I_2))$ with $\Psi_h(I_1, I_2) > \Psi_h'(I_1, I_2) \geq \hat{\Psi}_h(I_1, I_2)$,such that, by the intuitive criterion, a seller has to assign probability one that the proposer is a high-type bank.

Case 2:

If condition 6.16 does not hold, then the low-type bank which deviates and signs a first-to-default contract, can draw up a credit derivative contract in order to hedge its position. Let X be the amount a low-type bank receives in case of credit I_1 default and $(1-p_l)X$ the premium paid in equilibrium to hedge X. Then, in order to not incur in the cost of capital, the low-type signs a credit derivative contract such that:

$$\mu(I_1 + I_2) - \Psi_h(I_1, I_2) - (1-p_h)I_2(1+\mu) + X - (1-p_l)X = 0 \tag{6.17}$$

that is if:

$$X = \frac{(1-p_l)(I_1 + p_h I_2)(1+\mu) + (1-p_h)I_2(1+\mu) - \mu(I_1 + I_2)}{p_l}. \tag{6.18}$$

Since the low-type banks pay a fair premium, it turns out that the incentive-compatible constraint is the same as in Case 1.

Hence, in equilibrium, high-type banks decide to not hedge their position if and only if the expected cost of capital is lower than the cost of hedging, that is if:

$$\begin{aligned} &\rho((1-p_l)(I_1 + p_h I_2)(1+\mu) + (1-p_h)I_2(1+\mu) - \mu(I_1 + I_2)) \leq \\ &(p_h - p_l)\frac{(1-p_l)(I_1 + p_h I_2)(1+\mu) + (1-p_h)I_2(1+\mu) - \mu(I_1 + I_2)}{p_l} \end{aligned} \tag{6.19}$$

that is, only if:

$$\rho \leq \frac{(p_h - p_l)}{p_l} \equiv \tilde{\rho}. \tag{6.20}$$

Finally, the usual arguments apply to check the consistency of the beliefs and the uniqueness of the CK equilibrium . ■

Proof of Proposition 4.3: First of all note that if $\rho > \rho'$, then the CDO contract cannot be used to sustain a separating equilibrium. If $\mu \geq \widehat{\mu}$ and $\rho \leq \rho'$, then a high-type buyer makes larger profits signaling its own type by drawing a first-to default contract (and a plain vanilla conditioned on I_1 default) than by drawing a CDO if:

$$p_h(I_1+I_2)(1+\mu)-(I_1+I_2)+\rho\left(p_h(I_1+I_2)(1+\mu)-p_h^2(\frac{(I_1+I_2)(1+\mu)}{p_h+p_l})-(I_1+I_2)\right)$$
$$\leq \mu\,(I_1+I_2)-(1-p_l)(I_1+p_hI_2)(1+\mu)-(1-p_h)^2(1+\mu)I_2 \tag{6.21}$$

which can be rewritten as:

$$\rho(\frac{(I_1+I_2)\,(p_h+p_l)-p_hp_l(I_1+I_2)(1+\mu)}{(p_h+p_l)})\geq(p_h-p_l)(1+\mu)\,(I_1+p_hI_2) \tag{6.22}$$

or,

$$\rho\geq\frac{(p_h^2-p_l^2)(1+\mu)\,(I_1+p_hI_2)}{(I_1+I_2)\,(p_h+p_l)-p_hp_l(I_1+I_2)(1+\mu)}\equiv\bar{\rho}. \tag{6.23}$$

noticing that the denominator is positive if and only if $(1+\mu)<\frac{(p_h+p_l)}{p_hp_l}$, which holds by assumption. Finally it is easy to check that $\bar{\rho}<\rho'$ if and only if $p_h<1$.■

Proof of Proposition 4.4: The new VaR definition based on total loss is not affecting the second incentive-compatible constraint of the low-type (6.2) for the CDO contract and therefore the amount L that the protection buyer has to guarantee in order to signal its type is still $\hat{L}$. The same applies for the FTD menu; the premium paid for the FTD is $\hat{\Psi}_h\,(I_1,I_2)$ and the premium paid for the plain vanilla conditioned on I_1 default is $\Phi_h\,(I_2)$.

Under the new definition of capital requirement, the required capital for the FTD menu, (that is the maximum total loss) is:

$$VaR^{ftd}=\Phi_h\,(I_2)\equiv(1-p_h)I_2(1+\mu) \tag{6.24}$$

that is the fair premium on I_2 if the loan I_1 defaults.

The required capital for a CDO is simply the equity tranche:

$$VaR^{CDO}=L\equiv\frac{(I_1+I_2)(1+\mu)}{(p_h+p_l)} \tag{6.25}$$

and it is easy to show that $VaR^{ftd}<VaR^{CDO}$.

For the CDO menu we have that the high type prefers to not hedge the equity tranche $\hat{L}$ if:

$$p_h(I_1+I_2)(1+\mu)-(I_1+I_2)+\rho\left(\hat{L}\right)\geq \tag{6.26}$$
$$p_h(I_1+I_2)(1+\mu)+(1-p_h^2)\hat{L}-(1-p_l^2)\hat{L}-(I_1+I_2),$$

that is if:

$$\rho\leq p_h^2-p_l^2. \tag{6.27}$$

For the FTD menu the high-type banks decide to not hedge their position if and only if the expected cost of capital is lower than the cost of hedging, that is if:

$$\rho((1-p_h)I_2(1+\mu))\leq \tag{6.28}$$
$$(p_h-p_l)(1-p_h)I_2(1+\mu)$$

or:

$$\rho\leq(p_h-p_l). \tag{6.29}$$

If $\rho\leq(p_h-p_l)$, then a high-type buyer makes larger profits signaling its own type by drawing an FTD menu than by drawing a CDO menu if:

$$\begin{aligned} & p_h(I_1+I_2)(1+\mu)-(I_1+I_2)-\rho\left(\frac{(I_1+I_2)(1+\mu)}{p_h+p_l}\right) \\ \leq\ & \mu\left(I_1+I_2\right)-(1-p_l)(I_1+p_hI_2)(1+\mu)-(1-p_h)^2(1+\mu)I_2-\rho\left((1-p_h)(1+\mu)I_2\right)\end{aligned} \tag{6.30}$$

which can be rewritten as:

$$\rho\left(\frac{(I_1+I_2)(1+\mu)}{p_h+p_l}-(1-p_h)I_2(1+\mu)\right)\geq \tag{6.31}$$
$$(p_h-p_l)(1-p_l)(1+\mu)\left(I_1+p_hI_2\right)$$

or,

$$\rho\geq\frac{(p_h^2-p_l^2)(1-p_l)\left(I_1+p_hI_2\right)}{(I_1+I_2)-(p_h+p_l)(1-p_h)I_2} \tag{6.32}$$

■