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VOLATILITY THRESHOLD DYNAMIC CONDITIONAL CORRELATIONS: AN INTERNATIONAL ANALYSIS

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Volatility Threshold Dynamic Conditional Correlations: An International Analysis*

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Abstract

We extend the Dynamic Conditional Correlation multivariate GARCH specification to investigate the dynamic contemporaneous relationship between correlations and variances of the underlying assets. We present a generalization of the DCC model where the dynamic behavior depends on the assets variances through a threshold structure. Our purpose is to analyze the behavior of correlations in periods of high volatility. The application of the proposed specification to a sample of markets heterogeneous in the levels of their development allows the identification of market pairs whose correlations show low sensitivity to high underlying volatility.

JEL classification: C50, F37, G11, G15

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1. Introduction

Understanding the relationship between correlations and volatilities is crucial for risk management and optimal portfolio allocation strategies. Correlations that increase in volatile periods reduce the power of portfolio diversification when it is needed most. This paper extends the multivariate Dynamic Conditional Correlation (DCC) model of Engle (2002) and its generalization by Cappiello, Engle and Sheppard (2006) to investigate the dynamic relationship between the correlations and the volatilities of the underlying assets. In particular, we examine whether high volatility values of the assets, implied by the model, are associated with an increase in their correlation values. We could interpret the resulting specification as asymmetric in the level of volatility.

The early studies on the relationship of correlations and volatilities in international markets have often relied on the analysis of these measures computed over different sub-periods of the data sample. In particular, a range of studies focused on the comparison of the correlation coefficients during stable and volatile market periods (e.g., Bertero and Mayer 1990, King and Wadhwani 1990, Lee and Kim 1993, Erb et al. 1994, Calvo and Reinhart 1996). These papers present evidence that international correlations increase significantly in turbulent times.

Stambaugh (1995), Boyer et al. (1999) and Forbes and Rigobon (2002) show that tests of changing correlations based on correlation coefficients conditional on different levels of one or both return variables are biased due to heteroskedasticity of financial return series. A range of papers take into account the heteroskedasticity property of financial time series when testing for changing correlations in varying volatility regimes. Longin and Solnik (1995) and Ramchard and Susmel (1998) are examples of studies doing this in the framework of multivariate ARCH-type models.

Longin and Solnik (1995) test the hypothesis of higher correlation during volatile periods, using a bivariate Constant Conditional Correlation (CCC) GARCH model (Bollerslev, 1990) as a base specification. The authors allow the estimated correlation value for the turbulent market periods to differ from the constant correlation coefficient for the rest of the sample by introducing a threshold on the contemporaneous value of the volatility. Differently, Ramchard and Susmel (1998) propose a bivariate SWARCH model that makes correlations a function of variance regimes, with different correlations for periods of high and low volatility. While in Longin and Solnik the correlations depend on an exogenous volatility threshold (the unconditional variance of the process), in Ramchard and Susmel the volatility regime is endogenously determined within the model. Note that in both Longin and Solnik (1995) and Ramchard and Susmel (1998) the correlations are assumed to be constant within high and low volatility states. Both studies find that market correlations rise when the conditional volatilities are high. Edwards and Susmel (2001) present an analysis of the international stock market co-movements studying the co-dependence of volatility regimes[1]. They also use a bivariate SWARCH model with the purpose of investigating whether periods of high volatility are correlated across countries, and present empirical evidence that confirms this hypothesis.

In this paper we propose a generalization of the approach of Longin and Solnik (1995) in two directions: first, we model correlations in a dynamic way following the growing literature started by Engle (2002) with the Dynamic Conditional Correlation model and then generalized by Cappiello et al. (2006); second, taking advantage of the dynamic behavior of correlations, we explicitly include in the model the volatility thresholds.

[1] An alternative approach to the examination of this type of asymmetric codependence structure of asset returns is proposed in Longin and Solnik (2001). The paper uses the extreme value theory to model the asymptotic distribution of multivariate tail correlation, and shows that conditional correlations in the international markets increase in volatile bear markets, but not in bull markets. Further evidence that the correlations of international markets increase conditional on large negative returns, is presented, for example, in Karolyi and Stulz (1996), Solnik et al. (1996), De Santis and Gerard (1997), Ang and Bekaert (2002), Bae, Karolyi and Stulz (2003), Das and Uppal (2004), and Bekaert, Harvey and Ng (2005).

To demonstrate the practical relevance of our model we employ a sample of national stock indices from markets heterogeneous in the levels of their development and integration into international securities markets. While there is a considerable body of research investigating the Asian and Latin American emerging stock markets, the transition markets of Central and Eastern Europe (CEE) have seen much less attention so far. Our sample includes stock indices from the major developed markets as well as three largest transition stock markets of Central Europe: Hungary, Poland and the Czech Republic.

The empirical evidence indicates that the response of the transition markets to global market events is not always similar to that of the developed markets. The results of the application of the extended DCC specifications to our sample delivers strong evidence that the turbulent periods are associated with an increase in the correlations among the developed markets. For the cross-correlations of the transition markets (in particular of the Hungarian and the Czech markets) among each other and with the developed markets, however, this pattern is by far not as pronounced. The Polish market, on the other hand, in many respects behaves similar to the developed markets. Furthermore, we document that for the developed markets the observed increase in correlations associated with the extreme volatility is more evident for the negative innovations, and is less pronounced for the positive innovations. Thus, our results in general support the findings in Longin and Solnik (2001), among others, that correlations tend to increase in volatile bear markets, but not in bull markets. The identification of market pairs the correlations of which do not increase in volatile periods has potential implications for leveraging the benefits of international portfolio diversification. The investigation of these implications itself is, however, outside the scope of this paper.

We proceed as follows: Section 2 presents our modeling strategy evidencing the differences with respect to the actual approaches, while Section 3 presents the dataset used in the empirical analysis of Section 4; Section 5 concludes.

2. VT-DCC: Volatility Threshold Dynamic Conditional Correlations Models

The main innovation of this paper is represented by the introduction of a new class of Dynamic Conditional Correlation models that generalize the original contributions of Engle (2002) and Cappiello et al. (2006). We named the models belonging to this class as Volatility Threshold Dynamic Conditional Correlations models (with a VT- prefix henceforth), given that the correlation dynamic partially depends on variance values through a threshold structure. We first present the basic Variance Threshold generalization of the DCC model of Engle (2002) and in a following section, the additional representations we propose.

Consider an n-variate conditional process ε_t with zero mean and covariance matrix H_t, which is identically distributed following an unspecified density D(.):

$$\varepsilon_t \mid F_{t-1} \sim D\left(0, H_t\right) \tag{1}$$

where F_{t-1} denotes the conditioning information set including the information up to time *t-1*. The vector ε_t may represent either a zero mean returns vector or the residuals vector of a return mean model. The VT-DCC model has the following representation. We define the conditional covariance matrix with the usual decomposition as

$$H_t = D_t R_t D_t \tag{2}$$

where D_t is a diagonal matrix of conditional volatilities

$$D_t = diag\left\{\sqrt{h_{i,t}}\right\} \tag{3}$$

and $R_t = \{\rho_{ij,t}\}$ represents the time-varying conditional correlation matrix. Furthermore, denote by η_t the variance standardized residuals

$$\eta_t = D_t^{-1}\varepsilon_t \tag{4}$$

in addition, note that they are correlated and with a unit variance. Following Bollerslev (1990), Engle (2002) and the several contributions generalizing their models, the conditional variance $h_{i,t}$ could follow any univariate GARCH model. There are no reasons for requiring the use of a specific representation for all conditional variances, which can be specified on a series-specific case. Furthermore, note that R_t corresponds to the conditional covariance matrix of the variance standardized residuals η_t and if it is assumed to be time invariant the model collapses on the Constant Conditional Correlation model of Bollerslev (1990). The VT-DCC model specifies the dynamics of the correlation matrix as follows:

$$R_t = (diag(Q_t))^{-\frac{1}{2}} Q_t (diag(Q_t))^{-\frac{1}{2}} \tag{5}$$

$$Q_t = (1-\alpha-\beta)\bar{R} + \alpha\left(\eta_t\eta_t'\right) + \beta Q_{t-1} + \gamma\left(V_t - \bar{V}\right) \tag{6}$$

where α, β and γ are scalar coefficients, $\bar{R}$ is the unconditional correlation matrix of η_t, $\bar{R} = E\left[\eta_t\eta_t'\right]$, V_t is a dummy variable matrix related to the volatility threshold structure, $\bar{V} = E\left[V_t\right]$ and $E[\cdot]$ denotes an unconditional expectation.

The dummy variable matrix V_t has the following structure:

$$V_t = \left[v_{ij,t}\right] \quad v_{ij,t} = \begin{cases} 1 & \text{if } h_{i,t} > d\left(\{h_{i,t}\}_{t=1}^{T}\right) \text{ or } h_{j,t} > d\left(\{h_{j,t}\}_{t=1}^{T}\right) \\ 0 & otherwise \end{cases} \tag{7}$$

where $d\left(\left\{h_{i,t}\right\}_{t=1}^{T}\right)$ is a given threshold for the conditional variances of variable *i*, determined using the entire conditional variance series $\left\{h_{i,t}\right\}_{t=1}^{T}$ (and similarly for $\left\{h_{j,t}\right\}_{t=1}^{T}$). A following section will extensively discuss the definition of thresholds and their possible generalizations. At this stage, we only evidence that the V_t dummy matrix may be created using an 'and' condition instead of an 'or' condition, as follows:

$$V_t = \left[v_{ij,t}\right] \quad v_{ij,t} = \begin{cases} 1 \quad \text{if } h_{i,t} > d\left(\left\{h_{i,t}\right\}_{t=1}^{T}\right) \text{ and } h_{j,t} > d\left(\left\{h_{j,t}\right\}_{t=1}^{T}\right) \\ 0 \qquad\qquad\qquad otherwise \end{cases} \tag{8}$$

Note that the VT-DCC model collapses on the DCC model of Engle (2002) if the γ coefficient is zero. In order to avoid explosive patterns in the dynamic of Q_t we impose that $\alpha + \beta < 1$. Furthermore, unconditionally, the expectation of Q_t is still equal to the unconditional correlation matrix $\overline{R}$, implying that the VT-DCC model is subject to the unconditional correlation targeting constraint. This fact allows interpreting the VT-DCC model as a correlation model similarly to the DCC model of Engle (2002).

Given the quadratic structure in (5), R_t is guaranteed to be positive definite if Q_t is positive definite. Differently from the DCC model of Engle (2002), the choice of a suitable starting point Q_0 is not sufficient to guarantee the positive definiteness if the dummy matrix is defined as in (7). In this case, we must impose the positive definiteness in the estimation step of the model. If the V_t matrix follows (8) it will be positive semi-definite by construction and thus the choice of an appropriate Q_0 value guarantees the positive definiteness of Q_t (given that it will be the sum of positive definite matrices, Q_{t-1} and $\bar{R}$, and positive semi-definite matrices, V_t, $\bar{V}$ and $\left(\eta_t \eta_t'\right)$).

The VT-DCC model could be used in several financial areas. The varying relationship between volatility and correlation values of the different asset pairs in the portfolio, if present but ignored, could have serious consequences for portfolio hedging effectiveness. In particular, we may overstate the value of portfolio diversification if we do not account for the increase in co-movement of asset prices in the periods of high volatility, and, in particular, high downside volatility. The extension of the DCC model we propose tests the hypothesis whether high volatility values of the underlying assets are associated with an increase in their correlation values. An investor rearranging his portfolio would appreciate the possibility of identifying assets for which this association is relatively weak. In particular, other things being equal, one could consider such assets as potentially attractive targets for portfolio diversification.

Furthermore, the VT-DCC model could be useful in the contagion literature given that it will enable the distinction of correlation movements associated with volatility spillover effects from the changes in the correlation levels associated with pure contagion events. In fact, once the correlation dynamic has been estimated, we could filter out the effects of the volatility threshold component and analyze the remaining patterns in order to highlight jumps in the correlations that we could associate to contagion.

2.1 Model extensions

A drawback of the VT-DCC dynamic specified in (6) is that all the elements of the conditional correlation matrix are restricted to have the same behavior. One strand of the DCC-related literature has proposed extensions of the DCC model with richer dynamic, see Franses and

Hafner (2003) and Billio et al. (2006), among others. Cappiello et al. (2006) that propose a BEKK[2]-type generalization of the model have followed a second possible approach. In the Generalized DCC (GDCC) model of Cappiello et al. (2006), the following equation drives the correlation dynamic:

$$Q_t = (\bar{Q} - A\bar{Q}A' - B\bar{Q}B') + A(\eta_{t-1}\eta_{t-1}^{'})A' + BQ_{t-1}B' \tag{9}$$

where A and B are $n \times n$ diagonal matrices[3]. As a result, the dynamics of the individual elements of the matrix Q_t is specified as follows:

$$q_{ij,t} = (1 - \alpha_i\alpha_j - \beta_i\beta_j)\bar{q}_{ij} + \alpha_i\alpha_j\eta_{i,t-1}\eta_{j,t-1} + \beta_i\beta_j q_{ij,t-1} \tag{10}$$

Although this generalized model adds flexibility to Engle's specification, the number of parameters to be estimated increases considerably, but remains feasible (it is linear in the number of correlations, which is, however, quadratic in the number of assets)[4].

Within a Volatility Threshold framework, the approach of Cappiello et al. (2006) allows for the introduction of individual series specific volatility impact parameters. We propose the following extension of the GDCC models in (9) introducing a 'diagonal' Volatility Threshold component:

$$Q_t = (\bar{Q} - A\bar{Q}A' - B\bar{Q}B' - \Gamma\bar{V}\Gamma') + A(\eta_{t-1}\eta_{t-1}^{'})A' + BQ_{t-1}B' + \Gamma V_t\Gamma' \tag{11}$$

where $\bar{V} = E\left[V_t\right]$, and A, B and Γ are $n \times n$ diagonal matrices. Now, a sufficient condition ensuring the positive definiteness of the covariance matrix Q_t is that $(\bar{Q} - A\bar{Q}A' - B\bar{Q}B' - \Gamma\bar{V}\Gamma')$

[2] The BEKK multivariate GARCH model was firstly proposed by Engle and Kroner (1995).
[3] The most general model representation includes full parameter matrices, but this will raise the well know course of dimensionality.
[4] The Asymmetric Generalized DCC in Cappiello et al. (2006) includes an additional component accounting for the asymmetric impact of the past negative shocks on the correlation process.

is positive definite[5] if V_t is created as in equation (8), while if V_t follows (7) the positive definiteness of Q_t must be imposed in the estimation step.

Within the VT-GDCC specification, we specify the dynamics of the individual elements of Q_t as:

$$q_{ij,t} = (1-\alpha_i\alpha_j - \beta_i\beta_j)\overline{q}_{ij} + \alpha_i\alpha_j\eta_{i,t-1}\eta_{j,t-1} + \beta_i\beta_j q_{ij,t-1} + \gamma_i\gamma_j\left(v_{ij,t} - \overline{v}_{ij}\right) \qquad (12)$$

where we separately evidence the volatility threshold component. In the empirical applications, the VT-GDCC model allows for identification of heterogeneity in the response of the markets to, say, high volatility, given the introduction of different coefficients in the diagonal of matrix Γ. On the other hand, a restriction on the GARCH dynamics of the conditional correlations in some cases could be well justifiable, leading to a more parsimonious specification and/or making the model estimation feasible also in large dimensions.[6] As an example, we may impose that A and B diagonal elements are identical, or similarly we may transform the matrices into scalars, given that we are modeling a set of integrated financial markets with common correlation dynamic, because they react in a similar way to the shocks. Introducing the restrictions on the GARCH correlation dynamic in (11) (that is, imposing the diagonal elements of the matrices A and B to be identical), but maintaining the heterogeneity in the volatility threshold component, lead to the following correlation specific dynamic behavior:

$$q_{ij,t} = (1-\alpha^2 - \beta^2)\overline{q}_{ij} + \alpha^2\eta_{i,t-1}\eta_{j,t-1} + \beta^2 q_{ij,t-1} + \gamma_i\gamma_j\left(v_{ij,t} - \overline{v}_{ij}\right) \qquad (13)$$

In the following, we refer to the specification in (6) as the *Volatility Threshold DCC* (VT-DCC), the one in (11)-(12) as the *Volatility Threshold GDCC* (VT-GDCC), and to the specification in

[5] The sufficient condition is generally imposed in the optimization routines.

[6] As shown in Engle and Sheppard (2001), the scalar DCC model leads to sub-optimal portfolio selection in case of many assets (like 20 or 30) as it assumes the same GARCH-type dynamics for all the asset-specific conditional correlations. This assumption becomes, however, increasingly more likely to be satisfied in case of small number of assets.

(11)-(13) as the restricted VT-GDCC. In the generalized versions of the model the products of the coefficients, $\gamma_i\gamma_j$, measure the sensitivity of the correlations between markets i and j to the levels of volatility in the underlying markets. Therefore, they are of direct interest in the investigation of the relation between correlation and volatility. We suggest directly testing the significance of these products rather than simply analyzing the individual γ_i coefficients.

Note that, in general, we can add the VT- component to different dynamic correlation specifications proposed in the literature, creating, as we suggested, a new model class. Among the possible interesting specifications to be investigated in future contributions, we mention the works of Tse and Tsui (2002) and of Pelletier (2006). In particular, the joint use of the Volatility Threshold structure and the Markov switching dynamics may provide useful tools for the contagion analysis.

A further generalization of the VT-DCC models refers to the relation between the Volatility Threshold component and the matrix Q_t. In the previous dynamic equations, we have always assumed that the VT effect is contemporaneous to the correlation matrix, $Q_t = f\left(V_t\right)$. We can generalize this relation allowing for lagged effects, $Q_t = f\left(V_t, V_{t-1}, ...V_{t-K}\right)$, where K is the maximum lag. Note that by increasing the lags of V_t we may largely increase the parameter set. For this reason, we suggest the inclusion of lagged effects only in the VT-DCC models in (6) and in the restricted VT-GDCC of equation (13). Positive definiteness of the correlation matrix is achieved as in the cases without lags in the dummy variable matrix V_t. We discuss in the following section the model extensions related to the thresholds definition and to the thresholds structure.

2.2 Volatility thresholds

The Volatility Threshold model inherits its name from the presence of a threshold-based component affecting the correlation dynamic. We previously introduced the different model representations simply stating that the thresholds are functions of the conditional variance series. We now present alternative approaches that could be followed for the definition of the thresholds. The first method defines thresholds as fractiles of the conditional variance series. In this case, we may define the thresholds given an estimate of the conditional variances and we may choose to fix for each $h_{i,t}$ series a threshold that identifies conditional variances in the upper *k-th%* of the empirical density of $h_{i,t}$. However, this approach may raise a problem since the thresholds will be series specific, and the magnitude of the thresholds between countries may vary. A different approach is to provide thresholds based on fractiles but determined on standardized conditional variance sequences as follows:

i) compute the mean and the variances of each conditional variance sequence $\upsilon_i = \frac{1}{T}\sum_{t=1}^{T} h_{i,t}$, $\tau_i^2 = \frac{1}{T}\sum_{t=1}^{T}\left(h_{i,t} - \upsilon_i\right)^2$, and compute the standardized conditional variance sequences $\bar{h}_{i,t} = \left(h_{i,t} - \upsilon_i\right)\tau_i^{-1}$;

ii) compute the $\bar{d}$ threshold on a common basis as fractiles of the density of the entire set of standardized conditional variances $\left\{\bar{h}_{i,t}\right\}_{i=1}^{n}$;

iii) then get back to the specific thresholds $d_i = \bar{d}\tau_i + \upsilon_i$.

With this alternative approach, the thresholds are determined on a common basis, taking into account possible differences in terms of magnitude and dispersion of the conditional variance sequences. Both strategies propose thresholds based on fractiles in order to ensure the existence of a minimum number of threshold events. Note that the choice of the preferred fractile could be done by some calibration exercises. Finally, we evidence that the general approach we propose is close to the methods of Tong (1983).

In the empirical application, we will present a comparison of the two alternative ways for the threshold definition. A rather different method, which is not included in the present paper, is the endogenous estimation of the thresholds. We may define the series *i* threshold as an additional parameter to be estimated. In this last case, the model would require more computational intensive estimation methods.

We could further generalize the VT-DCC models by modifying the threshold component. In fact, we could consider the introduction of multiple thresholds in order to capture the possible changes in correlations associated with different changes in the variance levels. In fact, we may distinguish between correlation effects associated with moderate jumps in the volatilities and correlation movements due to severe volatility changes. In this case, if we introduce *L* thresholds, we may restate the VT component of the VT-DCC model in (8) as follows:

$$\gamma\left(V_t - \bar{V}\right) \rightarrow \sum_{l=1}^{L} \gamma_l \left(V_{l,t} - \bar{V}_l\right) \tag{14}$$

where $\bar{V}_l = E\left[V_{l,t}\right]$,

$$V_t = \left[v_{ij,t}\right] \quad v_{ij,t} = \begin{cases} 1 & \text{if } d_l\left(\left\{h_{i,t}\right\}_{t=1}^{T}\right) < h_{i,t} \le d_{l+1}\left(\left\{h_{i,t}\right\}_{t=1}^{T}\right) \\ & \text{or } d_l\left(\left\{h_{j,t}\right\}_{t=1}^{T}\right) < h_{j,t} \le d_{l+1}\left(\left\{h_{j,t}\right\}_{t=1}^{T}\right) \\ 0 & \textit{otherwise} \end{cases}. \tag{15}$$

in addition $d_1\left(\left\{h_{i,t}\right\}_{t=1}^{T}\right) < d_2\left(\left\{h_{i,t}\right\}_{t=1}^{T}\right) < \ldots < d_L\left(\left\{h_{i,t}\right\}_{t=1}^{T}\right)$. Note that when $l = L$ the conditions are one sided only ($d_L\left(\left\{h_{i,t}\right\}_{t=1}^{T}\right) < h_{i,t}$ or $d_L\left(\left\{h_{j,t}\right\}_{t=1}^{T}\right) < h_{j,t}$). Continuing our previous example, the introduction of two-sided conditions allows separating the effect of a moderate volatility increase on correlations from the effect of a severe increase. In fact, the correlation effect of a variance change between thresholds *l* and *l+1* is completely associated with coefficient γ_l and a direct significance test is available. Differently, we could define the dummy variable matrices as follows:

$$V_{l,t}=\left[v_{l,ij,t}\right] \quad v_{l,ij,t} = \begin{cases} 1 & \text{if } d_l\left(\left\{h_{i,t}\right\}_{t=1}^{T}\right) < h_{i,t} \text{ or } d_l\left(\left\{h_{j,t}\right\}_{t=1}^{T}\right) < h_{j,t} \\ 0 & \qquad\qquad otherwise \end{cases} \tag{16}$$

where the one-sided 'if' condition is used. In this second hypothesis, we can interpret the coefficient γ_l as an incremental correlation effect coming from variances above threshold *l* with respect to the effect coming from variance above the threshold *l-1* (because variances above *l* are also above *l-1*). The previous generalizations of the VT component were all presented with the 'or' condition. They can be adapted for the inclusion of the 'and' condition as in equation (8). The main difference between the two approaches ('or' against 'and' condition) is in the constraint needed for ensuring the positive definiteness of the correlation matrix: the 'or' condition requires a direct imposition or check of positive definiteness of Q_t in the estimation step; differently, the 'and' condition requires either a constraint on the parameters in VT-GDCC model or the choice of a suitable starting point in the VT-DCC model.

As emphasized in the introduction to this paper, a range of studies have identified that the correlations between assets increase for downside moves, especially for extreme downside moves, rather than for upside moves. Below we propose a further modification of the VT component that considers the case of "extreme" volatility associated with bear markets.[7] In the framework of the DCC model, we could define this, for example, as the case when the fitted volatility for the period t exceeds the pre-specified threshold and at the same time the observed return at time $t-1$ is negative (which is equivalent to the corresponding standardized residual being negative). To integrate this feature into our specification, we may redefine the dummy variables matrix, V_t, as follows:

$$v_{ij,t} = \begin{cases} 1 \quad \text{if } \left(h_{i,t} > d\left(\left\{h_{i,t}\right\}_{t=1}^{T}\right) \text{ } and \text{ } \varepsilon_{i,t-1} < 0 \right) \text{ or } \left(h_{j,t} > d\left(\left\{h_{j,t}\right\}_{t=1}^{T}\right) \text{ } and \text{ } \varepsilon_{j,t-1} < 0 \right) \\ 0 \qquad\qquad\qquad\qquad\qquad\qquad otherwise \end{cases} \quad (17)$$

Equation (17) conditions the VT effect only to the existence of a negative return, however it does not help in the distinction of this negative innovation effect from the existence of a baseline VT-effect (or from the relevance of a VT effect associated with positive shocks). We could generalize the approach by allowing for the presence of two V_t-like matrices: the first defined as in equation (17), while the second with a similar structure but conditioning the VT effect to a positive innovation. Similarly, we may include both (7) and (17) in a single model, where the coefficients associated with (7) could be interpreted as a baseline VT effect while the coefficients related to (17) capture the asymmetric behavior associated with negative shocks. We refer to the specifications in (6) and (11), with two V_t-like matrices (defined as in (17) or as in (7) and (17)) as the *Volatility Threshold Asymmetric DCC* (VT-ADCC) and the *Volatility Threshold Asymmetric GDCC* (VT-AGDCC), respectively.

All the models described in this section could be modified in such a way that the correlation values are conditioned on the observed past return series only (but not on the fitted volatility

[7] In this context, see CES (2006), who provide an extension of the GDCC model in (7), the Asymmetric Generalized DCC, to account for the asymmetric impact of the sign of the past innovations on the current correlation values.

values). In this further case, we could have defined the matrix V_t in order to condition the correlation values on the past returns or squared returns exceeding a pre-specified threshold. Note that if we define the V_t matrices using squared returns we may also add an asymmetric effect as in the VT-ADCC and VT-AGDCC specifications. The discussion on the 'or' and 'and' conditions and on the positive definiteness of the correlation matrix previously presented directly extend to these further generalizations of the VT-DCC and VT-GDCC models.

2.3 Model estimation

Dynamic conditional correlation multivariate GARCH models generally allow for two-stage estimation. Specifically, we can write the likelihood function of the DCC models as a sum of a volatility part and a correlation part. We can express the quasi-normal likelihood of (1) as follows:

$$
\begin{aligned}
L(\theta) &= \sum_{t=1}^{T} L_t \approx -\frac{1}{2}\sum_{t=1}^{T}\left(\ln\left|H_t\right| + \varepsilon_t H_t^{-1}\varepsilon_t'\right) \\
&= -\frac{1}{2}\sum_{t=1}^{T}\left(\ln\left|D_t R_t D_t\right| + \varepsilon_t D_t^{-1} R_t^{-1} D_t^{-1}\varepsilon_t'\right)
\end{aligned}
\tag{18}
$$

where θ denotes the entire parameter set (it includes both the variance and the correlation parameters). We may rewrite (18) using the following decomposition of the time t log-likelihood:

$$
\begin{aligned}
&\ln\left|D_t R_t D_t\right| + \varepsilon_t D_t^{-1} R_t^{-1} D_t^{-1}\varepsilon_t' = 2\ln\left|D_t\right| + \ln\left|R_t\right| + \varepsilon_t D_t^{-1} R_t^{-1} D_t^{-1}\varepsilon_t' \\
&= \left(2\ln\left|D_t\right| + \varepsilon_t D_t^{-2}\varepsilon_t'\right) + \left(\ln\left|R_t\right| + \eta_t R_t^{-1}\eta_t' - \varepsilon_t D_t^{-2}\varepsilon_t'\right) \\
&= L_{1,t} + L_{2,t}
\end{aligned}
\tag{19}
$$

and we obtain the final representation of the Quasi Log-Likelihood:

$$L(\theta)=\sum_{t=1}^{T}L_t \approx -\frac{1}{2}\sum_{t=1}^{T}\left(L_{1,t}+L_{2,t}\right)=L_1(\phi)+L_2(\theta) \tag{20}$$

where ϕ is a subset of θ, including only the coefficients entering in the variance equations. Following Engle (2002), we can determine the estimates of volatility parameters by maximizing L_1 in (20). Note that this first stage log-likelihood is simply the sum of the individual series volatility log-likelihoods. Given the standardized residuals and the parameter estimates from the first stage of estimation, we obtain the correlation parameters by maximizing the second-stage log-likelihood L_2. This second step will require the estimate of a subset of θ including only correlation parameters.

In our case, the second stage likelihood will have parameters and correlation dynamic depending on the first stage parameters both via the first stage standardized residuals and through the first stage estimated conditional variances. However, as in Engle (2002), the parameters of the volatility models are determined exclusively in the first step. Therefore, we could consider the fitted volatility series as given for the second step of the estimation, focusing on correlation specific parameters.

Furthermore, as we evidenced in the previous section, we may relate the researcher's interest to functions of parameters, like the product of the volatility-threshold coefficients. In this case, the coefficient function values will be determined using the estimation outputs while the standard errors will be evaluated using the delta method.

3. Data Description

The empirical part of this paper concentrates on the investigation of the time-varying correlations between some of the largest European, the major transition CEE and the United States financial markets. As we previously mentioned, Hungary, Poland and the Czech Republic represent the largest capital markets in the CEE area.

We develop the analysis on the log-returns of the blue-chip indices of the considered group of markets. The stock market indices are CAC 40 (for France), DAX 30 (for Germany), FTSE 100 (for the U.K.), S&P 500 (for the U.S.), BUX 30 (for Hungary), WGI 20 (for Poland), and the PX 50 (for the Czech Republic).. We collected all the indices at weekly frequency and expressed them in Euro. The sample spans the period from January 1995 to July 2007, constituting 655 weekly return observations. The use of weekly data is preferred because of the existence of market frictions, in particular for transition economies, and because of the different trading hours within European countries and between Europe and the US. Furthermore, we run all analysis in a common currency in order to include all effects going through the exchange rate channel. In fact, by using a common currency, the returns on a given market are equal to the sum of the local currency stock market return and of the exchange rate return with respect to the common currency. The choice of the Euro is arbitrary; results in US dollars are similar. An extensive analysis of the information transmission mechanism that allow separating the effects of exchange rates channel from that of the pure stock exchanges channel is beyond the scope of the current paper and is left for future researches.

[INSERT TABLE 1 ABOUT HERE]

Table 1 presents some descriptive statistics of log returns of the seven stock market indices considered. All series show the typical non-normality of financial time series. They are negatively skewed and display excess kurtosis. The Ljung-Box statistics suggest serial autocorrelation in the returns of most indices (with exception of German DAX). The squared returns of all series are highly autocorrelated; we can consider it as evidence of ARCH effects in the residual series. This unconditional analysis does not evidence any specific difference between developed and transition markets, apart a larger standard deviation of the latter, a somewhat expected result.

[INSERT TABLE 2 ABOUT HERE]

Table 2 shows unconditional correlations of the return series. The highest correlations are between the three developed European markets, CAC 40 and DAX 30 (0.88), CAC 40 and FTSE 100 (0.80), and DAX 30 and FTSE 100 (0.77). These are followed by the correlation between S&P 500 and FTSE 100 (0.74), S&P 500 and DAX 30 (0.71), and S&P 500 and CAC 40 (0.71). The correlations within the transition markets range from 0.4648 for the Polish and Czech indices to 0.5747 for the Hungarian and Polish indices. It is interesting to note that the correlations among the transition markets are higher than the correlation of these markets with the developed markets.

In order to model the mean correlation and the possible relation between markets we specify a VAR-type structure for the weekly indices returns. Our mean model also includes a number of additional explanatory variables, which have been useful to predict asset returns, see the works by Ait-Sahalia and Brandt (2001) and Pesaran and Timmerman (1995, 2000). The variables we include are: (i) the short term interest rates for the European market, measured by the German 3 month money rate for the period from January 1995 through December 1998, and the Euro Interbank Offered Rate (ERIBOR) for the rest of the sample[8]; (ii) the short term U.S. interest rates, measured by the 3 month U.S. treasury bill rate; (iii) the long term European interest rates,

[8] The Euro was introduced on the 1st of January 1999.

measured by the German 10 year government bond yield; (iv) the long term U.S. interest rates measured by the U.S. 10 year government bond yield; finally, (v) the OPEC oil price. We consider the weekly difference for the interest rates while we used log-returns for the OPEC oil price.

We do not include a world stock market index among the explanatory variables for a number of reasons. First, the inclusion of a world index requires additional discussions on the interpretation of its coefficients within a CAPM-like framework, in particular because we use some indices related to the top segment of the markets and not broad market indices. Despite the relevance of the topic (see Engle and Rangel, 2007, for an example), it does not represent the focus of our contribution, which is the study of heterogeneous impact of variances on correlation between transition and developed markets. Furthermore, the introduction of a world index may generate some doubts on the estimation approach that should be adopted, given the possible correlation between the world index and the residuals of the national indices equations, and on the existence of additional common factors. Engle and Rangel (2007) present some interesting observations on these aspects, evidencing the need for a specific treatment.

V. Empirical Results

In order to capture the lagged dependence structure in the returns of the analyzed data series, we specify the mean dynamics as a VAR model:

$$\begin{bmatrix} X_t \\ Z_t \end{bmatrix} = \begin{bmatrix} \mu_X \\ \mu_Z \end{bmatrix} + \begin{bmatrix} \phi_{xx}(L) & \phi_{xz}(L) \\ \phi_{zx}(L) & \phi_{zz}(L) \end{bmatrix} \begin{bmatrix} X_t \\ Z_t \end{bmatrix} + \begin{bmatrix} \varepsilon_{x,t} \\ \varepsilon_{z,t} \end{bmatrix} \tag{21}$$

where X_t is the set of stock market returns and Z_t is the set of additional economic and financial variables described in the previous section. To assess whether the considered VAR specification is adequate we perform a Granger-causality test on the matrix $\phi_{zx}(L)$, in order to verify the null hypothesis of no effect from the stock markets on the variables in Z_t. The Wald coefficient test indicates that the null hypothesis of non-causality is rejected. We therefore continue to use the full VAR specification above for the mean estimation.

The residuals from the mean model are then used for modeling the volatility of the considered stock market indices. On all seven residuals series we fit a standard GARCH(1,1) specification of Bollerslev (1986) as well as the asymmetric generalization of Glosten et al. (1993) (the GJR(1,1) model). We perform the choice of the volatility models based on the Schwarz Information Criterion (SIC). Interestingly, the estimates indicate that for all seven indices the model preferred by SIC is GARCH(1,1)[9].

[INSERT TABLES 3 AND 4 ABOUT HERE]

Table 3 presents the cross-country correlations of standardized residuals, once the conditional heteroskedasticity has been removed, while table 4 presents the correlations of the fitted volatility series. In both cases the correlations among the developed markets are higher than the correlations between developed and transition markets, and among the transition markets. As expected, Tables 2 and 3 are very similar. There is, however, an important difference between the two sets of correlations in Tables 2 and 3, compared to the correlations in Table 4. The cross-correlations between developed and transition markets are lower in variances (Table 4) than in the returns or residuals (Tables 2 and 3). Furthermore, the correlations among developed markets and among transition markets are higher between variances than between returns and between standardized residuals. While the correlations among the developed markets volatilities range

[9] Variance models estimated are not reported. They are available from the authors upon request.

from 0.85 for the pair S&P 500 and DAX 30, to 0.94 for the pair CAC 40 and DAX 30, the cross-market correlations of the developed and transition volatility series range from 0.08 for the pair BUX 30 and S&P 500, to 0.29 for WGI 20 and FTSE100. The volatility correlations among transition markets are relatively high: 0.47 for the pair WGI 20 and PX50, 0.65 for BUX 30 and PX50, and 0.69 for WGI 20 and BUX 30.

[INSERT FIGURE 1 ABOUT HERE]

Figure 1 illustrates the development of the volatilities over the considered sample period. We evidence that the volatilities of the developed markets commove, and react to significant international events in a similar manner.[10] In the case of transition markets, it is interesting to note that while the reaction of these markets to the Asian crisis in late 1997 and to the Russian default in August-September 1998 was very strong, other major international events like September 11, the new economy bubble burst (stock prices reached their lowest level in October 2002), or the beginning of the military operations in Iraq in spring 2003, did not have a pronounced effect[11].

Volatility Threshold Dynamic Conditional Correlation Estimates

In section 2.2 we have presented alternative approaches we could follow for the definition of the volatility thresholds. In the empirical application below, we consider two types of thresholds: (i) the series specific thresholds and (ii) the common thresholds based on the standardized conditional variances[12]. Table 5 presents the specific and common volatility thresholds estimated for the 90% and 75% fractiles of the empirical density of the conditional variances. We motivate the choice of these two fractiles by the fact that the 90-th% fractile would capture the cases of

[10] For the formal analysis of the cross-country volatility comovements, particularly focusing on the periods of high volatility, see Edwards and Susmel (2001).
[11] The volatility peak in June 2006 for the Czech Republic (PX) may be associated with the political elections.
[12] See section 2.2. for the procedure employed for the calculation of the thresholds.

extreme volatility in the markets, while the 75-th% fractile would involve cases of relatively high, but not only extreme volatility.

[INSERT TABLE 5 ABOUT HERE]

The reported estimates indicate only minor deviations between the thresholds calculated on the series specific and common basis. Therefore, for brevity, we limit our presentation below to the series specific thresholds[13]. We now turn to the estimation of the Volatility Threshold specifications discussed in section 2. As our major interest is to explore potentially heterogeneous impacts of high volatility on correlations of different market pairs, we concentrate on the two extensions of the basic/restricted model in (6). We consider two cases: (i) the unrestricted series specific GARCH correlation dynamic and series specific volatility impact parameters referred to as the VT-GDCC model (eq. (12)), and (ii) the restricted GARCH correlation dynamic but series specific volatility impact parameters referred to as the restricted VT-GDCC model (eq. (13)). All correlation models are estimated with contemporaneous or lagged variance threshold effects, and volatility thresholds defined with the 'and' or 'or' conditions (eq. (7) and (8)). We summarize the range of the estimated specifications in the table below. Note that in all cases the volatility impact parameters have not been restricted.

	GARCH correlation dynamic parameters	*Volatility Threshold with "or" condition*
Specification 1	Unrestricted	Contemporaneous
Specification 2	Unrestricted	Lagged
Specification 3	Restricted	Contemporaneous
Specification 4	Restricted	Lagged
		Volatility Threshold with "and" condition
Specification 5	Unrestricted	Contemporaneous
Specification 6	Unrestricted	Lagged
Specification 7	Restricted	Contemporaneous
Specification 8	Restricted	Lagged

[13] The results based on the common thresholds are qualitatively similar and are available upon request.

Table 6 reports the estimates of *Specification 1* for the 90% fractile of the conditional variances. This specification builds on unrestricted GARCH correlation dynamic and volatility impact parameters, a contemporaneous volatility threshold and the condition that the volatility exceeds the threshold at least in one of the two markets in the pair (“or” condition). Significant ARCH ($\alpha_i\alpha_j$) and GARCH ($\beta_i\beta_j$) effects are present in the correlation dynamic of all market pairs. It is important to note, however, that the GARCH effects are higher for the developed market pairs than for the pairs involving the transition markets. For pairs including only transition markets the GARCH coefficients are close to 0.98 while pairs involving at least one transition market have coefficients that vary between 0.81 and 0.91. This implies higher correlation persistence between developed markets, measured by the sum of ARCH and GARCH effects. This is a somewhat expected result, evidencing the higher stability of the correlation between developed markets. Turning to the analysis of the effects of high volatility on the correlation levels, captured by the parameter products $\gamma_i\gamma_j$, the following observations are worth noting: there is a significant increase in the correlations among the developed markets associated with high volatility (at least in one) of the underlying markets; for the market pairs involving the transition markets, the volatility impact effect appears only if we consider pairs involving the Polish (WGI) market and one developed market, while for pairs including the Hungarian (BUX) and the Czech (PX) indices this effect is not significant at conventional levels. Thus, there is an evidence of Volatility Thresholds effects for a number of correlations.

While *Specification 1* investigates the contemporaneous relation between correlations and volatilities in the underlying markets, *Specification 2* explores whether high volatility in the markets affects the level of their correlations with a lag. The t-statistics of the parameter products $\gamma_i\gamma_j$, reported in Table 7, indicate that the significance of the lagged volatility threshold effects in the considered sample is on average higher than that of the contemporaneous effects in Table 6.

The estimates for this specification show that the positive effects of lagged high volatility on the correlations of the Hungarian (BUX) market with the developed markets are marginally significant. On the other hand, the correlations of the Czech (PX) market with the other markets do not significantly increase following high volatility in the underlying markets. The remaining coefficients, related to the traditional GDCC dynamic, remain close to the ones reported in Table 6. Finally, we note that in both cases, the highest coefficient values are associated with the volatility threshold effects involving the Polish market. The correlations between a developed market and the Polish market are more sensitive to high volatility states than the correlations between developed markets. The comparison between Specifications 1 and 2 is however not conclusive, there is not a clear preference for one model. We can simply evidence that the use of a lagged VT component implies a dependence of the actual correlation on the previous period conditional variances, which in turn depend on market innovations with a further lag. Therefore, lagged VT effects imply a dependence on cross-market shocks with two lags. This may be in any case of interest given that we may presume that the correlation reaction is not immediate.

Specifications 3 and 4 in table 8 restrict the GARCH correlations dynamic as presented in (13), i.e. the diagonal elements of the parameter matrices A and B are set to be identical. *Specification 3* considers the contemporaneous and *Specification 4* the lagged volatility threshold effects. One observes some differences between the estimates of the volatility threshold parameters of the unrestricted and the restricted GARCH dynamic models. In the restricted specifications the significance of the volatility threshold effects for the pairs involving the Hungarian BUX and the Czech PX is much lower (in the Czech case this effect even changes its sign). In order to compare *Specifications 1* and *2* with *Specifications 3* and *4*, respectively, we run likelihood ratio tests, reported in Table 15, Panel A.

The specifications with the unrestricted dynamic are preferred. This result, as well as some variation in the estimates related to the volatility threshold effects, emphasize that in the heterogeneous sample, similar to ours, allowing for the series specific GARCH correlation dynamic is important.

Table 15, Panels A and B, reports also the likelihood ratio tests between the restricted and unrestricted versions of other basic model specifications considered in the empirical part of this paper. In all cases, the unrestricted version is preferred. To conserve space for the further specifications we report the estimates of the unrestricted versions only.[14]

We now repeat the estimation of S*pecifications 1* and *2*, however with the volatility threshold set at 75% fractile of the empirical density of the conditional variance series. The estimates, presented in Tables 9 and 10, respectively, indicate that for both specifications considered, in most cases there is no significant effect of volatility on correlations at the 75% fractile threshold level. A few exceptions include the correlations of the Polish (WGI) with the US (SP) market, and the UK (FTSE) with the French (CAC) market, where the correlations increase with the volatilities in at least one of the underlying markets exceeding the 75% threshold. We may interpret the fact that the Volatility Threshold effect appears with a threshold set at the 90% fractile, while it almost disappears at the 75% fractile, as a relevance of very high volatility values only. We could use higher fractiles, but the limited number of events covered by these fractiles may create convergence problems and distortions in the inference procedures.

Specifications 5 and *6* consider the relation between the correlation and the underlying volatilities with a condition that volatilities in both underlying markets exceed a certain threshold, the so-called ‘and’ condition. We report the results for the 90% fractile of the conditional variance series in Table 11 for the contemporaneous threshold (*Specification 5*) and in Table 12 for the lagged threshold (*Specification 6*). The results for these two specifications are similar. They indicate that for all market pairs involving the transition markets (including the Polish (WGI) market), the periods with very high volatility in both markets are not associated with an increase in the correlations between these markets. The t-statistics reflecting the significance of this effect range

[14] The estimates of the restricted versions are available upon request.

from 0.1547 for the Hungarian (BUX) and the Czech (PX) markets pair to 0.6742 for the French (CAC) and Polish (WGI) markets pair. On the other hand, the high volatility in both underlying markets is associated with a significant increase in the correlations between the developed markets. The traditional GDCC coefficients are not affected by the changes in the Volatility Threshold component and are comparable to those reported in Tables 6 and 7.

The following set of estimates refers to *Specifications 5* and *6*, with the volatility threshold set at 75% level (Tables 13 and 14). For the case of the contemporaneous threshold (*Specification 5*), one observes a highly significant association between high underlying volatilities and the correlations of the Polish (WGI) market with the developed markets. The analysis of this result in combination with the corresponding results for the 90% threshold level in Table 11 indicates that these significant effects are generated by the volatilities in the 75% - 90% range of the empirical density of the underlying conditional variance series. For the developed market pairs the effect of volatility threshold on the correlations weakens at the 75% as compared to the 90% level. For the case of the lagged threshold (*Specification 6*), the “Polish” effect is reduced to being now only marginally significant, with the effects for the developed market pairs being weaker than in the contemporaneous case as well. In both *Specifications 5* and *6*, for the pairs involving the Hungarian (BUX) and the Czech (PX) markets, there is no evidence of increased correlations associated with volatilities exceeding 75% threshold level. The other correlation parameters (associated with the basic GDCC components) are comparable to those reported in Tables 11 and 12. We note that there is an evident difference between the VT component defined using the ‘or’ condition and that based on the ‘and’ condition. In fact, using the ‘or’ condition the VT effect emerges at higher fractiles only (exceeding 90%), while with the ‘and’ condition it is also observed at lower fractiles. However, with the exception of the Polish index, 90% VT effects are associated with higher coefficients, which can be interpreted as a more evident impact on correlations. We must emphasize that the number of observations satisfying the ‘and’ condition is, by construction, lower than in the case of the more flexible ‘or’ condition. It is also likely that the ‘and’ condition, which is more stringent, may be associated with very extreme cases, where both markets are concordant in the violation of the variance threshold. However, in general, the empirical evidence shows that in both cases, of concordant or discordant violations, the results are quite similar, indicating the presence of a positive VT effect. This implies that high volatility is associated with an increase in the correlations.

Volatility threshold and asymmetry effects

We estimate an additional group of specifications exploring the association of the VT effect with the sign of the innovations. The purpose of these additional model specifications is to verify if the correlation increase we evidence in case of high volatility takes place in association with negative innovations, that is, in periods of bear markets (which we may interpret as a large negative market correction or as the presence of a diffuse financial crisis). We estimate the Asymmetric VT representations introducing two VT matrices, the baseline effect represented in equation (7) and the negative asymmetric effect of equation (17). We report results in Tables 16 and 17 for the thresholds defined at the 90% and 75% fractiles, respectively. The estimates evidence that there is a relevant link between the existence of the Volatility Threshold effect and the occurrence of high volatility associated with negative shocks. The effect is statistically significant for correlations among the developed markets. For the market pairs involving the Polish market and a developed market, this effect is significant at the 75% fractile only. Furthermore, we estimate similar specifications including the baseline model together with the dummy matrix in (17) but with positive innovations. In this second case, the asymmetric coefficients turn out to be lower (in absolute values) and less significant (the number of significant coefficients decreases as well as the overall significance), see Tables 18 and 19. These results show that our model is able to capture the asymmetric dependence of financial return series documented in the previous literature, namely higher return correlations in the volatile bear than in volatile bull markets (see footnote 1). The model, therefore, posses an important feature relevant for the implementation of the optimal portfolio allocation and risk management decisions. However, our results are only partial given that a complete model specification (with the inclusion of the baseline VT effect and the two dummy matrices associated with positive and negative shocks) cannot be consistently estimated. In fact, the larger parameter number, the overlap between the dummy matrices (which does not create in any case an identification problem) and the existence in our dataset of market pairs with only a small number of observed threshold violations (this is true for both positive and negative returns, but the inclusion of both matrices worsen the problem), generate convergence problems in the estimation algorithms. Furthermore, we must consider our results as partial with respect to the asymmetry evidence. In our sample, for some market pairs, we observe a very limited number of threshold violations, when the dummy variable matrix is as in (17), with the ‘and’ instead of the ‘or’ condition (i.e. high volatility states associated with negative/positive innovations in both markets). This fact does not allow consistent estimation of the VT coefficients.

VI. Conclusions

This paper introduces a class of Volatility Threshold Dynamic Conditional Correlation models, in which the correlation dynamic partially depends on variance values through a threshold structure. These models allow an analysis of the dynamic behavior of correlations between assets in the periods of high volatility, and, therefore, present a tool, which could be applied to several areas including optimal portfolio decisions, hedging and contagion analysis.

The empirical application of the proposed Volatility Threshold specifications to a sample of international stock markets comprising developed and transition markets (Hungary, Poland and the Czech Republic) reveals heterogeneity in the relation between correlations and high volatility values for different market pairs in the sample. For most of the considered specifications, high underlying volatility implies an increase in the correlations among the developed markets and in the correlations between the Polish and the developed markets. The effect of high volatility on the correlations of the market pairs involving the Hungarian and the Czech markets is typically insignificant. Additionally, for the market pairs involving developed markets, our model captures the previously documented form of asymmetric dependence of financial return series, expressed in higher return correlations associated with the volatile bear rather than the volatile bull markets. The cross-sectional determinants of the observed heterogeneity (in the response of correlations to high volatility values) are most likely related to factors driving the segmentation/integration of a particular market from/into world capital markets. The study of these cross-sectional determinants, as well as of the potential implications of our findings for international asset allocation and portfolio construction considerations are interesting topics for future research.

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Table 1.

Descriptive statistics

	SP500	DAX30	CAC40	FTSE100	BUX30	WIG20	PX50
Mean	0.0017 (1.5336)	0.0020 (1.5619)	0.0017 (1.5236)	0.0014 (1.4466)	0.0037 (2.2164)	0.0021 (1.0877)	0.0022 (1.6556)
Max	0.1289	0.1506	0.1358	0.0967	0.1989	0.1759	0.1155
Min	-0.1222	-0.1978	-0.1409	-0.1279	-0.2704	-0.3005	-0.1769
St.dev.	0.0276	0.0332	0.0293	0.0248	0.0432	0.0494	0.0332
Skewness	-0.1654	-0.7260	-0.5788	-0.6208	-0.7193	-0.6127	-0.5580
Kurtosis	5.1059	7.2547	7.0398	6.6250	8.0190	7.1458	5.8308
JB	124.0228	551.5894	481.9758	400.6999	743.9567	510.0736	252.6942
LB(6)	19.093	10.632	15.652	14.697	29.972	18.074	19.545
LBS(6)	91.83	155.51	187.81	57.718	165.46	58.869	200.15

Stock market indices descriptive analysis for developer markets and transition markets (BUX30, Hungary – WIG20, Poland – PX50 Czech Republic): below the mean we report t-statistics for the null hypothesis of the mean to be equal to zero. JB is the Jarque-Bera test statistic, distributed as a χ_2^2; LB(6) and LBS(6) are Ljung-Box test statistics with 6 lags for return levels and return squares, respectively, distributed as a χ_6^2. The upper 1 and 5 percentile points of the χ_2^2 distribution are 9.21 and 5.99, respectively. The upper 1 and 5 percentile points of the χ_6^2 distribution are 16.81 and 12.59, respectively.

Table 2.

The cross-correlations of stock market returns

	SP500	DAX30	CAC40	FTSE100	BUX30	WIG20	PX50
SP500	1						
DAX30	0.7144	1					
CAC40	0.7112	0.8747	1				
FTSE100	0.7411	0.7657	0.7945	1			
BUX30	0.4184	0.4360	0.4174	0.4660	1		
WIG20	0.4133	0.4631	0.4535	0.4201	0.5747	1	
PX50	0.2853	0.3875	0.3918	0.3900	0.5396	0.4648	1

Table 3.

The cross-market correlations of the standardized residuals

	SP500	DAX30	CAC40	FTSE100	BUX30	WIG20	PX50
SP500	1						
DAX30	0.6823	1					
CAC40	0.6806	0.8500	1				
FTSE100	0.7221	0.7436	0.7580	1			
BUX30	0.4112	0.4519	0.4347	0.4706	1		
WIG20	0.4043	0.4738	0.4902	0.4357	0.5644	1	
PX50	0.2841	0.3759	0.3828	0.3891	0.5136	0.4483	1

Table 4.

The cross-market correlations of the GARCH volatility

	SP500	DAX30	CAC40	FTSE100	BUX30	WIG20	PX50
SP500	1						
DAX30	0.8466	1					
CAC40	0.8864	0.9395	1				
FTSE100	0.8985	0.8898	0.9126	1			
BUX30	0.0763	0.1539	0.1310	0.2426	1		
WIG20	0.1906	0.1195	0.1526	0.2869	0.6916	1	
PX50	0.2049	0.2370	0.1974	0.2712	0.6460	0.4719	1

Figure 1.

GARCH standardized volatility

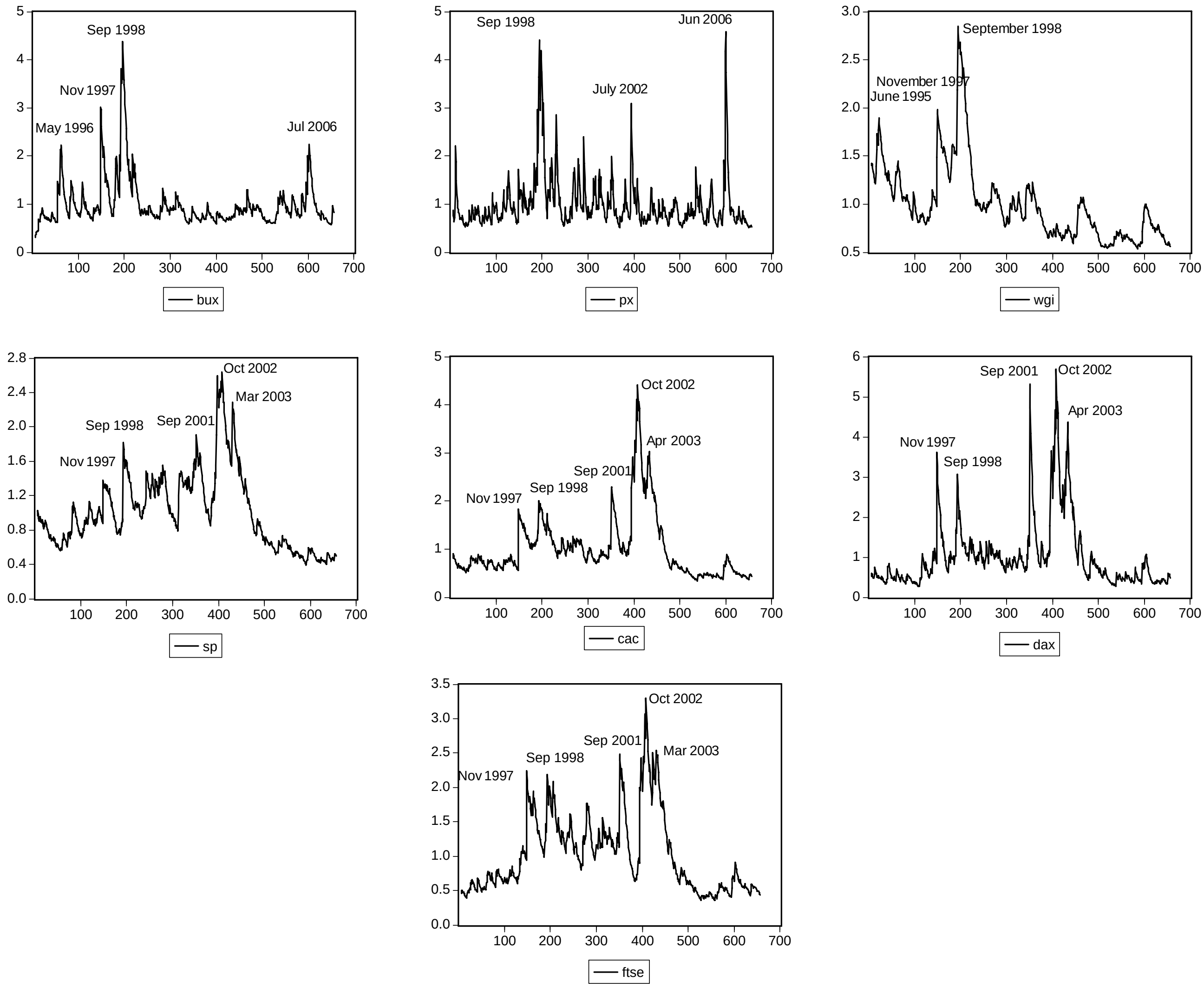

Table 5.

Volatility thresholds

	BUX30	PX50	WGI20	S&P500	CAC40	DAX30	FTSE100
Series specific thresholds							
75%	0.00163	0.00106	0.00239	0.00086	0.00086	0.00109	0.00070
90%	0.00235	0.00148	0.00326	0.00104	0.00136	0.00198	0.00100
Common thresholds							
75%	0.00185	0.00113	0.00250	0.00078	0.00087	0.00113	0.00062
90%	0.00265	0.00161	0.00356	0.00111	0.00124	0.00160	0.00088

Table 6

Specification 1:

$$q_{ij,t} = (1-\alpha_i\alpha_j - \beta_i\beta_j)\overline{q}_{ij} - \gamma_i\gamma_j\overline{v}_{ij} + \alpha_i\alpha_j\eta_{i,t-1}\eta_{j,t-1} + \beta_i\beta_j q_{ij,t-1} + \gamma_i\gamma_j v_{ij,t}$$

$$v_{ij,t} = \begin{cases} 1 \quad \text{if } h_{i,t} > d\left(\left\{h_{i,t}\right\}_{t=1}^{T}\right) \text{ or } h_{j,t} > d\left(\left\{h_{j,t}\right\}_{t=1}^{T}\right) \\ 0 \qquad \qquad \qquad \textit{otherwise} \end{cases}; \; d = 90\%$$

	Coef	t-stat		Coef	t-stat		Coef	t-stat
$\alpha_{px}\alpha_{bux}$	0.0109	1.4345	$\beta_{px}\beta_{bux}$	0.8110	8.7097	$\gamma_{px}\gamma_{bux}$	0.0132	1.0030
$\alpha_{wgi}\alpha_{bux}$	0.0142	1.8940	$\beta_{wgi}\beta_{bux}$	0.8510	15.6992	$\gamma_{wgi}\gamma_{bux}$	0.0196	1.1710
$\alpha_{sp}\alpha_{bux}$	0.0060	2.2535	$\beta_{sp}\beta_{bux}$	0.9182	19.5640	$\gamma_{sp}\gamma_{bux}$	0.0131	1.5518
$\alpha_{cac}\alpha_{bux}$	0.0100	2.3328	$\beta_{cac}\beta_{bux}$	0.9191	19.5302	$\gamma_{cac}\gamma_{bux}$	0.0092	1.4974
$\alpha_{dax}\alpha_{bux}$	0.0096	2.2287	$\beta_{dax}\beta_{bux}$	0.9194	19.5278	$\gamma_{dax}\gamma_{bux}$	0.0076	1.4679
$\alpha_{ftse}\alpha_{bux}$	0.0073	2.2963	$\beta_{ftse}\beta_{bux}$	0.9180	19.6259	$\gamma_{ftse}\gamma_{bux}$	0.0121	1.5869
$\alpha_{wgi}\alpha_{px}$	0.0251	2.0478	$\beta_{wgi}\beta_{px}$	0.8032	9.7637	$\gamma_{wgi}\gamma_{px}$	0.0268	1.2448
$\alpha_{sp}\alpha_{px}$	0.0107	2.1251	$\beta_{sp}\beta_{px}$	0.8666	10.5734	$\gamma_{sp}\gamma_{px}$	0.0180	1.4139
$\alpha_{cac}\alpha_{px}$	0.0178	2.2997	$\beta_{cac}\beta_{px}$	0.8675	10.5480	$\gamma_{cac}\gamma_{px}$	0.0126	1.3617
$\alpha_{dax}\alpha_{px}$	0.0170	2.2733	$\beta_{dax}\beta_{px}$	0.8678	10.5702	$\gamma_{dax}\gamma_{px}$	0.0105	1.3900
$\alpha_{ftse}\alpha_{px}$	0.0129	2.1543	$\beta_{ftse}\beta_{px}$	0.8665	10.5322	$\gamma_{ftse}\gamma_{px}$	0.0167	1.3424
$\alpha_{sp}\alpha_{wgi}$	0.0138	2.9835	$\beta_{sp}\beta_{wgi}$	0.9094	30.7645	$\gamma_{sp}\gamma_{wgi}$	0.0266	2.2033
$\alpha_{cac}\alpha_{wgi}$	0.0231	3.7710	$\beta_{cac}\beta_{wgi}$	0.9103	31.2494	$\gamma_{cac}\gamma_{wgi}$	0.0187	2.1018
$\alpha_{dax}\alpha_{wgi}$	0.0221	3.4971	$\beta_{dax}\beta_{wgi}$	0.9106	31.1137	$\gamma_{dax}\gamma_{wgi}$	0.0155	2.1222
$\alpha_{ftse}\alpha_{wgi}$	0.0167	3.3245	$\beta_{ftse}\beta_{wgi}$	0.9092	31.3356	$\gamma_{ftse}\gamma_{wgi}$	0.0246	2.6873
$\alpha_{cac}\alpha_{sp}$	0.0098	3.9722	$\beta_{cac}\beta_{sp}$	0.9821	158.1430	$\gamma_{cac}\gamma_{sp}$	0.0125	2.6719
$\alpha_{dax}\alpha_{sp}$	0.0094	3.6110	$\beta_{dax}\beta_{sp}$	0.9824	143.4225	$\gamma_{dax}\gamma_{sp}$	0.0104	2.6171
$\alpha_{ftse}\alpha_{sp}$	0.0071	3.0375	$\beta_{ftse}\beta_{sp}$	0.9810	127.7777	$\gamma_{ftse}\gamma_{sp}$	0.0165	2.7267
$\alpha_{dax}\alpha_{cac}$	0.0156	5.6871	$\beta_{dax}\beta_{cac}$	0.9834	262.6808	$\gamma_{dax}\gamma_{cac}$	0.0073	2.4556
$\alpha_{ftse}\alpha_{cac}$	0.0118	4.8034	$\beta_{ftse}\beta_{cac}$	0.9819	241.7162	$\gamma_{ftse}\gamma_{cac}$	0.0116	2.9984
$\alpha_{ftse}\alpha_{dax}$	0.0113	4.4443	$\beta_{ftse}\beta_{dax}$	0.9822	209.0443	$\gamma_{ftse}\gamma_{dax}$	0.0096	2.8735

LL -4946.410

Estimated coefficients, t-statistics and log-likelihood for the VT-DCC model above specified. T-statistics of the parameter functions are calculated using the delta method.

Table 7.

Specification 2:

$$q_{ij,t} = (1-\alpha_i\alpha_j-\beta_i\beta_j)\overline{q}_{ij}-\gamma_i\gamma_j\overline{v}_{ij}+\alpha_i\alpha_j\eta_{i,t-1}\eta_{j,t-1}+\beta_i\beta_j q_{ij,t-1}+\gamma_i\gamma_j v_{ij,t-1}$$

$$v_{ij,t}=\begin{cases}1 \quad \text{if } h_{i,t}>d\left(\left\{h_{i,t}\right\}_{t=1}^{T}\right) \text{ or } h_{j,t}>d\left(\left\{h_{j,t}\right\}_{t=1}^{T}\right) \\ 0 \qquad \qquad \qquad \textit{otherwise}\end{cases};\ d=90\%$$

	Coef	t-stat		Coef	t-stat		Coef	t-stat
$\alpha_{px}\alpha_{bux}$	0.0112	1.4927	$\beta_{px}\beta_{bux}$	0.8117	8.7121	$\gamma_{px}\gamma_{bux}$	0.0133	1.0300
$\alpha_{wgi}\alpha_{bux}$	0.0166	2.0523	$\beta_{wgi}\beta_{bux}$	0.8332	14.8548	$\gamma_{wgi}\gamma_{bux}$	0.0305	1.4435
$\alpha_{sp}\alpha_{bux}$	0.0059	2.2979	$\beta_{sp}\beta_{bux}$	0.9136	18.4626	$\gamma_{sp}\gamma_{bux}$	0.0136	1.7823
$\alpha_{cac}\alpha_{bux}$	0.0105	2.5249	$\beta_{cac}\beta_{bux}$	0.9099	18.4019	$\gamma_{cac}\gamma_{bux}$	0.0097	1.7490
$\alpha_{dax}\alpha_{bux}$	0.0101	2.4065	$\beta_{dax}\beta_{bux}$	0.9111	18.3940	$\gamma_{dax}\gamma_{bux}$	0.0088	1.6984
$\alpha_{ftse}\alpha_{bux}$	0.0075	2.4801	$\gamma_{ftse}\gamma_{bux}$	0.9103	18.4492	$\gamma_{ftse}\gamma_{bux}$	0.0134	1.8123
$\alpha_{wgi}\alpha_{px}$	0.0266	2.0927	$\beta_{wgi}\beta_{px}$	0.8028	9.9518	$\gamma_{wgi}\gamma_{px}$	0.0302	1.2367
$\alpha_{sp}\alpha_{px}$	0.0095	2.0948	$\beta_{sp}\beta_{px}$	0.8803	11.0291	$\gamma_{sp}\gamma_{px}$	0.0135	1.3785
$\alpha_{cac}\alpha_{px}$	0.0169	2.3420	$\beta_{cac}\beta_{px}$	0.8767	11.0098	$\gamma_{cac}\gamma_{px}$	0.0097	1.3576
$\alpha_{dax}\alpha_{px}$	0.0161	2.3050	$\beta_{dax}\beta_{px}$	0.8779	11.0167	$\gamma_{dax}\gamma_{px}$	0.0087	1.3720
$\alpha_{ftse}\alpha_{px}$	0.0120	2.1874	$\beta_{ftse}\beta_{px}$	0.8771	10.9977	$\gamma_{ftse}\gamma_{px}$	0.0133	1.3483
$\alpha_{sp}\alpha_{wgi}$	0.0141	2.8800	$\beta_{sp}\beta_{wgi}$	0.9035	30.2348	$\gamma_{sp}\gamma_{wgi}$	0.0310	2.7079
$\alpha_{cac}\alpha_{wgi}$	0.0251	4.0580	$\beta_{cac}\beta_{wgi}$	0.8999	30.0752	$\gamma_{cac}\gamma_{wgi}$	0.0222	2.5863
$\alpha_{dax}\alpha_{wgi}$	0.0240	3.7554	$\beta_{dax}\beta_{wgi}$	0.9011	30.0227	$\gamma_{dax}\gamma_{wgi}$	0.0201	2.5482
$\alpha_{ftse}\alpha_{wgi}$	0.0178	3.5916	$\beta_{ftse}\beta_{wgi}$	0.9003	30.1364	$\gamma_{ftse}\gamma_{wgi}$	0.0306	3.1242
$\alpha_{cac}\alpha_{sp}$	0.0089	3.5462	$\beta_{cac}\beta_{sp}$	0.9867	205.0052	$\gamma_{cac}\gamma_{sp}$	0.0099	3.1231
$\alpha_{dax}\alpha_{sp}$	0.0085	3.0969	$\beta_{dax}\beta_{sp}$	0.9881	194.0280	$\gamma_{dax}\gamma_{sp}$	0.0090	3.0522
$\alpha_{ftse}\alpha_{sp}$	0.0063	2.8146	$\beta_{ftse}\beta_{sp}$	0.9871	180.4744	$\gamma_{ftse}\gamma_{sp}$	0.0136	3.1817
$\alpha_{dax}\alpha_{cac}$	0.0152	5.6528	$\beta_{dax}\beta_{cac}$	0.9841	272.6420	$\gamma_{dax}\gamma_{cac}$	0.0064	2.6880
$\alpha_{ftse}\alpha_{cac}$	0.0113	4.9758	$\beta_{ftse}\beta_{cac}$	0.9831	270.2640	$\gamma_{ftse}\gamma_{cac}$	0.0098	3.2647
$\alpha_{ftse}\alpha_{dax}$	0.0108	4.6039	$\beta_{ftse}\beta_{dax}$	0.9845	250.4771	$\gamma_{ftse}\gamma_{dax}$	0.0089	3.0567

LL -4946.070

Estimated coefficients, t-statistics and log-likelihood for the VT-DCC model above specified. T-statistics of the parameter functions are calculated using the delta method.

Table 8.

$$\textit{Specification 3:}\ q_{ij,t} = (1-\alpha^2-\beta^2)\overline{q}_{ij} - \gamma_i\gamma_j\overline{v}_{ij} + \alpha^2\eta_{i,t-1}\eta_{j,t-1} + \beta^2 q_{ij,t-1} + \gamma_i\gamma_j v_{ij,t}$$

$$\textit{Specification 4:}\ q_{ij,t} = (1-\alpha^2-\beta^2)\overline{q}_{ij} - \gamma_i\gamma_j\overline{v}_{ij} + \alpha^2\eta_{i,t-1}\eta_{j,t-1} + \beta^2 q_{ij,t-1} + \gamma_i\gamma_j v_{ij,t-1}$$

$$v_{ij,t} = \begin{cases} 1 \quad \text{if } h_{i,t} > d\left(\left\{h_{i,t}\right\}_{t=1}^{T}\right) \text{ or } h_{j,t} > d\left(\left\{h_{j,t}\right\}_{t=1}^{T}\right) \\ 0 \qquad\qquad otherwise \end{cases};\ d = 90\%$$

	Specification 3 Coef	t-stat		*Specification 4* Coef	t-stat
α^2	0.0079	13.38004	α^2	0.00779	13.21574
β^2	0.98492	704.9643	β^2	0.98497	703.2758
$\gamma_{px}\gamma_{bux}$	-3.38E-05	-0.13131	$\gamma_{px}\gamma_{bux}$	0.00004	0.13201
$\gamma_{wgi}\gamma_{bux}$	5.80E-04	0.56855	$\gamma_{wgi}\gamma_{bux}$	0.00059	0.60582
$\gamma_{sp}\gamma_{bux}$	0.00130	0.65098	$\gamma_{sp}\gamma_{bux}$	0.00145	0.71265
$\gamma_{cac}\gamma_{bux}$	0.00115	0.64074	$\gamma_{cac}\gamma_{bux}$	0.00128	0.69796
$\gamma_{dax}\gamma_{bux}$	0.00116	0.64672	$\gamma_{dax}\gamma_{bux}$	0.00128	0.70483
$\gamma_{ftse}\gamma_{bux}$	0.00122	0.64319	$\gamma_{ftse}\gamma_{bux}$	0.00139	0.70336
$\gamma_{wgi}\gamma_{px}$	-1.60E-04	-0.12736	$\gamma_{wgi}\gamma_{px}$	0.00017	0.13954
$\gamma_{sp}\gamma_{px}$	-3.58E-04	-0.12557	$\gamma_{sp}\gamma_{px}$	0.00041	0.14201
$\gamma_{cac}\gamma_{px}$	-3.18E-04	-0.12623	$\gamma_{cac}\gamma_{px}$	0.00037	0.14115
$\gamma_{dax}\gamma_{px}$	-3.21E-04	-0.12631	$\gamma_{dax}\gamma_{px}$	0.00036	0.14104
$\gamma_{ftse}\gamma_{px}$	-3.38E-04	-0.12626	$\gamma_{ftse}\gamma_{px}$	0.00040	0.14117
$\gamma_{sp}\gamma_{wgi}$	0.00615	2.44187	$\gamma_{sp}\gamma_{wgi}$	0.00579	2.29744
$\gamma_{cac}\gamma_{wgi}$	0.00545	2.23219	$\gamma_{cac}\gamma_{wgi}$	0.00512	2.07220
$\gamma_{dax}\gamma_{wgi}$	0.00551	2.28065	$\gamma_{dax}\gamma_{wgi}$	0.00510	2.11857
$\gamma_{ftse}\gamma_{wgi}$	0.00580	2.34595	$\gamma_{ftse}\gamma_{wgi}$	0.00553	2.20055
$\gamma_{cac}\gamma_{sp}$	0.01218	3.65015	$\gamma_{cac}\gamma_{sp}$	0.01264	3.68494
$\gamma_{dax}\gamma_{sp}$	0.01232	3.40756	$\gamma_{dax}\gamma_{sp}$	0.01259	3.40159
$\gamma_{ftse}\gamma_{sp}$	0.01297	3.35723	$\gamma_{ftse}\gamma_{sp}$	0.01365	3.38016
$\gamma_{dax}\gamma_{cac}$	0.01092	3.25617	$\gamma_{dax}\gamma_{cac}$	0.01113	3.28140
$\gamma_{ftse}\gamma_{cac}$	0.01150	3.83578	$\gamma_{ftse}\gamma_{cac}$	0.01207	3.94264
$\gamma_{ftse}\gamma_{dax}$	0.01163	3.76627	$\gamma_{ftse}\gamma_{dax}$	0.01202	3.85061
LL	-4972.810		LL	-4972.930	

Volatility thresholds estimated coefficients, t-statistics and log-likelihood for the VT-DCC models above specified. T-statistics of the parameter functions are calculated using the delta method.

Table 9.

Specification 1:

$$q_{ij,t} = (1-\alpha_i\alpha_j - \beta_i\beta_j)\overline{q}_{ij} - \gamma_i\gamma_j\overline{v}_{ij} + \alpha_i\alpha_j\eta_{i,t-1}\eta_{j,t-1} + \beta_i\beta_j q_{ij,t-1} + \gamma_i\gamma_j v_{ij,t}$$

$$v_{ij,t} = \begin{cases} 1 \quad \text{if } h_{i,t} > d\left(\left\{h_{i,t}\right\}_{t=1}^{T}\right) \text{ or } h_{j,t} > d\left(\left\{h_{j,t}\right\}_{t=1}^{T}\right) \\ 0 \qquad \qquad \qquad \textit{otherwise} \end{cases}; \; d = 75\%$$

	Coef	t-stat		Coef	t-stat		Coef	t-stat
$\alpha_{px}\alpha_{bux}$	0.00925	1.4233	$\beta_{px}\beta_{bux}$	0.85254	11.04207	$\gamma_{px}\gamma_{bux}$	0.00510	0.91525
$\alpha_{wgi}\alpha_{bux}$	0.01322	1.86039	$\beta_{wgi}\beta_{bux}$	0.87061	17.73330	$\gamma_{wgi}\gamma_{bux}$	0.02093	1.49980
$\alpha_{sp}\alpha_{bux}$	0.00720	2.12278	$\beta_{sp}\beta_{bux}$	0.91762	20.62052	$\gamma_{sp}\gamma_{bux}$	0.00611	1.62095
$\alpha_{cac}\alpha_{bux}$	0.01134	2.21566	$\beta_{cac}\beta_{bux}$	0.92107	20.94038	$\gamma_{cac}\gamma_{bux}$	0.00355	1.38203
$\alpha_{dax}\alpha_{bux}$	0.01068	2.15591	$\beta_{dax}\beta_{bux}$	0.91959	20.88573	$\gamma_{dax}\gamma_{bux}$	0.00188	1.07419
$\alpha_{ftse}\alpha_{bux}$	0.00885	2.25235	$\gamma_{ftse}\gamma_{bux}$	0.92377	21.06308	$\gamma_{ftse}\gamma_{bux}$	0.00400	1.48548
$\alpha_{wgi}\alpha_{px}$	0.01888	1.99136	$\beta_{wgi}\beta_{px}$	0.85756	13.35561	$\gamma_{wgi}\gamma_{px}$	0.01418	1.14519
$\alpha_{sp}\alpha_{px}$	0.01028	2.06072	$\beta_{sp}\beta_{px}$	0.90387	14.06516	$\gamma_{sp}\gamma_{px}$	0.00414	1.19018
$\alpha_{cac}\alpha_{px}$	0.01619	2.24421	$\beta_{cac}\beta_{px}$	0.90727	14.12357	$\gamma_{cac}\gamma_{px}$	0.00241	1.09471
$\alpha_{dax}\alpha_{px}$	0.01525	2.20919	$\beta_{dax}\beta_{px}$	0.90581	14.07033	$\gamma_{dax}\gamma_{px}$	0.00127	0.90999
$\alpha_{ftse}\alpha_{px}$	0.01263	2.15465	$\beta_{ftse}\beta_{px}$	0.90993	14.13679	$\gamma_{ftse}\gamma_{px}$	0.00271	1.10807
$\alpha_{sp}\alpha_{wgi}$	0.01470	2.97865	$\beta_{sp}\beta_{wgi}$	0.92302	37.76439	$\gamma_{sp}\gamma_{wgi}$	0.01700	2.61146
$\alpha_{cac}\alpha_{wgi}$	0.02315	3.95398	$\beta_{cac}\beta_{wgi}$	0.9265	38.85957	$\gamma_{cac}\gamma_{wgi}$	0.00988	1.87305
$\alpha_{dax}\alpha_{wgi}$	0.02179	3.63272	$\beta_{dax}\beta_{wgi}$	0.92501	38.81414	$\gamma_{dax}\gamma_{wgi}$	0.00522	1.28742
$\alpha_{ftse}\alpha_{wgi}$	0.01806	3.76212	$\beta_{ftse}\beta_{wgi}$	0.92921	38.75320	$\gamma_{ftse}\gamma_{wgi}$	0.01113	2.16128
$\alpha_{cac}\alpha_{sp}$	0.01261	3.75121	$\beta_{cac}\beta_{sp}$	0.97653	140.4740	$\gamma_{cac}\gamma_{sp}$	0.00288	1.55296
$\alpha_{dax}\alpha_{sp}$	0.01187	3.39589	$\beta_{dax}\beta_{sp}$	0.97495	130.9339	$\gamma_{dax}\gamma_{sp}$	0.00153	1.16267
$\alpha_{ftse}\alpha_{sp}$	0.00983	3.05560	$\beta_{ftse}\beta_{sp}$	0.97939	121.0228	$\gamma_{ftse}\gamma_{sp}$	0.00325	1.58943
$\alpha_{dax}\alpha_{cac}$	0.01869	5.61381	$\beta_{dax}\beta_{cac}$	0.97863	204.8042	$\gamma_{dax}\gamma_{cac}$	8.87E-04	1.00408
$\alpha_{ftse}\alpha_{cac}$	0.01549	6.16084	$\beta_{ftse}\beta_{cac}$	0.98307	284.0230	$\gamma_{ftse}\gamma_{cac}$	0.00189	1.43528
$\alpha_{ftse}\alpha_{dax}$	0.01458	5.17516	$\beta_{ftse}\beta_{dax}$	0.98149	198.8599	$\gamma_{ftse}\gamma_{dax}$	9.99E-04	1.12241

LL -4949.990

Estimated coefficients, t-statistics and log-likelihood for the VT-DCC model above specified. T-statistics of the parameter functions are calculated using the delta method.

Table 10.

Specification 2:

$$q_{ij,t} = (1-\alpha_i\alpha_j - \beta_i\beta_j)\bar{q}_{ij} - \gamma_i\gamma_j\bar{v}_{ij} + \alpha_i\alpha_j\eta_{i,t-1}\eta_{j,t-1} + \beta_i\beta_j q_{ij,t-1} + \gamma_i\gamma_j v_{ij,t-1}$$

$$v_{ij,t} = \begin{cases} 1 \quad \text{if } h_{i,t} > d\left(\left\{h_{i,t}\right\}_{t=1}^{T}\right) \text{ or } h_{j,t} > d\left(\left\{h_{j,t}\right\}_{t=1}^{T}\right) \\ 0 \qquad \textit{otherwise} \end{cases};\ d = 75\%$$

	Coef	t-stat		Coef	t-stat		Coef	t-stat
$\alpha_{px}\alpha_{bux}$	0.00956	1.44791	$\beta_{px}\beta_{bux}$	0.84901	10.85755	$\gamma_{px}\gamma_{bux}$	0.00576	0.99560
$\alpha_{wgi}\alpha_{bux}$	0.01377	1.91785	$\beta_{wgi}\beta_{bux}$	0.86893	18.09595	$\gamma_{wgi}\gamma_{bux}$	0.02388	1.67368
$\alpha_{sp}\alpha_{bux}$	0.00737	2.15545	$\beta_{sp}\beta_{bux}$	0.91726	20.83406	$\gamma_{sp}\gamma_{bux}$	0.00581	1.72338
$\alpha_{cac}\alpha_{bux}$	0.01159	2.26367	$\beta_{cac}\beta_{bux}$	0.91946	21.16879	$\gamma_{cac}\gamma_{bux}$	0.00343	1.47164
$\alpha_{dax}\alpha_{bux}$	0.01088	2.20568	$\beta_{dax}\beta_{bux}$	0.91800	21.12538	$\gamma_{dax}\gamma_{bux}$	0.00200	1.16450
$\alpha_{ftse}\alpha_{bux}$	0.00908	2.29446	$\gamma_{ftse}\gamma_{bux}$	0.92227	21.30499	$\gamma_{ftse}\gamma_{bux}$	0.00356	1.54771
$\alpha_{wgi}\alpha_{px}$	0.01943	2.00432	$\beta_{wgi}\beta_{px}$	0.85539	13.03158	$\gamma_{wgi}\gamma_{px}$	0.01710	1.29214
$\alpha_{sp}\alpha_{px}$	0.0104	2.05783	$\beta_{sp}\beta_{px}$	0.90295	13.62728	$\gamma_{sp}\gamma_{px}$	0.00416	1.30465
$\alpha_{cac}\alpha_{px}$	0.01635	2.23929	$\beta_{cac}\beta_{px}$	0.90513	13.66656	$\gamma_{cac}\gamma_{px}$	0.00246	1.20157
$\alpha_{dax}\alpha_{px}$	0.01535	2.20777	$\beta_{dax}\beta_{px}$	0.90369	13.62231	$\gamma_{dax}\gamma_{px}$	0.00143	1.00639
$\alpha_{ftse}\alpha_{px}$	0.01281	2.14954	$\beta_{ftse}\beta_{px}$	0.90789	13.67076	$\gamma_{ftse}\gamma_{px}$	0.00255	1.19672
$\alpha_{sp}\alpha_{wgi}$	0.01497	3.11100	$\beta_{sp}\beta_{wgi}$	0.92415	38.52147	$\gamma_{sp}\gamma_{wgi}$	0.01726	2.65559
$\alpha_{cac}\alpha_{wgi}$	0.02354	4.09537	$\beta_{cac}\beta_{wgi}$	0.92637	39.69369	$\gamma_{cac}\gamma_{wgi}$	0.01019	1.94694
$\alpha_{dax}\alpha_{wgi}$	0.0221	3.73696	$\beta_{dax}\beta_{wgi}$	0.9249	39.48275	$\gamma_{dax}\gamma_{wgi}$	0.00594	1.41500
$\alpha_{ftse}\alpha_{wgi}$	0.01844	3.90854	$\beta_{ftse}\beta_{wgi}$	0.9292	39.61712	$\gamma_{ftse}\gamma_{wgi}$	0.01057	2.09703
$\alpha_{cac}\alpha_{sp}$	0.0126	3.88667	$\beta_{cac}\beta_{sp}$	0.97789	146.7829	$\gamma_{cac}\gamma_{sp}$	0.00248	1.54658
$\alpha_{dax}\alpha_{sp}$	0.01183	3.51311	$\beta_{dax}\beta_{sp}$	0.97634	134.3801	$\gamma_{dax}\gamma_{sp}$	0.00145	1.22118
$\alpha_{ftse}\alpha_{sp}$	0.00987	3.16210	$\beta_{ftse}\beta_{sp}$	0.98088	126.3844	$\gamma_{ftse}\gamma_{sp}$	0.00257	1.53758
$\alpha_{dax}\alpha_{cac}$	0.0186	5.66930	$\beta_{dax}\beta_{cac}$	0.97869	208.0575	$\gamma_{dax}\gamma_{cac}$	0.00085	1.05161
$\alpha_{ftse}\alpha_{cac}$	0.01552	6.28888	$\beta_{ftse}\beta_{cac}$	0.98324	290.2035	$\gamma_{ftse}\gamma_{cac}$	0.00152	1.38096
$\alpha_{ftse}\alpha_{dax}$	0.01457	5.30121	$\beta_{ftse}\beta_{dax}$	0.98168	203.1972	$\gamma_{ftse}\gamma_{dax}$	0.00089	1.14502

LL	-4949.700

Estimated coefficients, t-statistics and log-likelihood for the VT-DCC model above specified. T-statistics of the parameter functions are calculated using the delta method.

Table 11.

Specification 5:

$$q_{ij,t} = (1-\alpha_i\alpha_j - \beta_i\beta_j)\overline{q}_{ij} - \gamma_i\gamma_j\overline{v}_{ij} + \alpha_i\alpha_j\eta_{i,t-1}\eta_{j,t-1} + \beta_i\beta_j q_{ij,t-1} + \gamma_i\gamma_j v_{ij,t}$$

$$v_{ij,t} = \begin{cases} 1 & \text{if } h_{i,t} > d\left(\left\{h_{i,t}\right\}_{t=1}^{T}\right) \text{ and } h_{j,t} > d\left(\left\{h_{j,t}\right\}_{t=1}^{T}\right) \\ 0 & \textit{otherwise} \end{cases} ;\ d = 90\%$$

	Coef	t-stat		Coef	t-stat		Coef	t-stat
$\alpha_{px}\alpha_{bux}$	0.0111	1.4283	$\beta_{px}\beta_{bux}$	0.8383	9.1886	$\gamma_{px}\gamma_{bux}$	0.0024	0.1547
$\alpha_{wgi}\alpha_{bux}$	0.0159	2.0654	$\beta_{wgi}\beta_{bux}$	0.8555	14.6436	$\gamma_{wgi}\gamma_{bux}$	0.0037	0.1710
$\alpha_{sp}\alpha_{bux}$	0.0064	2.2915	$\beta_{sp}\beta_{bux}$	0.9167	18.0230	$\gamma_{sp}\gamma_{bux}$	0.0046	0.2038
$\alpha_{cac}\alpha_{bux}$	0.0130	2.5676	$\beta_{cac}\beta_{bux}$	0.9109	18.0138	$\gamma_{cac}\gamma_{bux}$	0.0040	0.2045
$\alpha_{dax}\alpha_{bux}$	0.0121	2.4655	$\beta_{dax}\beta_{bux}$	0.9125	18.0024	$\gamma_{dax}\gamma_{bux}$	0.0035	0.2043
$\alpha_{ftse}\alpha_{bux}$	0.0101	2.4924	$\gamma_{ftse}\gamma_{bux}$	0.9117	18.0694	$\gamma_{ftse}\gamma_{bux}$	0.0058	0.2041
$\alpha_{wgi}\alpha_{px}$	0.0195	1.8627	$\beta_{wgi}\beta_{px}$	0.8461	11.0421	$\gamma_{wgi}\gamma_{px}$	0.0094	0.2522
$\alpha_{sp}\alpha_{px}$	0.0078	1.9026	$\beta_{sp}\beta_{px}$	0.9066	12.0747	$\gamma_{sp}\gamma_{px}$	0.0118	0.2852
$\alpha_{cac}\alpha_{px}$	0.0159	2.0726	$\beta_{cac}\beta_{px}$	0.9009	12.0601	$\gamma_{cac}\gamma_{px}$	0.0103	0.2835
$\alpha_{dax}\alpha_{px}$	0.0148	2.0581	$\beta_{dax}\beta_{px}$	0.9024	12.0522	$\gamma_{dax}\gamma_{px}$	0.0089	0.2830
$\alpha_{ftse}\alpha_{px}$	0.0123	1.9709	$\beta_{ftse}\beta_{px}$	0.9017	12.0655	$\gamma_{ftse}\gamma_{px}$	0.0149	0.2838
$\alpha_{sp}\alpha_{wgi}$	0.0112	2.7454	$\beta_{sp}\beta_{wgi}$	0.9252	27.8202	$\gamma_{sp}\gamma_{wgi}$	0.0181	0.6686
$\alpha_{cac}\alpha_{wgi}$	0.0227	3.4219	$\beta_{cac}\beta_{wgi}$	0.9194	27.6695	$\gamma_{cac}\gamma_{wgi}$	0.0158	0.6742
$\alpha_{dax}\alpha_{wgi}$	0.0211	3.2461	$\beta_{dax}\beta_{wgi}$	0.9209	27.5362	$\gamma_{dax}\gamma_{wgi}$	0.0137	0.6671
$\alpha_{ftse}\alpha_{wgi}$	0.0176	3.3299	$\beta_{ftse}\beta_{wgi}$	0.9202	27.5699	$\gamma_{ftse}\gamma_{wgi}$	0.0229	0.6698
$\alpha_{cac}\alpha_{sp}$	0.0091	3.7337	$\beta_{cac}\beta_{sp}$	0.9851	255.9799	$\gamma_{cac}\gamma_{sp}$	0.0198	2.7147
$\alpha_{dax}\alpha_{sp}$	0.0085	3.2154	$\beta_{dax}\beta_{sp}$	0.9867	231.1695	$\gamma_{dax}\gamma_{sp}$	0.0171	2.4299
$\alpha_{ftse}\alpha_{sp}$	0.0071	3.0048	$\beta_{ftse}\beta_{sp}$	0.9860	232.9755	$\gamma_{ftse}\gamma_{sp}$	0.0286	2.9088
$\alpha_{dax}\alpha_{cac}$	0.0172	5.7668	$\beta_{dax}\beta_{cac}$	0.9805	234.1893	$\gamma_{dax}\gamma_{cac}$	0.0149	2.1850
$\alpha_{ftse}\alpha_{cac}$	0.0143	5.7767	$\beta_{ftse}\beta_{cac}$	0.9798	246.0274	$\gamma_{ftse}\gamma_{cac}$	0.0250	2.6160
$\alpha_{ftse}\alpha_{dax}$	0.0133	5.1133	$\beta_{ftse}\beta_{dax}$	0.9814	237.5877	$\gamma_{ftse}\gamma_{dax}$	0.0216	2.3989

LL -4951.569

Estimated coefficients, t-statistics and log-likelihood for the VT-DCC model above specified. T-statistics of the parameter functions are calculated using the delta method.

Table 12.

Specification 6:

$$q_{ij,t} = (1-\alpha_i\alpha_j - \beta_i\beta_j)\overline{q}_{ij} - \gamma_i\gamma_j\overline{v}_{ij} + \alpha_i\alpha_j\eta_{i,t-1}\eta_{j,t-1} + \beta_i\beta_j q_{ij,t-1} + \gamma_i\gamma_j v_{ij,t-1}$$

$$v_{ij,t} = \begin{cases} 1 & \text{if } h_{i,t} > d\left(\left\{h_{i,t}\right\}_{t=1}^{T}\right) \text{ and } h_{j,t} > d\left(\left\{h_{j,t}\right\}_{t=1}^{T}\right) \\ 0 & \textit{otherwise} \end{cases} ;\ d = 90\%$$

	Coef	t-stat		Coef	t-stat		Coef	t-stat
$\alpha_{px}\alpha_{bux}$	0.0111	1.4284	$\beta_{px}\beta_{bux}$	0.8363	9.0730	$\gamma_{px}\gamma_{bux}$	0.0038	0.2056
$\alpha_{wgi}\alpha_{bux}$	0.0157	2.0489	$\beta_{wgi}\beta_{bux}$	0.8540	14.3384	$\gamma_{wgi}\gamma_{bux}$	0.0048	0.2136
$\alpha_{sp}\alpha_{bux}$	0.0065	2.2978	$\beta_{sp}\beta_{bux}$	0.9147	17.6340	$\gamma_{sp}\gamma_{bux}$	0.0072	0.2977
$\alpha_{cac}\alpha_{bux}$	0.0129	2.5655	$\beta_{cac}\beta_{bux}$	0.9089	17.6228	$\gamma_{cac}\gamma_{bux}$	0.0063	0.2992
$\alpha_{dax}\alpha_{bux}$	0.0120	2.4615	$\beta_{dax}\beta_{bux}$	0.9105	17.6137	$\gamma_{dax}\gamma_{bux}$	0.0053	0.2984
$\alpha_{ftse}\alpha_{bux}$	0.0101	2.4951	$\gamma_{ftse}\gamma_{bux}$	0.9095	17.6683	$\gamma_{ftse}\gamma_{bux}$	0.0093	0.2980
$\alpha_{wgi}\alpha_{px}$	0.0192	1.8424	$\beta_{wgi}\beta_{px}$	0.8464	10.9468	$\gamma_{wgi}\gamma_{px}$	0.0076	0.2496
$\alpha_{sp}\alpha_{px}$	0.0079	1.9001	$\beta_{sp}\beta_{px}$	0.9065	11.9770	$\gamma_{sp}\gamma_{px}$	0.0114	0.2961
$\alpha_{cac}\alpha_{px}$	0.0158	2.0637	$\beta_{cac}\beta_{px}$	0.9008	11.9628	$\gamma_{cac}\gamma_{px}$	0.0100	0.2938
$\alpha_{dax}\alpha_{px}$	0.0147	2.0472	$\beta_{dax}\beta_{px}$	0.9024	11.9536	$\gamma_{dax}\gamma_{px}$	0.0085	0.2936
$\alpha_{ftse}\alpha_{px}$	0.0123	1.9651	$\beta_{ftse}\beta_{px}$	0.9014	11.9660	$\gamma_{ftse}\gamma_{px}$	0.0148	0.2948
$\alpha_{sp}\alpha_{wgi}$	0.0112	2.7374	$\beta_{sp}\beta_{wgi}$	0.9257	27.6247	$\gamma_{sp}\gamma_{wgi}$	0.0143	0.5043
$\alpha_{cac}\alpha_{wgi}$	0.0224	3.3875	$\beta_{cac}\beta_{wgi}$	0.9199	27.4637	$\gamma_{cac}\gamma_{wgi}$	0.0125	0.5059
$\alpha_{dax}\alpha_{wgi}$	0.0208	3.2162	$\beta_{dax}\beta_{wgi}$	0.9215	27.3311	$\gamma_{dax}\gamma_{wgi}$	0.0106	0.5002
$\alpha_{ftse}\alpha_{wgi}$	0.0175	3.3010	$\beta_{ftse}\beta_{wgi}$	0.9205	27.3565	$\gamma_{ftse}\gamma_{wgi}$	0.0186	0.5023
$\alpha_{cac}\alpha_{sp}$	0.0092	3.7653	$\beta_{cac}\beta_{sp}$	0.9852	255.9971	$\gamma_{cac}\gamma_{sp}$	0.0187	2.6649
$\alpha_{dax}\alpha_{sp}$	0.0086	3.2341	$\beta_{dax}\beta_{sp}$	0.9869	231.2399	$\gamma_{dax}\gamma_{sp}$	0.0160	2.3811
$\alpha_{ftse}\alpha_{sp}$	0.0072	3.0203	$\beta_{ftse}\beta_{sp}$	0.9859	230.3740	$\gamma_{ftse}\gamma_{sp}$	0.0279	2.8812
$\alpha_{dax}\alpha_{cac}$	0.0172	5.7897	$\beta_{dax}\beta_{cac}$	0.9807	236.8599	$\gamma_{dax}\gamma_{cac}$	0.0139	2.1324
$\alpha_{ftse}\alpha_{cac}$	0.0144	5.8193	$\beta_{ftse}\beta_{cac}$	0.9797	244.7662	$\gamma_{ftse}\gamma_{cac}$	0.0243	2.5904
$\alpha_{ftse}\alpha_{dax}$	0.0134	5.1272	$\beta_{ftse}\beta_{dax}$	0.9814	236.6741	$\gamma_{ftse}\gamma_{dax}$	0.0208	2.3611

LL -4951.946

Estimated coefficients, t-statistics and log-likelihood for the VT-DCC model above specified. T-statistics of the parameter functions are calculated using the delta method.

Table 13.

Specification 5:

$$q_{ij,t} = (1-\alpha_i\alpha_j-\beta_i\beta_j)\overline{q}_{ij} - \gamma_i\gamma_j\overline{v}_{ij} + \alpha_i\alpha_j\eta_{i,t-1}\eta_{j,t-1} + \beta_i\beta_j q_{ij,t-1} + \gamma_i\gamma_j v_{ij,t}$$

$$v_{ij,t} = \begin{cases} 1 \quad \text{if } h_{i,t} > d\left(\left\{h_{i,t}\right\}_{t=1}^{T}\right) \text{ and } h_{j,t} > d\left(\left\{h_{j,t}\right\}_{t=1}^{T}\right) \\ 0 \qquad\qquad \textit{otherwise} \end{cases} ;\ d = 75\%$$

	Coef	t-stat		Coef	t-stat		Coef	t-stat
$\alpha_{px}\alpha_{bux}$	0.0062	1.0191	$\beta_{px}\beta_{bux}$	0.8649	8.5512	$\gamma_{px}\gamma_{bux}$	0.0002	0.0959
$\alpha_{wgi}\alpha_{bux}$	0.0041	1.5144	$\beta_{wgi}\beta_{bux}$	0.9209	14.0374	$\gamma_{wgi}\gamma_{bux}$	0.0027	0.3333
$\alpha_{sp}\alpha_{bux}$	0.0081	1.7215	$\beta_{sp}\beta_{bux}$	0.9058	13.9145	$\gamma_{sp}\gamma_{bux}$	0.0031	0.3343
$\alpha_{cac}\alpha_{bux}$	0.0128	1.8485	$\beta_{cac}\beta_{bux}$	0.9110	14.1027	$\gamma_{cac}\gamma_{bux}$	0.0021	0.3354
$\alpha_{dax}\alpha_{bux}$	0.0121	1.8036	$\beta_{dax}\beta_{bux}$	0.9105	13.9900	$\gamma_{dax}\gamma_{bux}$	0.0017	0.3312
$\alpha_{ftse}\alpha_{bux}$	0.0123	1.7749	$\gamma_{ftse}\gamma_{bux}$	0.8971	14.1121	$\gamma_{ftse}\gamma_{bux}$	0.0033	0.3333
$\alpha_{wgi}\alpha_{px}$	0.0036	1.2873	$\beta_{wgi}\beta_{px}$	0.9340	12.5298	$\gamma_{wgi}\gamma_{px}$	0.0009	0.1045
$\alpha_{sp}\alpha_{px}$	0.0073	1.4109	$\beta_{sp}\beta_{px}$	0.9186	12.4634	$\gamma_{sp}\gamma_{px}$	0.0010	0.1046
$\alpha_{cac}\alpha_{px}$	0.0115	1.4502	$\beta_{cac}\beta_{px}$	0.9239	12.5081	$\gamma_{cac}\gamma_{px}$	0.0007	0.1042
$\alpha_{dax}\alpha_{px}$	0.0109	1.4382	$\beta_{dax}\beta_{px}$	0.9234	12.4482	$\gamma_{dax}\gamma_{px}$	0.0005	0.1040
$\alpha_{ftse}\alpha_{px}$	0.0111	1.3998	$\beta_{ftse}\beta_{px}$	0.9098	12.4422	$\gamma_{ftse}\gamma_{px}$	0.0011	0.1039
$\alpha_{sp}\alpha_{wgi}$	0.0048	2.5040	$\beta_{sp}\beta_{wgi}$	0.9781	110.9168	$\gamma_{sp}\gamma_{wgi}$	0.0144	3.1449
$\alpha_{cac}\alpha_{wgi}$	0.0076	2.8598	$\beta_{cac}\beta_{wgi}$	0.9837	369.7313	$\gamma_{cac}\gamma_{wgi}$	0.0094	3.7640
$\alpha_{dax}\alpha_{wgi}$	0.0072	2.9487	$\beta_{dax}\beta_{wgi}$	0.9832	247.3087	$\gamma_{dax}\gamma_{wgi}$	0.0077	2.8253
$\alpha_{ftse}\alpha_{wgi}$	0.0073	2.9567	$\beta_{ftse}\beta_{wgi}$	0.9687	132.5609	$\gamma_{ftse}\gamma_{wgi}$	0.0153	3.0994
$\alpha_{cac}\alpha_{sp}$	0.0152	3.7542	$\beta_{cac}\beta_{sp}$	0.9675	103.2159	$\gamma_{cac}\gamma_{sp}$	0.0110	2.3063
$\alpha_{dax}\alpha_{sp}$	0.0144	3.5491	$\beta_{dax}\beta_{sp}$	0.9670	96.1987	$\gamma_{dax}\gamma_{sp}$	0.0090	1.9487
$\alpha_{ftse}\alpha_{sp}$	0.0146	2.8488	$\beta_{ftse}\beta_{sp}$	0.9528	77.2625	$\gamma_{ftse}\gamma_{sp}$	0.0179	2.1783
$\alpha_{dax}\alpha_{cac}$	0.0226	6.2619	$\beta_{dax}\beta_{cac}$	0.9726	179.5774	$\gamma_{dax}\gamma_{cac}$	0.0059	1.9491
$\alpha_{ftse}\alpha_{cac}$	0.0230	4.9181	$\beta_{ftse}\beta_{cac}$	0.9583	117.4007	$\gamma_{ftse}\gamma_{cac}$	0.0117	2.1537
$\alpha_{ftse}\alpha_{dax}$	0.0218	4.6258	$\beta_{ftse}\beta_{dax}$	0.9578	111.2133	$\gamma_{ftse}\gamma_{dax}$	0.0096	1.8557

LL -4948.565

Estimated coefficients, t-statistics and log-likelihood for the VT-DCC model above specified. T-statistics of the parameter functions are calculated using the delta method.

Table 14.

Specification 6:

$$q_{ij,t} = (1-\alpha_i\alpha_j - \beta_i\beta_j)\overline{q}_{ij} - \gamma_i\gamma_j\overline{v}_{ij} + \alpha_i\alpha_j\eta_{i,t-1}\eta_{j,t-1} + \beta_i\beta_j q_{ij,t-1} + \gamma_i\gamma_j v_{ij,t-1}$$

$$v_{ij,t} = \begin{cases} 1 & \text{if } h_{i,t} > d\left(\left\{h_{i,t}\right\}_{t=1}^{T}\right) \text{ and } h_{j,t} > d\left(\left\{h_{j,t}\right\}_{t=1}^{T}\right) \\ 0 & \textit{otherwise} \end{cases};\ d = 75\%$$

	Coef	t-stat		Coef	t-stat		Coef	t-stat
$\alpha_{px}\alpha_{bux}$	0.0100	1.3459	$\beta_{px}\beta_{bux}$	0.8516	9.5585	$\gamma_{px}\gamma_{bux}$	0.0004	0.1835
$\alpha_{wgi}\alpha_{bux}$	0.0150	1.9682	$\beta_{wgi}\beta_{bux}$	0.8620	16.3821	$\gamma_{wgi}\gamma_{bux}$	-0.0036	-0.4408
$\alpha_{sp}\alpha_{bux}$	0.0087	2.2314	$\beta_{sp}\beta_{bux}$	0.9196	18.9322	$\gamma_{sp}\gamma_{bux}$	-0.0024	-0.3997
$\alpha_{cac}\alpha_{bux}$	0.0129	2.3233	$\beta_{cac}\beta_{bux}$	0.9214	19.0064	$\gamma_{cac}\gamma_{bux}$	-0.0014	-0.4043
$\alpha_{dax}\alpha_{bux}$	0.0122	2.2653	$\beta_{dax}\beta_{bux}$	0.9215	18.9140	$\gamma_{dax}\gamma_{bux}$	-0.0009	-0.4021
$\alpha_{ftse}\alpha_{bux}$	0.0104	2.2885	$\gamma_{ftse}\gamma_{bux}$	0.9233	19.1517	$\gamma_{ftse}\gamma_{bux}$	-0.0023	-0.4027
$\alpha_{wgi}\alpha_{px}$	0.0189	1.8861	$\beta_{wgi}\beta_{px}$	0.8456	11.9811	$\gamma_{wgi}\gamma_{px}$	-0.0029	-0.2421
$\alpha_{sp}\alpha_{px}$	0.0109	1.9504	$\beta_{sp}\beta_{px}$	0.9021	12.6371	$\gamma_{sp}\gamma_{px}$	-0.0019	-0.2347
$\alpha_{cac}\alpha_{px}$	0.0162	2.0411	$\beta_{cac}\beta_{px}$	0.9038	12.6594	$\gamma_{cac}\gamma_{px}$	-0.0012	-0.2372
$\alpha_{dax}\alpha_{px}$	0.0152	2.0343	$\beta_{dax}\beta_{px}$	0.9040	12.6172	$\gamma_{dax}\gamma_{px}$	-0.0007	-0.2381
$\alpha_{ftse}\alpha_{px}$	0.0130	1.9854	$\beta_{ftse}\beta_{px}$	0.9057	12.6902	$\gamma_{ftse}\gamma_{px}$	-0.0019	-0.2374
$\alpha_{sp}\alpha_{wgi}$	0.0164	2.9746	$\beta_{sp}\beta_{wgi}$	0.9132	31.0475	$\gamma_{sp}\gamma_{wgi}$	0.0159	1.8210
$\alpha_{cac}\alpha_{wgi}$	0.0243	3.7347	$\beta_{cac}\beta_{wgi}$	0.9149	31.5779	$\gamma_{cac}\gamma_{wgi}$	0.0097	1.6248
$\alpha_{dax}\alpha_{wgi}$	0.0228	3.4343	$\beta_{dax}\beta_{wgi}$	0.9150	31.2561	$\gamma_{dax}\gamma_{wgi}$	0.0061	1.3038
$\alpha_{ftse}\alpha_{wgi}$	0.0195	3.5245	$\beta_{ftse}\beta_{wgi}$	0.9168	31.1441	$\gamma_{ftse}\gamma_{wgi}$	0.0156	1.7631
$\alpha_{cac}\alpha_{sp}$	0.0141	4.0869	$\beta_{cac}\beta_{sp}$	0.9760	131.8774	$\gamma_{cac}\gamma_{sp}$	0.0064	2.0105
$\alpha_{dax}\alpha_{sp}$	0.0132	3.6682	$\beta_{dax}\beta_{sp}$	0.9762	117.3444	$\gamma_{dax}\gamma_{sp}$	0.0040	1.3620
$\alpha_{ftse}\alpha_{sp}$	0.0113	3.2988	$\beta_{ftse}\beta_{sp}$	0.9781	111.0406	$\gamma_{ftse}\gamma_{sp}$	0.0103	2.2195
$\alpha_{dax}\alpha_{cac}$	0.0196	5.9370	$\beta_{dax}\beta_{cac}$	0.9780	197.8272	$\gamma_{dax}\gamma_{cac}$	0.0024	1.2313
$\alpha_{ftse}\alpha_{cac}$	0.0168	6.0461	$\beta_{ftse}\beta_{cac}$	0.9799	226.4873	$\gamma_{ftse}\gamma_{cac}$	0.0062	1.9871
$\alpha_{ftse}\alpha_{dax}$	0.0158	5.1742	$\beta_{ftse}\beta_{dax}$	0.9801	160.2816	$\gamma_{ftse}\gamma_{dax}$	0.0039	1.3188

LL -4950.84

Estimated coefficients, t-statistics and log-likelihood for the VT-DCC model above specified. T-statistics of the parameter functions are calculated using the delta method.

Table 15.
Likelihood ratio tests

Panel A. 90% volatility threshold

Specifications	*LL Unrestricted*	*LL Restricted*	*LR*	*VT Component*
1 against 3	-4946.41	-4972.81	52.80	Contemporaneous
2 against 4	-4946.07	-4972.93	53.72	Lagged
5 against 7	-4951.57	-4970.60	38.06	Contemporaneous
6 against 8	-4951.95	-4970.74	37.58	Lagged

Panel B. 75% volatility threshold

Specifications	*LL Unrestricted*	*LL Restricted*	*LR*	*VT Component*
1 against 3	-4949.99	-4973.95	47.92	Contemporaneous
2 against 4	-4949.70	-4973.81	48.22	Lagged
5 against 7	-4948.57	-4969.91	42.68	Contemporaneous
6 against 8	-4950.84	-4969.81	37.94	Lagged

LR is the likelihood ratio test between the restricted (3, 4, 7 and 8) and the unrestricted (1, 2, 5 and 6) specifications. The test statistic follows a χ^2_{12}. The upper 1 and 5 percentile points of the χ^2_{12} distribution are 26.2 and 21.0, respectively.

Table 16

$$q_{ij,t} = (1-\alpha_i\alpha_j - \beta_i\beta_j)\bar{q}_{ij} - \gamma_i\gamma_j\bar{v}_{ij} - \xi_i\xi_j\bar{m}_{ij} + \alpha_i\alpha_j\eta_{i,t-1}\eta_{j,t-1} + \beta_i\beta_j q_{ij,t-1} + \gamma_i\gamma_j v_{ij,t} + \xi_i\xi_j m_{ij,t}$$

$$v_{ij,t} = \begin{cases} 1 \quad \text{if } h_{i,t} > d\left(\left\{h_{i,t}\right\}_{t=1}^{T}\right) \text{ or } h_{j,t} > d\left(\left\{h_{j,t}\right\}_{t=1}^{T}\right) \\ 0 \qquad\qquad otherwise \end{cases}$$

$$m_{ij,t} = \begin{cases} 1 \quad \text{if } \left(h_{i,t} > d\left(\left\{h_{i,t}\right\}_{t=1}^{T}\right) \; and \; \varepsilon_{i,t-1} < 0\right) \text{ or } \left(h_{j,t} > d\left(\left\{h_{j,t}\right\}_{t=1}^{T}\right) \; and \; \varepsilon_{j,t-1} < 0\right) \\ 0 \qquad\qquad otherwise \end{cases}$$

$$d = 90\%$$

	Coef	t-stat		Coef	t-stat		Coef	t-stat		Coef	t-stat
$\alpha_{px}\alpha_{bux}$	0.00867	1.32188	$\beta_{px}\beta_{bux}$	0.78973	8.46009	$\gamma_{px}\gamma_{bux}$	0.01218	1.00702	$\xi_{px}\xi_{bux}$	0.01367	0.55798
$\alpha_{wgi}\alpha_{bux}$	0.01112	1.76071	$\beta_{wgi}\beta_{bux}$	0.87201	18.94197	$\gamma_{wgi}\gamma_{bux}$	0.04149	1.86997	$\xi_{wgi}\xi_{bux}$	0.00375	0.25911
$\alpha_{sp}\alpha_{bux}$	0.00588	1.93910	$\beta_{sp}\beta_{bux}$	0.91135	22.21591	$\gamma_{sp}\gamma_{bux}$	0.00844	0.96393	$\xi_{sp}\xi_{bux}$	0.01731	0.75527
$\alpha_{cac}\alpha_{bux}$	0.00954	1.99827	$\beta_{cac}\beta_{bux}$	0.92011	22.19966	$\gamma_{cac}\gamma_{bux}$	0.00304	0.60947	$\xi_{cac}\xi_{bux}$	0.01051	0.75136
$\alpha_{dax}\alpha_{bux}$	0.00854	1.94940	$\beta_{dax}\beta_{bux}$	0.91717	22.12457	$\gamma_{dax}\gamma_{bux}$	0.00388	0.84887	$\xi_{dax}\xi_{bux}$	0.00829	0.73257
$\alpha_{ftse}\alpha_{bux}$	0.00717	2.01130	$\gamma_{ftse}\gamma_{bux}$	0.92099	22.31243	$\gamma_{ftse}\gamma_{bux}$	0.00599	1.02262	$\xi_{ftse}\xi_{bux}$	0.00808	0.73812
$\alpha_{wgi}\alpha_{px}$	0.02046	1.91100	$\beta_{wgi}\beta_{px}$	0.79697	8.91065	$\gamma_{wgi}\gamma_{px}$	0.04140	1.20869	$\xi_{wgi}\xi_{px}$	0.01142	0.29145
$\alpha_{sp}\alpha_{px}$	0.01082	2.01024	$\beta_{sp}\beta_{px}$	0.83292	9.40347	$\gamma_{sp}\gamma_{px}$	0.00842	0.74654	$\xi_{sp}\xi_{px}$	0.05267	1.12420
$\alpha_{cac}\alpha_{px}$	0.01755	2.12117	$\beta_{cac}\beta_{px}$	0.84094	9.38955	$\gamma_{cac}\gamma_{px}$	0.00303	0.52471	$\xi_{cac}\xi_{px}$	0.03195	1.06137
$\alpha_{dax}\alpha_{px}$	0.01571	2.09175	$\beta_{dax}\beta_{px}$	0.83825	9.39690	$\gamma_{dax}\gamma_{px}$	0.00387	0.69518	$\xi_{dax}\xi_{px}$	0.02521	1.03769
$\alpha_{ftse}\alpha_{px}$	0.01320	2.06054	$\beta_{ftse}\beta_{px}$	0.84174	9.39317	$\gamma_{ftse}\gamma_{px}$	0.00598	0.79379	$\xi_{ftse}\xi_{px}$	0.02457	1.03521
$\alpha_{sp}\alpha_{wgi}$	0.01388	2.90057	$\beta_{sp}\beta_{wgi}$	0.91969	38.54469	$\gamma_{sp}\gamma_{wgi}$	0.02869	1.09176	$\xi_{sp}\xi_{wgi}$	0.01446	0.32630
$\alpha_{cac}\alpha_{wgi}$	0.02252	3.88779	$\beta_{cac}\beta_{wgi}$	0.92854	40.17386	$\gamma_{cac}\gamma_{wgi}$	0.01033	0.64946	$\xi_{cac}\xi_{wgi}$	0.00878	0.32875
$\alpha_{dax}\alpha_{wgi}$	0.02015	3.53661	$\beta_{dax}\beta_{wgi}$	0.92558	39.83277	$\gamma_{dax}\gamma_{wgi}$	0.01318	0.95759	$\xi_{dax}\xi_{wgi}$	0.00692	0.32442
$\alpha_{ftse}\alpha_{wgi}$	0.01693	3.52546	$\beta_{ftse}\beta_{wgi}$	0.92943	39.76852	$\gamma_{ftse}\gamma_{wgi}$	0.02038	1.18081	$\xi_{ftse}\xi_{wgi}$	0.00675	0.32124
$\alpha_{cac}\alpha_{sp}$	0.01191	3.85123	$\beta_{cac}\beta_{sp}$	0.97043	135.1042	$\gamma_{cac}\gamma_{sp}$	0.00210	0.45407	$\xi_{cac}\xi_{sp}$	0.04048	3.07574
$\alpha_{dax}\alpha_{sp}$	0.01066	3.44671	$\beta_{dax}\beta_{sp}$	0.96733	127.3832	$\gamma_{dax}\gamma_{sp}$	0.00268	0.58391	$\xi_{dax}\xi_{sp}$	0.03193	2.29960
$\alpha_{ftse}\alpha_{sp}$	0.00895	2.95870	$\beta_{ftse}\beta_{sp}$	0.97136	123.0879	$\gamma_{ftse}\gamma_{sp}$	0.00415	0.65741	$\xi_{ftse}\xi_{sp}$	0.03112	2.26528
$\alpha_{dax}\alpha_{cac}$	0.01728	5.72796	$\beta_{dax}\beta_{cac}$	0.97664	220.2734	$\gamma_{dax}\gamma_{cac}$	9.66E-04	0.42594	$\xi_{dax}\xi_{cac}$	0.01937	3.08032
$\alpha_{ftse}\alpha_{cac}$	0.01452	5.71624	$\beta_{ftse}\beta_{cac}$	0.98070	279.4192	$\gamma_{ftse}\gamma_{cac}$	0.00149	0.46998	$\xi_{ftse}\xi_{cac}$	0.01888	3.06040
$\alpha_{ftse}\alpha_{dax}$	0.01299	4.63162	$\beta_{ftse}\beta_{dax}$	0.97757	230.8696	$\gamma_{ftse}\gamma_{dax}$	0.00190	0.60944	$\xi_{ftse}\xi_{dax}$	0.01489	2.95644

LL -4940.749

Estimated coefficients, t-statistics and log-likelihood for the VT-DCC model above specified. T-statistics of the parameter functions are calculated using the delta method.

Table 17

$$q_{ij,t} = (1-\alpha_i\alpha_j - \beta_i\beta_j)\bar{q}_{ij} - \gamma_i\gamma_j\bar{v}_{ij} - \xi_i\xi_j\bar{m}_{ij} + \alpha_i\alpha_j\eta_{i,t-1}\eta_{j,t-1} + \beta_i\beta_j q_{ij,t-1} + \gamma_i\gamma_j v_{ij,t} + \xi_i\xi_j m_{ij,t}$$

$$v_{ij,t} = \begin{cases} 1 \quad \text{if } h_{i,t} > d\left(\left\{h_{i,t}\right\}_{t=1}^{T}\right) \text{ or } h_{j,t} > d\left(\left\{h_{j,t}\right\}_{t=1}^{T}\right) \\ 0 \qquad\qquad\qquad \textit{otherwise} \end{cases}$$

$$m_{ij,t} = \begin{cases} 1 \quad \text{if } \left(h_{i,t} > d\left(\left\{h_{i,t}\right\}_{t=1}^{T}\right) \textit{ and } \varepsilon_{i,t-1} < 0\right) \text{ or } \left(h_{j,t} > d\left(\left\{h_{j,t}\right\}_{t=1}^{T}\right) \textit{ and } \varepsilon_{j,t-1} < 0\right) \\ 0 \qquad\qquad\qquad \textit{otherwise} \end{cases}$$

$$d = 75\%$$

	Coef	t-stat		Coef	t-stat		Coef	t-stat		Coef	t-stat
$\alpha_{px}\alpha_{bux}$	0.00728	1.22831	$\beta_{px}\beta_{bux}$	0.86353	12.44491	$\gamma_{px}\gamma_{bux}$	0.00198	0.26251	$\xi_{px}\xi_{bux}$	0.00845	0.60566
$\alpha_{wgi}\alpha_{bux}$	0.00380	1.50835	$\beta_{wgi}\beta_{bux}$	0.92164	23.01102	$\gamma_{wgi}\gamma_{bux}$	0.00702	0.60093	$\xi_{wgi}\xi_{bux}$	0.01733	0.80916
$\alpha_{sp}\alpha_{bux}$	0.00653	1.76149	$\beta_{sp}\beta_{bux}$	0.89753	22.40123	$\gamma_{sp}\gamma_{bux}$	-0.00243	-0.43086	$\xi_{sp}\xi_{bux}$	0.02194	1.38831
$\alpha_{cac}\alpha_{bux}$	0.01291	2.07821	$\beta_{cac}\beta_{bux}$	0.91314	23.05787	$\gamma_{cac}\gamma_{bux}$	-0.00151	-0.38679	$\xi_{cac}\xi_{bux}$	0.01395	1.33207
$\alpha_{dax}\alpha_{bux}$	0.01196	2.04642	$\beta_{dax}\beta_{bux}$	0.91329	23.01992	$\gamma_{dax}\gamma_{bux}$	0.00106	0.25359	$\xi_{dax}\xi_{bux}$	0.00895	1.04716
$\alpha_{ftse}\alpha_{bux}$	0.01408	1.95663	$\gamma_{ftse}\gamma_{bux}$	0.88557	22.17337	$\gamma_{ftse}\gamma_{bux}$	-0.00430	-0.81177	$\xi_{ftse}\xi_{bux}$	0.01998	1.37963
$\alpha_{wgi}\alpha_{px}$	0.00377	1.45536	$\beta_{wgi}\beta_{px}$	0.93124	16.92577	$\gamma_{wgi}\gamma_{px}$	0.00175	0.26837	$\xi_{wgi}\xi_{px}$	0.00900	0.71536
$\alpha_{sp}\alpha_{px}$	0.00649	1.62639	$\beta_{sp}\beta_{px}$	0.90688	16.90174	$\gamma_{sp}\gamma_{px}$	-6.07E-04	-0.31059	$\xi_{sp}\xi_{px}$	0.01139	0.97704
$\alpha_{cac}\alpha_{px}$	0.01284	1.82847	$\beta_{cac}\beta_{px}$	0.92265	16.89591	$\gamma_{cac}\gamma_{px}$	-3.76E-04	-0.30546	$\xi_{cac}\xi_{px}$	0.00724	0.94124
$\alpha_{dax}\alpha_{px}$	0.01189	1.80740	$\beta_{dax}\beta_{px}$	0.92280	16.80383	$\gamma_{dax}\gamma_{px}$	2.64E-04	0.18020	$\xi_{dax}\xi_{px}$	0.00465	0.83525
$\alpha_{ftse}\alpha_{px}$	0.01400	1.70016	$\beta_{ftse}\beta_{px}$	0.89479	16.43401	$\gamma_{ftse}\gamma_{px}$	-0.00107	-0.35019	$\xi_{ftse}\xi_{px}$	0.01038	0.94198
$\alpha_{sp}\alpha_{wgi}$	0.00339	2.02892	$\beta_{sp}\beta_{wgi}$	0.96792	96.43455	$\gamma_{sp}\gamma_{wgi}$	-0.00215	-0.44990	$\xi_{sp}\xi_{wgi}$	0.02336	2.23318
$\alpha_{cac}\alpha_{wgi}$	0.00670	2.49136	$\beta_{cac}\beta_{wgi}$	0.98475	357.5353	$\gamma_{cac}\gamma_{wgi}$	-0.00133	-0.40163	$\xi_{cac}\xi_{wgi}$	0.01485	2.08129
$\alpha_{dax}\alpha_{wgi}$	0.00620	2.50604	$\beta_{dax}\beta_{wgi}$	0.98491	317.9723	$\gamma_{dax}\gamma_{wgi}$	9.32E-04	0.25980	$\xi_{dax}\xi_{wgi}$	0.00953	1.37308
$\alpha_{ftse}\alpha_{wgi}$	0.00730	2.34095	$\beta_{ftse}\beta_{wgi}$	0.95502	85.86773	$\gamma_{ftse}\gamma_{wgi}$	-0.00379	-0.98705	$\xi_{ftse}\xi_{wgi}$	0.02127	2.57384
$\alpha_{cac}\alpha_{sp}$	0.01152	2.89974	$\beta_{cac}\beta_{sp}$	0.95899	88.63527	$\gamma_{cac}\gamma_{sp}$	4.61E-04	0.16928	$\xi_{cac}\xi_{sp}$	0.01880	2.43426
$\alpha_{dax}\alpha_{sp}$	0.01067	2.84152	$\beta_{dax}\beta_{sp}$	0.95915	91.30707	$\gamma_{dax}\gamma_{sp}$	-3.23E-04	-0.67724	$\xi_{dax}\xi_{sp}$	0.01206	2.87093
$\alpha_{ftse}\alpha_{sp}$	0.01256	2.37547	$\beta_{ftse}\beta_{sp}$	0.93003	56.75750	$\gamma_{ftse}\gamma_{sp}$	0.00131	0.22350	$\xi_{ftse}\xi_{sp}$	0.02693	1.96456
$\alpha_{dax}\alpha_{cac}$	0.02109	6.17323	$\beta_{dax}\beta_{cac}$	0.97583	213.4325	$\gamma_{dax}\gamma_{cac}$	-2.00E-04	-1.09853	$\xi_{dax}\xi_{cac}$	0.00767	2.77677
$\alpha_{ftse}\alpha_{cac}$	0.02485	4.60457	$\beta_{ftse}\beta_{cac}$	0.94621	79.78199	$\gamma_{ftse}\gamma_{cac}$	8.14E-04	0.21234	$\xi_{ftse}\xi_{cac}$	0.01712	2.27875
$\alpha_{ftse}\alpha_{dax}$	0.02301	4.38332	$\beta_{ftse}\beta_{dax}$	0.94637	83.44313	$\gamma_{ftse}\gamma_{dax}$	-5.71E-04	-0.52455	$\xi_{ftse}\xi_{dax}$	0.01099	3.17647

LL -4939.649

Estimated coefficients, t-statistics and log-likelihood for the VT-DCC model above specified. T-statistics of the parameter functions are calculated using the delta method.

Table. 18

$$q_{ij,t} = (1-\alpha_i\alpha_j - \beta_i\beta_j)\bar{q}_{ij} - \gamma_i\gamma_j\bar{v}_{ij} - \xi_i\xi_j\bar{m}_{ij} + \alpha_i\alpha_j\eta_{i,t-1}\eta_{j,t-1} + \beta_i\beta_j q_{ij,t-1} + \gamma_i\gamma_j v_{ij,t} + \xi_i\xi_j m_{ij,t}$$

$$v_{ij,t} = \begin{cases} 1 \quad \text{if } \; h_{i,t} > d\left(\left\{h_{i,t}\right\}_{t=1}^{T}\right) \text{ or } \; h_{j,t} > d\left(\left\{h_{j,t}\right\}_{t=1}^{T}\right) \\ 0 \qquad\qquad \textit{otherwise} \end{cases}$$

$$m_{ij,t} = \begin{cases} 1 \quad \text{if } \left(h_{i,t} > d\left(\left\{h_{i,t}\right\}_{t=1}^{T}\right) \; and \; \varepsilon_{i,t-1} > 0\right) \text{ or } \left(h_{j,t} > d\left(\left\{h_{j,t}\right\}_{t=1}^{T}\right) \; and \; \varepsilon_{j,t-1} > 0\right) \\ 0 \qquad\qquad \textit{otherwise} \end{cases}$$

$$d = 90\%$$

	Coef	t-stat		Coef	t-stat		Coef	t-stat		Coef	t-stat
$\alpha_{px}\alpha_{bux}$	0.01009	1.38292	$\beta_{px}\beta_{bux}$	0.80661	9.47891	$\gamma_{px}\gamma_{bux}$	0.0106	0.73843	$\xi_{px}\xi_{bux}$	0.02941	0.6246
$\alpha_{wgi}\alpha_{bux}$	0.01355	1.91856	$\beta_{wgi}\beta_{bux}$	0.86416	17.61620	$\gamma_{wgi}\gamma_{bux}$	0.03801	1.58589	$\xi_{wgi}\xi_{bux}$	0.00484	0.16624
$\alpha_{sp}\alpha_{bux}$	0.00540	2.09385	$\beta_{sp}\beta_{bux}$	0.92325	22.51858	$\gamma_{sp}\gamma_{bux}$	0.00545	0.8773	$\xi_{sp}\xi_{bux}$	0.01374	0.79172
$\alpha_{cac}\alpha_{bux}$	0.01004	2.27975	$\beta_{cac}\beta_{bux}$	0.91877	22.35891	$\gamma_{cac}\gamma_{bux}$	0.00322	0.58429	$\xi_{cac}\xi_{bux}$	0.01555	0.83952
$\alpha_{dax}\alpha_{bux}$	0.00968	2.20778	$\beta_{dax}\beta_{bux}$	0.92027	22.40568	$\gamma_{dax}\gamma_{bux}$	0.00361	0.73545	$\xi_{dax}\xi_{bux}$	0.01191	0.84827
$\alpha_{ftse}\alpha_{bux}$	0.00688	2.25756	$\gamma_{ftse}\gamma_{bux}$	0.91839	22.49454	$\gamma_{ftse}\gamma_{bux}$	0.00557	0.89911	$\xi_{ftse}\xi_{bux}$	0.01493	0.86858
$\alpha_{wgi}\alpha_{px}$	0.02217	1.92589	$\beta_{wgi}\beta_{px}$	0.81031	10.29852	$\gamma_{wgi}\gamma_{px}$	0.04057	1.03964	$\xi_{wgi}\xi_{px}$	0.01054	0.17328
$\alpha_{sp}\alpha_{px}$	0.00884	1.97555	$\beta_{sp}\beta_{px}$	0.86571	11.39239	$\gamma_{sp}\gamma_{px}$	0.00582	0.66271	$\xi_{sp}\xi_{px}$	0.02995	0.96915
$\alpha_{cac}\alpha_{px}$	0.01642	2.14609	$\beta_{cac}\beta_{px}$	0.86151	11.37122	$\gamma_{cac}\gamma_{px}$	0.00344	0.48294	$\xi_{cac}\xi_{px}$	0.03389	1.00926
$\alpha_{dax}\alpha_{px}$	0.01583	2.12176	$\beta_{dax}\beta_{px}$	0.86292	11.39047	$\gamma_{dax}\gamma_{px}$	0.00386	0.58743	$\xi_{dax}\xi_{px}$	0.02597	1.01641
$\alpha_{ftse}\alpha_{px}$	0.01125	2.03661	$\beta_{ftse}\beta_{px}$	0.86115	11.36713	$\gamma_{ftse}\gamma_{px}$	0.00594	0.67868	$\xi_{ftse}\xi_{px}$	0.03254	1.00012
$\alpha_{sp}\alpha_{wgi}$	0.01188	2.71443	$\beta_{sp}\beta_{wgi}$	0.92748	34.51223	$\gamma_{sp}\gamma_{wgi}$	0.02088	1.31577	$\xi_{sp}\xi_{wgi}$	0.00493	0.17965
$\alpha_{cac}\alpha_{wgi}$	0.02206	3.78942	$\beta_{cac}\beta_{wgi}$	0.92297	34.30976	$\gamma_{cac}\gamma_{wgi}$	0.01233	0.74551	$\xi_{cac}\xi_{wgi}$	0.00557	0.18151
$\alpha_{dax}\alpha_{wgi}$	0.02128	3.51117	$\beta_{dax}\beta_{wgi}$	0.92449	34.18555	$\gamma_{dax}\gamma_{wgi}$	0.01383	1.02165	$\xi_{dax}\xi_{wgi}$	0.00427	0.18281
$\alpha_{ftse}\alpha_{wgi}$	0.01511	3.22046	$\beta_{ftse}\beta_{wgi}$	0.92259	34.75556	$\gamma_{ftse}\gamma_{wgi}$	0.02131	1.38532	$\xi_{ftse}\xi_{wgi}$	0.00535	0.18244
$\alpha_{cac}\alpha_{sp}$	0.00879	3.48474	$\beta_{cac}\beta_{sp}$	0.98608	191.0750	$\gamma_{cac}\gamma_{sp}$	0.00177	0.50072	$\xi_{cac}\xi_{sp}$	0.01584	1.7701
$\alpha_{dax}\alpha_{sp}$	0.00848	3.04825	$\beta_{dax}\beta_{sp}$	0.98770	181.3594	$\gamma_{dax}\gamma_{sp}$	0.00199	0.61109	$\xi_{dax}\xi_{sp}$	0.01214	1.82448
$\alpha_{ftse}\alpha_{sp}$	0.00602	2.67870	$\beta_{ftse}\beta_{sp}$	0.98567	158.4562	$\gamma_{ftse}\gamma_{sp}$	0.00306	0.7196	$\xi_{ftse}\xi_{sp}$	0.0152	1.79622
$\alpha_{dax}\alpha_{cac}$	0.01576	5.71407	$\beta_{dax}\beta_{cac}$	0.98290	273.6369	$\gamma_{dax}\gamma_{cac}$	0.00117	0.447	$\xi_{dax}\xi_{cac}$	0.01373	2.09555
$\alpha_{ftse}\alpha_{cac}$	0.01119	4.52775	$\beta_{ftse}\beta_{cac}$	0.98089	248.2678	$\gamma_{ftse}\gamma_{cac}$	0.00181	0.50895	$\xi_{ftse}\xi_{cac}$	0.01721	2.15787
$\alpha_{ftse}\alpha_{dax}$	0.01079	4.20749	$\beta_{ftse}\beta_{dax}$	0.98250	244.9713	$\gamma_{ftse}\gamma_{dax}$	0.00203	0.62245	$\xi_{ftse}\xi_{dax}$	0.01319	2.48051

LL -4943.172

Estimated coefficients, t-statistics and log-likelihood for the VT-DCC model above specified. T-statistics of the parameter functions are calculated using the delta method.

Table. 19

$$q_{ij,t} = (1-\alpha_i\alpha_j - \beta_i\beta_j)\bar{q}_{ij} - \gamma_i\gamma_j\bar{v}_{ij} - \xi_i\xi_j\bar{m}_{ij} + \alpha_i\alpha_j\eta_{i,t-1}\eta_{j,t-1} + \beta_i\beta_j q_{ij,t-1} + \gamma_i\gamma_j v_{ij,t} + \xi_i\xi_j m_{ij,t}$$

$$v_{ij,t} = \begin{cases} 1 \quad \text{if } \; h_{i,t} > d\left(\left\{h_{i,t}\right\}_{t=1}^{T}\right) \text{ or } \; h_{j,t} > d\left(\left\{h_{j,t}\right\}_{t=1}^{T}\right) \\ 0 \qquad\qquad\qquad\qquad \textit{otherwise} \end{cases}$$

$$m_{ij,t} = \begin{cases} 1 \quad \text{if } \; \left(h_{i,t} > d\left(\left\{h_{i,t}\right\}_{t=1}^{T}\right) \; and \; \varepsilon_{i,t-1} > 0\right) \text{ or } \left(h_{j,t} > d\left(\left\{h_{j,t}\right\}_{t=1}^{T}\right) \; and \; \varepsilon_{j,t-1} > 0\right) \\ 0 \qquad\qquad\qquad\qquad\qquad\qquad \textit{otherwise} \end{cases}$$

$$d = 75\%$$

	Coef	t-stat		Coef	t-stat		Coef	t-stat		Coef	t-stat
$\alpha_{px}\alpha_{bux}$	0.00719	1.22641	$\beta_{px}\beta_{bux}$	0.85851	10.76087	$\gamma_{px}\gamma_{bux}$	0.00472	0.55555	$\xi_{px}\xi_{bux}$	0.00131	0.23826
$\alpha_{wgi}\alpha_{bux}$	0.00381	1.51041	$\beta_{wgi}\beta_{bux}$	0.91744	20.80499	$\gamma_{wgi}\gamma_{bux}$	0.01273	1.29302	$\xi_{wgi}\xi_{bux}$	0.00708	0.61959
$\alpha_{sp}\alpha_{bux}$	0.00761	1.83878	$\beta_{sp}\beta_{bux}$	0.88900	19.99073	$\gamma_{sp}\gamma_{bux}$	0.00273	0.39394	$\xi_{sp}\xi_{bux}$	0.01309	0.88042
$\alpha_{cac}\alpha_{bux}$	0.01348	2.07525	$\beta_{cac}\beta_{bux}$	0.90729	20.85118	$\gamma_{cac}\gamma_{bux}$	6.91E-04	0.14218	$\xi_{cac}\xi_{bux}$	0.00870	0.91405
$\alpha_{dax}\alpha_{bux}$	0.01248	2.05227	$\beta_{dax}\beta_{bux}$	0.90809	20.80117	$\gamma_{dax}\gamma_{bux}$	0.00332	0.86273	$\xi_{dax}\xi_{bux}$	0.00412	0.76287
$\alpha_{ftse}\alpha_{bux}$	0.01453	1.94185	$\gamma_{ftse}\gamma_{bux}$	0.87748	20.00412	$\gamma_{ftse}\gamma_{bux}$	-0.00120	-0.16347	$\xi_{ftse}\xi_{bux}$	0.01448	0.92361
$\alpha_{wgi}\alpha_{px}$	0.00375	1.42809	$\beta_{wgi}\beta_{px}$	0.93006	14.38762	$\gamma_{wgi}\gamma_{px}$	0.00406	0.61297	$\xi_{wgi}\xi_{px}$	0.00143	0.25289
$\alpha_{sp}\alpha_{px}$	0.00748	1.63716	$\beta_{sp}\beta_{px}$	0.90123	14.26922	$\gamma_{sp}\gamma_{px}$	8.70E-04	0.34388	$\xi_{sp}\xi_{px}$	0.00265	0.27049
$\alpha_{cac}\alpha_{px}$	0.01324	1.78541	$\beta_{cac}\beta_{px}$	0.91977	14.34466	$\gamma_{cac}\gamma_{px}$	2.20E-04	0.13791	$\xi_{cac}\xi_{px}$	0.00176	0.26739
$\alpha_{dax}\alpha_{px}$	0.01225	1.77599	$\beta_{dax}\beta_{px}$	0.92058	14.29057	$\gamma_{dax}\gamma_{px}$	0.00106	0.53560	$\xi_{dax}\xi_{px}$	8.33E-04	0.26013
$\alpha_{ftse}\alpha_{px}$	0.01427	1.66442	$\beta_{ftse}\beta_{px}$	0.88955	14.06688	$\gamma_{ftse}\gamma_{px}$	-3.83E-04	-0.15985	$\xi_{ftse}\xi_{px}$	0.00293	0.26741
$\alpha_{sp}\alpha_{wgi}$	0.00396	2.17148	$\beta_{sp}\beta_{wgi}$	0.96309	78.05776	$\gamma_{sp}\gamma_{wgi}$	0.00235	0.39996	$\xi_{sp}\xi_{wgi}$	0.01433	1.14309
$\alpha_{cac}\alpha_{wgi}$	0.00702	2.47918	$\beta_{cac}\beta_{wgi}$	0.98291	324.6962	$\gamma_{cac}\gamma_{wgi}$	5.94E-04	0.14200	$\xi_{cac}\xi_{wgi}$	0.00952	1.23977
$\alpha_{dax}\alpha_{wgi}$	0.00650	2.50586	$\beta_{dax}\beta_{wgi}$	0.98377	271.2574	$\gamma_{dax}\gamma_{wgi}$	0.00285	1.02912	$\xi_{dax}\xi_{wgi}$	0.00451	0.93586
$\alpha_{ftse}\alpha_{wgi}$	0.00756	2.35311	$\beta_{ftse}\beta_{wgi}$	0.95061	75.09906	$\gamma_{ftse}\gamma_{wgi}$	-0.00103	-0.16474	$\xi_{ftse}\xi_{wgi}$	0.01586	1.34764
$\alpha_{cac}\alpha_{sp}$	0.01401	3.33278	$\beta_{cac}\beta_{sp}$	0.95244	71.12899	$\gamma_{cac}\gamma_{sp}$	1.27E-04	0.11323	$\xi_{cac}\xi_{sp}$	0.01762	2.63267
$\alpha_{dax}\alpha_{sp}$	0.01297	3.29517	$\beta_{dax}\beta_{sp}$	0.95328	72.62044	$\gamma_{dax}\gamma_{sp}$	6.12E-04	0.34417	$\xi_{dax}\xi_{sp}$	0.00834	1.69252
$\alpha_{ftse}\alpha_{sp}$	0.01510	2.5606	$\beta_{ftse}\beta_{sp}$	0.92115	47.44277	$\gamma_{ftse}\gamma_{sp}$	-2.22E-04	-0.22333	$\xi_{ftse}\xi_{sp}$	0.02934	2.68201
$\alpha_{dax}\alpha_{cac}$	0.02297	6.08219	$\beta_{dax}\beta_{cac}$	0.97289	182.3539	$\gamma_{dax}\gamma_{cac}$	1.55E-04	0.13316	$\xi_{dax}\xi_{cac}$	0.00554	1.55098
$\alpha_{ftse}\alpha_{cac}$	0.02674	4.53615	$\beta_{ftse}\beta_{cac}$	0.94010	69.42914	$\gamma_{ftse}\gamma_{cac}$	-5.61E-05	-0.29043	$\xi_{ftse}\xi_{cac}$	0.01949	2.40771
$\alpha_{ftse}\alpha_{dax}$	0.02475	4.32067	$\beta_{ftse}\beta_{dax}$	0.94093	72.32132	$\gamma_{ftse}\gamma_{dax}$	-2.70E-04	-0.17413	$\xi_{ftse}\xi_{dax}$	0.00923	1.76899

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Estimated coefficients, t-statistics and log-likelihood for the VT-DCC model above specified. T-statistics of the parameter functions are calculated using the delta method.