# UNIVERSITÀ DEGLI STUDI DI PADOVA

# Dipartimento di Scienze Economiche ed Aziendali "Marco Fanno"

UNCERTAINTY AND MONETARY POLICY IN THE US:
A JOURNEY INTO NON-LINEAR TERRITORY

GIOVANNI PELLEGRINO
University of Verona

September 2014

*"MARCO FANNO" WORKING PAPER N.184*

# Uncertainty and Monetary Policy in the US:

# A Journey into Non-Linear Territory*

## GIOVANNI PELLEGRINO**

August 7, 2014

(I draft: March 31, 2014)

*Abstract*

This paper investigates the interaction between uncertainty and monetary policy by estimating a non-linear VAR with US post-WWII data. The uncertainty indicator is treated both as an endogenous variable in the VAR and as the transition indicator discriminating "high" vs. "low" uncertainty states. The impact of monetary policy shocks in different phases of the "uncertainty cycle" is assessed via the computation of Generalized Impulse Response Functions. Monetary policy shocks are found to be less effective when uncertainty is high, with the peak reactions of a battery of real variables being about two-thirds milder than those conditional on an initially low level of uncertainty. The framework is then put at work to investigate the effects of uncertainty shocks in the presence of the Zero Lower Bound. The "drop and rebound" response of real variables to uncertainty shocks documented by Bloom (2009) is found to be present only if the policy rate is a long way from its ZLB. Conversely, an uncertainty shock occurring when the economy is near the ZLB triggers longer-lasting recessions and does not lead to any significant "rebound".

*JEL-codes*: C32, E32, E52, E61

*Keywords*: Monetary policy shocks, Uncertainty shocks, non-linear Structural VAR, Interacted-VAR, Generalized impulse responses, Zero Lower Bound.

---

## 1. Introduction

Uncertainty is widely recognized as playing an important role in influencing economic activity. As documented by a number of recent studies, when an unexpected increase in economic uncertainty hits the economy, a negative response on real aggregate variables generally follows. A nonexclusive list of such works includes Bloom (2009), Baker, Bloom, and Davis (2013), Gilchrist, Sim and Zakrajsek (2013), Bachmann, Elstner and Sims (2013), Leduc and Liu (2013), Colombo (2013), and Nodari (2014). However, although the empirical relevance of uncertainty has been extensively investigated in the context of *linear* models, there has been little investigation of its relevance in the context of non-linear empirical models. At least in principle there are reasons to believe that *both* the impact of an uncertainty shock is state-dependent *and* the uncertainty level influences the impact of a monetary policy shock[1].

Although the present study will address both these issues, let us consider the latter first. Vavra (2014) and Bachmann, Born, Elstner and Grimme (2013), on the basis of some empirical facts on the frequency of price changes, develop competing structural models that predict (unforeseen) monetary policy changes to be less effective when uncertainty is high. Greater uncertainty should induce firms to change prices more frequently, therefore reducing the real effects of a monetary shock. However, although Vavra's calibrated DSGE model suggests that monetary policy is much less effective under high uncertainty, Bachmann et al.'s results suggest that the influence of uncertainty on the monetary policy transmission mechanism is almost negligible. Another theoretical reason for milder monetary policy effects under high uncertainty is provided in Aastveit, Natvik and Sola (2013). In their simple model featuring investment irreversibility and fixed investment costs, high uncertainty might reduce the influence of monetary policy on investment decisions.

This paper empirically assesses the importance of the interaction between uncertainty and monetary policy by estimating a non-linear VAR model with US post-WWII data. As first question we will ask whether the impact of monetary shocks can depend on the degree of uncertainty. An Interacted-VAR model is employed, in which the interaction terms between uncertainty and the policy rate are considered in each equation of the system. The Interacted-VAR model has been proposed recently by Towbin and Weber (2013) in the context of panel VAR models. This model (its cross-sectional correspondent) seems particularly well suited to the empirical questions approached in this paper. In principle, it allows one to have observation-driven coefficients that vary over time depending on the interactions that one judges important for the phenomenon under study. In particular, for my research, the interaction terms considered allow me to estimate marginal effects of a policy shock that are dependent on the value of the uncertainty variable, for each of the endogenous variables.

However, the procedure employed by existing studies to obtain impulse responses from an Interacted-VAR model does not seem well suited for application to the questions under investigation in the present work. This procedure consists in constructing the interaction terms as fully or partly composed of a number of exogenous variables (i.e. variables not modeled in the model). Of course, in general, this approach might be particularly helpful since it greatly simplifies the problem of constructing impulse response functions (IRFs). In fact, once a value

---

[1] Throughout the paper I will use the term "monetary policy" in the narrow sense of "interest rate policy."

for the exogenous variables is fixed, standard *linear* IRFs are obtained conditional on this exogenous value. However, in the context of the present study, since, as we will see, "uncertainty" moves (counter-cyclically) after a policy shock, this approach would cause an imprecise estimation of IRFs provided that no feedback is considered from the evolution of the system to its current behavior, in addition to concerns of simultaneous equation bias.

The present study simply accounts for these difficulties by modeling all the variables constituting the interaction terms inside the VAR as endogenous variables and computing Generalized Impulse Response Functions (GIRFs) (along the lines of Koop, Pesaran and Potter (1996)) to take account of the time-varying behavior of the system. As further advantage, empirical responses are interpreted more naturally and easily. Now starting conditions at the time of the shock *may* have a specific role in determining the response of a variable.

Following this strategy, we will be able to recover the state-dependent responses to a monetary policy shock of a battery of real variables, both for a high and a low uncertainty state. The high and low uncertainty states comprise initial histories with the uncertainty measure around its $90^{th}$ percentile and $10^{th}$ percentile respectively. GIRFs also allow us to compute the responses on a historical basis, which permit to go deeper on the way the transmission mechanism of monetary policy shocks might have changed over time. From these responses it emerges that the model, while being parsimonious, importantly seems able to capture an interesting portion of non-linearity in the underlying process.

The baseline Interacted-VAR employed comprises the following variables: uncertainty measure, CPI, real GDP, real investment, real consumption and FF rate. In the baseline analysis, economic uncertainty is proxied both by a credit spread measure (similar to that provided in Bachmann, Elstner and Sims (2013)) and by the stock market volatility index constructed by Bloom (2009). Other uncertainty proxies will be considered in the robustness checks sections.

The same model and framework are then applied to investigate the existence of a Zero Lower Bound (ZLB)-dependent effect of an uncertainty shock. This second question is particularly relevant in periods like the current one, of near-zero policy rate. A number of recent theoretical studies have investigated this issue (see Basu and Bundick (2012), Johannsen (2013), Nakata (2012)). All these studies suggest that the impact of an uncertainty shock will be greater and longer-lasting in the presence of the ZLB for the policy rate, at the very time when the monetary authority cannot perform its usual stabilizing function via conventional policy moves. In the present study I will test these theoretical results empirically. In addition to evidence from historical responses, we will compare the economy's state-conditional responses to an uncertainty shock for a near-ZLB state and for an unconditional state[2].

The main results I obtain are as follows. After an expansionary monetary shock, GDP, investment and consumption increase as expected, but they react less and more slowly if at the time of the shock uncertainty is high rather than low. Interestingly, in the case uncertainty is proxied by the credit spread, maximum average responses conditional on a high uncertainty state are almost two thirds (significantly) milder than those conditional on a low uncertainty state. These results are robust to different model specifications and uncertainty measures,

---

[2] For the "near-ZLB state" I mean a state where the policy rate is near zero at the time the shock hits. Details after.

although financial-related uncertainty measures appear to be the most important. Therefore the results suggest that uncertainty plays an important role in the transmission mechanism of monetary policy shock. Consequently, the monetary authority should be aware of the degree of economic uncertainty when designing policies. Moreover, since uncertainty is generally higher during recessions, it seems that unforeseen monetary policy is less effective exactly when it is most needed.

Secondly, a contractionary uncertainty shock is considered. After the shock real variables react immediately with a drop. Then, if the economy is far enough from the ZLB, such as in the unconditional state, they experience a quick rebound thanks to the aggressive reaction of the monetary authority; it takes just six months for consumption. These results are coherent with the "drop and rebound" effect found by Bloom (2009). However, Bloom's effect is not observed when the economy is in the ZLB state. In this case the recession induced by the shock lasts much longer, more than an year, and any overshooting is observed. Therefore, if the policy rate is kept low, large costs can expected to be paid, as the economy becomes more prone to "diseases" that are difficult to fight with standard "medicines". This might provide an argument in favor of Blanchard, Dell'Ariccia and Mauro's (2010) proposal of raising the inflation target, so that to deal better with recessionary events.

As far as I know, few empirical works shed light on issues similar to the ones approached in this paper. Relatively to the uncertainty-dependent effect of monetary policy shocks, the paper closest to the present one is Aastveit et al. (2013), which also uses an Interacted-VAR model to check empirically for that[3]. However, following previous Interacted-VARs work, they model uncertainty as exogenous and keep it fixed while recovering responses. The result is that monetary policy seems much less effective under high uncertainty. More specifically, responses for real variables - similar to mine - are four to five times weaker when uncertainty is fixed high at the $9^{th}$ decile, than when it is fixed low, at the $1^{st}$ decile. As explored below, the great difference between the results in Aastveit et al. (2013) and those in the present paper derives mainly from their lack of consideration of the endogeneity of uncertainty and their conditionally linear IRFs.

Regarding the empirical question of whether a contractionary uncertainty shock has a bigger impact on the economy in the presence of the ZLB, both Johannsen (2013) and Caggiano, Castelnuovo and Groshenny (2014) provide some evidence for this. Johannsen estimates two linear VARs on two different samples - one with and one without ZLB observations - and finds that the effects of an uncertainty shock are more recessionary in the whole-sample VAR. Caggiano et al. (2014), like the present study, also investigate the non-linear impact of uncertainty shocks, but answer a different question, namely whether the impact of a contractionary uncertainty shock is stronger in a state of recession. Their results suggest that the answer is affirmative. They use a Smooth Transition VAR and compute IRFs that are linear conditionally on a recession state. As a further result from their analysis they also find that when a "two-samples" approach is followed, Johannsen's result holds also conditional on a recession state. However, even if Johannsen's result let us imagine that real variables should react more to an uncertainty shock when the policy rate is close to zero, it does not permit us to inquire into *how and in which magnitude* definitively they react in this occurrence. In this study,

---

[3] It is actually the only paper I am aware of that deal with a similar question.

the use of GIRFs allow me to check this directly and this will permit to shed more light on the behavior of the economy near the ZLB.

The present paper is organized as follows. Section 2 describes the empirical methodology, the data and provides some empirical reasons for the use of a non-linear model, particularly of an Interacted-VAR model. In section 3 a monetary policy shock is considered and the relative results presented along with some robustness checks. In section 4 an uncertainty shock is considered. Section 5 concludes.

## 2. *Model, methodology, data and empirical motivation*

### 2.1. *The model, its features and identification*
#### 2.1.1. *The estimated model*

In order to study empirically how the transmission of monetary policy (uncertainty) shocks is influenced by the degree of economic uncertainty (the proximity to the ZLB), an Interacted Structural VAR model is employed. By interacting the monetary instrument with uncertainty it is possible to obtain – as will be clearer later - marginal effects of the policy rate (or of the uncertainty measure), that stochastically time-vary depending on the past level of uncertainty (or of the policy rate).

The Interacted-VAR model has been proposed recently for the panel case by Towbin and Weber (2013), and a number of studies employing the same methodology have followed [4]. Closer to the present work, Aastveit et al. (2013) have employed an Interacted VAR model to check the relevance of uncertainty in the transmission mechanism of monetary policy shocks.

The reduced form model I estimate can be written simply as follows :

$$Y_t = A^0 + D^0 Time_t + \sum_{l=1}^{L} A^l Y_{t-l} + \left( \sum_{l=1}^{L} C^l \, y_{t-l}^{(unc)} \cdot y_{t-l}^{(FF)} \right) + E_t \qquad (1)$$

Notice that without the terms in parentheses it corresponds to a standard linear reduced-form VAR model with $Y_t$ being the $(n \times 1)$ vector of the endogenous variables, $A^0$ the $(n \times 1)$ vector of constant terms, $D^0$ a similar vector of temporal slopes for the linear trend included, $A^l$ the usual $(n \times n)$ matrix of coefficients attached to $Y_{t-l}$, and $E_t$ the $(n \times 1)$ vector of reduced form residuals, where $n$ is the number of endogenous variables. The data employed will be discussed in details in section 2.3; for now it is enough to say that $Y_t$ will comprise six variables: an uncertainty proxy, and other macro variables for the price level, real output, real investment, real consumption and the policy rate.

What makes model 1 an interacted-VAR model is the part in parentheses though. Here $y_t^{(FF)}$ and $y_t^{(unc)}$ represent the scalar elements of the vector $Y_t$ referring to the policy rate and the uncertainty measure respectively. $C^l$ is a $(n \times 1)$ vector of coefficients for them. Thus, these parentheses contain the interaction terms appearing in each equation of the system, made

---

[4] In particular other works are Sa, Towbin and Wieladek (2013), Lanau, Wieladek (2012) and Nickel and Tudyka (2013). They use panel VARs and ask questions totally different from those dealt with in this paper.

by the products of the lags of the uncertainty variable and the corresponding lags of the policy rate.

As an illustration, in the tri-variate case with $L = 2$, $FF = 3$ (i.e. the third endogenous variable) and $unc = 2$, the regression associated to the n-th endogenous variable of model 1 is[5]:

$$y_t^{(th)} = a_{th}^0 + d_{th}^0 Time_t + a_{th,1}^1 y_{t-1}^{(1)} + a_{th,2}^1 y_{t-1}^{(2)} + a_{th,3}^1 y_{t-1}^{(3)} + a_{th,1}^2 y_{t-2}^{(1)} + a_{th,2}^2 y_{t-2}^{(2)} +$$
$$a_{th,3}^2 y_{t-2}^{(3)} + c_{th}^1 y_{t-1}^{(2)} \cdot y_{t-1}^{(3)} + c_{th}^2 y_{t-2}^{(2)} \cdot y_{t-2}^{(3)} + \varepsilon_{t,th} \ , \tag{2}$$

from which it is clear that the marginal effects of a change in the policy rate are time-varying depending on the value of uncertainty over time [6].

Notice that, unlike existing studies employing a similar Interacted-VAR model, my model considers all the variables constituting the interaction terms as endogenously determined variables. This turns out to be fundamental to my strategy's ability to recover the empirical responses of a shock.

Existing studies use interaction terms that are fully or partly composed by variables modeled as exogenous in the VAR model. These variables are then fixed to a given value when recovering IRFs, that are thus fully linear conditional on that imposed value. This approach is of course helpful and sensible in absence of endogeneity issues for the variables exogenously modeled. However, this ideal situation does not seem to hold for the present study: it is difficult to believe both that "uncertainty" remains constant after a policy shock and that the FF rate does not move after an uncertainty shock. In the absence of consideration of these issues a simultaneous equation bias might occur[7], in addition to the general concern that IRFs might be highly imprecise due to the fact that any feedback is considered from the evolution of the system to its current behavior[8].

However, once all the variables are modeled endogenously inside the VAR, the use of Generalized Impulse Responses (GIRFs) – according to Koop, Pesaran and Potter (1996) - allows us to simply account for these issues. GIRFs permit the Interacted-VAR model to behave like a fully non-linear model and to capture its behavior. This is the approach I follow. As further advantages over the existing approach to Interacted-VARs, any non-linear information

---

[5] $y_t^{(th)}$ is the n-th ordered scalar element in the vector $Y_t$, while $a_{th,j}^l$ is the coefficient in row $th$ and column $j$ in the matrix $A^l$.

[6] Notice indeed that equation 2 can be rewritten as $y_t^{(th)} = a_{th}^0 + d_{th}^0 Time_t + a_{th,1}^1 y_{t-1}^{(1)} + a_{th,2}^1 y_{t-1}^{(2)} + \left( a_{th,3}^1 + c_{th}^1 y_{t-1}^{(2)} \right) y_{t-1}^{(3)} + a_{th,1}^2 y_{t-2}^{(1)} + a_{th,2}^2 y_{t-2}^{(2)} + \left( a_{th,3}^2 + c_{th}^2 y_{t-2}^{(2)} \right) y_{t-2}^{(3)} + \varepsilon_{t,th}$. It exists also another similar representation that inverts the role between uncertainty and the policy rate.

[7] Think to the case the uncertainty measure was modeled as exogenous. If in reality uncertainty increased after a recessionary policy shock (as we will see it seems plausible) this would imply, in general, that the error in the structural equation for the FF rate we want to identify is positively correlated with the uncertainty measure.

[8] For example, to this respect, Ramey and Zubairy (2013) criticize the conditionally linear responses used by Auerbach and Gorodnichenko (2012) in their Smooth-Transition VAR to obtain the reaction of the economy to an expansionary fiscal spending shock. Indeed this approach not consider the feedbacks from the shock effects to the current state of the system. In Ramey and Zubairy's words: "the assumption implies that a positive shock to government spending during a recession does not help the economy escape the recession".

coming from the estimated model is wasted and the responses to a shock can be interpreted more easily and naturally, as is discussed below.

### 2.1.2. Key features of the model

Overall, the Interacted-VAR model 1 estimated in this paper is an observation-driven[9] state-dependent model whose dynamics depend on the past behavior of the process. Notice that any regime is imposed to the model. Conversely from some of the most commonly used non-linear models that reach non-linearity by combining two or more regime-specific linear VARs (e.g., Threshold VARs and Smooth Transition VARs), the Interacted-VAR model reach non-linearity for its second-order terms.

Furthermore, the model allows one to use simple linear least squares (OLS) methods for estimation and easy testing procedures for non-linearity because, although non-linear in variables, it is linear in parameters and does not depend on unobservable variables or nuisance parameters. OLS is in fact the estimation method I use.

Notice moreover that once all variables are endogenized, as in our case, the Interacted-VAR model I estimate can be regarded as a special case of a Vector Generalized Autoregressive (GAR) model. This model was launched by Mittnik (1990) as, in his words, a "natural generalization" of a linear VAR model. Provided that the underlying Data Generating Process is smooth, the model allows one to capture within it some general form of non-linearity. Indeed, it considers more than just the first order terms – to which the VAR model is restricted – from a Taylor series approximation of the unknown non-linear autoregressive model behind the data[10]. In spite of its theoretical appeal, however, the GAR model has had very little empirical success in economics, and none for the vector case[11]. One reason might be that they are very prone to problems of over-parameterization[12].

Therefore, a "fully endogenous" Interacted-VAR model can be regarded as a sensible second-order Taylor series approximation for an unknown underlying process. "Sensible", that is, in that theory is used both in order to make it as parsimonious as possible relative to a Vector GAR and to focus on the non-linear mechanism which is more relevant for the issue under examination. For example, in this study, I consider only the second order terms more likely to play a role in the transmission of a monetary policy / uncertainty shocks; their empirical relevance is shown in section 2.4.

We now turn to the choices made for the specification of the model and the identification of shocks.

---

[9] In observation-driven models the law of motion for the time-varying parameters is specified in terms of functions of observable variables, while in parameter-driven ones parameters are seen instead as stochastic processes with an error component (see Cox, 1981).

[10] That can be written as $Y_t = f(Y_{t-1}, \ldots, Y_{t-L}) + E_t$ (notice that the missed considerations of Moving Average terms of the error should be not a big problem, since it is the usual assumption used in VAR models).

[11] A study that estimates an univariate GAR is Rothman (1998). As Rothman writes, the class of GAR models "is an autoregressive analogue of a discrete Volterra series" (p. 166).

[12] Another problem for the general GAR model can be instability. This comes from the fact that the consideration of the squares of the endogenous variables might in some cases lead the model to resemble a chaotic process that might diverge for some shocks and some histories (see Aruoba, Bocola and Schorfheide (2012) and Granger (1998)). On the advantages of using GIRF for these kind of processes see Koop et al. (1996) section 3.3 .

### *2.1.3.Specification and identification*

Recognizing that the primary objective is recovering the structural impulse responses after a shock, I rely on Kilian and Ivanov (2005) for the choice of the lag order to be used. For quarterly data and sample size like in this work they suggest the use of the Hannan-Quinn Criterion. The lag used in the baseline specification is thus $L = 2$ (suggested for both the non-linear model and the nested linear one). In any event, the same results can be reached when higher lags are used. Robustness checks for that will be provided in sections 3.3 and 4.3 [13].

To identify the monetary policy (or uncertainty) shocks from the vector of reduced form residuals, I adopt the conventional short-run restrictions by means of the Cholesky decomposition. The ordering of the endogenous variables adopted for the baseline model is: (i) uncertainty, (ii) price index, (iii) output, (iv) investment, (v) consumption, and (vi) FF rate [14]. Thus, while the FF rate is allowed to react instantaneously to the price index, the real variables and uncertainty, these latter variables are not allowed to react instantaneously to the FF rate. Ordering the uncertainty measure as first variable is quite common among the studies on the impact of an uncertainty shock and implies that real variables are allowed to react to it on impact. In any case, in the robustness checks sections, we will consider also the case in which uncertainty is ordered last. Also the ordering of the FF rate as the last variable is very common: it permits to obtain policy shocks "purged" also by the influence of contemporaneous movements in the other variables, in addition to past movements.

To focus exclusively on the transmission mechanism of a given shock, I assume a constant covariance matrix $\Omega$. In this way, we can be sure that any estimated state-dependent effect arises not merely from a different identification of structural shocks at different points in time, but from a change in the propagation mechanism of the shock (a similar motivation is used by Fazzari, Morley and Panovska (2013)). A drawback of this assumption is that the model cannot provide different responses at the time of impact for any two different starting histories[15], though the responses are allowed to be different from the first period after the shock. Notice that in the case of a monetary policy shock this assumption gives no problem at all, since all variables will react with a lag to it. For the uncertainty shock, instead, this drawback applies. However, a robustness check that allows for different initial impacts will be provided.

### *2.2. Impulse responses computation*

Once the six-variate VAR model is estimated, what we are really interested in are the empirical responses to a monetary policy (uncertainty) shock. Since the estimated model is not linear the right way to proceed is that of the Generalized IRFs (GIRFs) described in Koop et al. (1996). Indeed, in general, for non-linear models, responses will depend on the starting history, the future shocks hitting the system and the size and sign of the initial shock. Following related

---

[13] The use of few lags is also a way to maintain the collinearity among regressors low, allowing for a more precise estimation.

[14] Notice that Sims, Stocks and Watson (1990) show that VAR in (log-)levels provide consistent estimates of the IRFs even in presence of integrated or co-integrated vectors.

[15] This of course holds also in other non-linear models (like Threshold VARs and Smooth Transition VARs) once the same assumption is used. In particular, this happens because the contemporaneous relation among variables is given by the Cholesky factor (also constant over time and capturing hence an average relation).

work in the literature, such as Fazzari et al. (2013), I provide state-conditional GIRFs that averages responses to an initial shock size across future shocks and a set of initial histories.

Specifically, for each of the two shocks considered above I will provide two GIRFs, referring respectively to two particular "states" of the economy. The difference between the two states is in the set of initial histories upon which they are conditioned. For example, in the case of a monetary policy shock, I will condition the system in turn both on histories with low uncertainty, around its 10th percentile, and histories with high uncertainty, around its 90th percentile[16], to obtain, respectively, an *average* response of the economy in the case of a low uncertainty state and of a high uncertainty state.

The bootstrap-based structural GIRFs are computed as follows; a detailed algorithm is presented in appendix A. First, a sequence of Gaussian shocks for periods 0 to 20 are extracted, according to the estimated variance-covariance matrix. On the basis of these, the system is iterated onward for a given initial history, i.e., for a given initial values of the variables. The result is a particular forecast of the endogenous variables conditional on the initial values and on a particular sequence of shocks. Then, a similar forecast is computed for the same initial values and sequence of shocks, with the exception that the structural shock in period 0 for the variable to be shocked is fixed at a given size, in our case, a 25 basis points reduction in the FF or 1 standard deviation increase in uncertainty. To obtain the response for this particular initial history and sequence of shocks it is sufficient to subtract the former expected evolution of the system from the latter one. The same computation is repeated for five hundred draws of the Gaussian shocks and averaged across them to get the response conditional on the particular history considered. Then, these responses are averaged across a subset of histories of interest to obtain the GIRFs conditional on that subset (i.e. a state in our case). Finally, to obtain estimates for the GIRFs along with their confidence intervals, a 2000-draws bootstrap procedure is used.

To test statistically the difference between the GIRFs under two states, a test statistic is needed, as GIRFs with their respective confidence bands are not enough. To this end I construct a simple test statistic recurring to the distribution of the difference between the two GIRFs, stemming from the 2000 draws used to implement the bootstrap; details provided in appendix A[17].

Along with the state-conditional responses, I will also provide the historical sequence of single history-conditional responses[18]. Each of them provide the expected change to be induced in the endogenous variables, over time, by a given shock hitting the system on a particular date. In this way we are able to evaluate output reaction to a shock in an historical perspective and to obtain deeper insights on the way the transmission mechanism of a monetary policy/uncertainty might have changed over time.

## 2.3. Data

The analysis needs both macroeconomic data and uncertainty data.

---

[16] Details provided in section 3.2.1 . Since GIRFs depend on the starting histories, we need to use a tolerance around the given thresholds to be sure that enough starting histories are selected and to obtain a state-conditional response.
[17] Aastveit et al. (2013) use a similar test.
[18] For details on the way they are computed refer to appendix A .

As for macroeconomic data, the variables used are: (i) (the log of) the Consumer Price Index (CPI), (ii) (the log of) real GDP, (iii) (the log of) real gross private domestic investment, (iv) (the log of) real personal consumption expenditures and (v) the effective FF rate as the instrument of monetary policy. The first four variables are multiplied by 100 for convenience. The source for these data is the Federal Reserve Bank of St. Louis' database (FRED2 database).

For uncertainty data I rely on two proxies in the baseline analysis: the stock market volatility index constructed by Bloom (2009), extended up to 2012Q4 [19], and a credit spread measure. Data for the first measure come from the Chicago Board Options Exchange VXO index for the period after 1986 and from the quarterly standard deviation of the daily S&P500 for the period before. The VXO is an index of percentage implied volatility on a hypothetical at the money S&P100 option 30 days to expiration. It is therefore a measure for the degree of uncertainty considered by financial markets. I will refer to this index as the VIX henceforth (Volatility IndeX).

The second measure is based on the credit spread on corporate bonds. As supported by Bachmann, Elstner and Sims (2013), it constitutes an important proxy for uncertainty, with macroeconomic effects comparable to those from a stock market volatility shock (see their results from a Structural VAR (SVAR) in figure 6). Moreover, Gilchrist et al. (2013) compare the macroeconomic consequences of a credit spread shock and an idiosyncratic uncertainty shock[20], when two different identification schemes of the shocks are used in their SVAR model. It emerges that the macroeconomic effects of both shocks are very similar, but also that the credit spread is an essential driver for the transmission of idiosyncratic uncertainty shocks[21] (see their figures 2 and 3).

The baseline credit spread measure I use is given by the difference between the 30-year Baa-rated corporate bond yield and the 10-year treasury bond yield (i.e. the common measure often referred to as "Baa–GS10 spread")[22]. It is available up to the end of 2013 and is constructed similarly to the measure constructed by Bachmann, Elstner and Sims (2013) which however is available only up to the end of 2011. This measure is thought to reflect both a liquidity premium and a safety premium (Krishnamurthy and Vissing-Jorgensen (2011), Nodari (2014)).

Figure 1 plots baseline uncertainty measures against recessionary periods by NBER. Notice that uncertainty and recessions appear to be correlated. Indeed, the highest local maxima for the uncertainty measures occurred mostly during recessions. The global maximum for both measures is reached during the peak of the recent Great Recession. However, there is not a perfect match between the two concepts. Just as an example, neither Black Monday at the end of 1987, nor the Worldcom and Enron scandals in 2002 occurred during a recession. Notice also that although the two measures of uncertainty commove strongly (the correlation coefficient is 61%), they behave differently in some episodes, like the episodes just mentioned, where the

---

[19] The monthly series can be obtained from http://www.stanford.edu/~nbloom/R.zip . Quarterly data is obtained by taking the quarter average of monthly data.
[20] They construct a proxy for idiosyncratic uncertainty using high-frequency firm level stock market data
[21] In their words (p. 2), "credit spreads are an important conduit through which fluctuations in uncertainty are propagated to the real economy."
[22] The series is available in the FRED2 database.

VIX were relatively higher and during the recession in '81-82, where instead the credit spread was so. The purpose of the horizontal lines in the figure will be explained below.

Other uncertainty proxies will be considered for space reasons in the robustness checks. These include a firm-level uncertainty measure by Jurado, Ludvinson and Ng (2013), the Economic Policy Uncertainty index (EPU) by Baker, Bloom and Davis (2013) in its historical version, and the credit spread by Bachmann et al. (2013) [23].

The sample period uses quarterly data and covers the longest period available starting from 1971Q1[24] (namely, up to 2013Q4 for the case uncertainty is proxied by the credit spread) .

### 2.4. Empirical motivation

In this section some empirical evidence at the multivariate level is presented in favor of non-linearity, in particular in favor of the Interacted-VAR model used.

Firstly, a Wald test on the overall exclusion of the interaction variables from model 1 was performed[25]. When uncertainty is proxied by the credit spread, a p-value virtually of 0 allows us to reject the null hypothesis that all the coefficients attached to the interaction terms are zero, while a p-value of 0.22 does not allow us to do the same when the VIX index is used as proxy. However, using the notation of equation 2, the term $y_{t-1}^{(VIX)} \cdot y_{t-1}^{(FF)}$ cannot be singularly excluded from the model at a 10 percent significance level (p-value 0.08)[26]. Notice that, in general, rejecting the null hypothesis in such tests is equivalent to rejecting the hypothesis of linearity. In particular, in the context of the present study, it also corresponds to a test that accepts the specific non-linear model used.

In addition, information criteria also suggest that our baseline Interacted-VAR specification is to be preferred to the nested linear one (i.e. when no interaction terms are considered). Indeed, when the credit spread is used to proxy uncertainty, the Akaike Information Criterion (AIC), the Schwartz Bayesian Criterion (SBC) and the Hannan-Quinn Criterion (HQC) all suggest our baseline six-variate Interacted-VAR model 1 to the nested linear model[27]. When, in turn, the VIX index is modeled as endogenous, one out of the three criteria, the AIC, still suggests the Interacted-VAR specification.

Finally, the parsimonious model employed seems not disregarding other important second-order interaction pieces. Firstly, if we add the "cross interaction terms" in equation 2 (i.e. $y_{t-1}^{(2)} \cdot y_{t-2}^{(3)}$ and $y_{t-2}^{(2)} \cdot y_{t-1}^{(3)}$) , we have no indication from any information criteria for their

---

[23] These measures will be defined in section 3.3 .

[24] The starting date is dictated in order to have a common starting date for all the uncertainty measures used. A robustness check for the case of a longer sample period will be provided in the robustness checks sections.

[25] To perform this hypothesis testing 3 lags are used for model 1. These allows to get residuals behaving like white noises at the 1% significance level (a Breusch-Godfrey LM test for residual autocorrelation is used, which remain correct also for possibly integrated variables (Brüggemann, Lütkepohl and Saikkonen (2006)).

[26] Here a note on the appropriateness of the test used. In general it is appropriate provided that the coefficients involved in the test refer to (or can be written as coefficients referring to) stationary variables (Sims, Stock and Watson (1990)). In our case there is no evidence that both the FF rate series, the uncertainty series and their products are non-stationary (ADF tests used) .

[27] In the details AIC=11.01, SBC=12.78 and HQC=11.73 for the Interacted-VAR and instead AIC=11.27, SBC=12.82 and HQC=11.9 for the corresponding linear VAR.

further consideration in the model. Furthermore, once considered, they do no help us to capture a larger deal of non-linearity in the process and results are very similar[28]. Secondly, a further concern might derive from the fact that in model 1 the uncertainty measure is interacted only with the policy rate. In the robustness checks section we will consider also the case in which it is interacted with the other endogenous variables in the system.

## 3. *Uncertainty-dependent effects of a policy shock*
### 3.1. *Related literature*

This section sheds some light on the following question: is unforeseen monetary policy less effective under conditions of high uncertainty? A brief review of the relevant literature, both theoretical and empirical, will both elucidate the reasons for the empirical analysis conducted and help to explain in more depth what the results obtained will tell us and what they will not.

Vavra (2014) finds that the frequency of price changes and the range of price changes commove together, in particular they strongly increase during recessions. In order to accommodate these facts in a general equilibrium Menu Cost model, he shows that uncertainty shocks - in particular volatility shocks - are needed. He theorizes that greater uncertainty should induce firms to change prices more frequently, therefore lowering the real effects of a monetary shock. In the most realistic calibrated version of his DSGE he finds that "The cumulative output IRF [of a nominal shock] is now 45% larger at the 10th percentile of volatility than at the 90th percentile of volatility". This means that "achieving a given increase in real output requires a greater increase in inflation during times of high volatility."

Bachmann, Born, Elstner and Grimme (2013) also investigate the same subject. On the basis of a micro-econometric analysis, they use a New Keynesian business cycle model to assess the importance of uncertainty in the transmission mechanism of a monetary policy shock. They capture the change in the frequency of price adjustment through a one-off change in the Calvo parameter. Looking at the Great Recession, they conclude that "it does not appear to be the volatility channel that is at the heart of the increase in price flexibility and the subsequent loss in effectiveness of monetary policy during the 08/09-recession." . . . . therefore . . . . "the role of heightened volatility (and of uncertainty) might be of minor concern for the conduct of traditional monetary policy".

Other reasons why monetary policy is less effective under high uncertainty derive from the "caution effect" that higher uncertainty might induce in households' and firms' decisions, especially investment ones. Aastveit et al. (2013) develop a simple deterministic three-periodical model in which they show how investment irreversibility and fixed investment costs might reduce the influence of monetary policy on investment decisions.

To my knowledge, Aastveit et al. (2013) is also the only empirical study dealing with the issue being addressed in this section. In the work closest to the present one they also use an Interacted-VAR model estimated with Bayesian methods and find that monetary policy is less

---

[28] For details see footnote 30, where are considered also some third-order terms that might be relevant. I do not consider instead squares of the endogenous variables, since as explained in footnote 12 , they can create instability problems.

effective under high uncertainty. In particular, maximum responses for real variables – similar to mine - to a contractionary policy shock are four to five times weaker when uncertainty is fixed high (9th decile) than when it is fixed low (1st decile). Although my results will also suggest that responses are weaker in the high than in the low uncertainty state, they give a somewhat smaller difference between the two, at "only" two thirds weaker. The reason behind the difference will be explored in the next section.

## 3.2. Baseline results

In this sub-section I present first some evidence on the historical responses to a monetary policy shock. These responses are a powerful instrument as they permit us to observe how the transmission mechanism of monetary policy shocks evolved over time, according to the degree of uncertainty. Then I present average responses for the low and high uncertainty states. These responses permit us to test both for their statistical significance and for the statistical difference between them in the two states.

Throughout the section results come from the estimation of the six-variate baseline Interacted-VAR model described in section 2.1 .

### 3.2.1. Historical evidence

Figure **2** presents the sample historical GIRFs for (the log of) real GDP to a 1% unexpected increase in the FF rate [29]. The figure provides a tridimensional picture, giving on the horizontal axis the date when the system is shocked (responses are conditional on initial conditions). Uncertainty is proxied by the credit spread.

Suggestively, despite the model's simplicity and parsimony, there is a lot of non-linearity in the responses[30]. It is evident that monetary policy shocks seem less effective in periods of high uncertainty. Three considerations are worth particular examination. First, GDP weakest reactions occurred during the recessions characterized by the highest levels of uncertainty, like the ones in '74-75, in '81-82, in 2001 and during the recent Great Recession[31]. In these periods the real effects of the monetary shock are both smaller and less persistent. Second, the weakening role of uncertainty is not only present during the major recessions: the GDP reaction appears "jagged" with local maxima during periods of local maxima in uncertainty (see also figure **1** ). Third, it is not only the level of uncertainty *per se* that determines the intensity of the responses. The particular "local history" also matters: see for example the responses during the recession in 2001. Notwithstanding that uncertainty was

---

[29] Here I use a positive shock because the resulting plot is easier to be observed. Below I will turn to an expansionary shock.

[30] Notice that the figure is very similar if we consider the cross interaction terms previously mentioned in section 2.4 (i.e. , using the notation of equation (2), $y_{t-1}^{(unc)} \cdot y_{t-2}^{(FF)}$ and $y_{t-2}^{(unc)} \cdot y_{t-1}^{(FF)}$). Moreover, the degree of non-linearity captured does not increase – in the sense that the overall shape of the response is preserved - neither if I consider some third-order terms that allow the marginal effects of the monetary policy shock to depend also on the square of the uncertainty level, in addition to its level (namely, , $\left(y_{t-1}^{(unc)}\right)^2 \cdot y_{t-1}^{(FF)}$ and $\left(y_{t-2}^{(unc)}\right)^2 \cdot y_{t-2}^{(FF)}$ ) . Figures for both cases are not reported for space reasons, but are available on request.

[31] Notice that in first half of 2009 the monetary policy shock seems to need more time before to produce the effect hoped (that is a reduction in GDP). This might suggest that in the peak of the Great Recession monetary policy waswere almost ineffective in the short-run.

somewhat higher between the end of 2002 and the beginning of 2003 for the WorldCom and Enron scandals and the Gulf War II, monetary shocks seem to be less effective in the medium run after the terrorist attack.

Now, in order to prepare the ground for the next sub-section, we turn to those GDP responses that are relevant for our high and low uncertainty states. Recall briefly that the high uncertainty state is designed to contain initial histories with uncertainty (here proxied by the credit spread) around its $9^{th}$ decile, with a tolerance of five percentiles, while the low uncertainty state contains those around its $1^{st}$ decile. As seen above, this definition follows the common practice in both theoretical and empirical works. When such a definition is used, 17 histories are selected for each of the two uncertainty states. Figure **1** helps to identify visually the histories selected: those for which the uncertainty measure is inside the upper and lower band defined by horizontal solid lines.

Next we consider a 25 basis points unexpected decrease in the FF rate (expansionary policy). Figure **3** displays the selected GDP responses, distinguishing them according to the state they belong to. The blue solid line is used for the low uncertainty state and the red dashed line for the high uncertainty state. Bold lines represent the corresponding GIRFs conditional on a high or low uncertainty state, i.e., they are the average across histories of the previous same-colored lines. The figure suggests that, although there are differences between responses depending on the initial history, it is clear that responses under a particular uncertainty state behave similarly. Indeed, overall, average (bold) responses show that the expansionary reaction in output is slower and smaller under high uncertainty.

### 3.2.2. Average evidence

Now I present the bootstrapped GIRFs conditional on each of the two uncertainty states, for all the endogenous variables in our baseline Interacted-VAR. As above, an expansionary monetary policy shock (equal to 25 basis points reduction in the FF rate) is considered.

First, we continue to proxy uncertainty with the credit spread. Figure **4. a** presents the GIRFs conditional on low or high uncertainty state with their corresponding 90% confidence intervals. Looking first at the real variables we see that after the shock, in both uncertainty states, they start to increase as expected. For example, in the low uncertainty state, investment increases by a maximum of more than 0.5%, consumption by more than 0.1% and GDP by around 0.2%. However, notice that the real variables responses appear weaker for the high uncertainty state. Maximum reactions for real variables are around two-thirds weaker in a high uncertainty state rather than in a low uncertainty one. Specifically, at their peak reactions, GDP, investment and consumption react respectively by around the 40, 46 and 50 percent more in the low uncertainty state relatively to the high uncertainty state.

Notice that while consumption is the variable reacting first to the shock, investment is the variable reacting more, and last. The strong, lagged reaction of investment is consistent with the real-option effects of models with non-convex adjustment costs, and the quicker response of consumption is consistent with the forward looking nature of this variable and the permanent income hypothesis.

Figure 4.$b$ plots the test statistic[32] for the difference between the responses conditional on the high and the low uncertainty state. The exterior light grey bands refer to the 90% confidence interval and the interior dark grey bands refer to the 68% one. The figure allows us to conclude that the difference between the real variables responses is highly significant and lasting from just after the shock (mostly for consumption) to around two years later.

The FF rate and the credit spread responses help us to explain this difference between uncertainty states. Indeed, in the high uncertainty state we observe both an initial lower persistence of the FF rate and a slower reaction of the credit spread. Regarding the latter, it seems that an expansionary monetary shock decreases economic uncertainty sooner when uncertainty is initially low than when it is high[33] [34].

On the other hand the empirical response of the price index presents a "price puzzle" in the response. Indeed, the responses predict, contrary to standard theory, a decrease in inflation following a monetary policy expansion[35]. In the robustness checks in section 3.3 we will use a measure of inflation expectations to sharpen the identification of the monetary policy shocks.

Unfortunately, the presence of the price puzzle prevents us from ascertaining whether prices are an important reason for the lower effectiveness of unforeseen monetary policy under high uncertainty, as theorized by Vavra (2014). In fact, Vavra's model suggests that, after an expansionary monetary policy shock, the higher the initial uncertainty the more inflation should increase.

Now it seems noteworthy a brief comparison with Aastveit et al.'s (2013) different results. What seems key behind the difference is my endogenous modeling of the uncertainty measure along with the use of GIRFs that allow us to consider, while recovering responses, the feedbacks from the shock impact to the evolution of the system. Figures $\mathbf{5}.\mathbf{a}$ and $\mathbf{5}.\mathbf{b}$ report the responses and the test statistic when, in my framework, I model uncertainty as an exogenous variable and I use IRFs conditionally linear to the value fixed for uncertainty. Uncertainty is in turn fixed to its 9th decile to obtain the response for the high uncertainty state and to its 1st decile for the low uncertainty state. More details on the model used and the algorithm to recover responses are provided in appendix B.

From a glance to figure $\mathbf{5}.\mathbf{a}$ it is evident that this time responses become much more distant among them. In particular, coherently with Aastveit, Natvik and Sola's (2013) results, maximum reactions for real variables are around four to five times weaker when uncertainty is in its upper decile than when it is in its lower one. The statistical difference between responses turns out to be much greater and lasts throughout the 20 quarters considered. Hence, taking

---

[32] See appendix A for details on how it is constructed.

[33] This result holds also when different uncertainty measures are employed and when different causal orderings are employed (allowing also for an immediate response of uncertainty to the policy shock). Figures available on request.

[34] Notice that the decrease in the credit spread (that here proxies uncertainty) after an expansionary policy shock is consistent with the findings by Beckworth, Moon and Toles (2010). Bekaert, Hoerova and Lo Duca (2014) find that the VIX index – in particular both sub-components in which they divide it – also decreases after an expansionary monetary shock.

[35] This fact is generally attributed to an omitted variables problem, meaning that variables considered from the Federal Reserve when deciding the target federal funds rate are not included in the VAR (see Sims (1992), Castelnuovo and Surico (2010)).

account of the endogenous role of uncertainty seems to be crucial to deal with the question under consideration.

Turning to our baseline analysis, figures $6.a$ and $6.b$ show the results for the case in which uncertainty is proxied by the VIX. For the histories selected refer again to figure **1** (see the horizontal bands defined by dashed lines). Comparing the figures with figure **4**, it is evident that exactly the same considerations apply. Although in this case responses get a bit closer between the two states, enough statistical significance remains for us to conclude that the responses are different across states[36].

To sum up, the results suggest that uncertainty is likely to play an important role in the transmission mechanism of monetary policy shocks. In the light of these results, when preparing policy the monetary authority should take account of the degree of uncertainty in the economy, since if it is high for any reason a given policy shock is going to produce smaller effects than what would be expected in normal or quiet times. Furthermore, since uncertainty is generally higher during recessions, monetary policy shocks seem likely to be less effective just when they are most needed.

### 3.3 Robustness checks

Figure **7** displays some robustness checks for the results outlined above. Only the responses for GDP, investment and consumption are reported[37]. The upper plots display the responses for the low uncertainty state and the lower plots those for the high uncertainty state. Different specifications that account for the use of different lag orders, Cholesky ordering, uncertainty proxies, a potential omitted variable, a longer sample period and a more general Interacted-VAR model are provided. At a glance the responses in most of the specifications used look very similar to the baseline ones, at least qualitatively. They are inside the 90 percent confidence band for the baseline case. The baseline case corresponds to the case in which uncertainty is proxied by the credit spread.

When three, or four, lags are used - rather than two - the responses are similar to those for the baseline case. Exactly the same conclusions can be reached and the test statistic (not reported) for the difference of real variables responses across states looks virtually the same as in the baseline case. This also holds if we order our uncertainty measure last in the Cholesky decomposition, so that its reaction to the policy shock is immediate.

In addition, a longer sample period is used, namely one starting in 1962:Q3. This is the date from which the VIX by Bloom (2009) is available and for this reason has been a widely used starting date for several works. As it is evident from the figure, here responses in the two states get even more distant among them[38].

---

[36] Now, at their peak reactions, GDP, investment and consumption react respectively by around the 32, 37 and 40 percent more in the low uncertainty state relatively to the high uncertainty state.

[37] The entire set of results regarding the robustness checks is available upon request. The figure is best viewed online in color. The responses presented in the robustness check sections are based on 500 bootstrap replications for the specifications different from the baseline one.

[38] Part of this is due to the fact that credit spread (that here works as a "transition" variable) reached its minima in '60s , and hence many of the starting histories selected for the low uncertainty state come from this period. In the

In order to sharpen the identification of the policy shock, I use also a measure of inflation expectations, as first-ordered variable[39]. This variable is forward looking in nature and can be thought of as a useful omitted variable to be considered (see Castelnuovo and Surico (2010)). However, as is clear from the figure, the estimated responses remain qualitatively similar for this specification too.

Another measure of credit spread is also used: the spread provided in Bachmann, Elstner and Sims (2013)[40]. In this case the responses under low and high uncertainty become again more distant from each other.

As a further robustness check another measure of uncertainty was considered: the Economic Policy Uncertainty (EPU) index[41]. In this case, responses get more similar between states. From the test statistic (not reported) it might be observed, however, that responses under the high uncertainty state start to be significantly smaller than the ones in the low uncertainty state after almost one year and a half from the shock, at the time of peak reaction in the low uncertainty state.

Furthermore, a firm-level uncertainty measure, constructed by Jurado, Ludvingson and Ng (2013), is used[42] . Here responses get closer just for the period immediately after the shock. Consumption response, starting from four quarters after the shock, is still significantly more reactive to the shock in the high uncertainty state, but to conclude the same for GDP and Investment  the 68 percent significance level has to be used.

Finally, it appears worth to consider also a more general Interacted-VAR model. In the baseline analysis we motivated to interact uncertainty just with the FF rate for several reasons: because it seems the most important non-linear piece to consider for the issues under consideration in this paper, because statistical evidence favors the model and to be as parsimonious as possible. Here instead, as a robustness check, we consider the case in which uncertainty is interacted also with the other variables - CPI, GDP, investment, consumption - for a total of 10 interaction variables[43]. As can be seen, responses for this more general Interacted-VAR are virtually on top of the baseline ones for the first two years after the shock.

---

baseline sample, instead, as seen, histories are selected from all over the sample for both states, and hence this might contribute to provide a better average evidence.

[39] In particular I use expectations for one-year-ahead annual average inflation, measured by the GDP price index, available  in the Survey of Professional Forecaster (SPF) by the Federal Reserve Bank of Philadelphia.

[40] It is constructed by the difference between the 30-year Baa-rated corporate bond yield and the 30-year treasury bond yield – where 20-year data are used when the 30-year one is missing.

[41] In particular I use the "beta" (i.e. preliminary) historical version of the EPU index available at http://www.policyuncertainty.com . The sample used is 1971:Q1 – 2013:Q4 (quarterly data is obtained by taking the quarter average of monthly data) . This index is constructed on the basis of the newspaper coverage of policy-related economic uncertainty (for details refers to the website). The "standard" EPU index is instead available since 1985.

[42] This is index is not an observable measure but rather it is the common factor across many series of unforecastable components coming from observations of firm-level profit growth (normalized by sales). It is available from the starting of our sample to 2011:Q2. I use the uncertainty factor for uncertainty horizon of one quarter.   I detrend the series with a cubic polynomial and further restrict the sample till 2008:Q3 to avoid instability problems due to the highly non-stationary features of the series.

[43] In particular, the model I am using here is the following (for notation refer to model 1): $Y_t = A^0 + D^0 Time_t + \sum_{l=1}^{L} A^l Y_{t-l} + \left( \sum_{j \neq unc} \sum_{l=1}^{L} C^{l,j} y_{t-l}^{(unc)} \cdot y_{t-l}^{(j)} \right) + E_t$ . Notice that, in this case of many interaction variables, the responses showed in the figure are the sample responses (I preferred to not perform bootstrap since it might increase the estimation error to which sample responses are already subject for the increased multicollinearity among regressors).

After that, while responses for the high uncertainty state remain so, responses for the low uncertainty state appear to be more persistent than baseline estimates.

We can conclude therefore that results in the baseline analysis are robust with respect to the use of many different model specifications and different uncertainty proxies. Admittedly, when using non-financial proxies for uncertainty, responses get closer among the two states. However, also in this case, still enough statistical evidence remains to conclude that high uncertainty lowers the impact of a monetary policy shock, particularly at the peak reactions of real variables.

## 4. ZLB-dependent effects of an uncertainty shock

### 4.1. Related literature

Some recent studies have investigated theoretically the impact of an uncertainty shock when the economy is constrained at the Zero Lower Bound (ZLB) for the policy rate. Among these are Basu and Bundick (2012), Nakata (2012) and Johannsen (2013)[44], all of whose calibrated DSGE models suggest that the recessionary effects of an uncertainty shock should be greater and longer-lasting in the presence of the ZLB, since the monetary authority cannot perform its usual stabilizing function. This holds irrespective of the volatility/uncertainty shock considered[45]. Basu and Bundick (2012) argue that the sharp increase in uncertainty (calibrated using the VIX index) in late 2008 may have played a significant role in worsening the Great Recession. In this period indeed the Fed had a policy rate near zero.

Empirically, the impact of an unconditional uncertainty shock has been extensively studied in the literature after the seminal work by Bloom (2009); see Baker, Bloom, and Davis (2013), Bachmann, Elstner and Sims (2013), Gilchrist, Sim, and Zakrajsek (2013), Leduc and Liu (2013), Colombo (2013), Nodari (2014) among others. Generally it is found that an uncertainty shock induces a contraction in real economic activity, with macroeconomic effects resembling those from a negative demand shock: inflation and employment also generally follow. Bloom (2009) finds that, starting from 7 months after the shock, in the VIX index that I also use, industrial production and employment exhibit a subsequent recovery and rebound: see his figures 2 and 3. All of these studies, however, use *linear* VAR models, thus obtaining state-independent responses of aggregate variables.

A study which considers the non-linear effects of an uncertainty shock is Caggiano, Castelnuovo and Groshenny (2014). They examine, in the context of a Smooth Transition VAR, the impact of an uncertainty shock conditional on a recession state. Focusing mostly on U.S. unemployment, they find that the impact of an uncertainty shock (proxied by a VIX shock) on unemployment is underestimated in a linear VAR. Notice that my empirical question is related to their question only to a minor extent, since recession episodes and ZLB episodes are not the same thing[46]. Another linked work is Benati (2013). He investigates the role of economic *policy*

---

[44] Other studies have approached the question from a more theoretical point of optimal monetary policy at the ZLB (e.g. Bundick (2013), Adam and Billi (2007) and Nakov (2008) ).

[45] Basu and Bundick (2012) and Nakata (2012) use an increase in uncertainty about the household discount factor, whereas Johannsen uses an increase in fiscal policy uncertainty.

[46] Indeed ZLB episodes happened only recently (for post WWII samples) and furthermore they are not only confined to the NBER recent recessionary periods.

uncertainty by means of a time-varying VAR model. In particular he examines whether policy uncertainty shocks (proxied by the EPU index) had any peculiar role during the Great Recession for a number of countries, among which U.S. His study suggests that the answer to the question is not totally clear-cut, since it appears to depend on the empirical identification strategy employed. However, when IRFs are scrutinized - for both identification strategies he uses - these "did not exhibit any peculiar pattern, during the Great Recession, compared to previous years". My study deals mostly with economic uncertainty (particularly with financial-related uncertainty in the baseline analysis). However, in the robustness checks section I will also consider a policy uncertainty shock to see whether results are robust also to this case.

The empirical question whether a contractionary uncertainty shock has more impact on the economy in the presence of the ZLB has received recently some attention; to my knowledge the only attempts are by Johannsen (2013) and Caggiano et al. (2014) . Using the EPU index and estimating two linear VARs, one with sample ending in 2007, before the ZLB started, and another ending in mid 2012, Johannsen finds that after the shock consumption, industrial production and employment show a deeper and longer lasting recessionary phase in the extended sample VAR. Caggiano et al. (2014), in a robustness check for their main results, reach the same conclusion conditional on recession: when ZLB observations are present in their sample the macroeconomics effects of uncertainty shocks are magnified with respect to the case they are absent.

As explained in the introduction, this study aims to shed more light on this topic, using the ability of the GIRFs to capture the state-conditional effects of an uncertainty shock more directly and quantitatively. In this way my empirical analysis might also confirm the results found from the aforementioned more structural models.

## 4.2. Baseline results
### 4.2.1. Historical evidence

This section offers evidence, on a historical perspective, on the impact of a contractionary uncertainty shock (one standard deviation increase) on real GDP. Uncertainty is proxied by the credit spread. Responses are obtained from the baseline Interacted-VAR model presented in section 2.1.

Figure **8** plots the sample estimated responses for each initial date in our sample. Notice that while historically real GDP has exhibited the "drop and rebound" effect that Bloom (2009) documents, the same does not seem to hold in the recent past, when the policy rate has been constrained by the ZLB. In particular since late 2008 real GDP seems to be experiencing a deeper and longer lasting recession after the uncertainty shock. The periods in which an uncertainty shock would have been most harmful are those around the first half of 2009[47].

This glance at the results makes clear that the average response under a ZLB state will be different from that in an unconditional state.

---

[47] Notice that although the response is obtained inevitably by an out-of-the sample iteration for the last starting dates in the sample, for the responses in the mid 2009 it still is possible to obtain an in-sample GIRF.

### 4.2.2. Average evidence

In order to compute the Generalized IRFs for the (near-)ZLB state all the histories for which the FF rate is between 0 and 20 basis points are selected. There are 19 selected histories for the credit spread case and 15 for the VIX case[48] It seems useful to compare the ZLB-dependent response with the response for an unconditional state, obtained setting the initial history to the sample mean of each variable. This latter unconditional GIRF provides an average response of the economy to an uncertainty shock, as in normal times.

Figures 9.$a$ and 9.$b$ plot these GIRFs and the corresponding test statistic for the case uncertainty is proxied by the credit spread. Notice that now, differently from the policy shock considered above, the reaction at time zero for a given variable is equal across states because of the assumption of a constant covariance matrix (see section 2.1). However, in the next sub-section I will provide a robustness check in which a different initial reaction is allowed.

In figure 9.$a$, notice first that after the uncertainty shock real variables react negatively, significantly and strongly, on impact. Investment falls soon - in the same quarter - by around 1% while GDP and consumption fall by almost 0.25%. From the first period after the shock onward the responses are dependent on whether the economy is in an average state or near the ZLB. In the unconditional state, the uncertainty shock is soon offset by an increasingly aggressive monetary policy reaction which, on average, reduces the policy rate by a maximum of over 60 basis points. The peak of the recession is reached after six months for GDP and investment, after which a rebound is observed, while consumption returns to its initial level after just six months. These results are consistent with those found by Bloom (2009). Notice in addition that, after the shock, prices fall consistently with the resemblance of the uncertainty shock to a negative demand shock (Leduc and Liu (2013)).

In the ZLB state, however, the monetary authority cannot react with the aggressiveness she would like: remember again that the response in period zero reflects an average reaction for the whole sample. Here the true result is the flatness of the policy rate reaction from the first period after the shock onward. Therefore the policy implemented is not sufficient to offset quickly the negative effects of the uncertainty shock which in this case are stronger and more persistent. Indeed, after the initial drop, real variables continue to decrease until the peak of the recession is reached after about a year for GDP and investment and a year and a half for consumption. Investment at the peak reaction has fallen by around 4%. Recession this time lasts longer, and takes longer to recover, especially consumption. Thus, contrary to Bloom's results, this time no significant rebound is observed.

Finally, from figure 9.$b$ notice that the responses of real variables are very significantly different between states: the significance starts straight after the shock and lasts around three years. The credit spread and the FF rate reactions are also statistically different initially.

---

[48] In particular all the histories selected are those from 2009:Q1 onward (the last date in the sample is not considered). In these histories the FF rate varies from the minimum of 0.07 (in 2011:Q4) to the maximum of 0.19 in different periods. Hence my threshold just permit me to select all the sample histories where the ZLB problem has become relevant for standard policy moves of 25 basis points.

Figures $10.a$ and $10.b$ report the same plots with uncertainty proxied by the VIX index. This time, the VIX shock does not have an impact as strong as at the previous credit spread shock. Also the monetary authority takes a while to react strongly in the unconditional state, whereas its reaction is insignificant in the ZLB state. These considerations apart, we have exactly the same outcomes as before. The difference of the responses between the two states gets very significant after three to six quarters and it persists for a long time. Also the difference between reactions of the FF rate is highly significant.

Thus, the findings suggest that an uncertainty shock is much more harmful in periods in which the policy rate is very low and consequently the monetary authority has little freedom to counteract it effectively. Therefore the economy is likely to pay a heavy cost in these situations.

### 4.3. Robustness checks

Figure $11.a$ presents some robustness checks for the results obtained in the previous section. The baseline response refers to the credit spread uncertainty case. Briefly, we see first that the baseline results do not depend in any way on the lag order considered: responses are virtually the same. Also, although when uncertainty is ordered last we obviously lose the impact effect at time zero, responses still behave similarly. Results are robust to the use of a larger sample period too (for details refer to section 3.3). Results are also similar when the Bachmann, Elstner and Sims (2013) spread is used. When the historical EPU index is used, however, any significant rebound is observed in real variables, neither in the unconditional state. In any case there is still enough evidence of difference between state-dependent responses for GDP and consumption, starting from around an year and a half and an year after the shock respectively. Notice that the lack of a very peculiar role for a *policy* uncertainty shock in shaping real variables responses during periods with very low policy rate relative to normal periods is compatible with the results of Benati (2013) (see section 4.1 for a discussion of these). [49]

Let us now present a robustness check for figure 9. This time I will allow the responses to differ between states at time zero. Specifically, I use two different variance-covariance matrices, one for recovering the GIRFs in the ZLB state and one to recover them for the unconditional state. In particular, the variance-covariance matrix, and the corresponding Cholesky factor, used to obtain the ZLB responses derive from the estimation of the usual baseline VAR, but this time over the restricted sample from 2007:Q4 to 2013:Q4 (25 observations). The observations of the FF rate near the ZLB are indeed concentrated in this sample period[50]. In order, whereas, to recover the unconditional responses the same variance-covariance matrix is used as was used in figure 9, since this reassumes average relations in variables in the sample.

---

[49] There is no robustness check to the use of the firm-level uncertainty measure by Jurado et al. (2013) due to the fact that, when the series is restricted till 2008:Q3 to deal with instability problems (as done in section 3.3), there are anymore ZLB observations in the sample.

[50] Actually I should start from 2009:Q1 to have the first observation below the threshold of 20 basis point for the FF rate, but the sample would be too short to permit a sensible estimation of the contemporaneous relationship among variables. The addition of more observation in order to better estimate the VCV matrix has though the cost that the FF rate is 4.5 in the first observation. However the figure is robust in its findings to the use of close starting dates. Figures available on request.

Figure 11.$b$ provides the two sample responses (corresponding to those reported in figure 9.$a$) when we use the variance-covariance matrices explained above. A normalized unforeseen uncertainty increase is considered (equal to the standard deviation in the unconditional state). Here the responses are allowed to have a different immediate impact, from which two considerations follow. First, this time the contemporaneous decrease of the FF rate is somewhat less in the ZLB state and is coherent with the Fed's possible actions (given our definition of the ZLB state). Second, for the other variables the possibility of reacting differently on impact does not seem so important. Indeed they react in almost the same way in both states. The reason for this seems that these variables will react to the Fed's immediate reaction only after a lag: thus the ZLB influences the impact of an uncertainty shock just for the fact that it will influence the reaction of the monetary authority.

Although this robustness check is admittedly a simple one, it allows us to conclude that our main result does not depend on the use of a constant variance-covariance matrix. So, again, after an uncertainty shock we expect a much longer recession in the ZLB state and hence no rebound effect in real variables, due, again, to fact that the monetary authority's capacity to react is constrained.

## 5. *Conclusion*

While the empirical relevance of economic uncertainty as a factor unconditionally influencing economic activity is well documented by a number of studies, its relevance as a factor interacting with monetary policy is still not fully ascertained empirically. It is here that the present paper contributes, examining the issue by means of an Interacted–VAR model which, thanks to GIRFs, leaves the interaction variables completely free to be determined endogenously while computing responses.

What I find is, firstly, that monetary policy shocks are generally less effective under high uncertainty. Maximum responses for real variables under a high uncertainty state are around two thirds of those under a low uncertainty state. The variables which most explains this impact on the effectiveness of monetary policy seem in general financial-related uncertainty measures. Secondly, when an uncertainty shock is considered, I find that the recessionary effects of the shock are far greater when the economy is around the ZLB, as suggested theoretically by several papers. The monetary authority indeed cannot substantially offset the shock. This sheds more light on the "drop and rebound effect" found by Bloom (2009) .

There are important policy implications, which are interrelated. First, the monetary authority - in the US, the Fed - should be aware of the level of uncertainty when designing policies, particularly if she wishes to foresee more precisely the results of a policy action. When uncertainty is high more aggressiveness is needed. Second, policy makers need to bear in mind the trade-off involved in maintaining the policy rate low, since although it helps the economy in fostering a rebound it also makes the economy more vulnerable to the incurable[51] negative effects of an unexpected uncertainty increase. This might provide empirical evidence in favor of an argument raised by Blanchard, Dell'Ariccia and Mauro (2010), i.e. to raise the inflation target to have - in their words - "higher nominal interest rates to start with".

---

[51] At least to the use of conventional monetary policy.

The present study of course does not permit us to enquire into the causal role of uncertainty or why some measures of uncertainty seem more important than others in influencing the propagation of monetary shocks. Research based on micro-founded structural model is needed. The empirical importance of the credit spread might suggest that financial frictions play a role in the transmission mechanism of monetary policy. Another argument left to future research is whether a more aggressive reaction of the monetary authority could *totally* counteract the influence of an high uncertainty state – particularly in the very short run - , and, if so, how[52].

---

[52] What this paper might suggest is that the answer does not seem to be clear cut. Specifically, referring just to the baseline analysis, when credit spread is used to proxy uncertainty it seems that even a more aggressive shock by the Fed in the high uncertainty state cannot be as effective in the short-run as in the low uncertainty state. The same *peak* reaction as in the low uncertainty state can be obtained under an high uncertainty state (by just calibrating the initial size of the shock – meaningful in a non-linear model – in the high uncertainty state), but this reaction arrives with around an year of delay (figure not reported, but to have an intuition it might be helpful to look at the *shape* of real variables responses in figure 4 for both states). Instead, in the case uncertainty is proxied by the VIX, empirical responses suggest that a more aggressive exogenous expansionary movement in the policy rate can counteract effectively the influence of high uncertainty also in the short-run. More research is needed in this regard. Here just a question: could this depend on the fact that the credit spread proxies uncertainty shocks with "first-order" shocks and the VIX with "second-order"(or volatility) shocks?

*References*

**Aastveit, K. A., G. J. Natvik, and S. Sola** (2013): "Macroeconomic Uncertainty and the Effectiveness of Monetary Policy." Norges Bank W.P. n.17

**Adam, K., and R. Billi** (2006): "Optimal Monetary Policy Under Commitment with a Zero Bound on Nominal Interest Rates," *Journal of Money, Credit and Banking*.

**Aruoba, S. B., L. Bocola, and F. Schorfheide** (2012): "A New Class of Nonlinear Time Series Models for the Evaluation of DSGE Models," available at http://www.philadelphiafed.org/research-and-data/events/2011/dsge-models-workshop/papers/schorf.pdf

**Auerbach, A., and Y. Gorodnichenko** (2012): "Measuring the Output Responses to Fiscal Policy","*American Economic Journal: Economic Policy*, 4(2), 1-27.

**Bachmann, R., B. Born, S. Elstner, and C. Grimme** (2013): "Time-varying business volatility, price setting, and the real effects of monetary policy," *Mimeo*.

**Bachmann, R., S. Elstner, and E. Sims** (2013): "Uncertainty and Economic Activity: Evidence from Business Survey Data," *American Economic Journal: Macroeconomics*, 5(2), 217–249.

**Bekaert, G., M. Hoerova and M. Lo Duca** (2014): "Risk, Uncertainty and Monetary Policy", *Journal of Monetary Economics*, forthcoming.

**Baker, S., N. Bloom, and S. Davis** (2013): "Measuring Economic Policy Uncertainty." Stanford University and the University of Chicago Booth School of Business, *Mimeo*.

**Basu, S., and B. Bundick** (2012): "Uncertainty Shocks in a Model of Effective Demand," NBER Working Paper No. 18420.

**Beckworth, D., P. M Kenneth, and J. H. Toles** (2010): "Monetary policy and corporate bond yield spreads" , *Applied Economics Letters*, 17, 1139–1144.

**Blanchard, O. J. , G. Dell'Ariccia and P. Mauro** (2010). "Rethinking Macroeconomic Policy," *IMF Staff Position Notes* 2010/03, International Monetary Fund

**Bloom, N.** (2009): "The Impact of Uncertainty Shocks." *Econometrica*, 77(3): 623–685.

**Brüggemann, R., H. Lütkepohl and P. Saikkonen** (2006): "Residual Autocorrelation Testing for Vector Error Correction Models," *Journal of Econometrics*, 134, 579-604.

**Bundick, B.** (2013): "Forward Guidance Under Uncertainty.". Working Paper, available at https://www2.bc.edu/brent-bundick/papers/BundickForwardGuidanceUncertainty.pdf .

**Caggiano, G., E. Castelnuovo, and N. Groshenny** (2014): "Uncertainty Shocks and Unemployment Dynamics: An Analysis of Post-WWII U.S. Recessions," *Journal of Monetary Economics*, forthcoming.

**Castelnuovo, E., and P. Surico** (2010): "Monetary Policy, Inflation Expectations and the Price Puzzle." *Economic Journal*, 120(549), 1262-1283

**Christiano, L. J., M. Eichenbaum, and C. L. Evans**. (1999): "Monetary policy shocks: What have we learned and to what end?" In J. B. Taylor and M. Woodford (Eds.), *Handbook of Macroeconomics*, Volume 1 of Handbook of Macroeconomics, Chapter 2, pp. 65–148. Elsevier.

**Clark, T. and K. West** (2007): "Approximately Normal Tests for Equal Predictive Accuracy in Nested Models," *Journal of Econometrics*, 135, 155-186.

**Colombo, V.** (2013): "Economic Policy Uncertainty in the US: Does it matter for the Euro Area?," *Economics Letters*, volume 121(1) , 39–42.

**Fazzari, S. M., J. Morley, and I. Panovska**, (2013): "State Dependent Effects of Fiscal Policy." UNSW Australian School of Business Research Paper No. 2012-27A.

**Gilchrist, S., J. Sim, and E. Zakrajšek** (2013): "Uncertainty, Financial Frictions, and Irreversible Investment," Boston University and Federal Reserve Board, *Mimeo*.

**Granger, C. W. J.** (1998): "Overview of nonlinear time series specifications in economics," available                                                                                           at http://www.google.it/url?sa=t&rct=j&q=&esrc=s&frm=1&source=web&cd=5&cad=rja&uact=8&ved=0CFkQFjAE&url=http%3A%2F%2Femlab.berkeley.edu%2Fsymposia%2Fnsf98%2Fgranger.ps&ei=1MI1U4DOLZTZ4QSF2IGADg&usg=AFQjCNHnajkFnr4rD1QAhyUJLiAVvUlAmA&bvm=bv.63808443,d.bGQ

**Hamilton**, **J. D.** (1994): "Time Series Analysis," Princeton University Press

**Johannsen, B. K.** (2013): "When are the Effects of Fiscal Policy Uncertainty Large?," Job Market Paper, available at http://gradstudents.wcas.northwestern.edu/~bkj463/paper.pdf

**Kilian, L. and R.J. Vigfusson** (2011): "Are the Responses of the U.S. Economy Asymmetric in Energy Price Increases and Decreases?" *Quantitative Economics* 2, 419-453.

**Kilian, L. and V. Ivanov** (2005): "A Practitioner's Guide to Lag Order Selection For VAR Impulse Response Analysis," *Studies in Nonlinear Dynamics & Econometrics*, Volume 9, Issue 1, Article 2.

**Koop, G. M.H. Pesaran, and S.M. Potter** (1996): "Impulse Response Analysis in Nonlinear Multivariate Models." *Journal of Econometrics* 74, 119-147.

**Krishnamurthy, A., and A. Vissing-Jorgensen** (2011): "The Effects of Quantitative Easing on Long-Term Interest Rates ," *Brooking Papers on Economic Activity*

**Lanau S., and  T. Wieladek** (2012) : "Financial Regulation and the Current Account," IMF WP/12/98

**Leduc, S., and Z. Liu** (2013): "Uncertainty Shocks are Aggregate Demand Shocks," Federal Reserve Bank of San Francisco, Working Paper 2012-10.

**Mittnik**, **S.** (1990): "Modeling Nonlinear Processes With Generalized Autoregressions," *Applied Mathematic Letters*, Vol.  3, No.  4. pp.  71-74.

**Nakata, T.** (2012): "Uncertainty at the Zero Lower Bound," *Mimeo*

**Nakov, A.** (2008): "Optimal and Simple Monetary Policy Rules with Zero Floor on the Nominal Interest Rate," *International Journal of Central Banking*.

**Nickel, C. and A. Tudyka** (2013): "fiscal stimulus in times of high debt: reconsidering multipliers and twin deficits," ECB wp no 1513

**Nodari, G.** (2014): "Uncertainty, Financial Regulation and Credit Spreads in the U.S. ," University of Padova, *Journal of Macroeconomics,* forthcoming

**Ramey, V. A., and S. Zubairy** (2013): "Government Spending Multipliers in Good Times and in Bad: Evidence from U.S. Historical Data",University of California at San Diego and Texas A&M University,  *Mimeo*.

**Rothman, P.**    (1998): "Forecasting asymmetric unemployment rates," *The Review of Economics and Statistics*, 80, pp. 164-164

**Sá, F., P. Towbin and T. Wieladek** (2014): "capital inflows, financial structure and housing booms," *Journal of the European Economic Association*, 12: 522–546.

**Sims, C., J. Stock and M. Watson** (1990)**:** "Inference in linear time series models with some unit roots." *Econometrica*,  58(1), pp. 113-144

**Sims, C.A..** (1992): "Interpreting the macroeconomic time series facts: the effects of monetary policy." *European Economic Review*, vol. 36(5), pp. 975–1000.

**Towbin, P. and S. Weber** (2013): "Limits of floating exchange rates: The role of foreign currency debt and import structure," *Journal of Development Economics* 101(1), 179–101

**Vavra, J.** (2014): "Inflation dynamics and time-varying volatility: new evidence and an Ss interpretation," *Quarterly Journal of Economics*, Volume 129, No. 1, Feb. 2014

Figure 1: baseline uncertainty proxies for the period from 1971:Q1
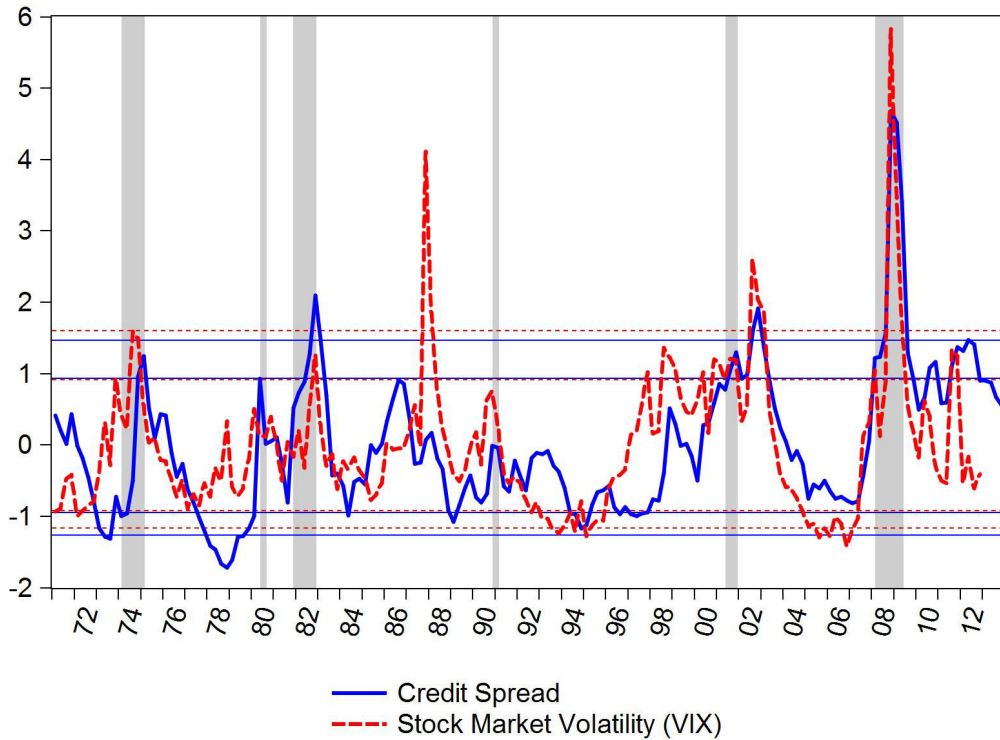


**Figure 1:** The baseline uncertainty proxies are displayed (both are standardized). Refer to section 2.4 for their description. Shaded area represent NBER recessions. The horizontal lines define the 5-percentiles tolerance bands around the 90th and 10th percentiles of the corresponding distributions (solid lines are used for the credit spread case and dashed lines for the VIX case). Episodes of credit spread identified by its upper decile selection band are the Franklin National financial crisis at the end of 1974, the monetary cycle turning point in 1982, the 9/11 terroristic attack in 2001, Worldcom and Enron scandals in 2002, the Gulf War II at the starting of 2003, the Credit crunch in 2007-2008 (the episodes are taken from Bloom (2009)). Recently it has been still high too. The VIX index instead is inside the upper band for the same episodes of before and in addition the Black Monday at the end of 1987, the Asian crisis in 1998 and the Russian and LTCM Default in 1999.

sample response of GDP to a +1 FF shock
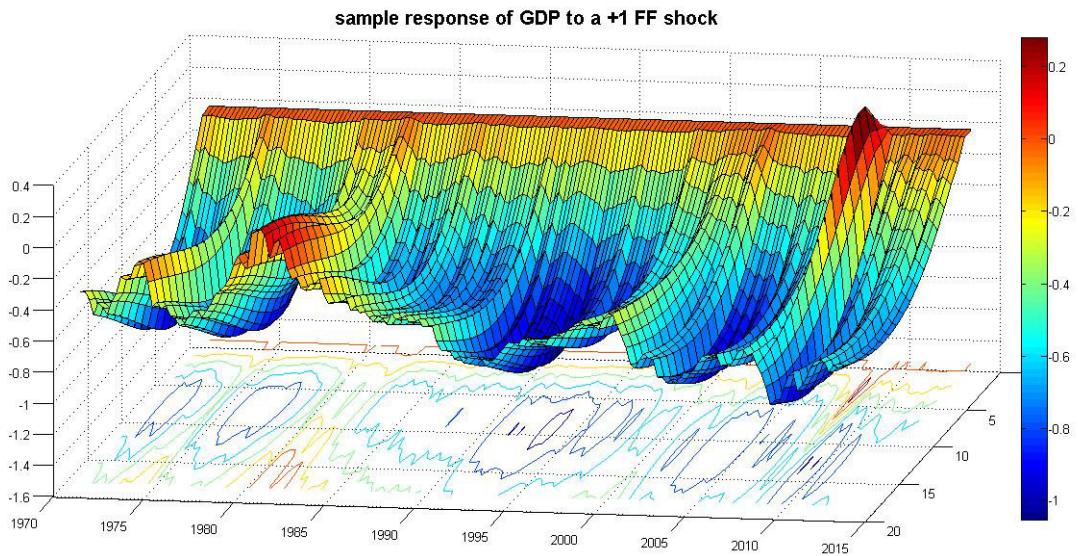
**Figure 2**: There are reported the sample responses for (log) real GDP to an unexpected 1% increase of the FF rate for all starting histories in the sample. The horizontal axis with the dates indicate the quarter in which the system is shocked. The scale of colors permit to have an intuition of the size of the response. The algorithm employed is explained in appendix A.
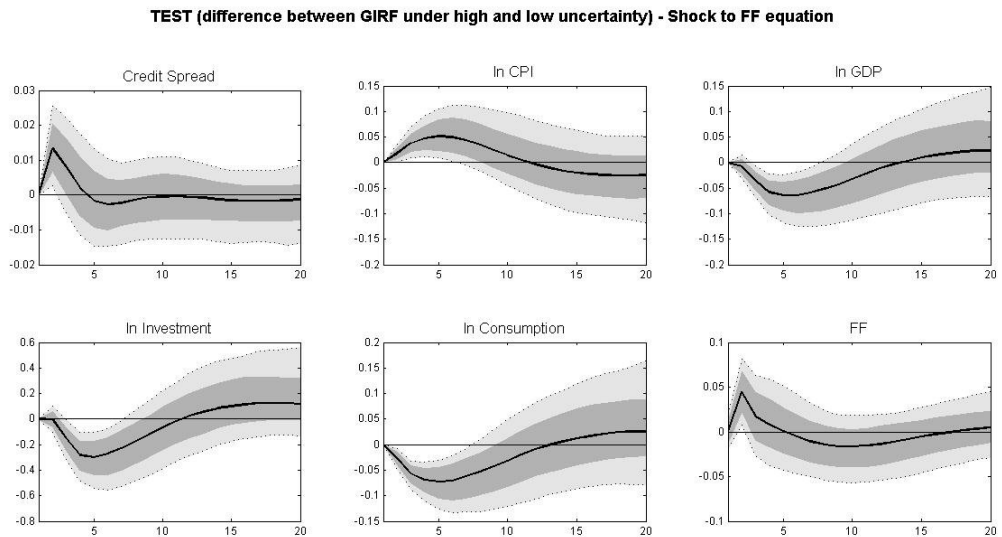


IRF - Shock to FF equation. bold solid blue: sample average if low uncertainty, bold dashed red: sample average if high uncertainty

ln GDP
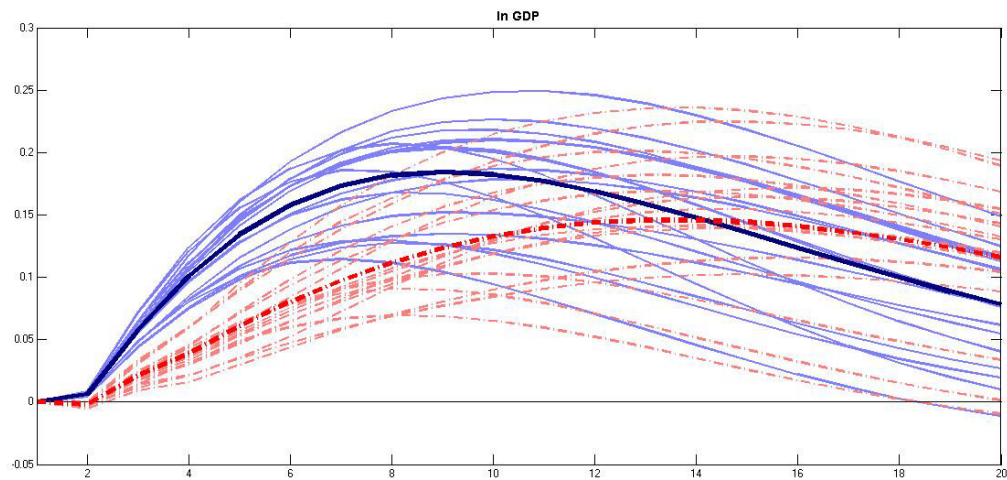
**Figure 3.** Sample single-history GIRFs of (log) real GDP to a 25 basis unexpected reduction in the FF rate, differentiated according to the uncertainty state they belong to. Two uncertainty states are defined: "low uncertainty" is referred to histories with uncertainty measure (here proxied by the credit spread) near to its low decile, while "high uncertainty" is referred to histories with the uncertainty measure around its upper decile. Bold lines refer to the state-conditional responses for a low or high uncertainty state.
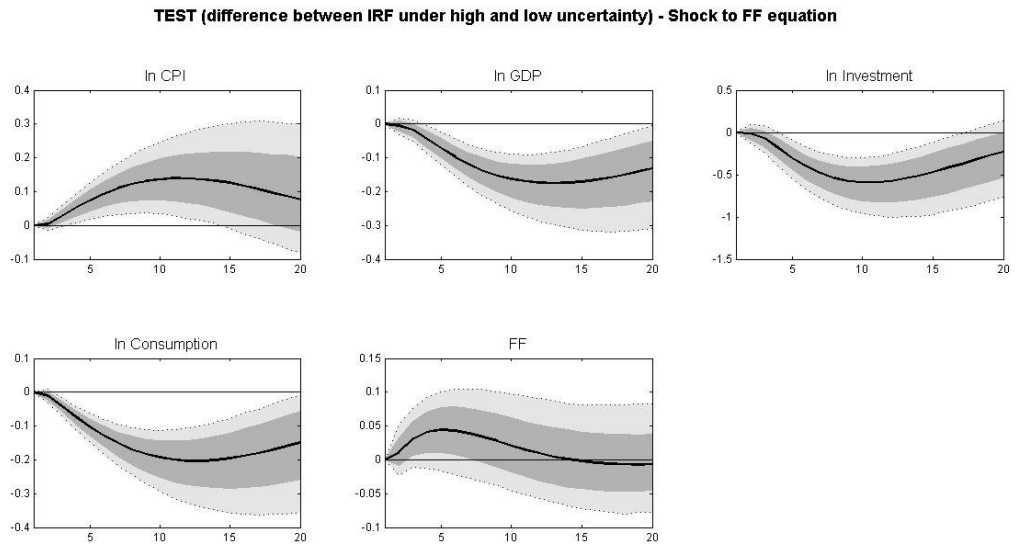
29
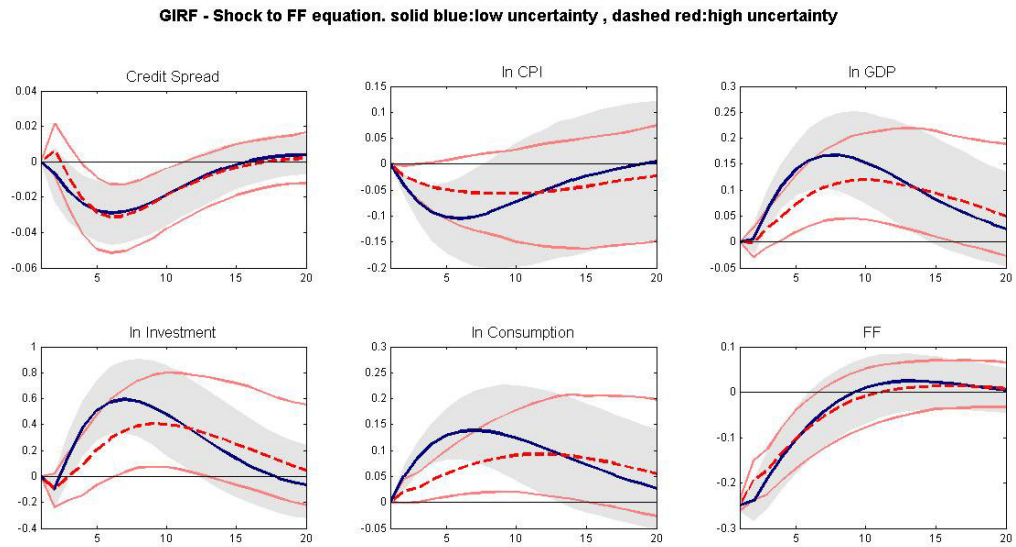
**Figure 4.a.**    Impulse responses to a 25 basis points unexpected decrease in the FF rate, for two different states of uncertainty (proxied by Credit Spread in the figure):  low vs. high. "Low uncertainty" is referred to histories with uncertainty measure near to its low decile, while "high uncertainty" is referred to histories with the uncertainty measure around its upper decile.   Confidence bands are referred to the 90% confidence level. Responses are computed using the algorithm descript in appendix A and from the estimation of the baseline Interacted-VAR descript in section 2.1.3 .Cholesky decomposition is used and the ordering of the variables is [credit spread, ln CPI, ln GDP, ln Investment, ln Consumption, FF rate].



**Figure 4.b**. Here a test statistic is reported for the difference between states of the responses from the just above figure.   Solid bold lines represent the difference between the responses under high and low uncertainty. Light grey areas represent the 68% probability bands for the significance of the difference while outside dark grey areas represent the 90% probability bands. Details are given in appendix A.
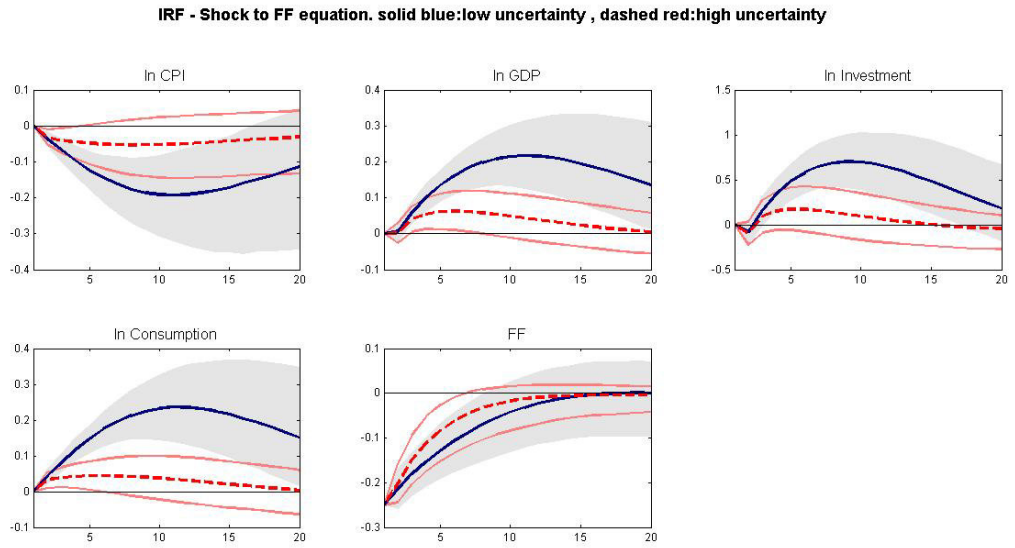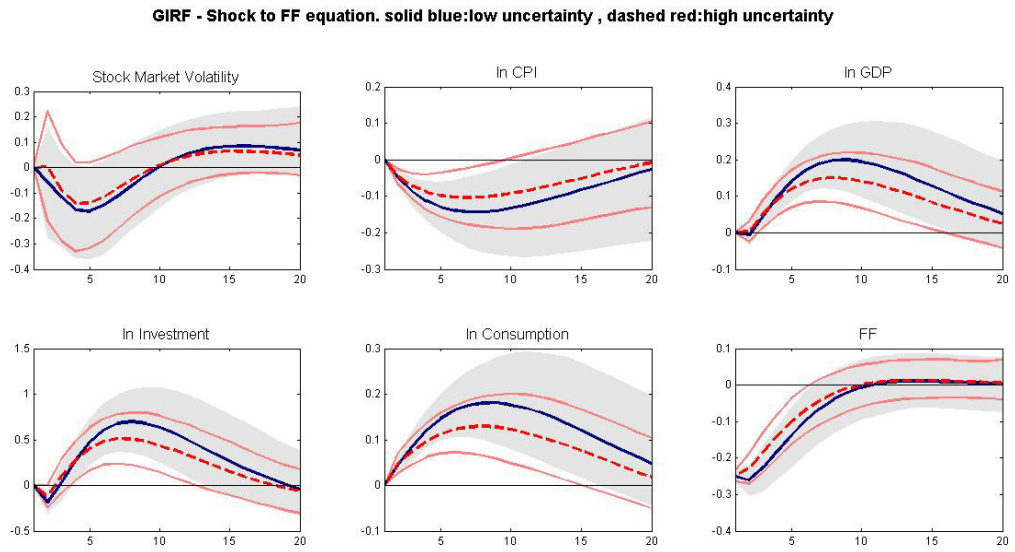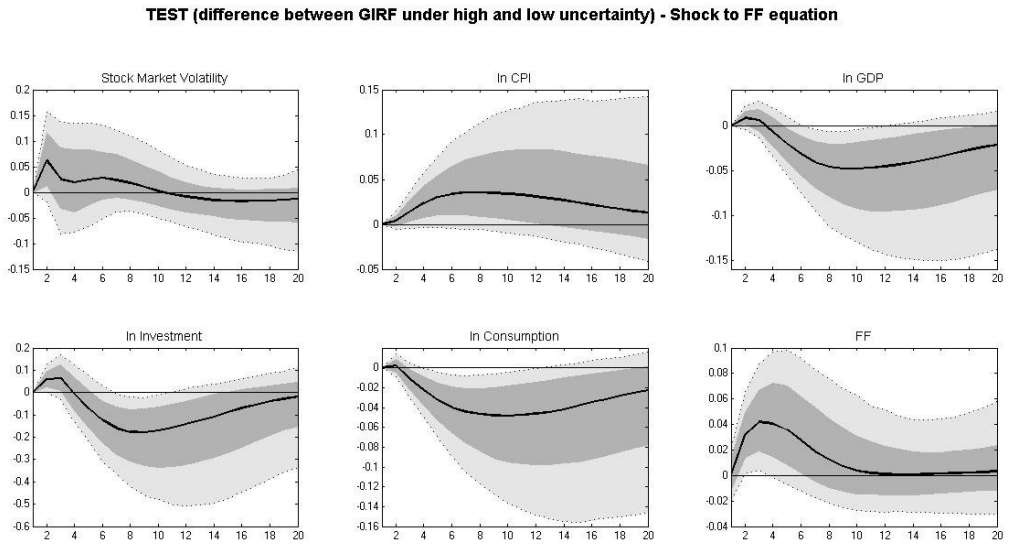
**Figure 5.a:** Impulse responses to a 25 basis points unexpected decrease in the FF rate, for two different states of uncertainty (proxied by Credit Spread in the figure): low vs. high. "Low uncertainty" is referred to the case uncertainty value is fixed to its low decile, while "high uncertainty" is referred to the case uncertainty is fixed to its upper decile. Confidence bands are referred to the 90% confidence level. Responses are computed using the algorithm descript in appendix A and from the estimation of the Interacted-VAR descript in this appendix. Cholesky decomposition is used and the ordering of the variables is [ln CPI, ln GDP, ln Investment, ln Consumption, FF rate].



**Figure 5.b:** Here a test statistic is reported for the difference between states of the responses from the just above figure. Solid bold lines represent the difference between the responses under high and low uncertainty. Light grey areas represent the 68% probability bands while outside dark grey areas represent the 90% probability bands. Details are given in appendix A.
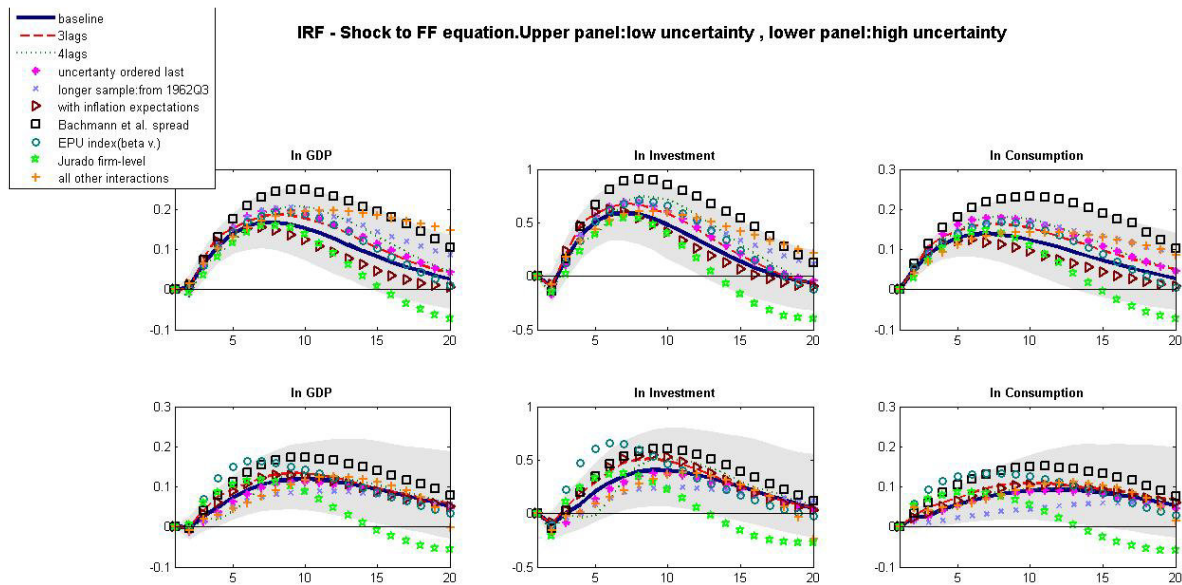
**Figure 6.a.** Impulse responses to a 25 basis points unexpected decrease in the FF rate, for two different states of uncertainty (proxied by VIX index in the figure): low vs. high. "Low uncertainty" is referred to histories with uncertainty measure near to its low decile, while "high uncertainty" is referred to histories with the uncertainty measure around its upper decile. Confidence bands are referred to the 90% confidence level. Responses are computed using the algorithm descript in appendix A and from the estimation of the baseline Interacted-VAR descript in section 2.1.3 .Cholesky decomposition is used and the ordering of the variables is [VIX, ln CPI, ln GDP, ln Investment, ln Consumption, FF rate].

TEST (difference between GIRF under high and low uncertainty) - Shock to FF equation



**Figure 6.b** Here a test statistic is reported for the difference between states of the responses from the just above figure. Solid bold lines represent the difference between the responses under high and low uncertainty. Light grey areas represent the 68% probability bands for the significance of the difference while outside dark grey areas represent the 90% probability bands. Details are given in appendix A.

32

**Figure 7.** Impulse responses to a 25 basis points unexpected decrease in the FF rate, for two different states of uncertainty (proxied by Credit Spread in the figure): low vs. high. "Low uncertainty" is referred to histories with uncertainty measure near to its low decile, while "high uncertainty" is referred to histories with the uncertainty measure around its upper decile. Confidence bands are referred to the 90% confidence level. Responses are computed using the algorithm descript in appendix A. For details on the specifications used and the uncertainty measures used refer to section 3.3 . The baseline case refers to the estimation of the baseline Interacted-VAR descript in section 2.1.3 for the Cholesky ordering: [Credit Spread, ln CPI, ln GDP, ln Investment, ln Consumption, FF rate].
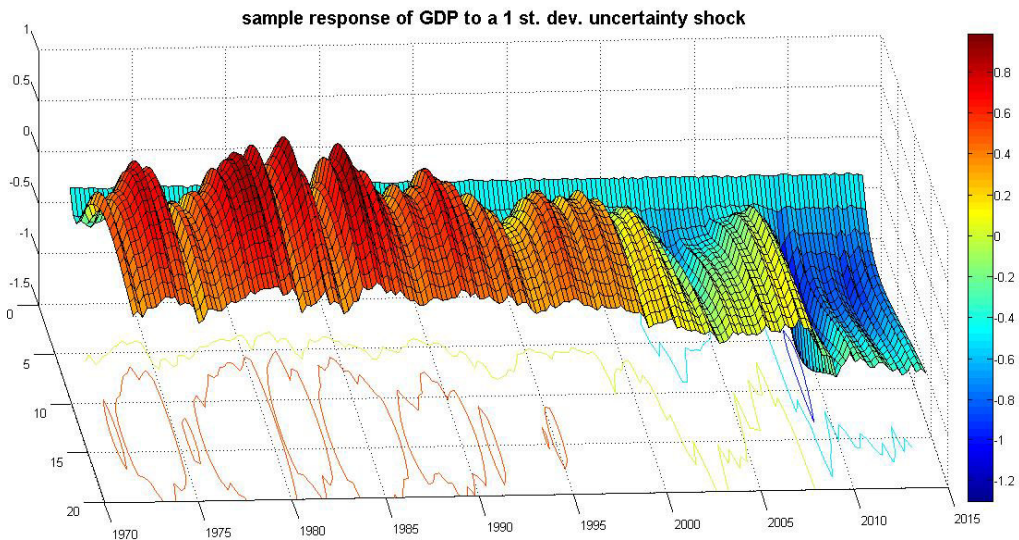


**Figure 8:** there are reported the sample responses for (log) real GDP to an 1 standard deviation increase uncertainty (here proxied by the credit spread) for all starting histories in the sample. The horizontal axis with the dates indicate the quarter in which the system is shocked. The scale of colors permit to have an intuition of the size of the response. The algorithm employed is explained in appendix A.
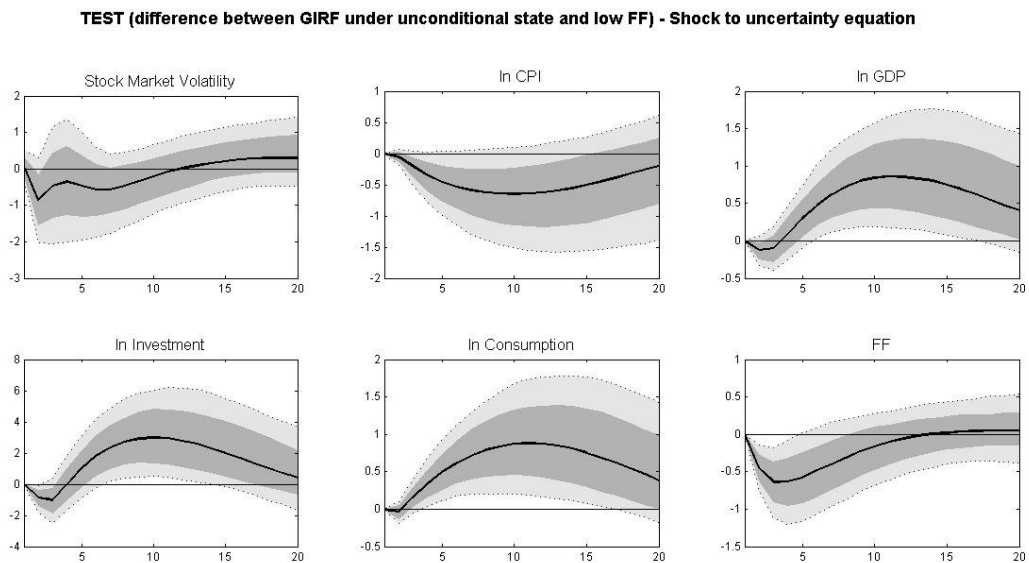
33
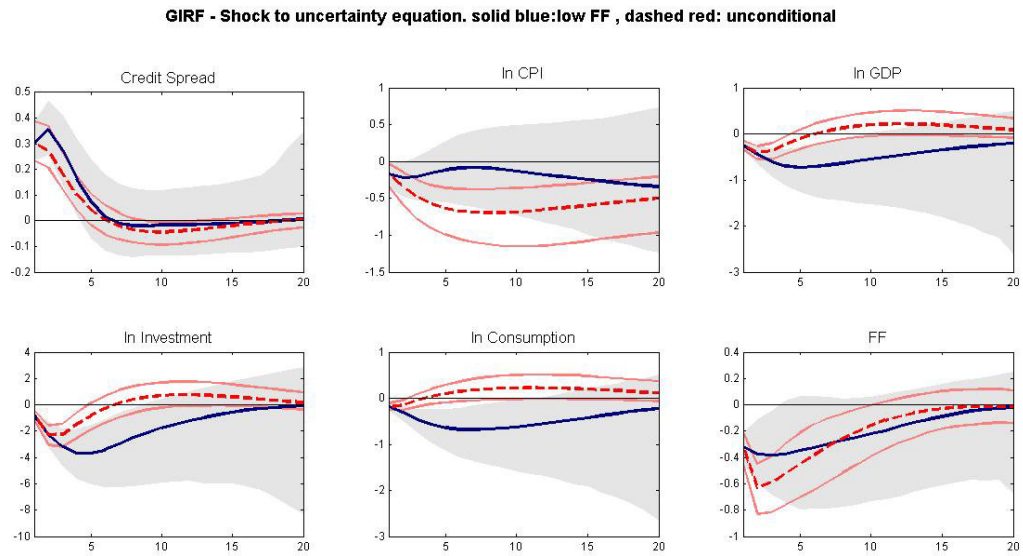
**Figure 9.a**. Impulse responses to a one standard deviation unexpected increase in uncertainty (proxied by Credit Spread in the figure) , for two different states of the policy rate:  Zero Lower Bound  vs. unconditional. "Low FF" is referred to histories with FF rate between 0 and 20 basis points, while "unconditional" is referred to an hypothetical history with all variables that start from their average. Responses are computed using the algorithm descript in appendix A and from the estimation of the baseline Interacted-VAR descript in section 2.1.3 .Cholesky decomposition is used and the ordering of the variables is [credit spread, ln CPI, ln GDP, ln Investment, ln Consumption, FF rate].
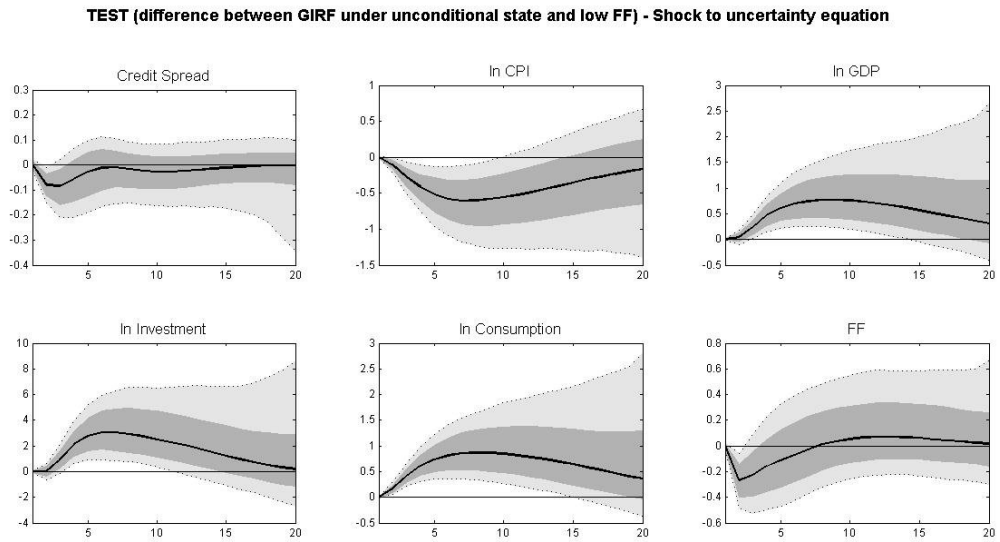


**Figure 9.b**. Here a test statistic is reported for the difference between states of the responses from the just above figure.   Solid bold lines represent the difference between the responses under an unconditional and a ZLB state. Light grey areas represent the 68% probability bands while outside dark grey areas represent the 90% probability bands. Details are given in appendix A.
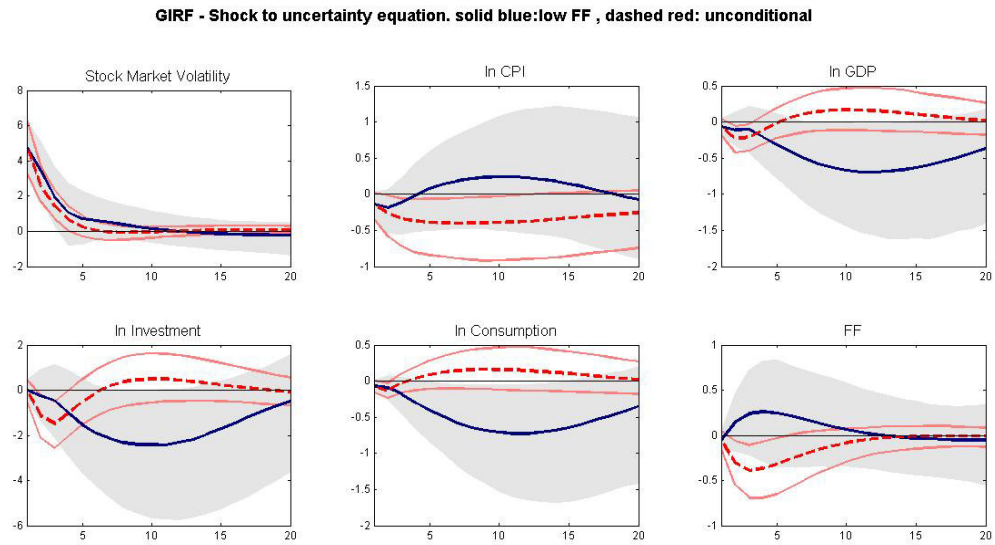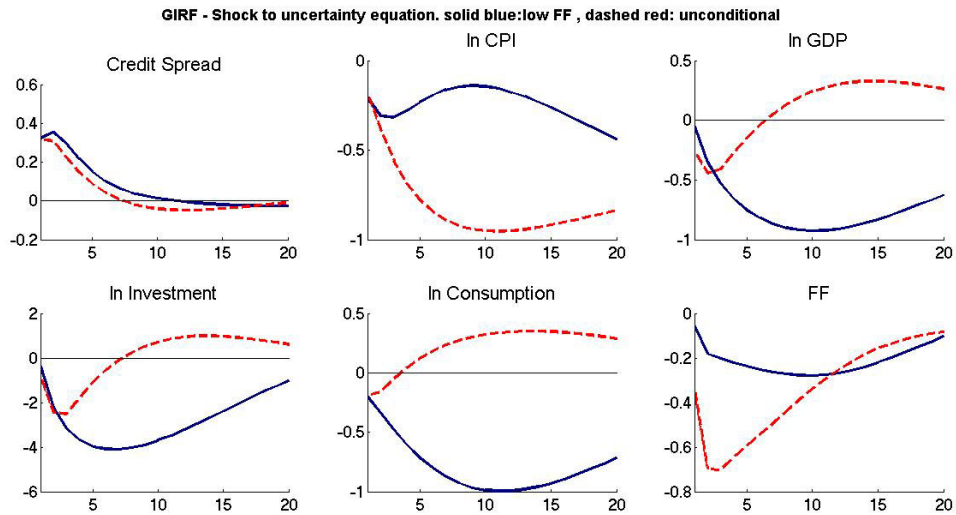
**Figure 10.a**. Impulse responses to a one standard deviation unexpected increase in uncertainty (proxied by VIX in the figure) , for two different states of the policy rate: Zero Lower Bound vs. unconditional. "Low FF" is referred to histories with FF rate between 0 and 20 basis points, while "unconditional" is referred to an hypothetical history with all variables that start from their average. Responses are computed using the algorithm descript in appendix A and from the estimation of the baseline Interacted-VAR descript in section 2.1.3 .Cholesky decomposition is used and the ordering of the variables is [VIX, ln CPI, ln GDP, ln Investment, ln Consumption, FF rate].

TEST (difference between GIRF under unconditional state and low FF) - Shock to uncertainty equation



**Figure 10.b**. Here a test statistic is reported for the difference between states of the responses from the just above figure.   Solid bold lines represent the difference between the responses under an unconditional and a ZLB state. Light grey areas represent the 68% probability bands while outside dark grey areas represent the 90% probability bands. Details are given in appendix A.

35
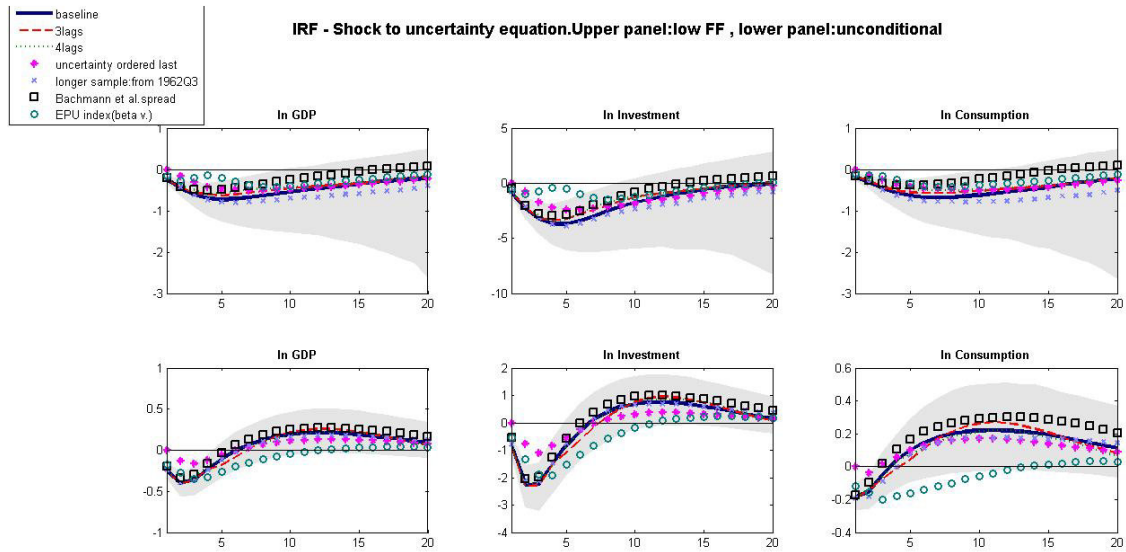
**Figure 11.a**: Robustness checks: Impulse responses to a one standard deviation unexpected increase in uncertainty (proxied by Credit Spread in the figure) , for two different states of the policy rate: Zero Lower Bound vs. unconditional. "Low FF" is referred to histories with FF rate between 0 and 20 basis points, while "unconditional" is referred to an hypothetical history with all variables that start from their average. Responses are computed using the algorithm descript in appendix A. For details on the specifications used and the uncertainty measures used refer to section 4.3 . The baseline case refers to the estimation of the baseline Interacted-VAR descript in section 2.1.3 for the Cholesky ordering: [Credit Spread, ln CPI, ln GDP, ln Investment, ln Consumption, FF rate].



**Figure 11.b**: Sample Impulse responses to a one standard deviation unexpected increase in uncertainty (proxied by Credit Spread in the figure) , for two different states of the policy rate: Zero Lower Bound vs. unconditional state. The only difference respect to the previous figure 9.a (a part the fact that here sample averages responses are reported (see appendix A for the algorithm)) is that in the ZLB state the variance-covariance used to get the responses comes still from the same baseline VAR, but when estimated on the sample period from 2007:Q3 - 2013:Q4 . See section 4.3 for details.

*Appendix A: Further technical details for section 2.2*

- ### *GIRFs computation algorithm*

The algorithm used to obtain the bootstrap-based GIRFs and their relative confidence intervals is presented here, focusing for simplicity on the monetary policy shock. It follows Koop, Pesaran and Potter (1996), with the modification of considering an orthogonal structural shock, as in Kilian and Vigfusson (2011)[53]. Before of that, notice that the theoretical GIRF of a of the vector of endogenous variables $Y$, $h$ periods ahead, for a starting history $\omega_{t-1} = \{Y_{t-1}, \dots, Y_{t-p}\}$, and an initial structural shock $\delta_t$ can be expressed – following Koop et al. – as:

$$GIRF_Y(h, \delta_t, \omega_{t-1}) = E[Y_{t+h}|\delta_t, \omega_{t-1}] - E[Y_{t+h}|\omega_{t-1}] \,,$$

where $E[\cdot]$ represents the expectation operator. Here the algorithm for our state-conditional estimated GIRFs:

1. The reduced-form model 1 is estimated through OLS
2. (i) A sequence of $N$[54] residuals is extracted from the empirical distribution of residuals resulting from point 1, and (ii) a simulated series for the endogenous variables is obtained by iterating on the estimated model $1$[55]. In iterating on the system, the interactions terms are constructed from the simulated series.
3. A reduced form VAR is estimated on the simulated series obtained;
4. Initial histories (i.e. lagged values for the endogenous variables) with the uncertainty measure around the $10^{th}$ (and $90^{th}$) percentiles for low (and high) uncertainty state are selected from the simulated series obtained, with tolerance of five percentiles around the designed percentiles;
5. One history $\omega_{t-1}$ is considered. A series of $S = 21$ six-dimensional residuals is extracted from a Gaussian distribution with mean zero and variance-covariance matrix equal to that resulting from estimation in step 3. To obtain responses to a 0.25% expansionary FF shock in the $nth$ variable the following two evolutions of the system are considered. Firstly, compute the 20 horizons-ahead evolution of the model when iterated on, on the basis of the previous Gaussian residuals and the starting history considered. Secondly, consider the evolution that would have followed if, at time $t$, a residual corresponding to a structural shock $\delta_t$ with 0.25% impact impulse in the FF rate replaces the corresponding Gaussian residual extracted for that time for the $nth$ variable[56]. To obtain the responses we can now simply subtract, at each horizon ahead and for each variable, the first evolution of the system from the second.

---

[53] Fazzari et al. (2013) use a similar approach to obtain GIRFs for a Threshold VAR (T-VAR) model (see their supplementary appendix for details), although they do not use the bootstrap procedure - my most external cycle in the algorithm – since they use Bayesian techniques to obtain confidence bands.

[54] Equal to the number of observations.

[55] The Matlab code for generating bootstrap artificial draws for the endogenous variables is built on that provided in the VAR Toolbox by Ambrogio Cesa-Bianchi https://sites.google.com/site/ambropo/MatlabCodes. The bootstrap used is similar to the one used by Christiano, Eichenbaum and Evans (1999) (see their note 23). The code repeats the explosive artificial draws to be sure that exactly 2000 draws are used. This happens from never to 1‰ times of the 2000 draws .

[56] In the details to get the impulse requested, using the Cholesky decomposition of the variance-covariance matrix of the VAR at point 3, a reduced-form residual corresponding to an orthogonalized shock of $[-1/(\text{st.\,dev. of the structural FF shock})] * 0.25$ replaces the Gaussian residual at time $t$ for the n-th variable.

6. (i) Point 5 is repeated, for the same history, for number $D = 500$ different Gaussian extractions[57]. (ii) An average across these extractions is taken to obtain a consistent estimate of the responses conditional on the history considered, that is $\widehat{GIRF}_Y(\delta_t, \omega_{t-1}) = \left\{\hat{E}[Y_{t+h}|\delta_t, \omega_{t-1}] - \hat{E}[Y_{t+h}|\omega_{t-1}]\right\}_{h=0}^{20}$.
7. (i) Points 5 and 6 are repeated for each initial history of both states among those histories selected at point 4 and then (ii) the empirical responses obtained are averaged across histories for each given state, to get a state-conditional response, $\widehat{GIRF}_Y(\delta_t, state)$.
8. Points 2 to 7 are repeated 2000 times (bootstrap procedure). For the 2000 state-conditional impulse responses the median and the 90% confidence bands are presented.

This procedure thus provides us with a bootstrap-based state-conditional GIRF and its confidence intervals. For the uncertainty shock the same algorithm is used, but the starting histories will be selected according to the FF value.

- ***Test statistic***

To test statistically the difference between the GIRFs under two different states a test statistic is computed. It is built on the distribution of the difference between the two GIRFs that stems from the 2000 bootstrap draws. Specifically, I subtract the median response under the first state e.g., low uncertainty state, from the median response under the other state e.g., high uncertainty state, for each of the twenty horizons ahead and for each of the 2000 simulated responses. From the resulting distribution I plot the median difference together with the 90% and 68% probability bands. If the horizontal axes are outside the probability bands this means that the difference between the two responses is significant at the 90% or 68% confidence level respectively.

- ***Historical responses***

In the work also sample responses on a historical perspective are provided, i.e. a set of single-history conditional GIRFs ordered according to the quarter they refer to. The steps of the previous algorithm used to get these responses are the following: **1**; (instead of **4 :**) "*all* histories in the sample are selected in historical order"; **5** ; **6**. ; **7.**i .

---

[57] If an history is revealed to bring an explosive response (namely if this is explosive for most of the Gaussian draws, in the sense that the response of the variable shocked diverges instead than reverting to zero) is discarded. This happens from never (for sample responses) to very rarely (for simulated responses in the bootstrap).

*Appendix B: more details behind the comparison with Aastveit – Natvik – Sola results*

This appendix complete section 3.2 by providing more details on the way figure **5** is obtained. The aim of this figure was to show the reader that I am are able to obtain results similar to Aastveit, Natvik and Sola (2013) once I follow assumptions similar to them. More in general, this appendix illustrates how the methodology and results of Aastveit et al. (2013) can be nested in the model and procedure used in this paper. A reduced-form interacted-VAR model similar to that estimated by the authors can be written as the following (the same notation is used as for model **1** ):

$$Y_t = A^0 + D^0 Time_t + \sum_{l=1}^{L} A^l Y_{t-l} + \sum_{l=1}^{L} B^l x_{t-l} + \left( \sum_{l=1}^{L} C^l x_{t-l} \cdot y_{t-l}^{(FF)} \right) + E_t \ ^{58} \tag{A.1}$$

where $x_{t-l}$ is the scalar for the exogenous uncertainty measure at time $t-l$ , $B^l$ a $(n \times 1)$ vector for the coefficients attached to $x_{t-l}$ in each regression and for the other terms the same notation as before is followed.

Although the VAR in (A.1) is in principle non-linear – allowing for uncertainty time-dependent marginal effects – the fact that uncertainty is modeled as exogenous in the VAR makes it difficult to compute IRFs that take this into account. Hence it should be fixed while recovering responses. Thus the responses obtained are fully linear, in the sense that they do not depend on both the initial history, future shocks, size of the shock.

For example notice that the generic n-th equation of model $A.1$ , while being iterated on to compute the responses, becomes the following one (consider the example of a bivariate model with the same notation of equation 2 in the main text [59]):

$$y_t^{(th)} = a_{th}^0 + d_{th}^0 Time_t + a_{th,1}^1 y_{t-1}^{(1)} + \left( a_{th,2}^1 + c_{th}^1 \bar{x} \right) y_{t-1}^{(2)} + b_{th,1}^1 x_{t-1} + a_{th,1}^2 y_{t-2}^{(1)} +$$
$$\left( a_{th,2}^2 + c_{th}^2 \bar{x} \right) y_{t-2}^{(2)} + b_{th,2}^2 x_{t-2} + \varepsilon_{t,th} \tag{A.2}$$

where $\bar{x}$ is the fixed value for the uncertainty measure in the interaction variables.

To obtain IRFs under these assumptions it is sufficient to substitute the following point **4'** for points **4**, **5** , **6** and **7** of the algorithm in appendix A: 'The system is shocked and dynamic responses are obtained for 20 steps ahead by iterating on the model once the uncertainty measure is fixed and all the endogenous variables are set initially to zero. Via a Cholesky factor orthogonalized responses are obtained'. Notice that this procedure is similar to the one employed for a standard linear VAR[60].

To obtain figure **5**, following Aastveit et al. (2013), I fixed uncertainty to its 90th percentile to obtain the response in case of high uncertainty, and to its 10th percentile to obtain the response in case of low uncertainty.

---

[58] Actually this model is a bit different from Aastveit et al. (2013) model, although is anyway able to get very similar results. The true model they use can be written in my notation as $Y_t = A^0 + D^0 Time_t + \sum_{l=1}^{L} A^l Y_{t-l} + B^0 x_t^{MA} + \left( \sum_{l=1}^{L} C^l y_{t-l}^{(FF)} \cdot x_t^{MA} \right) + E_t$ , where $x_t^{MA}$ is a four quarter moving average for the exogenous uncertainty measure. I prefer to use the model in $A.1$ given that it is a special case of model 1 . In this way it is possible to show that the difference of my results with respect to Aastveit et al.'s ones depends just on the endogenously or exogenously modeling of uncertainty and the use of GIRFs (rather than from using a partially different model).

[59] Suppose that the variable referred to the policy rate is $y_t^{(2)}$.

[60] This iterated procedure to get IRFs from a linear VAR is illustrated in Hamilton (1994, p. 319 and around).