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INVESTING IN BIOGAS: TIMING, TECHNOLOGICAL CHOICE AND THE VALUE OF FLEXIBILITY FROM INPUTS MIX

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Investing in biogas: timing, technological choice and the value of flexibility from inputs mix*

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Abstract

In a stochastic dynamic frame, we study the technology choice problem of a continuous co-digestion biogas plant where input factors are substitute but need to be mixed together to provide output. Given any initial rule for the composition of the feedstock, we consider the possibility of revising it if economic circumstances make it profitable. Flexibility in the mix is an advantage under randomly fluctuating input costs and comes at a higher investment cost. We show that the degree of flexibility in the productive technology installed depends on the value of the option to profitably re-arrange the input mix. Such option adds value to the project in that it provides a device for hedging against fluctuations in the input relative convenience. Accounting for such value we discuss the trade-off between investment timing and profit smoothing flexibility.
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1 Introduction

Consider a product produced by mixing together some input factors according to a given rule. Suppose that such product may be provided using forever the initial productive mode or, as soon as future changes in the relative input convenience make it worthwhile, by switching to a revised mode while keeping the option to revert to the initial mode.

A similar problem characterizes the technology for the production of biogas through anaerobic digestion of biomasses.[1] A biogas plant consists, in general, of two main components: a digester (or several digesters) and a gas holder. The digester is a waterproof container where the fermentable mixture is introduced in the form of slurry. Technological improvements today allow for the installation of digesters able to process almost any biodegradable material and simultaneously process two or more different types of raw material. Input factors for the production of biogas may be, for instance, livestock and poultry wastes, night soil, paper wastes, aquatic weeds, water hyacinth

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[1]On anaerobic digestion and methane production see for instance Chynoweth et al. (2001) and Lazarus and Rudstrom (2007).

and seaweed, hay, corn maize and crop residuals such as cane trash and plant stubble. However, as reported by several authors,[2] the design of the plant operations and the final biogas yield depends on the composition of the feedstock to be fermented in the digester and, needless to say, the choice of the feeding mixture (the diet) plays a crucial role in reducing of the cost of the biogas produced (Callaghan et al., 2002; Gerin et al., 2008; Schievano et al., 2009, Banzato et al. 2010).

The relative economic convenience of a factor is affected by different sources of uncertainty such as market, regulatory and technological uncertainty. It follows that, under changing circumstances, a flexible technology allowing for a revision of the diet is an advantage. However, such technology comes at a greater cost which is sunk in nature and increasing in the level of diet flexibility (i.e. on the "distance" between the initial and the revised diet). In fact, in addition to the initial instalment cost due to the design, optimization and maintenance of the production facility, we account also for other set-up costs related to the availability of biomass. A continuous flow of biomass is crucial for the production process and to guarantee continuity to the input provision biogas producers must often sign multi-year contracts with agro-zoo technical firms. In turn, these contractual arrangements entail several costs due to search for input suppliers close enough to the plant to reduce transport costs, input quality and delivery timing monitoring, and contract enforcement.

Once the investment has been undertaken the plant produces biogas using the most convenient diet in the feasible range defined by the degree of flexibility chosen. Hence, the value of the plant sums the value of producing biogas with a defined initial diet plus the value of the option to switch, as soon as it is worthwhile, to a revised one.

The novelty in this paper is represented by the set-up of a general model where the relationship between the value added by investment timing flexibility and diet flexibility can be analysed as a compound option.[3] In a continuous time framework, we let the firm choose the optimal diet revision to hedge against input price volatility. Then, playing backward, we determine the timing of investment in the plant where such flexible technology has been installed.

We show that benefits from diet flexibility can be traded off with the investment cost magnitude and, depending on profit volatility, the firm may opt for the highest feasible level of flexibility or for lower ones. As uncertainty about future profits increases, the firm prefers to have a more flexible biogas production technology. The project itself is more valuable but, consistently with most literature on the investment-uncertainty relationship, under irreversibility a firm undertaking a more expensive investment should further postpone the decision in order to gather information on future prospects and reduce regret.

However, while in general investment timing and investment cost are negatively related, we obtain the opposite for reasonably realistic investment scenarios. This result has important implications for investment timing and plant design. The intuition behind it is straightforward. Since the firm is able to choose the degree of diet flexibility then higher investment costs can be balanced by adjusting the diet revision in order to guarantee a net present value high enough to sustain earlier investment.

Our paper can be included in the literature stream using a real options approach to study the value of flexibility and its role on investment under uncertainty and irreversibility. Previous papers taking this approach, such as Kulatilaka (1988, 1993), Triantis and Hodder (1990) and He and Pindyck (1992), apply option theory to assess the value of flexibility on manufacturing. In particular, Kulatilaka (1988, 1993) uses a stochastic dynamic programming model to evaluate the options available in a mutual exclusive flexible production process where switching is costly.

[2] See for instance Chynoweth (2004) and Amon et al. (2007a, 2007b).

[3] See Stokes et al. (2008) where the standard investment model in Dixit and Pindyck (1994, p. 136) is used to study the timing of investment in a biogas plant.

Triantis and Hodder (1990) analyse the investment in a technology able to provide a k-variety of products with no cost at the switching nodes. Capacity constraints are considered and the firm may also mothball and later restart the operations. He and Pindyck (1992) highlight the relationship between technology and capacity choice. From this perspective, they study output flexibility and determine in a stochastic frame the desired degree of flexibility in the technology and the capacity to be installed.[4] However, these scholars do not consider the analysis of production processes where a different input mix may be used and do not model the optimal choice of the degree of resulting flexibility. We believe that our contribution fills in this gap in this literature.

Finally, our paper provides a solid theoretical frame to the rising literature[5] on the economics of energy production with fuel blends which has, up to our knowledge, completely neglected the crucial impact that the flexibility in the blend may have on the profitability of the productive system.

The remainder of the paper is organized as follows. In the next section the basic set-up is presented. In section 3 and 4 we respectively determine the value of diet flexibility and the optimal adjustment policy. In section 5 we solve for the timing of the investment. We discuss the relationship between diet and investment timing flexibility supporting our analysis through comparative statics and assessing the value added to the investment project by flexibility. In Section 6, we conclude by summarizing our results and discussing other applications of our frame. For instance, to production based on the co-combustion of different input factors, i.e. flexible-fuel engines[6] (FFE) and co-firing[7]power stations.

2 The basic set-up

We aim to study the decision to invest in a plant that produces biogas using different input mixtures (diets). We will derive the value of the flexibility attached to the option to switch between different diets and then the value of the plant. The option to switch represents an important strategic tool for the firm since it may allow to hedge against input price volatility by rearranging the input mix.[8] This flexibility is valuable and must be accounted for when assessing the plant Net Present Value (NPV). Operationally, we first determine the optimal diet adjustment on the basis of future cost fluctuations and then, moving backward, we derive the value of the plant where such optimally characterized flexible technology would be installed.

We begin by introducing the following assumptions:

Assumption 1 Once installed, the biogas plant is never mothballed, nor abandoned.

Assumption 2 The plant produces a fixed output flow normalized for simplicity to 1 m^3 of biogas and every cost is expressed per unit of output.

[4] To characterize the operating technology, the scholars quoted use the concept of a "mode of operation" to describe a mutual exclusive flexibility, i.e. "invest" vs. "wait to invest", "use gas" vs. "use oil", or " continuous operation" vs. "shut down" or vs. "abandon project" and so on (Brennan and Schwartz, 1985; Kulatilaka, 1988, 1993).

[5] See among others Rubab and Kandpal, (1996), Mæng et al. (1999), Hughes (2000), Lazarus and Rudstrom (2007), Stokes et al. (2008), Schievano et al. (2009), Banzato et al. (2010).

[6] See Nichols (2003) and http://en.wikipedia.org/wiki/Flexible_fuel.

[7] On co-firing see among others Hughes (2000), Sami et al. (2001) and Nussbaumer (2003).

[8] On the assessment of biogas yield with different diets see for instance Callaghan et al. (2002), Raven and Gregersen (2007), Bouallagui et al. (2009) and Banzato et al. (2010).

Assumption 3 The firm can rearrange operations by switching back and forward between two different production regimes. Under each regime, a different diet is used to feed the biogas digester.

Assumption 4 A diet that guarantees a positive NPV always exists.

While Assumption 2 is standard, Assumptions 1, 3 and 4 deserve some comments. By Assumption 1, we abstract from other operative options since, according to empirical evidence,[9] their exercise represents only a remote possibility.[10] By Assumption 3, we simplify the analysis assuming that to produce 1 m^3 of biogas the digester must be fed using a mixture of two composite inputs:[11] one formed by dry material and the other by liquid material. Finally, by Assumption 4, we focus only on the investment timing problem taking away considerations related to the investment choice itself. Note that the cost advantages of changing diet are not affected by this assumption and that, technically speaking, a positive (forward) time trigger for investment always exists.

2.1 A flexible input mixture technology

Denote by D^1 the starting diet and assume that it is initially composed by a share,[12] $\alpha \in (0, 1)$, of c and a share, $1 - \alpha$, of d where c and d are perfectly substitutable composite input factors.[13] The unit cost, c_t, for input c is stochastic while the cost for the other input is constant and equal

[9] As a matter of fact, Kramer (2004) mentions some cases of digesters idle or abandoned, but in each case this was due to mechanical and structural problems and/or owners' lack of the knowledge needed to properly manage the system.

[10] Note that this assumption is not crucial since, as shown by Di Corato and Moretto (2009), if maintenance and scrapping costs are comparatively small, the impact of the options to mothball and/or abandon is negligible. On the magnitude of those costs see for instance Kramer (2004, 2008) and Banzato et al. (2010).

[11] Succulent plant material performs better than dried material in terms of biogas yield. Hence, semi-drying may be needed when, for instance, brushes and weeds are processed. In this respect, experience has shown that the raw material ratio to water must be 1:1 (National Academy of Sciences, 1977; Da Silva, 1979; Amon et al., 2007a, 2007b). On the basis of this evidence, we abstract, for the sake of simplicity, from a multi-input problem by grouping materials into two main bundles. However, note that although the model can be extended to allow for multiple diets and more input factors, complexity would not add more insight to our main results.

[12] The level of α could be justified by the existence of regulative or technological constraints which impose starting with a certain diet. However, as we will show below, assuming a certain starting α does not impose any restriction on the set of possible diet revisions that the manager may consider desirable to hedge against input cost volatility.

[13] As argued above, production of biogas is inefficient if the mixture used is too diluted or too concentrated. To maintain the right total solid concentration, water may be added to the slurry before the anaerobic action starts. Therefore, without losing in generality, we may assume perfect substitutability between the inputs, adding the cost of water to the cost of one of them (Singh, 1971; National Academy of Sciences, 1977; Da Silva, 1979). However, as argued by Callaghan et al. (2002), research is needed to investigate the adverse effects that some waste types may have when used in conjunction with another waste type. In this respect, in the appendix A.3, we allow for imperfect substitutability and show that in this case $(\alpha, 1 - \alpha)$ represents the share on total cost covered by each input.

to d.[14] Let the stochastic dynamic of c_t over time follow a drift-less geometric Brownian motion[15]

$$\frac{dc_t}{c_t} = \sigma dz_t \tag{1}$$

where σ is the volatility parameter and dz_t is the increment of the standard Wiener process satisfying $E\left[dz_t\right] = 0$, $E\left[dz_t^2\right] = dt$.[16]

In order to reduce the complexity of the analysis we assume that the market price of 1 m^3 is constant[17] and equal to $p > d$ (Assumption 4). Hence, the instantaneous profit function under D^1 is given by:

$$\begin{aligned} p - C_1 &= p - \left[\alpha c_t + (1-\alpha)\, d\right] \\ &= p - d + \alpha\,(d - c_t) \end{aligned}$$

Further, depending on the relative economic convenience of each material with respect to the other, we allow for a successive revision of D^1 obtained by switching to D^2 if the latter turns out to be more profitable.[18] In D^2 the shares are adjusted and are respectively given by α' and $1-\alpha'$, where $\alpha' = \gamma\alpha$ with $\gamma \in [0, 1/\alpha]$.[19]

The adjustment coefficient γ measures the "distance" between the two diets in terms of input shares and completely characterizes our flexible technology. Hence, for each γ-production technology, the instantaneous profit function under D^2 is:

$$\begin{aligned} p - C_2 &= p - \left[\alpha' c_t + \left(1-\alpha'\right) d\right] \\ &= p - d + \gamma\alpha\,(d - c_t) \end{aligned}$$

[14] The alternative disposal of some raw materials may be strictly regulated. For instance, this is the case with manure (Commission Regulation No. 208/2006, EC, 2006). In this case, the input cost is represented by the opportunity cost that the plant holder would bear. On the impact of regulation see for instance Raven and Gregersen, 2007, Gerin et al., 2008, Schievano et al., 2009 and Banzato et al., 2010.

[15] Note that one may equivalently consider the input priced d as numeraire ($d = 1$) and illustrate by (1) the stochastic fluctuations in the input price ratio (see for instance Dixit, 1989b).

[16] In our model, we assume that the uncertainty driven by technological and regulative change is processed on the market place and then captured by the input prices dynamics. A more general GBM with Poisson jumps, capturing for instance sudden technological and regulative shocks affecting c_t, may be considered without adding substantial insight to our results.

[17] The price of 1 m^3 of biogas may be considered constant due to regulation and/or to the trading of Renewable Energy Certificates (RECs). See Directive 2001/77/EC on the promotion of electricity from renewable energy sources in the internal electricity market. Note that with the simultaneous inclusion of two stochastic variables, the problem has no closed form solution and must be solved numerically. However, this characterization would not impact significantly on our results.

[18] Some switching costs may arise when changing diet. They are mainly represented by the profit foregone during the time period needed for the production process to resume to the standard performance level. We abstract from those costs at no cost in terms of additional insight.

[19] Note that by this assumption one may rearrange over the entire feasible range:

$$\gamma = 0 \rightarrow \alpha' = 0 \rightarrow C_2 = d$$
$$\gamma = \frac{1}{\alpha} \rightarrow \alpha' = 1 \rightarrow C_2 = c_t$$

Taking D^1 as starting diet and accounting for the option to switch between diets, the instantaneous profit function is equal to:

$$\begin{aligned} \pi_t &= \max\{\max[(p - C_1), (p - C_2)]\} \\ &= \max\{p - d + \max[\alpha(d - c_t), \alpha'(d - c_t)]\} \end{aligned} \quad (2)$$

By (2), we get that D^1 is adopted only if $C_1 < C_2$. Conditionally on the choice of γ, this relation holds when $c_t > d$ if $\gamma > 1$ or when $c_t < d$ if $\gamma < 1$. In both cases, the firm produces biogas with the initial diet D^1 and keeps open the option to switch to D^2. On the contrary, if $C_1 > C_2$ it is optimal to adopt D^2 (this holds when $c_t < d$ with $\gamma > 1$ or when $c_t > d$ with $\gamma < 1$, knowing that however it is possible to switch back to D^1.

Therefore, assessing the value of investing in a biogas plant with diet flexibility, we must distinguish between two scenarios where a flexible diet can be valuable. Under both scenarios, by holding the option to switch to an alternative diet the firm can hedge against input price volatility by increasing the share of the relatively less expensive input. Under the first scenario, if c_t falls under d then a larger share of the input c should be used ($\gamma \in [1, 1/\alpha]$), whereas the presence of such input factor must be reduced if c_t rises above d ($\gamma \in [0, 1]$). As shown by Figure 1 and 2, the two scenarios may be seen as symmetric. For instance, suppose that $c_t > d$ and assume $D_1 = (\frac{1}{2}, \frac{1}{2})$ as starting diet and $\gamma = \frac{3}{2}$ as optimal correction, i.e. the firm holds the option to switch to $D_2 = (\frac{3}{4}, \frac{1}{4})$. Now, suppose that $c_t < d$ and assume $D_1 = (\frac{3}{4}, \frac{1}{4})$ and $\gamma = \frac{2}{3}$. It follows that the firm may switch to $D_2 = (\frac{1}{2}, \frac{1}{2})$. Summing up, in both states the firm may switch back and forth between the same two diets. Thus, without loss of generality, we proceed in the following section deriving the value of the flexible technology only for $c_t > d$.[20]

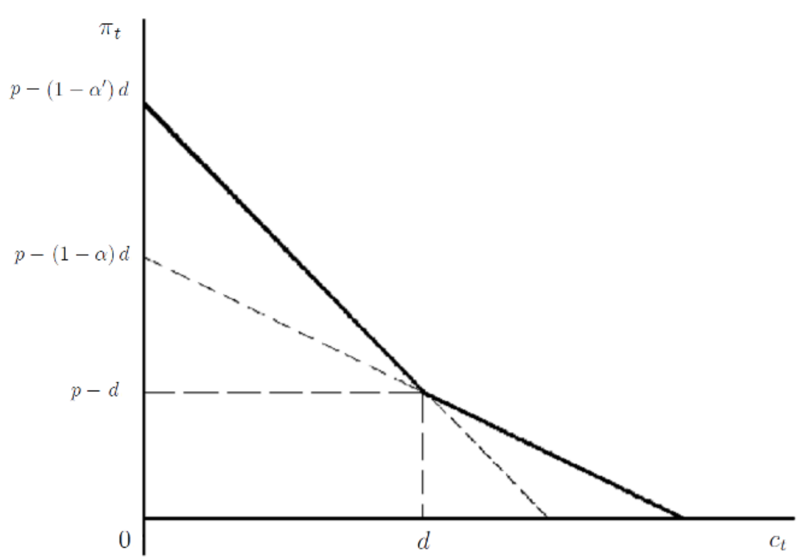

Figure 1: profit function with $\gamma \in [1, 1/\alpha]$

[20]We prove this symmetry in the appendix (see section A.3).

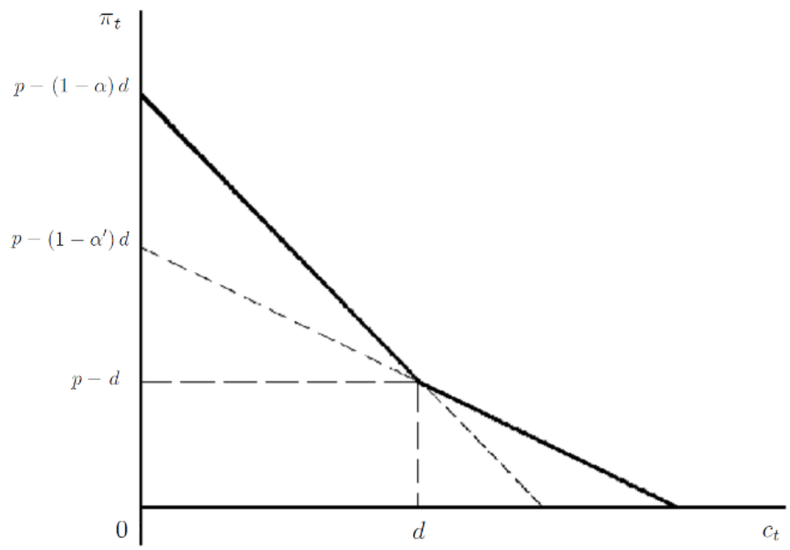

Figure 2: profit function with $\gamma \in [0,1]$

3 The value of the flexible technology

Denote by $V^{(D^1)}$ and $V^{(D^2)}$ the value of the biogas plant under diet D^1 and D^2 respectively. Since for $\gamma \in [1, 1/\alpha]$, the condition $C_1 < C_2$ holds when $c_t > d$, then $V^{(D^1)}$ and $V^{(D^2)}$ solve the following dynamic programming problem (Dixit, 1989a, pp. 624-628):

$$\Gamma V^{(D^1)}(c_t, \gamma; \alpha) = -(p - C_1) \quad \text{for } d < c_t < \infty \tag{3}$$

$$\Gamma V^{(D^2)}(c_t, \gamma; \alpha) = -(p - C_2) \quad \text{for } c_t < d \tag{4}$$

where $\Gamma = \frac{1}{2}\sigma^2 c_t^2 \frac{\partial^2}{\partial c_t^2} - r$ is the differential operator with the riskless interest rate r.[21]

Solving the problem [3-4] yields[22]

$$V^{(D^1)}(c_t, \gamma; \alpha) = \tfrac{p-C_1}{r} + (\gamma - 1) A c_t^{\beta_2} \quad \text{for } d < c_t < \infty \tag{5}$$

$$V^{(D^2)}(c_t, \gamma; \alpha) = \tfrac{p-C_2}{r} + (\gamma - 1) B c_t^{\beta_1} \quad \text{for } c_t < d \tag{6}$$

where $\beta_1 > 1$ and $\beta_2 < 0$ are the roots of the characteristic equation $\phi(\beta) = \frac{1}{2}\sigma^2 \beta (\beta - 1) - r = 0$ and

$$A = \frac{\alpha}{r(\beta_1 - \beta_2)} d^{1-\beta_2} > 0, \;\; B = \frac{\alpha}{r(\beta_1 - \beta_2)} d^{1-\beta_1} = A d^{\beta_2 - \beta_1} > 0.$$

[21] An interest rate incorporating a proper risk adjustment can be used taking the expectation with respect to a distribution of c_t adjusted for risk neutrality (Cox and Ross, 1976).

[22] See appendix A.1.

In (5) the term $\frac{p-C_1}{r}$ indicates the present value of producing biogas forever using D^1 while $(\gamma - 1)Ac_t^{\beta_2}$ represents the value of the option to switch to D^2. In (6), $\frac{p-C_2}{r}$ is the present value of producing biogas forever adopting D^2, while $(\gamma-1)Bc_t^{\beta_1}$ is the value of the option to switch back to D^1. Note that, as discussed in the introduction, under both diets, the value of the option to switch increases with the adjustment γ. This makes sense considering that, as conditions for switching occur, the higher the adjustment to the starting diet, the higher the profit from switching. Finally, for $\gamma = 1$, the options to switch back and forth between D^1 and D^2 are not available and, rightly, their value is null.

4 Optimal diet adjustment

Suppose that the firm is producing biogas by using the initial diet D^1 and holding the option to switch to D^2. Such option is valuable in that it allows for adjustment of the production regime to benefit from a change in the input price relative convenience. Clearly, its value depends on the magnitude of the feasible revision. Hence, the question is: which γ should be chosen in order to optimally revise D^1?

The plant investment cost depends on the characteristics of the digestion process, on the dimensions and on the materials used for digestion. However, since the cost of setting up a flexible technology should mainly depend on the initial diet, and be increasing on the "distance" between D^1 and the alternative, D^2,[23] we model such cost, $I(\gamma)$, as a function of γ. In addition, we normalize the investment cost to zero, i.e. $I(1) = 0$, and characterize it by some reasonable assumptions: convexity in γ with $I_\gamma(\gamma) > 0$ when $\gamma > 1$ and $I_\gamma(\gamma) < 0$ for $\gamma < 1$ and symmetry when adopting extreme diets, i.e. $I\left(\frac{1}{\alpha}\right) = I(0)$. To accommodate these requirements, we model the cost function as an increasing cost-to-scale Cobb-Douglas quadratic in $(\gamma - 1)$, i.e.:

$$I(\gamma) = \begin{cases} k\frac{\alpha}{1-\alpha}\frac{(\gamma-1)^2}{2} & \text{for } \gamma \in [1, 1/\alpha] \\ k\frac{\alpha'}{1-\alpha'}\frac{(\gamma-1)^2}{2} & \text{for } \gamma \in [0, 1] \end{cases} \tag{7}$$

where k is a dimensional term adjusting the investment cost for the component merely dependent on both gas-holder and digester capacity.[24] The terms $\frac{\alpha}{1-\alpha}$ and $\frac{\alpha'}{1-\alpha'}$ introduce a normalization in the cost function. They capture the greater cost of operating with a digester when the initial diet is mainly based on one of the two input factors.[25] This requires, for symmetry $I\left(\frac{1}{\alpha}\right) = I(0)$ to hold, that $\alpha = 1 - \alpha'$. Finally, the assumption of a quadratic cost function is a matter of realism since it allows accommodating non-modal choices. In fact, without convexity we would get extreme outcomes, i.e., either the use of d or c as sole input ($\gamma = 0$ or $\gamma = 1/\alpha$).

[23] The digester reactors can be constructed by using brick, cement, concrete and steel, while the gas holder is normally an airproof steel container. As pointed out by Rubab and Kandpal (1996) and Banzato et al. (2010) the diet composition influences considerably the cost of the storage capacity and, in particular, the cost of the digester capacity. A progressive reduction in the costs of the plant can be observed as the installed power increases.

[24] A fixed investment cost K_0 independent of γ may be included in (9). However, many authors report that scale economies and technological progress have importantly reduced such cost over the last decade (Maeng et al., 1999; Devenuto and Ragazzoni, 2008, Banzato et al. 2010).

[25] An efficient anaerobic digestion requires both liquefaction and gasification steps to be properly balanced. In fact, only by liquefying the feedstock the bacteria can activate and start the digestion process. However, if liquefaction occurs at a faster rate, the resulting accumulation of acids may inhibit the process (National Academy of Sciences, 1977; Da Silva, 1979). This effect may be due to the use of extreme diets and we capture it by introducing the term $\frac{\alpha}{1-\alpha}$, where $\lim_{\alpha\to1}\frac{\alpha}{1-\alpha} = +\infty$ and $\lim_{\alpha\to1}\frac{d\left(\frac{\alpha}{1-\alpha}\right)}{d\alpha} = +\infty$.

Having defined the cost of setting up a generic γ-production technology, we can determine the γ by which D^1 should be optimally revised. By the symmetry of the two scenarios, we only determine the optimal revision for $c_t > d$.[26]

The optimal γ must maximize the value of the flexible technology, $V^{(D^1)}(c_t, \gamma; \alpha)$, minus the instalment cost $I(\gamma)$. That is,

$$\gamma^* = \arg\max NPV(c_t, \gamma; \alpha) \quad s.t.\ \gamma > 1 \quad \text{for } d < c_t < \infty \tag{8}$$

where $NPV(c_t, \gamma; \alpha) = V^{(D^1)}(c_t, \gamma; \alpha) - I(\gamma)$.

Solving the maximization problem in (8) it follows that

Proposition 1 *The optimal flexible technology when investing at $c_t \in (d, \infty)$ is*

$$\gamma^*(c_t; \alpha) = \begin{cases} 1 + \frac{1-\alpha}{\alpha}(\frac{c_t}{\widehat{c}})^{\beta_2} & \text{for } \widehat{c} \leq c_t < \infty \\ \frac{1}{\alpha} & \text{for } d < c_t < \widehat{c} \end{cases} \tag{9}$$

where $\widehat{c} = \left(\frac{k}{A}\right)^{\frac{1}{\beta_2}}$ and $A = \frac{\alpha}{r(\beta_1 - \beta_2)} d^{1-\beta_2}$.

Proof. See section A.2 in the appendix. ∎

By Proposition (1) $\gamma^*(c_t; \alpha)$ is decreasing in c_t and it tends to 1 as $c_t \to \infty$. This makes sense since as c_t increases, the probability of a future fall diminishes and the value of investing in flexibility progressively decreases. Put differently, it is not economically meaningful to invest in a technology the flexibility of which is probably not going to be exploited. On the contrary, as c_t tends to d then the probability of a fall increases and having a flexible technology makes economic sense. Note that the threshold $\widehat{c}$ indicates the cost level below which the probability of a fall in c_t is so high that the diet should be optimally revised the diet by the highest feasible $\gamma^*(c_t; \alpha) = \frac{1}{\alpha}$ in order to use exclusively input c.[27]

Further, by Proposition (1) it follows that the net present value of the adopted technology is state contingent. In fact, substituting (9) into $NPV(c_t, \gamma; \alpha)$ yields

$$NPV(c_t, \gamma^*(c_t); \alpha) = \begin{cases} \frac{p - C_1}{r} + \frac{1}{2} k \frac{1-\alpha}{\alpha} (\frac{c_t}{\widehat{c}})^{2\beta_2} & \text{for } \widehat{c} \leq c_t < \infty \\ \frac{p - C_1}{r} + k \frac{1-\alpha}{\alpha} \left[(\frac{c_t}{\widehat{c}})^{\beta_2} - \frac{1}{2} \right] & \text{for } d < c_t < \widehat{c} \end{cases} \tag{10}$$

where the second term on the r.h.s. of (10) represents the value of the option to switch to D^2. It is immediate to verify that since $(\frac{c_t}{\widehat{c}})^{\beta_2} < 1$ for $c_t \geq \widehat{c}$ then the value of the diet flexibility is higher when $c_t < \widehat{c}$, i.e. when it is possible to switch from $D^1 = (\alpha, 1 - \alpha)$ to $D^2 = (1, 0)$.

5 Investment timing vs. diet flexibility

In this section we derive the value of the option to invest in the plant producing biogas with flexible diet for the digester as well as the optimal timing rule. More specifically we assume that, once installed, the plant produces biogas using the diet D^1, while maintaining the flexibility to move to a new one, D^2, every time c_t fluctuates above (below) the cost of the other input d. Hence, for any given diet D^1, it makes sense to assume that the firm fixes the optimal diet revision (i.e. how much to diverge from D^1), at the time the investment in the flexible biogas plant is undertaken.

[26] See section A.3 in the appendix where we determine the optimal $\gamma < 1$ when $c_t < d$ and verify the symmetry between the two scenarios.

[27] Note that for $\widehat{c} \leq d$ the plant manager always chooses $\gamma^*(\alpha, c_t) < \frac{1}{\alpha}$.

Let's consider the option to invest in the region $d < c_t < \infty$ where $\gamma^* > 1$. We denote by $F(c_t)$ the value of such option. $F(c_t)$ solves the following dynamic problem:

$$\Gamma F(c_t) = 0 \tag{11}$$

where $\Gamma = \frac{1}{2}\sigma^2 c_t^2 \frac{\partial^2}{\partial c_t^2} - r$.

The solution to this differential equation is

$$F(c_t) = H c_t^{\beta_2} \quad \text{for } d < c^* < c_t \tag{12}$$

where c^* is the threshold where the exercise of the option to invest is optimal.[28] The constant H_2 and the optimal investment trigger c^* can be derived attaching to (12) the following matching value and smooth pasting conditions:

$$F(c^*) = NPV(c^*, \gamma^*(c^*); \alpha) \tag{13}$$
$$F'(c^*) = NPV_c(c^*, \gamma^*(c^*); \alpha) \tag{14}$$

where NPV is given by (10).[29]

We know, by (9), that the optimal diet adjustment parameter γ^* depends on c_t within the subset $\widehat{c} \leq c_t < \infty$ while it takes the maximum feasible level, $\frac{1}{\alpha}$, for $d < c_t < \widehat{c}$. This implies that the value of the option, $F(c_t)$ and the optimal exercise threshold c^* are state-contingent and must be determined for each subset. The solution to the system [13-14] yields the following results:

Proposition 2 *1) The optimal investment trigger for the flexible technology is given by*

$$c^* = \frac{\beta_2}{\beta_2 - 1} \frac{p - (1-\alpha)d - rI(\gamma^*(c^*))}{\alpha} \tag{15}$$

where

$$I(\gamma^*(c^*)) = \begin{cases} \frac{1}{2}k\frac{1-\alpha}{\alpha}(\frac{c^*}{\widehat{c}})^{2\beta_2} & \text{for } \widehat{c} \leq c^* < \infty \\ \frac{1}{2}k\frac{1-\alpha}{\alpha} & \text{for } d < c^* < \widehat{c} \end{cases}$$

2) The value of the option to invest is given by

$$F(c_t) = \begin{cases} NPV(c^*, \gamma^*(c^*); \alpha)(\frac{c_t}{c^*})^{\beta_2} & \text{for } c_t > c^* \\ NPV(c_t, \gamma^*(c^*); \alpha) & \text{for } c_t \leq c^* \end{cases} \tag{16}$$

Proof. See section A.4 in the appendix. ■

According to Proposition (2), it is worth investing in the plant when the cost of the input c_t is lower or equal to c^*. Rearranging (15) we obtain

$$\frac{p - C_1(c^*)}{r} - I(\gamma^*(c^*)) = -\frac{1}{\beta_2}\frac{\alpha c^*}{r} > 0 \tag{17}$$

That is, the investment should occur when the NPV of the initial cash flow, $\frac{p-C_1(c^*)}{r}$, covers the investment cost, $I(\gamma^*(c^*))$. However, unlike the standard Marshallian rule where investment

[28] The general solution to (11) takes the form $F(c_t) = H_1 c_t^{\beta_1} + H_2 c_t^{\beta_2}$. Since for $c_t \to \infty$ the value of the option to invest, $F(c_t)$, should vanish, the boundary condition $\lim_{c_t \to \infty} F(c_t) = 0$ is required. It follows that for such condition to hold $H_1 = 0$.

[29] Totally differentiating $F(c^*)$ we obtain $F'(c^*) = NPV_c(c^*, \alpha, \gamma^*(c^*)) + NPV_\gamma(c^*, \alpha, \gamma^*(c^*))\frac{d\gamma^*}{c_t}$. But since γ is optimally chosen, then $NPV_\gamma(c^*, \alpha, \gamma^*(c^*)) = 0$.

should be undertaken when $NPV(c^*, \gamma^*(c^*); \alpha) \geq 0$, here $NPV(c^*, \gamma^*(c^*); \alpha)$ must be higher than $-\frac{1}{\beta_2}\frac{\alpha c^*}{r}$. Put differently, since the firm holds the option to postpone the investment and collect information on the project profitability, the full Marshallian cost which should be imputed to the plant is $I(\gamma^*(c^*)) - \frac{1}{\beta_2}\frac{\alpha c^*}{r} > I(\gamma^*(c^*))$. See figures[30] 3.

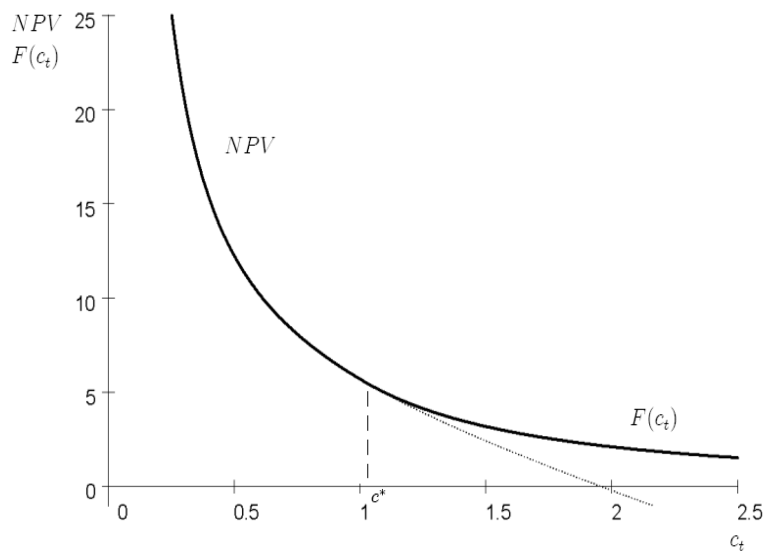

Figure 3: Value of investment with $c^*= 1.018652524$, $p = 1.3$, $\alpha = 0.3$, $r = 0.07$, $\sigma = 0.2$, $k = 1$, $d = 1$.

Condition (17) highlights also the trade-off between investment timing and diet flexibility. Note in fact that the wedge, $-\frac{1}{\beta_2}\frac{\alpha c^*}{r}$, which corresponds to the opportunity cost of investing as soon as the NPV is positive, depends on the optimal investment trigger c^*. To hold the option to switch from diet D^1 to diet D^2 increases the NPV and allows the firm to invest earlier. To discuss this important trade-off and disentangle the impact of volatility and investment cost on the investment decision, we present in the next section some useful comparative statics.

5.1 Comparative statics

Let's now characterize analytically the solution in (15) and its properties by some comparative statics with respect to the volatility, σ, of the input price c_t and the dimensional term of the investment cost. k. As shown in the appendix

Proposition 3 *1) The optimal investment trigger, c^*, is monotonically decreasing in the volatility, σ, i.e.*

$$\frac{\partial c^*}{\partial \sigma} < 0 \quad for\ d < c^* < \infty \tag{18}$$

[30]For figures and tables in this section we consider a biogas plant under a $1MW$ capacity and a Combined Heat and Power (CHP) system to burn biogas. Burning 1 m^3 of biogas provides 21 MJ which, by the equivalence $3.6MJ = 1kWh$ and allowing for a thermal efficiency of 89%, corresponds to 5.18 kWh electricity (see http://en.wikipedia.org/wiki/Cogeneration). Consistently with evidence from Italy, the output price is set equal to 0.3 $euro/kWh$ while maintenance/operating costs are in the range [0.025, 0.040] $euro/kWh$ (see for instance Boschetti, 2006 and Banzato et al., 2010). Finally, following Callaghan et al. (2002) we set $D^1 = (0.3; 0.7)$.

2) The optimal investment trigger, c^*, is decreasing in the investment cost dimensional term, k when $d < c^* < \widehat{c}$ and increasing otherwise , i.e.

$$\frac{\partial c^*}{\partial k} \begin{cases} < 0 & for\ d < c^* < \widehat{c} \\ > 0 & for\ \widehat{c} \leq c^* < \infty \end{cases} \tag{19}$$

Proof. See section A.6 in the appendix ■

The first part of Proposition (3) illustrates a standard finding in the literature on the investment-uncertainty relationship. Under uncertainty about future prospects a firm undertaking an irreversible investment may prudentially postpone the investment decision in order to collect further information on c_t. By plugging (15) into the wedge, $-\frac{1}{\beta_2}\frac{\alpha c^*}{r}$, in (17) and using Proposition (3) it is straightforward to show some limit results.[31] First, note that as the frame becomes deterministic ($\sigma \to 0$) then $\beta_2 \to -\infty$ and (17) reduces to the standard Marshallian rule. Second, as uncertainty soars ($\sigma \to \infty$) then $\beta_2 \to 0$, the firm adopts the maximum degree of flexibility, $\gamma = \frac{1}{\alpha}$, and invests at the lowest feasible level $c^* = d$. Both these results make sense, in fact, if uncertainty disappears it is not worth to wait for additional information before investing, while under higher uncertainty the firm prefers a higher degree of flexibility and trades it off with a delay in the investment.

In addition, it is worth to note that since the threshold $\widehat{c}$ ($\gamma^*(\widehat{c};\alpha) = \frac{1}{\alpha}$) increases in the level of uncertainty then as $\sigma \to 0$ the domain $d < c^* < \widehat{c}$ collapses. As shown in the first column of table 2 for $\sigma = 0.1$, $\widehat{c}$ is close to $d = 1$ or even lower. This implies that for low levels of uncertainty, it is worth investing in a level of flexibility lower than $\frac{1}{\alpha}$ since it increases the probability of not benefiting from it. On the contrary, as $\sigma \to \infty$ the domain $\widehat{c} < c^* < \infty$ shrinks and by (15), the investment threshold reduces to $c^* = \frac{\beta_2}{\beta_2 - 1}\frac{p-(1-\alpha)d - r\frac{1}{2}k\frac{1-\alpha}{\alpha}}{\alpha}$. In this case, as σ increases, even if the benefits from flexibility must be traded off with the investment cost magnitude (k), the firm always finds it desirable to install a more flexible technology (See table 2).

The second part of Proposition (3) provides instead a counterintuitive result. While in a standard frame, investment timing and investment cost are negatively related (i.e. the higher the investment cost, the lower the investment threshold and the probability of hitting it), we obtain the opposite effect in the domain $\widehat{c} < c^* < \infty$. This means that the firm can choose a degree of flexibility which, optimally managing the switch between D^1 and D^2, can balance the higher investment cost and guarantee a NPV high enough to sustain earlier investment. This is evident in table 2: as k increases γ decreases and, by reducing $I(\gamma^*(c^*))$, results in a higher c^*. Note also that since the threshold $\widehat{c}$ is decreasing in k, as the cost of the plant k increases, the domain $d < c^* < \widehat{c}$ shrinks and $\frac{\partial c^*}{\partial k} > 0$ over the entire set (d, ∞).

This effect does not occur in the domain $d < c^* \leq \widehat{c}$ where the firm "must" choose the maximum degree of flexibility (i.e. the corner solution). The required initial investment becomes so high that the firm prefers to postpone the investment in order to gather information on c_t. As may be expected, the sub-optimal choice of diet flexibility prevents the firm from balancing the increasing investment cost with an optimal reduction of γ. This is evident in table 2 for $\sigma = 0.2$ and 0.3. The level of uncertainty is so high that the firm installs (sub-optimally) the highest feasible level

[31] See section A.7 in the appendix.

of flexibility. Therefore, as k increases γ remains stuck to $\frac{1}{\alpha}$ and c^* must decrease.

Table 1: Betas and option multiple for $r = 0.07$

	σ		
	0.1	0.2	0.3
β_1	4.274917218	2.436491673	1.843709624
β_2	-3.274917218	-1.436491673	-0.843709624
$\beta_2 / (\beta_2 - 1)$	0.7660773416	0.5895738077	0.457615241

Table 2: Investment timing vs. diet flexibility for $\alpha = 0.3$, $p = 1.3$, $d = 1$, $r = 0.07$

	σ					
	0.1		0.2		0.3	
Range	$d < c_t \leq \hat{c}$	$\hat{c} < c_t$	$d < c_t \leq \hat{c}$	$\hat{c} < c_t$	$d < c_t \leq \hat{c}$	$\hat{c} < c_t$
k=0.5						
$\hat{c}$	1.03951		1.7385		3.95392	
c^*	-	1.52363	1.089	-	1	-
$\pi(c^*)$	-	0.14291	0.27033	-	0.3	-
$\gamma(c^*)$	-	1.66708	3.333	-	3.333	-
NPV	-	2.08925	5.53338	-	7.42342	-
k=1						
$\hat{c}$	0.84122		1.07304		1.73874	
c^*	-	1.52797	1.01865	1.07304	1	-
$\pi(c^*)$	-	0.14161	0.2944	0.27809	0.3	-
$\gamma(c^*)$	-	1.33044	3.3333	3.3333	3.3333	-
NPV	-	2.04637	5.55345	5.13937	6.84009	-
k=2.5						
$\hat{c}$	0.63591		0.56701		0.58692	
c^*	-	1.5305	-	1.12279	-	1
$\pi(c^*)$	-	0.14085	-	0.26316	-	0.3
$\gamma(c^*)$	-	1.13146	-	1.8745	-	2.48842
NPV	-	2.0214	-	4.16918	-	5.47253
k=5						
$\hat{c}$	0.51461		0.34997		0.2581	
c^*	-	1.53133	-	1.15304	-	1
$\pi(c^*)$	-	0.1406	-	0.25409	-	0.3
$\gamma(c^*)$	-	1.06561	-	1.42087	-	1.74421
NPV	-	2.0132	-	3.81961	-	4.87912

5.2 The value of diet flexibility

To conclude our discussion on the relationship between investment timing and diet flexibility, we firstly isolate the value added by investing in a flexible diet technology for the production of biogas and secondly we provide a measure for the corresponding effect in terms of investment timing. We determine the value added by diet flexibility by comparing the net present values of the investment project when investing without and with diet flexibility. That is,[32]

$$\Delta\%NPV = \frac{NPV(c^{+},\gamma^{*}(c^{+});\alpha) - NPV(c^{+},1;\alpha)}{NPV(c^{+},\gamma^{*}(c^{+});\alpha)} \tag{20}$$

where c^{+} is the optimal investment threshold for a non flexible biogas production, i.e. $\gamma = 1$.

In addition, we derive the expected optimal anticipation or delay with respect to the investment at c^{+}, that is (Dixit 1993, pp. 54-57):

$$E\left[T^{*} - T^{+}\right] = -\frac{\ln\left[\frac{c^{*}}{c^{+}}\right]}{\frac{\sigma^{2}}{2}} \tag{21}$$

where $T^{i} = \inf\left(t > 0 \mid c_{t} = c^{i}\right)$ with $i = ^{*},\ ^{+}$.

The numerical results are presented in table 3. In particular, it is evident that the benefit in terms of NPV is decreasing in the magnitude of investment cost and can be substantial, even for a high k, when profits are highly uncertain (see third column). Yet higher gains in terms of NPV may correspond to higher implicit cost in terms of cash flows given up during the time period $T^{*} - T^{+}$. This is evident in the first two columns of table 3 where uncertainty is low or medium. However, note that in both columns as k increases the investment delay reduces when it is possible to install a less flexible technology. This is consistent with the discussion contained in the previous section where we noticed the increasing effect of k on the optimal threshold c^{*}. Obviously as k increases, $\Delta\%NPV$ reduces. On the contrary, when the firm must invest in the highest feasible level of flexibility ($\sigma = 0.2$, $k = 0.5$ and $k = 1$), a higher k implies more prudence and a delay in the investment. Finally, since highly volatile inputs prices make flexibility valuable then $\Delta\%NPV$ must be high enough to profitably sustain an earlier investment.

Summing up, these results provide some useful guidelines characterizing the decision to install a biogas plant, i.e.:

Remark 1 Higher diet flexibility always guarantees a benefit in terms of NPV.

Remark 2 Under low-medium uncertainty about future profits, the investor trades off higher diet flexibility and investment delay.

Remark 3 Under high uncertainty about future profits, higher diet flexibility is always preferred

[32] Note that we take the timing of investing without flexibility as reference point.

and the investment is anticipated.

Table 3: The value of diet flexibility for $\alpha = 0.3$, $p = 1.3$, $d = 1$, $r = 0.07$

	σ					
	0.1		0.2		0.3	
Range	$d < c_t \leq \hat{c}$	$\hat{c} < c_t$	$d < c_t \leq \hat{c}$	$\hat{c} < c_t$	$d < c_t \leq \hat{c}$	$\hat{c} < c_t$
k=0.5						
$\hat{c}$	1.03951		1.7385		3.95392	
c^*	1.5322		1.1791		0.91523	
$\pi(c^*)$	-	1.52363	1.089	-	1	-
$\gamma(c^*)$	-	2.2414%	29.25%	-	42.431%	-
NPV	-	1.1215	3.9745	-	-1.9684	-
k=1						
$\hat{c}$	0.84122		1.07304		1.73874	
c^+	1.5322		1.1791		0.91523	
c^*	-	1.52797	1.01865	1.07304	1	-
Δ%NPV	-	1.1332%	19.848%	20.188%	37.948%	-
$E[T^* - T^+]$	-	0.55263	7.3135	4.7129	-1.9684	-
k=2.5						
$\hat{c}$	0.63591		0.56701		0.58692	
c^+	1.5322		1.1791		0.91523	
c^*	-	1.5305	-	1.12279	-	1
Δ%NPV	-	0.45636%	-	9.1883%	-	22.866%
$E[T^* - T^+]$	-	0.22207	-	2.4469	-	-1.9684
k=5						
$\hat{c}$	0.51461		0.34997		0.2581	
c^+	1.5322		1.1791		0.91523	
c^*	-	1.53133	-	1.15304	-	1
Δ%NPV	-	0.22875%	-	4.8154%	-	12.907%
$E[T^* - T^+]$	-	0.11358	-	1.1175	-	-1.9684

6 Conclusions

In this paper we analyze the effect of flexibility on the decision to invest in a biogas plant. In a stochastic dynamic frame, we model a problem where input factors are substitute but, unlike Kulatilaka (1993) and He and Pindyck (1992), need to be mixed together to provide output. Given any initial rule for the composition of the feedstock, we consider the possibility of revising it if economic circumstances make it profitable. Flexibility in the mix is an advantage under randomly fluctuating input costs and comes at a higher investment cost. We show that the degree of flexibility in the productive technology installed depends on the value of the option, implicitly provided with the initial investment, to profitably re-arrange the input mix. Such option adds value to the project in that it provides a device for hedging against fluctuations in the input relative convenience. Accounting for such value we discuss the trade-off between investment timing and profit smoothing flexibility.

We believe that our analysis may apply also to other frames. First, to FFEs which are engines capable of burning blends where gasoline and either ethanol or methanol are present in different proportions. Under uncertain fossil fuel prices and changes in transport regulations due to environmental concerns, flexibility is an advantage and may justify the adoption of more costly FFEs and push R&D investment in FFEs. Second, it may also apply to investment in co-firing power stations where co-combustion of coal, waste and/or biomass may provide cheaper and more environmentally friendly energy. Also in this case, flexibility in the operations allows reduction in greenhouse gas emissions to meet new energy and environmental policy requirements and hedging against volatile fossil fuel prices. Apart from higher investment costs due to the design and optimization of coal and biomass blend and burning facilities, another issue, affecting also biogas production, is represented by the availability of biomass. This requires setting up an ample network of providers close enough to the plant to reduce transport costs. The set-up of such network impacts also on the degree of vertical integration chosen and involves additional sunk costs due to the organizational design, i.e. search cost for providers, monitoring input quality and delivery timing, and contract enforcement. In this specific respect but also in a more general setting, our model is a valid instrument for analysis of the role of flexibility in vertical arrangements affecting firm integration.[33]

[33] The initial rule can also be thought of as a determined vertical structure where only part of the input factor required is outsourced. Alvarez and Stenbacka (2007), Wang et al. (2007) and Moretto and Rossini (2008) provide a theoretical analysis of partial outsourcing where levels of vertical integration vary continuously over the unit interval. In particular, our model can be seen as a generalization of the model in Moretto and Rossini (2008) where, departing from the previous contributions, the switch to a new organizational design is not seen as irreversible and the firm can switch back to the original set-up.

A Appendix

A.1 The value of the flexible technology

The general solution to the differential equations (3) and (4) takes the form:[34]

$$V^{(D^1)}(c_t,\gamma;\alpha) = \tfrac{p-C_1}{r} + \widehat{A}_2 c_t^{\beta_2} \quad \text{for } d < c_t < \infty \tag{A.1.1}$$

$$V^{(D^2)}(c_t,\gamma;\alpha) = \tfrac{p-C_2}{r} + \widehat{B}_1 c_t^{\beta_1} \quad \text{for } c_t < d \tag{A.1.2}$$

where $\beta_1 > 1$ and $\beta_2 < 0$ are the roots of the characteristic equation $\phi(\beta) = \frac{1}{2}\sigma^2\beta(\beta-1) - r = 0$. In (5) and (6) the first term indicates the present value of producing biogas using forever $D^1$and D^2 respectively. The second term represents the value of the option to switch to the alternative diet. Note that since the availability of strategic options always increases the value of a project, the constants $\widehat{A}_2$ and $\widehat{B}_1$ must be non-negative.[35]

At $c_t = d$, the standard pair of conditions for optimal exercise must hold, the value-matching condition

$$V^{(D^1)}(d,\gamma;\alpha) = V^{(D^2)}(d,\gamma;\alpha) \tag{A.1.3}$$

and the smooth-pasting condition

$$V_c^{(D^1)}(d,\gamma;\alpha) = V_c^{(D^2)}(d,\gamma;\alpha). \tag{A.1.4}$$

Solving the system [A.1.3-A.1.4] yields

$$\begin{aligned}\hat{A}_2 &= (\gamma-1)A = (\gamma-1)\frac{\alpha}{r(\beta_1-\beta_2)}d^{1-\beta_2} \geq 0 \\ \hat{B}_1 &= (\gamma-1)B = (\gamma-1)\frac{\alpha}{r(\beta_1-\beta_2)}d^{1-\beta_1} \geq 0\end{aligned}$$

By plugging the two constants into (A.1.1) and (A.1.2) we obtain (5) and (6).

A.2 Proof of Proposition 1

Suppose $c_t > d$ and $\gamma > 1$. The optimal level of flexibility is given by

$$\begin{aligned}\gamma^* &= \arg\max NPV^{(\gamma>1)}(c_t,\gamma;\alpha) \\ &= \arg\max\left[\frac{p-C_1}{r} + (\gamma-1)Ac_t^{\beta_2} - k\frac{\alpha}{1-\alpha}\frac{(\gamma-1)^2}{2}\right]\end{aligned} \tag{A.2.1}$$

where $A = \frac{\alpha}{r(\beta_1-\beta_2)}d^{1-\beta_2} > 0$.

From the FOC for (A.2.1) we obtain

$$\gamma^* - 1 = \frac{Ac_t^{\beta_2}}{k\frac{\alpha}{1-\alpha}} \tag{A.2.2}$$

[34] Note that under D^1 the general solution to (5) should take the form $V^{(D^1)}(c_t,\alpha,\gamma) = \frac{p-C_1}{r} + \widehat{A}_1 c_t^{\beta_1} + \widehat{A}_2 c_t^{\beta_2}$ where the second and third terms stand for the value of the option to switch to D^1. However, since the value of the option vanishes as $c_t \to \infty$ then we set $\widehat{A}_1 = 0$. Similarly, under D^2 the general solution to (6) should take the form $V^{(D^2)}(c_t,\alpha) = \frac{p-C_2}{r} + \widehat{B}_1 c_t^{\beta_1} + \widehat{B}_2 c_t^{\beta_2}$. However, the option to switch to D^1 is valueless as $c_t \to 0$ and then we set $\widehat{B}_2 = 0$.

[35] See Dixit and Pindyck (1994, chps. 6 and 7) for a thorough discussion.

and it is easy to prove that the SOC is always satisfied. From A.2.2 it turns out that:

$$\gamma^*\left(c_t;\alpha\right)=\begin{cases} 1+\frac{1-\alpha}{\alpha}\left(\frac{c_t}{\widehat{c}}\right)^{\beta_2} & \text{for } \widehat{c}\le c_t \\ \frac{1}{\alpha} & \text{for } d<c_t<\widehat{c} \end{cases} \tag{A.2.3}$$

where $\widehat{c}=\left(\frac{k}{A}\right)^{1/\beta_2}$.

A.3 Symmetry

Suppose $c_t<d$ and $\gamma<1$. The optimal level of flexibility is given by

$$\begin{aligned}\gamma^* &= \arg\max NPV^{(\gamma<1)}(c_t,\gamma;\alpha) \qquad\qquad (A.3.1)\\ &= \arg\max\left[\frac{p-C_1}{r}+(1-\gamma)Bc_t^{\beta_1}-k\frac{\alpha'}{1-\alpha'}\frac{(\gamma-1)^2}{2}\right]\end{aligned}$$

where $B=\frac{\alpha'}{r(\beta_1-\beta_2)}d^{1-\beta_1}>0$.

The FOC for (A.3.1) yields

$$(\gamma^*-1)=-\frac{Bc_t^{\beta_1}}{k\frac{\alpha'}{1-\alpha'}} \tag{A.3.2}$$

and the SOC is always satisfied. From (A.3.2) we obtain

$$\gamma^*\left(c_t;\alpha'\right)=\begin{cases} 1-\left(\frac{c_t}{\widetilde{c}}\right)^{\beta_1} & \text{for } 0<c_t\le\widetilde{c} \\ 0 & \text{for } \widetilde{c}<c_t<d \end{cases} \tag{A.3.3}$$

where $\widetilde{c}=\left(\frac{k\frac{\alpha'}{1-\alpha'}}{B}\right)^{1/\beta_1}$.

To prove the symmetry between the two scenarios it is sufficient to show that $\alpha=1-\alpha'$. First, suppose that $\widehat{c}=d$. It follows that

$$\begin{aligned}\left(\frac{k}{A}\right)^{1/\beta_2} &= d\\ k &= Ad^{\beta_2}\\ k &= \frac{\alpha}{r\left(\beta_1-\beta_2\right)}d\end{aligned}$$

Second, impose $\widetilde{c}=d$. This yields

$$\begin{aligned}\left(\frac{k\frac{\alpha'}{1-\alpha'}}{B}\right)^{1/\beta_1} &= d\\ k &= \frac{1-\alpha'}{\alpha'}Bd^{\beta_1}\\ k &= \frac{1-\alpha'}{r\left(\beta_1-\beta_2\right)}d\end{aligned}$$

Clearly, both equalities hold only if $\alpha=1-\alpha'$. Note that by setting $\widetilde{c}=\widehat{c}=d$ we are actually considering the entire set of feasible diets $\gamma\in[0,1/\alpha]$. Finally, note also that if $\alpha=1-\alpha'$ then the investment cost function in (7) is symmetric, i.e.

$$I\left(\frac{1}{\alpha}\right)=\frac{1}{2}\frac{1-\alpha}{\alpha}=\frac{1}{2}\frac{\alpha'}{1-\alpha'}=I(0)$$

A.4 Proof of Proposition 2

We consider first the case where $d < c^* \leq \widehat{c}$. In this domain, the firm invests in a technology with $\gamma^* = 1/\alpha$. Substituting and solving the system [13-14] we obtain

$$c^* = \frac{\beta_2}{\beta_2 - 1}\frac{p - (1-\alpha)\, d - \frac{1}{2} rk \frac{1-\alpha}{\alpha}}{\alpha} \tag{A.4.1}$$

$$H = \left\{ \frac{p - (1-\alpha)\, d - \alpha c^*}{r} + k\frac{1-\alpha}{\alpha}\left[\left(\frac{c^*}{\widehat{c}}\right)^{\beta_2} - \frac{1}{2} \right] \right\} c^{*-\beta_2} \tag{A.4.1bis}$$

If $c^* \geq \widehat{c}$, the firm chooses a technology with $\gamma^* = 1 + \frac{1-\alpha}{\alpha}(\frac{c_t}{\widehat{c}})^{\beta_2}$. Substituting this value into (13) and (14) and solving the system yields:

$$c^* = \frac{\beta_2}{\beta_2 - 1}\frac{p - (1-\alpha)\, d - \frac{1}{2} rk \frac{1-\alpha}{\alpha}\left(\frac{c^*}{\widehat{c}}\right)^{2\beta_2}}{\alpha} \tag{A.4.2}$$

$$H = \left[\frac{p - (1-\alpha)\, d - \alpha c^*}{r} + \frac{1}{2}k\frac{1-\alpha}{\alpha}\left(\frac{c^*}{\widehat{c}}\right)^{2\beta_2} \right] c^{*-\beta_2} \tag{A.4.2bis}$$

where $c^* \geq \widehat{c}$, is the solution of the implicit equation in (A.4.2). By plugging H_2 into (12) we obtain (16).

Let's now check for the existence of c^*. Define $f(c_t) = \frac{p-(1-\alpha)d-\alpha c_t(1-\frac{1}{\beta_2})}{\frac{1}{2}rk\frac{1-\alpha}{\alpha}}$ and $g(c_t) = \left(\frac{c_t}{\widehat{c}}\right)^{2\beta_2}$. If $d < \widehat{c}$ then $f(\widehat{c}) \leq 1$ is a sufficient and necessary condition for $c^* \leq \widehat{c}$. It is then easy to verify that a sufficient condition for $c^* > \widehat{c}$ is $f(\widehat{c}) \geq 1$. Equation (A.4.2) may have two solutions, c^* and $\overline{c}$ with $c^* > \overline{c}$. However, since optimality requires $F''(c^*) > 0$ then it can be easily checked that the second order condition holds only for c^*. Finally, note that if $\widehat{c} \leq d$ then $\gamma^*(c_t; \alpha) \leq \frac{1}{\alpha}$ and $f(\widehat{c}) \geq 1$ is still a sufficient condition for $c^* > \widehat{c}$.

A.5 Imperfect substitutability

Consider a plant using a Cobb-Douglas technology mixing two different types of materials to produce 1 m^3 of biogas. Solve the following cost minimization problem:

$$c(w, x) = \min\left[w_1 x_1 + w_2 x_2\right] \quad \text{such that } x_1^{\alpha} x_2^{1-\alpha} = 1$$

where x_1 and x_2 are the quantity of the two inputs, α, $1-\alpha$ the output elasticities and w_1 and w_2 the unit input prices.

The solution yields the conditional demand functions for both factors and the cost function:

$$\begin{aligned} x_1(w_1,w_2) &= \left[\frac{\alpha}{1-\alpha}\frac{w_2}{w_1}\right]^{1-\alpha} \\ x_2(w_1,w_2) &= \left[\frac{\alpha}{1-\alpha}\frac{w_2}{w_1}\right]^{-\alpha} \\ c(w_1, w_2) &\equiv K w_1^{\alpha} w_2^{1-\alpha} \end{aligned}$$

where $K = \alpha^{-\alpha}(1-\alpha)^{\alpha-1}$.

Denote by q the unit output price and note that profits maximization is equivalent to the maximization of the ratio $\frac{q}{c(w_1,w_2)}$. Taking its logarithm and rearranging, we obtain the following objective function

$$\begin{aligned} \pi &= p - [\alpha c + (1-\alpha) d] \\ &= p - C_1 \end{aligned}$$

where $p = \ln \frac{q}{K}$, $c = \ln w_1$ and $d = \ln w_2$. Thus, our frame may easily apply also to the case of imperfect substitutable input factors.

A.6 Proof of Proposition 3

Let's rearrange (15) as follows

$$(1 - \frac{1}{\beta_2})\alpha c^* = p - (1-\alpha)\, d - rI(\gamma^*(c^*)) \tag{A.6.1}$$

Differentiating on both sides with respect to σ

$$(1 - \frac{1}{\beta_2})\alpha \frac{\partial c^*}{\partial \sigma} + \frac{\alpha c^*}{\beta_2^2}\frac{\partial \beta_2}{\partial \sigma} = -r\frac{\partial I(\gamma^*(c^*))}{\partial \sigma}$$

Note that

$$\frac{\partial I(\gamma^*(c^*))}{\partial \sigma} = \begin{cases} \frac{1}{2}k\frac{1-\alpha}{\alpha}\frac{\partial\left[(\frac{c^*}{\widehat{c}})^{2\beta_2}\right]}{\partial \sigma} > 0 & \text{for } \widehat{c} < c^* < \infty \\ 0 & \text{for } d < c^* \leq \widehat{c} \end{cases}$$

It follows that in the domain $\widehat{c} < c^* < \infty$

$$\frac{\partial c^*}{\partial \sigma} = -\frac{\frac{\alpha c^*}{\beta_2^2}\frac{\partial \beta_2}{\partial \sigma} + rk\frac{1-\alpha}{\alpha}(\frac{c^*}{\widehat{c}})^{2\beta_2}[\ln(\frac{c^*}{\widehat{c}})\frac{\partial \beta_2}{\partial \sigma} - \frac{\beta_2}{\widehat{c}}\frac{\partial \widehat{c}}{\partial \sigma}]}{(1-\frac{1}{\beta_2})\alpha + rk\frac{\beta_2}{c^*}\frac{1-\alpha}{\alpha}(\frac{c^*}{\widehat{c}})^{2\beta_2}} < 0$$

in that,

$$\frac{\partial \beta_2}{\partial \sigma} > 0, \frac{\partial \widehat{c}}{\partial \sigma} = -\left\{\frac{\ln\left[\frac{rk(1-2\beta_2)}{\alpha d}\right]}{\beta_2} + \frac{2}{1-2\beta_2}\right\}\frac{\widehat{c}}{\beta_2}\frac{\partial \beta_2}{\partial \sigma} > 0$$

and, by the second order condition for the optimality of c^*,

$$(1-\frac{1}{\beta_2})\alpha + rk\frac{\beta_2}{c^*}\frac{1-\alpha}{\alpha}(\frac{c^*}{\widehat{c}})^{2\beta_2} > 0$$

For $d < c^* \leq \widehat{c}$ it is straightforward to show that

$$\frac{\partial c^*}{\partial \sigma} = (\frac{1}{\beta_2} - 1)c^*\frac{\partial \beta_2}{\partial \sigma} < 0$$

Differentiating (A.6.1) with respect to k

$$(1 - \frac{1}{\beta_2})\alpha\frac{\partial c^*}{\partial k} = -r\frac{\partial I(\gamma^*(c^*))}{\partial k}$$

where

$$\frac{\partial I(\gamma^*(c^*))}{\partial k} = \begin{cases} \frac{1}{2}\frac{1-\alpha}{\alpha}\left\{(\frac{c^*}{\widehat{c}})^{2\beta_2} + k\frac{\partial\left[(\frac{c^*}{\widehat{c}})^{2\beta_2}\right]}{\partial k}\right\} < 0 & \text{for } \widehat{c} < c^* < \infty \\ \frac{1}{2}\frac{1-\alpha}{\alpha} > 0 & \text{for } d < c^* \leq \widehat{c} \end{cases}$$

Rearranging, it is easy to show that $\frac{\partial \widehat{c}}{\partial k} = \frac{\widehat{c}}{k\beta_2} < 0$ and

$$\frac{\partial c^*}{\partial k} = \begin{cases} \frac{\frac{1}{2}r\frac{1-\alpha}{\alpha}(\frac{c^*}{\widehat{c}})^{2\beta_2}}{(1-\frac{1}{\beta_2})\alpha + rk\frac{\beta_2}{c^*}\frac{1-\alpha}{\alpha}(\frac{c^*}{\widehat{c}})^{2\beta_2}} > 0 & \text{for } \widehat{c} < c^* < \infty \\ -\frac{1}{2}r\frac{1-\alpha}{\alpha} < 0 & \text{for } d < c^* \leq \widehat{c} \end{cases}$$

A.7 Some limit results

Plugging (15) into $-\frac{1}{\beta_2}\frac{\alpha c^*}{r}$ yields

$$-\frac{1}{\beta_2}\frac{\alpha c^*}{r} = \frac{\frac{p-(1-\alpha)d}{r} - I(\gamma^*(c^*))}{1-\beta_2} \tag{A.7.1}$$

Differentiating (A.7.1) with respect to σ and k, we obtain

$$\frac{\partial\left[\frac{\frac{p-(1-\alpha)d}{r}-I(\gamma^*(c^*))}{1-\beta_2}\right]}{\partial\sigma} = \frac{\frac{p-(1-\alpha)d}{r} - I(\gamma^*(c^*))}{(1-\beta_2)^2}\frac{\partial\beta_2}{\partial\sigma} - \frac{1}{(1-\beta_2)}\frac{\partial I(\gamma^*(c^*))}{\partial\sigma} > 0 \tag{A.7.2}$$

and

$$\frac{\partial\left[\frac{\frac{p-(1-\alpha)d}{r}-I(\gamma^*(c^*))}{1-\beta_2}\right]}{\partial k} = -\frac{1}{1-\beta_2}\frac{\partial I(\gamma^*(c^*))}{\partial k} \begin{array}{ll} < 0 & \text{for } \widehat{c} < c^* < \infty \\ > 0 & \text{for } d < c^* \leq \widehat{c} \end{array} \tag{A.7.3}$$

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