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FINANCING FLEXIBILITY: THE CASE OF OUTSOURCING

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May 2016

"MARCO FANNO" WORKING PAPER N.206

Financing flexibility: the case of outsourcing*

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May 16, 2016

Abstract

We investigate the relationship between the extent and timing of vertical flexibility and the financial choices of a firm. By vertical flexibility we mean partial/total and reversible outsourcing of a necessary input. A firm simultaneously selects its vertical setting and how to finance it. We examine debt and venture capital. Debt is provided by a lender that requires the payment of a fixed coupon over time and, as a collateral, an option to buy out the firm in certain circumstances. Debt leads to the same level of flexibility which would be acquired by an unlevered firm. Yet investment occurs earlier. With venture capital less outsourcing may be is adopted with respect to the unlevered case and the firm invests mostly later. Hence, as the injection of venture capital may reduce the need of vertical flexibility, a novel relationship can be established for the substitutability between a real and a financial variable.

Keywords: vertical integration, flexible outsourcing, debt, equity and venture capital, real options.
JEL Classification: C61; G31; G32; L24.

*We acknowledge the financial support of the Universities of Bologna and Padova within the 2014-15 RFO scheme and of Fondazione Cassa dei Risparmi di Forlì. We thank Jonathan Lhost, Andrea Mantovani and Riccardo Martina, the audiences at the 2014 ASSET Conference (Aix-en-Provence, France), at the 2015 Economic Design conference (Istanbul, Turkey), at the SIE 2015 conference (Naples, Italy), at the IIOC 2016 conference (Philadelfia,Usa) and at seminars held at the University of Trieste and Bologna for comments and suggestions.

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1 Introduction

Our aim is to analyze the influence of external financial sources on the extent and the timing of investment in vertical flexibility of a corporate enterprise, i.e., buying inputs from the market (outsourcing) in a variable and reversible manner, going back to internal production whenever economically convenient.

Outsourcing and flexibility are crucial for most firms which apparently buy inputs in variable proportions changing often the span of activity along the vertical chain of production. Vertical flexibility improves the ability to cope with uncertain scenarios and impinges on competitiveness, scale of production and social efficiency. Unfortunately flexibility does not come for free. Procurement of inputs from the market calls for the set-up of a supply chain with specific logistic investment. A vertically flexible firm must be ready to substitute an internally produced input with an externally acquired one and viceversa. It must be equipped to bring back in-house partly or entirely at any time (backsourcing or reshoring) input production. This vertical flexibility entails keeping alive and paying for a dedicated internal facility and for the associated know-how. As a result vertical flexibility may be quite dear.

The costs of flexibility may vary over time and industries since they may depend on technical progress in production and logistic services, efficiency of external input markets and available financial tools. Indeed a firm may finance vertical flexibility in many ways such as a mix of equity, debt and other external sources such as venture capital. Unfortunately, the financial side of flexibility is most of the times sidestepped in current studies since funding and organizational issues are studied separately in financial,[1] managerial, industrial organization and operations research literatures.[2] Our purpose is to jointly analyze finance and corporate organization to see whether the amount of flexibility acquired may vary according to specific financial arrangements. On the real side we shall explore extent and type of vertical flexibility that can be secured by arms' length outsourcing of inputs while maintaining in-house production facilities. On the financial side we shall see how the mix of equity[3] and convertible debt[4] or the participation of a venture capitalist[5] affect the commitment and the timing of investment in flexibility.

Our investigation is solicited by broad casual observation, literature and press reports[6] showing that firms change over time their vertical production structure, expanding and/or subsequently reducing (or the other way round) the extent of outsourcing. For instance in the automotive industry most brands adopt partial outsourcing, i.e., concomitant internal production and purchase of engines and other intermediate products from external sources. Moreover, the extent of outsourcing is frequently revised as witnessed by the variable level of value of purchased inputs over revenue found in most balance sheets.[7] From the point of view of the value of a firm, different

[1] See, for a good survey of main related issues, Tirole (2006).

[2] See Van Mieghen (1999), Wang et al. (2007), Moretto and Rossini (2012).

[3] We exclude from our investigation new equity raised through a capital increase since it tends to reduce the price of existing stock and may open the way to a loss of control. See Eckbo et al. (2007).

[4] In general, issuing convertible bonds is one way for a firm to minimize negative investor interpretation of its corporate actions (http://www.entrepreneur.com/article/159520).

[5] Recent empirical literature emphasizes the weight of venture capital in the growth of infant firms (Hellmann and Puri, 2002; Jørgensen et al. 2006; Da Rin et al. 2011).

[6] For instance Apple has recently increased the outsourcing of some inputs while reducing and bringing back home other inputs. See for further examples: *The Economist* (2011, 2013), *Forbes* (2012). See also empirical assessments in Klein (2005) and Rossini and Ricciardi (2005). Specific examples may found in Benaroch et al. (2012). An interesting textbook case relating to the vertical flexibility story of toy giant Lego is narrated in Larsen et al. (2010). The historical evolution of outsourcing in Ford and GM may be read in Shih (2013). A web site containing updated news about reshoring in the U.S. is http://www.reshorenow.org/news/.

[7] This ratio may change also for technology reasons - a new input is added or an existing input is abandoned- and

degrees of outsourcing and vertical flexibility may be associated to distinct degrees of risk born and, hence, distinct firm stock value. Then, it seems consequential examining how financial choices affect the riskiness and the quantity of flexibility acquired. As flexible technologies reduce risk (profit volatility) they may be considered a kind of (real) option and their price should reflect their (option) value (Amran and Kulatilaka, 1999, Ch. 16). As a result, a vertically flexible firm may have a value larger than the corresponding non flexible enterprise. However, as we shall see, this is not always the case whenever the cost of flexibility and the related financial aspects are properly accounted for.

In the ensuing pages we consider two alternative cases. In the first the control right over the investment decision is allocated to the firm (i.e., the shareholders), while in the second the control belongs with an outside investor (i.e., a venture capitalist). While the timing of the investment is set by one party the terms of the investment are determined by both parties. In both circumstances the level of outsourcing is always fixed by the operating party. As to the financial sources of the investment, in the first case we deal with debt financing, while the second contains a pure equity offer: ownership is shared with an outside investor without side payments (i.e., no debt service by the firm).

The investigation shows that with debt the firm rushes to adopt flexibility. The debt we consider is warranted to dodge principal agent pitfalls as it may be hard to finance it unless the lender gets a fair collateral. This is represented by an option to buy the firm in case outsourcing makes production in-house worthless. The result is that debt makes a firm invests in the vertically flexible technology earlier than in the pure equity case. When debt is insured by the associated option covenant, the extent of flexibility acquired is equal to that adopted with equity. Only the timing can be affected since shareholders rush to reap profits from flexibility as soon as possible since they know that future may be gray due to the Damocles' sword of the buyout. When we move to the alternative case of a venture capitalist involvement outsourcing is lower than with equity. Risk sharing provided by venture capital somehow makes the firm less willing to adopt higher outsourcing as an insurance against uncertainty. Venture capital (a financial resource) turns out to be a substitute for outsourcing (a real instrument). The firm gets outside capital instead of giving out a share of the vertical chain of production.[8] We establish a novel substitutability between a real internal organization choice and a financial variable, proving that finance and industrial setting are intertwined decisions.

The paper roadmap is the following: in section 2 we go through some literature, in section 3 we see the basic model, in section 4 we study the value of a vertically flexible firm in the control case without debt, in section 5 we introduce debt with collateral, in section 6 we examine the case of venture capital. The epilogue is in section 7. The Appendix contains the proofs omitted from the text.

2 Literature review

Literature has examined vertical flexibility (Shy and Stenbacka, 2005; Alvarez and Stenbacka, 2007; Moretto and Rossini, 2012; Yoshida, 2012) scantly going into the relationship with capital structure. Contributions on the link between generic industrial decisions and financial structure may be found in Lederer and Singhal (1994), in Leland (1998), in Mauer and Sarkar (2005), Benaroch et al. (2012), Nishihara and Shibata (2013), Banerjee et al. (2014), Teixeira (2014 a, b), Lambrecht et al.

because of changes in relative prices along the vertical chain of production.

[8] A growing empirical literature shows the importance that venture capital may have for new firms (Hellmann and Puri, 2002; Jørgensen et al. 2006; Da Rin et al. 2011).

(2015). A few contributions show that inefficiency arises if organization-strategic decisions are not taken simultaneously with financial choices. Mauer and Sarkar (2005) focus on the agency cost of financing investment with debt in a dynamic stochastic framework. In a similar framework Leland (1998) digs the same topic raised in the seminal paper of Jensen and Meckling (1976) without examining flexibility. Unlike Leland (1998), Mauer and Sarkar (2005) emphasize the inefficiency of debt. In the traditional Modigliani and Miller (1958) scenario the value of a firm is given by the sum of its liabilities. Equity and debt turn out to be almost perfect substitutes. However, equity holders and debtholders never coincide and each group has a different objective function. Shareholders maximize the equity value while debtholders maximize the debt value. The consequence is a sub additive result. Only a "social planner" would rather maximize the sum of debt and equity pursuing a first best. Mauer and Sarkar (2005) calculate the agency cost of debt as the difference between the total value of a firm where each group of stakeholders optimizes separately and the case where the whole value of the firm is jointly maximized. equity holders, due to limited liability, tend to overinvest if they do not face the proper agency cost of debt confirming the old Jensen and Meckling (1976) wisdom. Mainstream literature has only partially investigated whether financial structure makes a difference as to the amount of vertical flexibility and outsourcing. Benaroch and al. (2012) analyze the particular case of service production. Outsourcing may allow a firm facing volatile demand to avoid the risk of bearing fixed costs that cannot be easily covered. Thanks to outsourcing of capital intensive services the firm translates a fixed into a variable cost, reducing risk. In Banerjee et al. (2014) the investment in a new technology, such as a flexible vertical process, financed by an external subject, is seen as a joint option. Timing of the exercise of the option and the rule concerning the sharing of returns of the investment have to be established jointly by the firm and by the financial investor. According to Banerjee et al. (2014) it is inefficient to specify a sharing mode before the venture is carried out.[9] In Teixeira (2014, a, b) outsourcing with risky debt is associated with high profit. The effect of product market competition and capital structure is examined when a firm can choose between outsourcing based either on spot or long-term contracts. Competition between buyers may make outsourcing less desirable and more costly. Lambrecht et al. (2015) adopt a real options approach for outsourcing under product demand uncertainty and in house production with decreasing returns. Financial costs drive to partial or total outsourcing and affect the firm's financial beta. In Yoshida (2012) the extent of flexibility chosen by one agent affects the level of uncertainty of the scenario. In a two-agent symmetric framework more flexibility adopted by one agent calls for a similar move by the rival making flexibility a strategic complement. The increase in (endogenous) uncertainty induces an investment delay. In our framework this kind of symmetry is absent. The extent of flexibility is set (asymmetrically) only by the operating party to hedge against cost uncertainty.

3 The model

In our endeavor we couple two streams of contributions: the first on vertical flexibility and the timing of adoption of a specific technology to carry out outsourcing (Shy and Stenbacka, 2005; Alvarez and Stenbacka, 2007; Moretto and Rossini, 2012; Alipranti et al., 2014); the second on financial choices of a firm in an uncertain dynamic framework (Leland, 1994; Lederer and Singhal,

[9] In a tiny empirical literature Bakhtiari and Breunig (2014) assess the role of outsourcing as a device to smooth demand uncertainty at firm level on longitudinal data. They find an asymmetric link with demand fluctuations, i.e., outsourcing increases substantially during slumps while does not respond much to demand increases. Some scanty data investigation on the financial counterpart of outsourcing is attempted but it is fairly inconclusive. In Moon and Phillips (2014) a higher level of outsourcing makes the firm less risky in terms of cash flows. The result is a capital structure with less debt and more equity mainly in high value-added industries.

1994; Triantis and Hodder, 1990; Banerjee et al., 2014). We investigate the internal organization of a firm that manufactures a final good at a constant pace. A unit of a perfectly divisible input is needed for producing each unit of output (perfect vertical complementarity). For the input provision, the firm relies on a vertically flexible technology allowing for:[10]

i) internal production,

ii) (total or partial) outsourcing when the market price of the input decreases,

iii) backsourcing if market conditions change.

Once the technology has been installed, the input may be produced in-house at the marginal cost d, or purchased on the market. The input market price, c_t, fluctuates according to the following geometric Brownian motion:[11]

$$dc_t/c_t = \gamma dt + \sigma d\omega_t, \text{ with } c_0 = c \tag{1}$$

where γ is the drift parameter, $\sigma > 0$ is the instantaneous volatility of the market input price and $d\omega_t$ is the standard increment of a Wiener process (or Brownian motion) uncorrelated over time.[12]

By using the technology introduced above the firm may, at any t, partially or fully purchase the input from another source at the price c_t. Hence, denoting by $\alpha \in (0, 1]$ the outsourced share, it follows that:

1. the firm produces the input totally in-house when $d < \hat{c}_t = \alpha c_t + (1 - \alpha)d$, where $(1 - \alpha)$ is the share of input produced in-house,
2. the firm outsources when $d \geq \hat{c}_t$.

We assume that the decision of the level of outsourcing is irreversible (i.e. once chosen α cannot be changed), while changing regime is fully reversible (i.e. there are no switching costs). As $\alpha > 0$, the condition $\hat{c}_t > d$ holds whenever $c_t > d$, the instantaneous profit function is:[13]

$$\begin{aligned} \pi_t &= \max[0,\ p - d + \max(d - \hat{c}_t, 0)] \\ &= \max[0,\ p - d + \alpha * \max(d - c_t, 0)] \end{aligned} \tag{2}$$

where p is the output market price.

With $\alpha < 1$ the firm uses a linear combination of produced and procured input when $c_t \leq d$. It can go back to vertical integration if $c_t > d$. Notice that, with $\alpha = 1$, the firm buys the input

[10] For the sake of exposition, in the following, we will consider as "Technology" the arrangement chosen by the firm in terms of optimal combination of in-house and potentially outsourceable input.

[11] The dynamic setting adopted implies that the input market is perfectly competitive or that the forces moving the price over time do not depend on the market structure. A different approach is adopted by Billette de Villemeur et al. (2014) where an imperfect market for the input in the upstream section of production makes the firm delay entry.

[12] When the input is purchased abroad, the price uncertainty may be due to the fluctuating exchange rate (see Kogut and Kulatilaka, 1994; Dasu and Li, 1997; Kouvelis et al. 2001).

[13] Switching costs do not change qualitatively our conclusions. They give rise to a hysteresis interval in the option to switch from producing the input in-house to outsourcing it. See Benaroch et al. (2012) for the consideration of switching costs.

entirely from an independent provider, while keeping the option of returning to complete internal manufacturing. Finally, to exclude default, we assume that $p - d > 0$.[14]

The sunk cost of the flexible technology is given by:[15]

$$I(\alpha) = k_1 + (k_2/2)\alpha^2 \quad \text{for } \alpha \in (0, 1] \tag{3}$$

where k_1 is the direct cost to install and to keep internal facilities working (i.e., the cost of maintaining and updating the process for the internal production of the input) with total or partial outsourcing. The term $(k_2/2)\alpha^2$ is the organizational cost to design and run a flexible system combining in-house production and outsourcing of a specific input (Simester and Knez, 2002). That requires setting up a logistically sustainable supply chain of subcontractors, monitoring input quality and contract enforcement and so on.[16]

We do not consider investment in capacity expansion and assume that the new facility is already optimally employed to meet demand producing the input in-house. This requires $(p - d)/r > k_1$ where r is a positive constant interest rate. Therefore, we explicitly exclude the case $\alpha = 0$ with $k_1 > 0$.[17]

As anticipated in the introduction, the firm may finance the adoption of the flexible technology in two alternative ways, that is: 1) by issuing a perpetual debt paying a yearly coupon D that debtholders may convert into company's equity at certain times or 2) by venture capital. In both cases we suppose that the capital markets are frictionless and that there are no information asymmetries between shareholders, lenders and venture capitalists. All agents are assumed to be risk neutral. Last, to assure convergence, we require that $r > \gamma$.[18]

4 The benchmark case: the unlevered vertically flexible firm

In this section we derive the value of the operating facility, the optimal outsourcing share and the optimal investment policy of a firm entirely financed by equity holders.

4.1 The operating value

We examine the firm's operating value allowing for two potential scenarios. First, if $c_t > d$ we have a *vertically integrated* firm manufacturing the input in house but keeping the option of buying it

[14] Vertical flexibility, as maintained in the introduction, is an insurance against risk based on updating the know-how and keeping the facilities to produce the input in house. This assumption allows us to focus on differential financial arrangements and see how they affect the decision as to whether, when and how much to invest in the flexible technology and outsourcing. The consideration of the option to default would not affect the quality of our conclusions.

[15] The investment cost is assumed quadratic only for the sake of simplicity. None of the results is altered if one allows for a more general functional form such as $I(\alpha) = k_1 + (k_2/\delta)\alpha^\delta$ with $\delta > 1$ (see Alvarez and Stenbacka, 2007).

[16] The increasing cost of recurring to outsourcing may be seen as the mirror image of a (specificity based) hold-up which grows with the share of outsourcing as in Transaction Cost Economics (TCE) that emphasizes how hold-up in outsourcing relationships make input markets less efficient than internal production (Williamson, 1971; Joskow, 2005; Whinston, 2003). Of course generic inputs like, for instance, janitorial services do not require specific know how and cannot be modeled in this way (Anderson and Parker, 2002; Holmes and Thornton, 2008) while for other services flexibility of outsourcing may matter a lot (Benaroch et al., 2012).

[17] The case where $k_1 > 0$ and $\alpha = 0$ represents the standard case where the firm invests in a plant with exclusive in-house input provision. Note that in this paper we will abstract from the analysis of this case.

[18] Alternatively, under the assumption of complete capital markets, we may assume that there are some traded assets that can be used to hedge the input cost uncertainty z_t of (1). These traded assets together with a riskless asset allow to construct a continuously re-balanced self-financing portfolio that replicates the value of the firm (Constantinides, 1978; Harrison and Kreps, 1979; Cox and Ross, 1976).

on the market whenever convenient, i.e., when $c_t < d$. Second, if $c_t < d$ the firm outsources a share α of the input while making in-house the remaining $1-\alpha$, keeping the option to manufacture the whole input in-house whenever convenient.

A standard pricing argument leads to the following general solution for the unlevered operating firm's value (see Appendix A):

$$F^U(c_t;\alpha) = \begin{cases} \frac{p-d}{r} + \tilde{A}c_t^{\beta_2} & \text{if } c_t > d, \\ [\frac{p-(1-\alpha)d}{r} - \alpha\frac{c_t}{r-\gamma}] + \tilde{B}c_t^{\beta_1} & \text{if } c_t < d. \end{cases} \tag{4}$$

where $\beta_2 < 0$ and $\beta_1 > 1$ are, respectively, the negative and the positive roots of the characteristic equation $\Phi(\beta) = (1/2)\sigma^2\beta(\beta-1) + \gamma\beta - r$ and

$$\begin{aligned} \widetilde{A}(\alpha) &= \alpha A \equiv \alpha\frac{r-\gamma\beta_1}{r(\beta_1-\beta_2)(r-\gamma)}d^{1-\beta_2}, \\ \widetilde{B}(\alpha) &= \alpha B \equiv \alpha\frac{r-\gamma\beta_2}{r(\beta_1-\beta_2)(r-\gamma)}d^{1-\beta_1}. \end{aligned} \tag{4.1}$$

Notice that $F^U(c_t;\alpha)$ is a convex function of c_t, with $\lim_{c\to\infty} F^U(c_t;\alpha) = (p-d)/r$ and $\lim_{c\to 0} F^U(c_t;\alpha) = [p-(1-\alpha)d]/r$. The terms, $\frac{p-d}{r}$ and $[\frac{p-(1-\alpha)d}{r} - \alpha\frac{c_t}{r-\gamma}]$ are the present values of the firm associated to the two distinct vertical arrangements and, as it appears from Eq. (4), viable in-house production rules out any closure option or default. The additional terms $\tilde{A}c_t^{\beta_2}$ and $\tilde{B}c_t^{\beta_1}$ represent the value of the option to switch from vertical integration to outsourcing and the other way round, respectively. Notice that if $\alpha \to 0$, i.e., the firm is vertically integrated, both $\widetilde{A}(\alpha)$ and $\widetilde{B}(\alpha)$ tend to 0. In this limit case, due to the extreme level of vertical integration, the value of productive flexibility is, as illustrated by the corresponding two options, null. In contrast, if $\alpha \to 1$, as the input may be purchased entirely from an independent provider, the value associated to the underlying productive flexibility is, ceteris paribus, the highest possible.

4.2 Optimal outsourcing share and investment timing

Since equity is a perpetual claim, the optimal investment timing can be expressed equivalently in terms of the optimal input market price, c^{*U} , triggering investment in the flexible technology by the equity holders. Working backward, we first determine the optimal α, that is, the outsourcing share maximizing the firm's NPV once the new technology has been installed. Then, by maximizing the ex-ante value of the firm, we get the optimal investment threshold c^{*U}.

Consider the firm manufacturing the input in-house, while holding the option to switch to outsourcing, at a future date, if c_t becomes lower than d.[19] We determine the optimal α by solving the following problem:

$$\alpha^{*U} = \arg\max[\frac{p-d}{r} + \alpha A c_t^{\beta_2} - I(\alpha)], \tag{5}$$

i.e., by maximizing the value of the firm in Eq. (4) minus the cost of setting up a dedicated production organization allowing for outsourcing.

Solving Problem (5) we obtain:

$$\alpha^{*U}(c_t) = \begin{cases} 1 & \text{if} \quad c_t \leq \tilde{c}^U \\ (A/k_2)c_t^{\beta_2} & \text{if} \quad c_t > \tilde{c}^U \end{cases} \tag{6}$$

[19] In this paper we focus on the problem of adopting a new technology. Taking a different starting point would not make sense, as the option to switch to outsourcing in the future exists only if the firm is not outsourcing now.

where $\tilde{c}^U = (k_2/A)^{1/\beta_2}$. Note that the optimal α is decreasing in c_t, i.e., $\partial\alpha^{*U}/\partial c_t < 0$. This relationship reads the current and future value that one may attach to flexibility. In fact, the higher is c_t the less likely is its fall. As a consequence, the less likely one may benefit from having invested in flexibility. On the contrary, as c_t decreases, α rises and one may find optimal, for relatively low values of c_t, investing massively in flexibility, i.e., $\alpha^{*U} = 1$.

Let's now turn to the optimal investment policy. The value of the option to invest, i.e., the ex-ante value of the firm, is given by:

$$O^U(c_t, c^{*U}) = \max_{T^{U*}} E_t[e^{-r(T^{*U}-t)}][F^U(c^{*U}, \alpha^{*U}(c^{*U})) - I(\alpha^{*U}(c^{*U}))] \qquad (7)$$

where $T^{U*} = \inf\{t \geq 0 \mid c_t = c^{U*}\}$ is the optimal investment timing and $\alpha^{*U}(c^{*U})$ is the optimal outsourcing share to be chosen at $t = T^{U*}$.

The standard pricing arguments used for determining $F^U(c_t; \alpha)$ can be applied to solve Problem (7) and determine the optimal investment threshold c^{*U}. Consider a firm setting up a productive organization allowing for partial or even total outsourcing, i.e., $\alpha^{*U} \leq 1$. If the current input market price, $c_0 = c$, is such that immediate investment is not optimal, we can show that:[20]

Proposition 1 *Provided that $c_0 = c \geq \tilde{c}^U$ and $\tilde{c}^U \geq d$ (or $Ad^{\beta_2} \geq k_2$), the optimal investment thresholds and the corresponding levels of outsourcing are*

$$c^{*U} = \begin{cases} \{\frac{[2k_2(\frac{p-d}{r}-k_1)]^{1/2}}{A}\}^{1/\beta_2} \geq \tilde{c}^U & \text{if } \frac{p-d}{r} \leq k_1 + \frac{k_2}{2} \\ c, & \text{if } \frac{p-d}{r} > k_1 + \frac{k_2}{2} \end{cases} \qquad (8.1)$$

and

$$\alpha^{*U}(c^{*U}) = \begin{cases} (2\frac{\frac{p-d}{r}-k_1}{k_2})^{1/2} \leq 1, & \text{if } \frac{p-d}{r} \leq k_1 + \frac{k_2}{2} \\ Ac^{\beta_2}/k_2 \leq 1 & \text{if } \frac{p-d}{r} > k_1 + \frac{k_2}{2} \end{cases} \qquad (8.2)$$

Proof See Appendix A.

The balance between the expected present value of the net cash flows from manufacturing the input in house, $\frac{p-d}{r}$, and the investment cost of a flexible technology, $k_1 + \frac{k_2}{2}$, is crucial for the optimal timing. The investment is postponed when $\frac{p-d}{r} \leq k_1 + \frac{k_2}{2}$, otherwise it is always optimal investing immediately. The firm puts off investment only when the net present value of future cash flows (from in-house input production) does not cover the investment cost. By waiting, the gap is bridged thanks to the increased value of the option to outsource as c_t goes down.

Once identified the investment threshold, the value of the option to invest can be determined by plugging c^{*U} into Eq. (7), yielding:

$$O^U(c_t; c^{*U}) = (c_t/c^{*U})^{\beta_2}[F^U(c^{*U}, \alpha^{*U}(c^{*U})) - I(\alpha^{*U}(c^{*U}))], \text{ for } c_t \geq c^{*U} \qquad (10)$$

where $E_t[e^{-r(T^{*U}-t)}] = (c_t/c^{*U})^{\beta_2}$ is the stochastic discount factor based on the probability that the investment will be carried out.[21] If $\alpha^{*U}(c^{*U}) \leq 1$, as the firm maintains the ability to produce the

[20] In Appendix A (Scenario 2.A), similar results are obtained when $\tilde{c}^U < d$ (or $Ad^{\beta_2} < k_2$). The firm always opts for a production organization allowing for partial outsourcing. In Scenario 1.2.A the current value of c is such that $\tilde{c}^U \geq c \geq d$. The degree of flexibility degenerates to $\alpha^{*U} = 1$ and the investment timing problem has no interior solution.

[21] The expected present value $E_t[e^{-r(T^{*U}-t)}] = (c_t/c^{*U})^{\beta_2}$, can be determined by using dynamic programming (see e.g. Dixit and Pindyck, 1994, pp. 315-316).

input in-house, the value of the option to invest coincides with the value of the option to outsource, i.e.,

$$O^U(c_t; c^{*U}) = \alpha^{*U}(c^{*U})Ac_t^{\beta_2}. \tag{10.1}$$

The intuition? Once the investment is undertaken, the new flexible technology allows to produce the input in-house holding the option to outsource. The firm will find optimal to invest when the profit from producing the input in-house is sufficiently high to cover the investment cost adjusted by the degree of flexibility that maximizes the net present value of the investment project. Ex-ante, this explains why the value of the option to invest coincides with the value of the option to outsource.

5 Debt funding with a takeover option (warrant)

The firm negotiates a contract with an (financial) investor to get the funds to cover part of the cost of the initial investment paying a fixed coupon $D > 0$ per year. The contract makes a provision for a call option to be handed over to debtholders who can exercise it to buy out the equity should outsourcing make it very profitable and internal facility useless. This call option is a collateral for debt, i.e., a kind of (costly) "sweetener" for the investor.[22] The contract indicates a specific covenant (the collateral) allowing the lender to buy out the firm. Then, a rational shareholder signs the contract only if the coupon $D < p - d$.

Further, if a takeover occurs, the lender keeps producing shutting down the in-house input production facility. This restructuring decision is costly, yet it may entail a potential revenue, through the recovery of part of the initial fixed investment. We denote the relative cash flow by k_3 and assume that $k_3 > -k_1$.[23]

The sequence of moves, in case of debt funding, is the following: first the firm and the lender decide the terms of the deal (i.e., the coupon and the buyout option in the covenant). Then, the firm optimally sets both the level of flexibility α and the investment timing while the lender chooses how much to lend and when to buy out the firm.[24]

5.1 The operating value

In this case, the instantaneous profit is:

$$\pi_t = p - d - D + \max(d - \hat{c}_t, 0) \tag{2bis}$$

5.1.1 The value of debt

Let $D(c_t; \alpha)$ be the market value of debt. Since it has no stated maturity we obtain:

[22] The loan is a convertible (into equity) debt. To some extent all kinds of debt may be liable to be considered as convertible into a collateral. There are infinite types of conversion of debt according to the financial rules and the legal framework of the contract specifying the collateral. After all each debt implies a collateral, i.e., some kind of pawn.

[23] There may be several ways to restructure the firm after the buyout. We opted for the simplest mode given that the buyout may occur when outsourcing is much more profitable than in-house production. In those circumstances the lender considers internal production of the input unnecessary expensive and sets $\alpha = 1$. None of our results depends on this assumption.

[24] Notice the relevance of the point concerning who sets the timing of the investment. The evaluation of debt may take place in different scenarios. We confine to a simple, realistic, framework where the lender buys out the entire equity and adopts the outsourcing setting chosen by incumbent shareholders. We may alternatively consider cases in which the option is not to buy the entire equity but just a chunk or cases in which the lender decides to keep flexibility without restricting to outsourcing forever.

Lemma 1 *The value of debt is:*

$$D\left(c_t;\alpha\right)=\begin{cases} \frac{D}{r}+(\frac{c_t}{c^l})^{\beta_2}[(\frac{p}{r}-\frac{c^l}{r-\gamma})-(k_3+\frac{D}{r})] & \text{if} \quad c_t>c^l, \\ \frac{p}{r}-\frac{c_t}{r-\gamma}-k_3 & \text{if} \quad c_t\leq c^l. \end{cases} \tag{11.1}$$

where

$$c^l=\frac{\beta_2}{\beta_2-1}(r-\gamma)(\frac{p-D}{r}-k_3) \tag{11.2}$$

is the buyout threshold.

Proof See Appendix B.

The multiple $\frac{\beta_2}{\beta_2-1}$ is the wedge between the debtholders' actual investment cost and the benefit. The cost is made by the foregone flow of coupons plus the switching cost $(\frac{D}{r}+k_3)$. The benefit is the cash flows $(\frac{p}{r})$ that embody the uncertainty and the irreversibility of the decision to restructure. Since $\frac{\beta_2}{\beta_2-1}<1$, the buyout occurs when the market input price has gone substantially low. Hence it is better to buy the input forever and scrap the option to backsourcing. Some comparative statics say that:

$$\frac{\partial c^l}{\partial k_3}=-\frac{rc^l}{p-D-rk_3}<0 \text{ and } \frac{\partial c^l}{\partial D}=-\frac{c^l}{p-D-rk_3}<0\ .$$

In the first the higher is the cost ($k_3>0$), or the lower is the associated benefit ($0\geq k_3>-k_1$), the later the buyout occurs. The second maintains that an increase in the coupon (the benefit for the lender) induces a decrease in the threshold, i.e., making the buyout less likely. In other words, a larger coupon makes the lender less eager to buy out the firm by converting debt into equity. The assumption $p-D-rk_3>0$ guarantees that c^l is positive. For $p-D-rk_3\leq 0$ the buyout will never occur (i.e., $c^l\leq 0$) and, consequently, the relative option would be worthless. Hence, we assume that $p-D-rk_3>0$.

5.1.2 The value of equity

Letting $E\left(c_t;D\right)$ be the market value of the levered equity, standard pricing arguments yield:

Lemma 2 *The value of levered equity (for incumbent shareholders) is:*

$$E\left(c_t;\alpha\right)=\begin{cases} \frac{p-d-D}{r}+\widetilde{A}c_t^{\beta_2}-(\frac{c_t}{c^l})^{\beta_2}M(c^l,\alpha) & \text{if} \quad c_t>d, \\ [\frac{p-(1-\alpha)d-D}{r}-\alpha\frac{d}{r-\gamma}]+\tilde{B}c_t^{\beta_1}-(\frac{c_t}{c^l})^{\beta_2}M(c^l,\alpha) & \text{if} \quad c^l<c_t<d \\ 0 & \text{if} \quad c_t\leq c^l \end{cases} \tag{11}$$

where

$$M(c^l,\alpha)=[\frac{p-(1-\alpha)d-D}{r}-\alpha\frac{c^l}{r-\gamma}+\tilde{B}c^{l\beta_1}].$$

Proof See Appendix B.

As above, the terms $\tilde{A}c_t^{\beta_2}$and $\tilde{B}c_t^{\beta_1}$ indicate the value of the option to switch from vertical integration to outsourcing and the other way round, respectively. The term $E_t[e^{-r(T^l-t)}]*M(c^l,\alpha)$, where $E_t[e^{-r(T^l-t)}]=(c_t/c^l)^{\beta_2}$ is the stochastic discount factor and $T^l=\inf\{t\geq 0\mid c_t=c^l\}$ is the buyout timing,[25] indicates the loss for incumbent shareholders due to the potential buyout. Thus, the presence of this option reduces the market value of equity. This loss can be interpreted as a kind of agency cost that the equity has to pay to the lender (Mauer and Sarkar, 2005) since shareholders maximize only the equity value and not the entire value of the firm made by debt plus equity.[26]

5.1.3 The value of the levered firm

By Lemma 1 and 2, the market value of the levered firm is given by:

$$F^L(c_t;\alpha) = E\left(c_t;\alpha\right) + D\left(c_t;\alpha\right) = \begin{cases} F^U(c_t;\alpha) - Z(c_t;c^l) & \text{if } c_t > c^l, \\ (\frac{p}{r} - \frac{c_t}{r-\gamma}) - k_3 & \text{if } c_t \leq c^l, \end{cases} \tag{14}$$

where $Z(c_t;c^l) = (c_t/c^l)^{\beta_2}[\tilde{B}c^{l\beta_1} - (1-\alpha)(\frac{d}{r} - \frac{c^l}{r-\gamma}) + k_3]$.

Notice that the Modigliani-Miller irrelevance theorem does not hold here. The value of the levered firm is equal to the value of the unlevered firm, $F^U(c_t;\alpha)$, minus the present value of the payoff associated with the buyout, which accrues to the debtholders converting their debt. This payoff comes from the restructuring of the firm's operations. It is the present value of future incremental cash flows, due to the decision of fresh owners to set $\alpha = 1$, $-(1-\alpha)(\frac{d}{r} - \frac{c^l}{r-\gamma})$, minus the implicit cost of the operation, i.e., the foregone value of the option to produce the input in-house, $\tilde{B}c^{l\beta_1}$, plus the switching cost $k_3 > 0$ (or benefit if $0 \geq k_3 > -\ k_1$).

If $c^l \rightarrow 0$ the firm is never bought by the lender. Then $Z(c_t;c^l) \rightarrow 0$.We are back to the unlevered firm as illustrated in Section 4. The value of the firm does not depend on debt but on the collateral. Without it, the value of the firm would be the sum of debt and equity and the use of debt would not reduce the value of the firm, i.e., $F^L(c_t;\alpha) = F^U(c_t;\alpha)$.

Once again, the term $Z(c_t;c^l)$ can be viewed as the agency cost of debt and it may proxy the distance between a perfectly competitive complete debt market and an imperfect one where collaterals are required.

5.2 The optimal outsourcing share and the investment timing

Since equity holders control both the decision about the outsourcing share and the timing of the investment, we proceed as above by determining first α^{*L} and then c^{*L}. To get α^{*L}, equity holders maximize Eq. (11) minus the cost of setting up the production organization:

$$\alpha^{*L} = \arg\max\left[E(c_t;\alpha) - (I(\alpha) - K^L)\right] \tag{17}$$

where $K^L \leq I(\alpha)$ is the share of the investment expenditure paid by the lender who controls the amount to loan and the buyout timing. Since a rational investor will not agree to finance the firm

[25] Note that c^l must be lower than the internal cost of production d, otherwise, a buyout does not make sense. Why should equityholders borrow money to invest in a flexible technology which would be bought out before it pays off? This holds if (See Appendix B):

$$c^l \leq d \rightarrow \frac{p-D}{r} \leq (1 - \frac{1}{\beta_2})\frac{d}{r-\gamma} + k_3.$$

[26] In the absence of any agency fee shareholders would excessively increase debt since they are protected by limited liability that sets a boundary on losses which do not exceed equity while leaving to shareholders the opportunity of getting the upside cream, i.e., profits, in bonanza times. This occurs in markets in which there is some degree of asymmetric information.

unless K^L is a (financially) fair price for the debt, we set $K^L = D(c_t;\alpha)$ for $c_t > c^l$.[27] Then, substituting in Eq. (17), we obtain:

$$\alpha^{*L} = \arg\max[F^L(c_t;\alpha) - I(\alpha)] \tag{18}$$

where $F^L(c_t;\alpha)$ is given by Eq. (14).

As before, let's consider a firm manufacturing in-house the input, while holding the option to switch to outsourcing. By the first-order condition for Problem (18), the optimal outsourcing share is the solution of the following equation:

$$Ac_t^{\beta_2} - (\frac{c_t}{c^l})^{\beta_2}[Bc^{l\beta_1} + (\frac{d}{r} - \frac{c^l}{r-\gamma})] - k_2\alpha^{*L}(c_t) = 0 \tag{19}$$

where A and B are as in Eq. (4.1).

Since $\partial\alpha^{*L}/\partial c_t < 0$, if c_t is low it is optimal to choose $\alpha^{*L} = 1$, while, as c_t increases α goes down and tends to zero for high values of c_t. Further, if $c^l \to 0$, then $\alpha^{*L} \to \alpha^{*U}$. The value of the option to invest in the vertically flexible technology is equal to:

$$O^L(c_t, c^{*L}) = \max_{T^{L*}} E_t[e^{-r(T^{*L}-t)}][F^U(c^{*L}, \alpha^{*L}(c^{*L})) - I(\alpha^{*L}(c^{*L}))].$$

where $T^{*L} = \inf\{t \geq 0 \mid c_t = c^{*L}\}$ is the optimal investment timing.

Then, going through the same steps as before, we can prove that:

Proposition 2 *Provided that $c_0 = c \geq \tilde{c}^L$ and $\tilde{c}^L \geq d$ (or $Ad^{\beta_2} \geq k_2$), the optimal investment thresholds and the corresponding levels of outsourcing are:*[28]

$$c^{*L} = \begin{cases} \{\frac{[2k_2(\frac{p-d}{r}-k_1)]^{1/2}}{A-(\frac{1}{c^l})^{\beta_2}[Bc^{l\beta_1}+(\frac{d}{r}-\frac{c^l}{r-\gamma})]}\}^{1/\beta_2} \geq \tilde{c}^L & \text{if } \frac{p-d}{r} \leq k_1 + \frac{k_2}{2} \\ c, & \text{if } \frac{p-d}{r} > k_1 + \frac{k_2}{2} \end{cases} \tag{20.1}$$

and

$$\alpha^{*L}(c^{*L}) = \begin{cases} (2\frac{\frac{p-d}{r}-k_1}{k_2})^{1/2} \leq 1, & \text{if } \frac{p-d}{r} \leq k_1 + \frac{k_2}{2} \\ Ac^{\beta_2}/k_2 \leq 1 & \text{if } \frac{p-d}{r} > k_1 + \frac{k_2}{2} \end{cases} \tag{20.2}$$

Proof See Appendix B.

Substituting Eqs. [20.1-20.2] in $O^L(c_t)$, we can rearrange the value of the option to invest as follows:

$$O^L(c_t; c^{*L}) = \alpha^{*L}(c^{*L})Ac_t^{\beta_2} - Z(c_t; c^l) \ \text{ for } c_t > c^{*L} \tag{21}$$

Unlike the case of pure equity, the value of investing is reduced by the term that captures, among others, the value of the option to buy out held by debtholders. We summarize the comparison with respect to the unlevered firm in the following proposition:

[27] Note that the lender chooses the amount of the loan as a function of c_t. That is, as in Mauer and Sarkar (2005), the contract may be seen as a revolving credit line where the firm decides when to use it.

[28] We propose in Appendix B the analysis of two cases: *i)* $\tilde{c}^L < d$ (or $Ad^{\beta_2} < k_2$) and $\alpha^{*L} < 1$ (Scenario 2.B) and *ii)* c is such that $\tilde{c}^L \geq c \geq d$ and $\alpha^{*U} = 1$ (Scenario 1.2.B).

Proposition 3 *The levered firm invests earlier than the unlevered firm, i.e.,*

$$c^{*L} \geq c^{*U} \tag{22.1}$$

but adopts the same proportion of outsourced input, i.e.,

$$\alpha^{*L} = \alpha^{*U}. \tag{22.2}$$

The value of the option to invest is lower for the levered firm than for the unlevered firm, i.e.,

$$O^L(c_t) < O^U(c_t), \quad \textit{for} \quad c_t > c^{*L} \geq c^{*U} \tag{22.3}$$

Since the levered firm decides both α^{*L} and c^{*L} by maximizing only the value of equity, it has no reason to change the level of outsourcing. Part of the investment is paid by the lender and the risk born by the equity holders is just represented by the buyout option in the hands of the lender. Then, the equity holders have an incentive to invest earlier to reap profits as soon as possible.

6 Venture capital

In this section we examine a second financial arrangement involving risk capital. We consider a firm offering to an outside investor, a venture capitalist, a share of profits $\psi \in (0, 1)$ in exchange for partially funding the investment in the flexible technology. The venture capitalist decides when the deal should be implemented by setting the optimal investment threshold c^{*V}. The equity holders control the investment technological design by tuning the optimal outsourcing share α^{*V}.

The decision frame is modified with respect to debt where equity holders set both investment timing and the share of outsourced input. Now the sequence of moves is the following:

i) the equity holders offer ψ

ii) the venture capitalist observes the realizations of c_t and decides when to accept the offer ψ and invest,

iii) the equity holders set the optimal share of outsourcing.[29]

As before we proceed backwards. First, the equity holders decide $\alpha^{*V}(c_t)$. Then, the venture capitalist, given the optimal reaction function $\alpha^{*V}(c_t)$, sets the optimal investment threshold c^{*V}. equity holders may anticipate their offer ψ that could be announced even before investment takes place, i.e., at $t < T^{*V}$.

equity holders select the optimal α as follows:

$$\alpha^{*V} = \arg\max[(1-\psi)F^U(c_t;\alpha) - (I(\alpha) - K^V)] \tag{23}$$

where $(1-\psi)F^U(c_t;\alpha)$ is the share of value compensating the equity holders and $K^V \leq I(\alpha)$ is the transfer set by the venture capitalist in order to co-fund the investment.

Solving Problem (23) yields:

$$\alpha^{*V}(c_t) = \begin{cases} 1 & \text{if} \quad c_t \leq \tilde{c}^V \\ (1-\psi)(A/k_2)c_t^{\beta_2} & \text{if} \quad c_t > \tilde{c}^V \end{cases} \tag{24}$$

[29]We can model the above framework as a sequential game where, at each time $s \geq t$, equity holders offer ψ and the venture capitalist may accept or reject the offer. Thus, at every point of time, the external investor has the action set [Accept, Reject] that can be seen as a perpetual call option. See Lukas and Welling (2014) for an application of this game to supply chains.

where $\tilde{c}^V = [k_2/(1-\psi)A]^{1/\beta_2} \leq \tilde{c}^U$ for $\psi \in (0,1]$.

Let's now turn to the optimal investment policy set by the venture capitalist. The value of the option to invest held by the venture capitalist is:

$$O^V(c_t) = \max_{T^{V*}} E_t[e^{-r(T^{*V}-t)}][\psi F^U(c^{*V}, \alpha^{*V}(c^{*V})) - K^V] \tag{25}$$

where $T^{*V} = \inf\{t \geq 0 \mid c_t = c^{*V}\}$ is the optimal investment timing and $\alpha^{*V}(c^{*V})$ is the corresponding optimal outsourcing share. Then, going through the same steps as before, we can prove that:

Proposition 4 *Provided that $c_0 = c \geq \tilde{c}^V$ and $\tilde{c}^V \geq d$ (or $Ad^{\beta_2} \geq k_2/(1-\psi)$), the optimal investment thresholds and the corresponding levels of outsourcing are:*[30]

$$c^{*V} = \begin{cases} \{\frac{[\frac{k_2}{1-\psi}(\frac{p-d}{r} - \frac{K^V}{\psi})]^{1/2}}{A}\}^{1/\beta_2} > \tilde{c}^V & \text{if } \frac{p-d}{r} \leq \frac{K^V}{\psi} + \frac{k_2}{1-\psi} \\ c, & \text{if } \frac{p-d}{r} > \frac{K^V}{\psi} + \frac{k_2}{1-\psi} \end{cases} \tag{26.1}$$

and

$$\alpha^{*V}(c^{*V}) = \begin{cases} [\frac{1-\psi}{k_2}(\frac{p-d}{r} - \frac{K^V}{\psi})]^{1/2} \leq 1, & \text{if } \frac{p-d}{r} \leq \frac{K^V}{\psi} + \frac{k_2}{1-\psi} \\ (1-\psi)Ac^{\beta_2}/k_2 \leq 1 & \text{if } \frac{p-d}{r} > \frac{K^V}{\psi} + \frac{k_2}{1-\psi} \end{cases} \tag{26.2}$$

Proof See Appendix C.

Note that for $\frac{p-d}{r} > \frac{K^V}{\psi} + \frac{k_2}{1-\psi}$, the option to invest $O^V(c_t, c^{*V})$ is always increasing in c^{*V} which implies that the venture capitalist invests immediately, i.e., at $c_0 = c$, and sets $\alpha^{*V}(c^{*V}) = (1-\psi)Ac^{\beta_2}/k_2$. In contrast, for $\frac{p-d}{r} \leq \frac{K^V}{\psi} + \frac{k_2}{1-\psi}$ the investment is always postponed, i.e., $c^{*V} < c$.

Substituting Eqs. (26.1) and (26.2) into Eq. (25) the value of the option to invest is equal to

$$O^V(c_t; c^{*V}) = 2\psi\alpha^{*V}(c^{*V})Ac_t{}^{\beta_2} \tag{25.1}$$

i.e., 2ψ times the value of the option to outsource. Hence, the value of the firm is split in equal parts between the shareholders and the venture capitalist only when $\psi = 1/2$.

Unlike previous cases, the condition for the existence and the finiteness of the optimal trigger becomes $\psi > \underline{\psi} = K^V/(\frac{p-d}{r})$, while the necessary condition for having $c^{*V} > \tilde{c}^V$ and then $\alpha^{*V} < 1$, is now $\frac{p-d}{r} \leq \frac{K^V}{\psi} + \frac{k_2}{1-\psi}$. For values of ψ tending to $\underline{\psi}$ it may be optimal for equity holders to give up investing in the flexible technology, i.e. $\alpha^{*V} \to 0$. If $\psi \to 1$ equity holders are instead implicitly "selling" the firm to the venture capitalist.

An interesting result comes from $\frac{\partial\alpha^{*V}(c^{*V})}{\partial K^V} < 0$ that shows that there is a substitutability between outsourcing and the extent of the venture capital involvement, i.e., a substitutability between a real and a financial variable. A delayed investment is the immediate effect of this substitutability as $\frac{\partial c^{*V}}{\partial K^V} > 0$.

The comparison with respect to the unlevered firm is summarized in the following proposition:

Proposition 5 *If the flexible technology is partially financed by a venture capitalist, then:*

$$\begin{array}{ll} c^{*V} < c^{*U} & \text{for } \psi \in (\widehat{\psi}, 1) \\ c^{*V} \geq c^{*U} & \text{for } \psi \in (\underline{\psi}, \widehat{\psi}] \end{array} \tag{27.1}$$

[30] We provide in Appendix C the analysis of the cases: *i)* $\tilde{c}^V < d$ (or $Ad^{\beta_2} < k_2/(1-\psi)$) and $\alpha^{*V} < 1$ (Scenario 2.C) and *ii)* c is such that $\tilde{c}^V \geq c \geq d$ and $\alpha^{*V} = 1$ (Scenario 1.2.C).

where $\widehat{\psi}$ is the cutoff value with respect to the timing of investment, i.e., the positive root of $J(\psi) = 2\psi^2(\frac{p-d}{r} - k_1) - \psi(\frac{p-d}{r} - 2k_1) - K^V = 0$.

Further, if $\frac{p-d}{r} - k_1 \geq k_1 + K^V$, then:

$$\alpha^{*V} < \alpha^{*U} \quad \textit{for all } \psi \in (\underline{\psi}, 1); \tag{27.2}$$

otherwise, i.e., $\frac{p-d}{r} - k_1 < k_1 + K^V$, we get:

i)

$$\alpha^{*V} < \alpha^{*U} \textit{ when } \frac{p-d}{r} - k_1 \leq k_1 + K^V. \tag{27.3}$$

and

ii)

$$\begin{array}{ll} \alpha^{*V} \geq \alpha^{*U} & \text{for } \psi \in [\psi_1, \psi_2] \\ \alpha^{*V} < \alpha^{*U} & \textit{otherwise} \end{array} \tag{27.4}$$

where ψ_1 and ψ_2 are the two positive roots of the equation $Q(\psi) = \psi^2\frac{p-d}{r} + \psi(\frac{p-d}{r} - 2k_1 - K^V) + K^V = 0$.

Proof See Appendix C.

When ψ is low, i.e., $\psi \in (\underline{\psi}, \widehat{\psi})$, the venture capitalist enters earlier than the unlevered firm. As argued previously, with a low ψ, an outside investor is better off anticipating the time he will receive the "sure" profits from producing in-house. In contrast, if ψ is high, i.e., $\psi \in (\widehat{\psi}, 1)$, the venture capitalist prefers to hold longer the option to wait for "expected" higher profits and invests later than the unlevered firm. The equity holders choose a lower level of outsourcing with respect to the unlevered firm whenever $\frac{p-d}{r} - k_1 \geq k_1 + K^V$ (comparison between the return from vertical integration and the cost of flexibility in the presence of venture capital).

Using Eq. (10.1) and Proposition 5, we find that:

$$\begin{aligned} \Phi(\psi) = O^V(c_t)/O^U(c_t) = 2\psi\frac{\alpha^{*V}(c^{*V})}{\alpha^{*U}(c^{*U})} &= [2(1-\psi)\frac{\psi(p-d) - rK^V}{(p-d) - rk_1}\psi]^{1/2} \\ &= [2(1-\psi)\frac{\psi - \underline{\psi}}{1 - r\frac{k_1}{p-d}}\psi]^{1/2}, \text{ for } c_t > \max(c^{*V}, c^{*U}) \end{aligned} \tag{28}$$

With venture capital, unlike debt, the impact of the funding source on the option to invest is ambiguous. The ratio $\Phi(\psi)$ may in fact be even higher than 1, which implies that a higher value is associated to this option. Note, however, that $O^V(c_t) < O^U(c_t)$ for $K^V \geq k_1$, i.e., when the venture capitalist's commitment is higher than the fixed cost.

This implies that the value of the option to invest for the venture capitalist is higher than the option for the unlevered firm, when $K^V < k_1$ and the share of profits is, vis à vis the capital injection K^V, sufficiently high.

This combination provides the conditions for associating a higher value to the option to invest in the firm. If the venture capitalist is granted a high share of profits (above the cutoff level) the venture capitalist involvement turns out to be an actual alternative to vertical flexibility in terms of risk for the shareholders. This result shows the difference between the incentives to invest in the flexible technology by two different agents, the lender and the venture capitalist. For the latter the investment commitment and the expected reward may make the value of the option to enter the project much higher than for the lender, who is constrained to a fixed coupon and participates to the risk of the project only if flexibility becomes useless.

How large is the gain for the venture capitalist depends on its ability to negotiate a favorable ψ vis à vis a small K^V. The venture capitalist may obtain a large benefit if shareholders are foreclosed from the credit market and badly need funds for investing.

6.1 Optimal sharing rule

An open question is the determination of the share parameter ψ set by the equity holders maximizing the expected net present value of the project payoff.[31] Hence, they announce, before reaching the investment timing c^{*V}, the ψ solving the following problem:

$$\max_{\psi}(\frac{c_t}{c^{*V}})^{\beta_2}[(1-\psi)F^U(c^{*V};\alpha^{*V}(c^{*V}))-(k_1+\frac{k_2}{2}\alpha^{*V}(c^{*V})^2-K^V)]. \tag{29}$$

As shown in Appendix B, we can rearrange Problem (29) as follows:

$$\max_{\psi} G(\psi)=(\frac{1-\psi}{\frac{p-d}{r}-\frac{K^V}{\psi}})^{1/2}[3(1-\psi)\frac{p-d}{r}+(3-\frac{1}{\psi})K^V-2k_1]. \tag{29.1}$$

We resort to numerical simulations in order to identify the optimal share ψ^* under different scenarios as reported in the ensuing tables.

In Table 1 we see the parameters set we adopt.

Parameters	**Value**
p	3.5
d	3.47
k_1	0.6
k_2	$0.8, 1, 1.2$
K^V	0.1, 0.25, 0.5
r	0.03, 0.04
γ	0, -0.005, -0.01
σ	0.05, 0.1, 0.15

Table 1: Parameters' values

Table 2 presents the computed values relative to the optimal share ψ^*, the cutoff $\widehat{\psi}$ and $\Phi(\psi^*)$.

		$(p-d)/r = 0.75$			$(p-d)/r = 1$		
		ψ^*	$\widehat{\psi}$	$\Phi(\psi^*)$	ψ^*	$\widehat{\psi}$	$\Phi(\psi^*)$
	0.1	83.867%	19.648%	97.691%	88.283%	25.000%	63.631%
K^V	0.25	87.245%	43.145%	77.456%	91.177%	44.782%	51.593%
	0.5	95.863%	74.304%	34.027%	97.255%	67.539%	25.115%

Table 2: The optimal share of profits ψ^*, the cutoff $\widehat{\psi}$ and the ratio $\Phi(\psi^*)$.

[31] In a different environment Banerjee et al. (2014) introduce a bargaining as to the share parameter and find that it is inefficient to set it before the investment, due to time inconsistency. Only a bargaining carried out after the investment may assure intertemporal efficiency. In our case efficiency comes from the backward induction solution whereby ψ is set at the end of the decision chain and from the fact that ψ does not depend on c_t.

From Table 2 we can see that ψ^* is always above the cutoff value $(\widehat{\psi})$ (discriminating between early and delayed investment with respect to the case of an unlevered project) and goes up with the commitment of the venture capitalist, i.e., K^V. At the same time the value of the option to invest by the venture capitalist decreases with respect to the unlevered firm since it becomes more expensive.

Table 3 contains values for the optimal outsourcing levels and the corresponding investment thresholds[32] for the cases of venture capital and the (benchmark) case of an unlevered firm. We study the impact of changes in drift (γ) and volatility (σ) of the input price diffusion process for different optimal ψ^* (taken from Table 2).

	$(p-d)/r = 0.75$; $\psi^* = 0.87245$		
	$\sigma = 0.05$	$\sigma = 0.1$	$\sigma = 0.15$
$\alpha^{*U} = 61.237\%$	c^{*U}	c^{*U}	c^{*U}
$\gamma = 0$	5.897	14.722	48.130
$\gamma = -0.005$	8.448	21.767	73.375
$\gamma = -0.01$	13.230	33.812	115.472
$\alpha^{*V} = 27.183\%$	c^{*V}	c^{*V}	c^{*V}
$\gamma = 0$	4.635	8.703	20.375
$\gamma = -0.005$	6.022	11.668	28.052
$\gamma = -0.01$	8.376	16.247	39.571
	$(p-d)/r = 1$; $\psi^* = 0.91177$		
	$\sigma = 0.05$	$\sigma = 0.1$	$\sigma = 0.15$
$\alpha^{*U} = 100\%$	c^{*U}	c^{*U}	c^{*U}
$\gamma = 0$	6.368	18.660	77.105
$\gamma = -0.005$	10.318	31.464	135.380
$\gamma = -0.01$	18.914	56.841	248.335
$\alpha^{*V} = 28.293\%$	c^{*V}	c^{*V}	c^{*V}
$\gamma = 0$	4.894	10.420	29.383
$\gamma = -0.005$	6.997	15.499	45.253
$\gamma = -0.01$	11.006	24.316	72.080

Table 3: Optimal outsourcing shares and investment timing for $K^V = 0.25$.

In Table 3 we notice that with venture capital α is always lower than in the unlevered case. This illustrates the presence of the substitutability effect discussed above. The "real" hedging device which may be set up by properly combining in house and outsourced input production can be substituted by the "financial" device implicitly purchased once allowed for funding raised through venture capital. We also notice that α^{*V} grows in $(p-d)/r$ since the higher investment cost may

[32] See Appendix D for the simulation background.

be covered by the higher expected present value of net cash flow with in house input production. Focusing on timing, investment occurs always later than in the unlevered case. The impact instead of a change in the parameters γ and σ is similar. Investment occurs earlier as γ gets lower since it becomes higher the probability of exploiting outsourcing. In both cases, earlier investment is the response to a more volatile environment. This is a remarkable result in the context of the literature on real options. Notice that the standard effect inducing delay in the exercise of the option to invest is more than balanced by the presence of the option to outsource and backsource. These two instruments provide two hedging tools against the fluctuations of the relative profitability of buying rather than making and the other way around.

	$K^V = 0.1$; $\psi^* = 83.867\%$		$K^V = 0.25$; $\psi^* = 87.245\%$		$K^V = 0.5$; $\psi^* = 95.863\%$	
	α^{*V}	α^{*U}	α^{*V}	α^{*U}	α^{*V}	α^{*U}
$k_2 = 0.8$	31.828%	61.237%	27.183%	61.237%	15.905%	61.237%
$k_2 = 1$	28.468%	54.772%	24.313%	54.772%	14.226%	54.772%
$k_2 = 1.2$	25.988%	50.000%	22.195%	50.000%	12.987%	50.000%

Table 4: Optimal outsourcing shares α^{*V} and α^{*U} for $(p-d)/r = 0.75$, $\gamma = 0$, and $\sigma = \{0.05, 0.1, 0.15\}$

In table 4, we check for the impact on flexibility of different levels of commitments of the venture capitalist. We observe that α^{*V} is decreasing in K^V. Hence, the substitutability effect discussed above becomes stronger as commitment increases. Finally, in Table 4 we see the effect of a higher k_2 on flexibility. As expected, less flexibility is adopted when its relative impact on the investment cost increases.

7 Epilogue

We have investigated how the financial choices of a firm affect the extent and timing of investment in vertical flexibility. To this purpose we have considered a firm that must decide simultaneously the internal vertical setting and the corresponding financial structure in a dynamic stochastic framework. In our frame the firm viewed as vertically flexible since it has an option to outsource entirely or partially a necessary input and to reverse its choice by going back to in-house production, i.e., vertical integration.

Flexibility calls for a costly investment, partly fixed and partly dependent upon the extent of outsourcing. The goal is to set up a suitable supply chain and to keep alive the know-how and the facilities to backsource the input in case market circumstances require to do so. Two quite common external financial sources for the investment in the vertically flexible firm are: fixed price finance, i.e., debt and risk capital, i.e., venture capital. So far the latter has never been investigated together with flexibility. In the former case a lender may be willing to finance the project if she gets a fair collateral. This requirement may be fulfilled by an option to buy the company's equity in case the production in-house becomes worthless. This collateral makes the lender willing to finance the corporate since limited liability may otherwise induce the incumbent equity holders to overinvest. The levered firm decides the level of outsourcing and the timing of the investment while the lender sets only the size of the investment and the buyout time. With collateralized debt the shareholders rush to invest earlier with respect to a corresponding pure equity unlevered firm. Debt induces the firm to invest earlier since shareholders are eager to reap expected profits, consistently with common observation suggesting that debt may accelerate innovation in organizational flexibility.

When we consider venture capital involvement, the sharing of risk that the participation of the venture capitalist implies may make the firm less eager to adopt much outsourcing as an insurance against uncertainty. Further, we find that the higher is the commitment in terms of venture capital the lower is the extent of outsourcing. We may then conclude that outsourcing and venture capital may be viewed as a kind of substitute. This result establishes a fresh substitutability between a real and a financial decision of a firm. In addition if the share of profits ψ is high, the venture capitalist prefers holding longer the option to invest so that investment occurs when higher profits are expected.

A Appendix A: Benchmark case

A.1 The operating value

The standard arbitrage and hedging arguments require that the vertically flexible firm value, $F^U(c_t;\alpha)$, is the solution of the following dynamic programming problems:

$$\Gamma F^U(c_t;\alpha) = -(p-d), \text{ for } c_t > d \tag{A.1}$$

and

$$\Gamma F^U(c_t;\alpha) = -[p-\alpha c_t-(1-\alpha)d], \text{ for } c_t < d, \tag{A.2}$$

where Γ is the differential operator: $\Gamma = -r + \gamma c\frac{\partial}{\partial c} + \frac{1}{2}\sigma^2 c^2 \frac{\partial^2}{\partial c^2}$. The solution of Eq. (A.1) and Eq. (A.2) requires the following boundary conditions:

$$\lim_{c\to\infty}[F^U(c_t;\alpha)-(p-d)/r] = 0 \qquad \text{if } c_t > d$$

and

$$\lim_{c\to 0}\{F^U(c_t;\alpha) - [\frac{p-(1-\alpha)d}{r} - \alpha\frac{c_t}{r-\gamma}]\} = 0, \quad \text{if } c_t < d$$

where $\frac{p-d}{r}$ is the present value of the firm "making" the input, while $[\frac{p-(1-\alpha)d}{r} - \alpha\frac{c_t}{r-\gamma}]$ is the present value when "buying" a share α of the input. Then, from the assumptions and the linearity of Eq. (A.1) and Eq. (A.2), using the above boundary conditions, we get:

$$F^U(c_t;\alpha) = \begin{cases} \frac{p-d}{r} + \tilde{A}c_t^{\beta_2} & \text{if } c_t > d \\ [\frac{p-(1-\alpha)d}{r} - \alpha\frac{c_t}{r-\gamma}] + \tilde{B}c_t^{\beta_1} & \text{if } c_t < d. \end{cases} \tag{A.3}$$

where $\beta_2 < 0$ and $\beta_1 > 1$ are, respectively, the negative and the positive root of the characteristic equation: $\Phi(\beta) \equiv \frac{1}{2}\sigma^2\beta(\beta-1)+\gamma\beta-r$. By the value matching and the smooth pasting conditions at $c_t = d$ we obtain the two constants (Dixit and Pindyck, 1994, p. 189):

$$\begin{aligned} \tilde{B} &= \alpha B \equiv \alpha\frac{r-\gamma\beta_2}{r(\beta_1-\beta_2)(r-\gamma)}d^{1-\beta_1}, \\ \tilde{A} &= \alpha A \equiv \alpha\frac{r-\gamma\beta_1}{r(\beta_1-\beta_2)(r-\gamma)}d^{1-\beta_2}, \end{aligned} \tag{A.4}$$

which are always nonnegative and linear in α.

A.2 Optimal outsourcing share

Since $\tilde{A} = \alpha A$, the optimal vertical arrangement is given by:

$$\begin{aligned} \alpha^{*U} &= \arg\max\left[F^U(c_t;\alpha) - I(\alpha)\right] \\ &= \arg\max[\frac{p-d}{r} + \alpha A c_t^{\beta_2} - (k_1 + \frac{k_2}{2}\alpha^2)]. \end{aligned} \tag{A.5}$$

Then, the FOC is:

$$Ac_t^{\beta_2} - k_2\alpha = 0 \tag{A.5.1}$$

while the SOC is always satisfied. From Eq. (A.5.1) we obtain:

$$\alpha^{*U}(c_t) = \begin{cases} 1 & \text{if} \quad c_t \le \tilde{c}^U \\ (A/k_2)c_t^{\beta_2} & \text{if} \quad c_t > \tilde{c}^U \end{cases} \tag{A.6}$$

where $\tilde{c}^U = (k_2/A)^{1/\beta_2}$, which corresponds to Eq. (6) in the text.

A.3 Investment timing

The value function of the option to invest is

$$O^U(c_t) = \max_{T^{U*}} E_t[e^{-r(T^{*U}-t)}](F^U(c^{*U}, \alpha^{*U}(c^{*U})) - I(\alpha^{*U}(c^{*U})) \quad \text{(A.7)}$$

where $T^{U*} = \inf\{t \geq 0 \mid c_t = c^{U*}\}$ is equivalent to the optimal investment timing and $\alpha^{*U}(c^{*U})$ is the optimal outsourcing share at $t = T^{U*}$.

Equation (A.7) is equivalent to

$$O^U(c_t, c^{*U}) = \max_{c^{*U}} (\frac{c_t}{c^{*U}})^{\beta_2} (F^U(c^{*U}, \alpha^{*U}(c^{*U})) - I(\alpha^{*U}(c^{*U})) \quad \text{(A.7.1)}$$

Let's solve the maximization problem allowing for the two potential scenarios, that is,

$$\begin{array}{lll} \text{Scenario 1.A} & \alpha^{*U} \leq 1, & \text{if } \tilde{c}^U \geq d \rightarrow Ad^{\beta_2} \geq k_2 \\ \text{Scenario 2.A} & \alpha^{*U} < 1, & \text{if } \tilde{c}^U < d \rightarrow Ad^{\beta_2} < k_2 \end{array}$$

A.3.1 Scenario 1.A

At $c_0 = c$, when evaluating the investment decision and the relative optimal timing, two potential investment scenarios may arise, that is,

$$\begin{array}{lll} \text{Scenario 1.1.A} & \alpha^{*U} \leq 1, & \text{if } c \geq \tilde{c}^U \\ \text{Scenario 1.2.A} & \alpha^{*U} = 1, & \text{if } \tilde{c}^U \geq c > d \end{array}$$

Scenario 1.1.A By substituting Eqs. (3), (A.3) and (A.6) into Eq. (A.7) we have

$$O^U(c, c^{*U}) = \max_{c^{*U}} (\frac{c}{c^{*U}})^{\beta_2} [\frac{p-d}{r} + \frac{1}{2} \frac{(Ac^{*U\beta_2})^2}{k_2} - k_1] \quad \text{(A.8)}$$

Optimality requires:

$$-\frac{\beta_2}{c^{*U}} (\frac{c}{c^{*U}})^{\beta_2} [\frac{p-d}{r} - \frac{1}{2} \frac{(Ac^{*U\beta_2})^2}{k_2} - k_1] = 0 \quad \text{(A.8.1)}$$

Solving for c^{*U} yields

$$c^{*U} = \{\frac{[2k_2(\frac{p-d}{r} - k_1)]^{1/2}}{A}\}^{1/\beta_2} \quad \text{(A.8.2)}$$

Substituting c^{*U} into Eq. (3) gives

$$\alpha^{*U}(c^{*U}) = (2\frac{\frac{p-d}{r} - k_1}{k_2})^{1/2} \quad \text{(A.8.3)}$$

Let's check if the investment threshold is consistently set, that is, if $c^{*U} \geq \tilde{c}^U$. Note that

$$c^{*U} = \alpha^{*U}(c^{*U})^{1/\beta_2} \tilde{c}^U ,$$

hence, it follows that

$$c^{*U} \geq \tilde{c}^U \text{ for } \alpha^{*U}(c^{*U}) \leq 1$$

or

$$c^{*U} \geq \tilde{c}^U \text{ for } \frac{p-d}{r} \leq k_1 + \frac{k_2}{2} \quad \text{(A.8.4)}$$

Note in fact that for

$$\frac{p-d}{r} > k_1 + \frac{k_2}{2} > k_1 + \frac{k_2}{2}\alpha^{*U}(c^{*U})^2$$

$O^U(c, c^{*U})$ is increasing in c^{*U} and Eq. (A.8.1) has no solution. This implies that the firm invests immediately, i.e., at $c_0 = c$, and sets

$$\alpha^{*U}(c^{*U}) = Ac^{\beta_2}/k_2 \tag{A.8.5}$$

Scenario 1.2.A By substituting Eqs. (3), (A.3) and (A.6) into Eq. (A.7) we have

$$O^U(c, c^{*U}) = \max_{c^{*U}}(\frac{c}{c^{*U}})^{\beta_2}[\frac{p-d}{r} + Ac^{*U\beta_2} - (k_1 + \frac{k_2}{2})] \tag{A.9}$$

By taking the first derivative of the objective with respect to c^{*U} we have:

$$\frac{\partial O^U(c_t, c^{*U})}{\partial c^{*U}} = -\frac{\beta_2}{c^{*U}}(\frac{c}{c^{*U}})^{\beta_2}[\frac{p-d}{r} - (k_1 + \frac{k_2}{2})] \tag{A.9.1}$$

This implies that

$$c^{*U} = \begin{cases} \tilde{c}^U, & \text{if } \frac{p-d}{r} > k_1 + \frac{k_2}{2} \\ d, & \text{if } \frac{p-d}{r} \leq k_1 + \frac{k_2}{2} \end{cases} \tag{A.9.2}$$

A.3.2 Scenario 2.A

For $\tilde{c}^U < d$, the firm always invest in a technological frame where $\alpha^{*U} < 1$. The analysis is then identical to the one provided for scenario 1.1.A. We only need to check if the investment threshold is consistently set, that is, if $c^{*U} \geq d$. It is immediate to show that

$$\begin{aligned} c^{*U} &= \alpha^{*U}(c^{*U})^{1/\beta_2}\tilde{c}^U \quad \geq d \\ &\rightarrow \\ \frac{p-d}{r} &\leq k_1 + \frac{k_2}{2}(\frac{d}{\tilde{c}^U})^{2\beta_2} = k_1 + \frac{k_2}{2}(\frac{Ad^{\beta_2}}{k_2})^2 < k_1 + \frac{k_2}{2} \end{aligned}$$

As above, if $\frac{p-d}{r} > k_1 + \frac{k_2}{2}(\frac{Ad^{\beta_2}}{k_2})^2$, the firm invests immediately, i.e., at $c_0 = c$, and chooses

$$\alpha^{*U}(c^{*U}) = Ac^{\beta_2}/k_2.$$

B Appendix B: Debt and Equity

B.1 Debt

The value function for $c_t > c^l$ is

$$\max_{\tau>0} D(c_t; \alpha) = \frac{D}{r} + E[e^{-r\tau}][(\frac{p}{r} - \frac{c^l}{r-\gamma}) - (k_3 + \frac{D}{r})] \tag{B.1}$$

where $\tau = \min\{t > 0 : c_t = c^l\}$ and $k_3 \geq -k_1$. The problem can be rearranged as follows

$$\max_{c^l} D\left(c_t;\alpha\right) = \frac{D}{r} + (\frac{c_t}{c^l})^{\beta_2}[(\frac{p}{r} - \frac{c^l}{r-\gamma}) - (k_3 + \frac{D}{r})] \tag{B.1.1}$$

Optimality requires:

$$-\frac{\beta_2}{\beta_2 - 1}(\frac{p-D}{r} - k_3) + \frac{c^l}{r-\gamma} = 0 \tag{B.1.2}$$

which gives

$$c^l = \frac{\beta_2}{\beta_2 - 1}(r-\gamma)(\frac{p-D}{r} - k_3) \tag{B.1.3}$$

where

$$\frac{\partial c^l}{\partial D} = -\frac{c^l}{p - D - rk_3} < 0 \tag{B.1.3.1}$$

$$\frac{\partial c^l}{\partial k_3} = -\frac{rc^l}{p - D - rk_3} < 0. \tag{B.1.3.2}$$

Last note that

$$c^l \leq d \rightarrow \frac{p-D}{r} \leq (1 - \frac{1}{\beta_2})\frac{d}{r-\gamma} + k_3 \tag{B.1.4}$$

B.2 Equity

The dynamic programming problem underlying the definition of the market value of equity is similar to the one solved above for the determination of the operating value in the benchmark case. One in fact simply needs to adjust for the periodic cash flow which would be now $p - d - D$ for $c_t > d$ and $p - D - \alpha c_t - (1-\alpha)d$ for $c^l \leq c_t < d$.

Conditions for an optimal switch, i.e., value matching plus smooth pasting condition, at $c_t = d$ between the two productive frames require

$$\frac{p-d-D}{r} + \widehat{A}d^{\beta_2} = [\frac{p-(1-\alpha)d - D}{r} - \alpha\frac{d}{r-\gamma}] + \widehat{B}d^{\beta_1} + \tag{B.2.1}$$

$$-(\frac{d}{c^l})^{\beta_2}[\frac{p-(1-\alpha)d-D}{r} - \alpha\frac{c^l}{r-\gamma} + \widehat{B}c^{l\beta_1}]$$

$$\widehat{A}\beta_2 d^{\beta_2 - 1} = -\alpha\frac{1}{r-\gamma} + \widehat{B}\beta_1 d^{\beta_1 - 1} + \tag{B.2.2}$$

$$-\frac{\beta_2}{c^l}(\frac{d}{c^l})^{\beta_2 - 1}[\frac{p-(1-\alpha)d-D}{r} - \alpha\frac{c^l}{r-\gamma} + \widehat{B}c^{l\beta_1}].$$

Solving the system [B.2.1-B.2.2] yields

$$\widehat{A} = \widetilde{A} - (\frac{1}{c^l})^{\beta_2}[\frac{p-(1-\alpha)d-D}{r} - \alpha\frac{c^l}{r-\gamma} + \tilde{B}c^{l\beta_1}] \tag{B.3.1}$$

$$\widehat{B} = \alpha\frac{r - \beta_2\gamma}{r(r-\gamma)(\beta_1 - \beta_2)}d^{1-\beta_1} = \tilde{B} \tag{B.3.2}$$

Note that:

$$\frac{\partial\widehat{A}}{\partial c^l} = c^{l-(\beta_2+1)}[\beta_2\frac{p-d-D}{r}(1-\alpha) + \alpha\beta_2 k_3 + (\beta_2 - \beta_1)\tilde{B}c^{l\beta_1}] < 0$$

and

$$\lim_{c^l\rightarrow 0}\widehat{A} = \widetilde{A}, \quad \lim_{c^l\rightarrow d}\widehat{A} = -(\frac{1}{d})^{\beta_2}(\frac{p-d-D}{r})$$

B.3 Value of the levered firm

The market value of the levered firm is given by:

$$V^L(c_t;\alpha) = E\,(c_t;\alpha) + D\,(c_t;\alpha)$$

Using our results above, it is immediate to show that

$$E\,(c_t;\alpha)+D\,(c_t;\alpha) = \begin{cases} \frac{p-d}{r} + \widetilde{A}c_t^{\beta_2} - (\frac{c_t}{c^l})^{\beta_2}[\tilde{B}c^{l\beta_1} - (1-\alpha)(\frac{d}{r} - \frac{c^l}{r-\gamma}) + k_3] & \text{if } c_t > d, \\ [\frac{p-(1-\alpha)d}{r} - \alpha\frac{c_t}{r-\gamma}] + \tilde{B}c_t^{\beta_1} - (\frac{c_t}{c^l})^{\beta_2}[\tilde{B}c^{l\beta_1} - (1-\alpha)(\frac{d}{r} - \frac{c^l}{r-\gamma}) + k_3] & \text{if } c^l < c_t < d \\ (\frac{p}{r} - \frac{c_t}{r-\gamma}) - k_3 & \text{if } c_t \le c^l \end{cases}$$

which, in turn, implies that

$$F^L(c_t;\alpha) = \begin{cases} F^U(c_t;\alpha) - Z(c_t;c^l) & \text{if } c_t > c^l, \\ (\frac{p}{r} - \frac{c_t}{r-\gamma}) - k_3 & \text{if } c_t \le c^l \end{cases} \tag{B.4}$$

where $Z(c_t;c^l) = (c_t/c^l)^{\beta_2}[\tilde{B}c^{l\beta_1} - (1-\alpha)(\frac{d}{r} - \frac{c^l}{r-\gamma}) + k_3]$.

B.4 Optimal outsourcing share and investment timing

Since equity holders control both the decision about the outsourcing share and the timing of the investment, we proceed as above by determining first α^{*L} and then c^{*L}. In order to determine α^{*L}, the equity holders solve the following problem

$$\alpha^{*L} = \arg\max\left[E(c_t;\alpha) - (I(\alpha) - K^L)\right] \tag{B.5.1}$$

where $K^L \le I(\alpha)$ is the share of the investment expenditure paid by the lender who controls the amount to loan and the buyout timing. Since a rational investor will not agree to finance the firm unless k is a (financially) fair price for the debt, we set $K^L = D\,(c_t;\alpha)$ for $c_t > c^l$. Then, substituting, we obtain:

$$\alpha^{*L} = \arg\max[F^L(c_t;\alpha) - I(\alpha)] \tag{B.5.2}$$

where, as shown above, $F^L(c_t;\alpha) = F^U(c_t;\alpha) - Z(c_t;c^l)$.

Substituting for $F^L(c_t;\alpha)$ and $I(\alpha)$ the problem can be rearranged as follows

$$\alpha^{*L} = \arg\max\{\frac{p-d}{r} + \alpha A c_t^{\beta_2} - (\frac{c_t}{c^l})^{\beta_2}[\alpha B c^{l\beta_1} - (1-\alpha)(\frac{d}{r} - \frac{c^l}{r-\gamma}) + k_3] - (k_1 + \frac{k_2}{2}\alpha^2)\}. \tag{B.5.3}$$

The relative FOC is:

$$A c_t^{\beta_2} - (\frac{c_t}{c^l})^{\beta_2}[B c^{l\beta_1} + (\frac{d}{r} - \frac{c^l}{r-\gamma})] - k_2\alpha = 0 \tag{B.5.4}$$

while the SOC is always satisfied.

Eq. (B.5.4) yields

$$\alpha^{*L}(c_t) = \{A - c^{l-\beta_2}[B c^{l\beta_1} + (\frac{d}{r} - \frac{c^l}{r-\gamma})]\}\frac{c_t^{\beta_2}}{k_2}. \tag{B.5.5}$$

Now, we must identify conditions for having

$$0 < \alpha^{*L}(c_t) \le 1.$$

In order to prove that $\alpha^{*V}(c_t) > 0$ it suffices to show that

$$H(c^l) = Ac^{l\beta_2} - [Bc^{l\beta_1} + (\frac{d}{r} - \frac{c^l}{r-\gamma})] > 0.$$

Note that $H(c^l)$ is convex in c^l and

$$\lim_{c^l \to 0} H(c^l) = \infty, \; H(d) = 0, \; \left.\frac{\partial H(c^l)}{\partial c^l}\right|_{c^l=0} = 0$$

It follows that $\alpha^{*L}(c_t) > 0$ for any $c_t \in (0, d]$. Further, we have $\alpha^{*V}(c_t) \leq 1$ if

$$c_t \leq \tilde{c}^L = \{\frac{k_2}{A - c^{l-\beta_2}[Bc^{l\beta_1} + (\frac{d}{r} - \frac{c^l}{r-\gamma})]}\}^{1/\beta_2}$$

Summing up, we have

$$\alpha^{*L}(c_t) = \begin{cases} 1 & \text{if} \quad c_t \leq \tilde{c}^L \\ \frac{A - c^{l-\beta_2}[Bc^{l\beta_1} + (\frac{d}{r} - \frac{c^l}{r-\gamma})]}{k_2} c_t^{\beta_2} & \text{if} \quad c_t > \tilde{c}^L \end{cases} \tag{B.6}$$

and it is immediate to show that

$$\tilde{c}^L \leq \tilde{c}^U$$
$$\alpha^{*L}(c_t) = \alpha^{*U}(c_t) - \frac{Bc^{l\beta_1} + (\frac{d}{r} - \frac{c^l}{r-\gamma})}{k_2} (\frac{c_t}{c^l})^{\beta_2} \leq \alpha^{*U}(c_t)$$

B.5 Investment timing

The value function of the option to invest for the case of a levered firm is

$$O^L(c_t) = \max_{c^{*L}} (\frac{c_t}{c^{*L}})^{\beta_2} (F^L(c^{*L}, \alpha^{*L}(c^{*L})) - I(\alpha^{*L}(c^{*L})) \tag{B.7}$$

where $T^{L*} = \inf\{t \geq 0 \mid c_t = c^{L*}\}$ is the optimal investment timing and $\alpha^{*L}(c^{*L})$ is the optimal outsourcing share at $t = T^{L*}$.

Let's solve the maximization problem allowing for the two potential scenarios, that is,

Scenario 1.B $\quad \alpha^{*L} \leq 1, \quad$ if $\tilde{c}^L \geq d \to Ad^{\beta_2} \geq k_2 + (\frac{d}{c^l})^{\beta_2}[Bc^{l\beta_1} + (\frac{d}{r} - \frac{c^l}{r-\gamma})]$
Scenario 2.B $\quad \alpha^{*L} < 1, \quad$ if $\tilde{c}^L < d \to Ad^{\beta_2} < k_2 + (\frac{d}{c^l})^{\beta_2}[Bc^{l\beta_1} + (\frac{d}{r} - \frac{c^l}{r-\gamma})]$

B.5.1 Scenario 1.B

At $c_0 = c$, when evaluating the investment decision and the relative optimal timing, two potential investment scenarios may arise, that is,

Scenario 1.1.B $\quad \alpha^{*L} \leq 1, \quad$ if $c \geq \tilde{c}^L$
Scenario 1.2.B $\quad \alpha^{*L} = 1, \quad$ if $\tilde{c}^L \geq c < d$

Scenario 1.1.B Using Eqs. (3), (A.3), (B.4) and (B.6), the problem (B.7) can be rearranged as follows

$$\begin{aligned} O^L(c_t, c^{*L}) &= \max_{c^{*L}} (\frac{c_t}{c^{*L}})^{\beta_2} [F^U(c^{*L}, \alpha^{*L}(c^{*L})) - Z(c^{*L}; c^l) - (k_1 + \frac{k_2}{2}\alpha^{*L}(c^{*L})^2)] \\ &= \max_{c^{*L}} (\frac{c_t}{c^{*L}})^{\beta_2} [\frac{p-d}{r} + \alpha^{*L}(c^{*L}) A c^{*L\beta_2} - (k_1 + \frac{k_2}{2}\alpha^{*L}(c^{*L})^2)] + \\ &- (\frac{c_t}{c^l})^{\beta_2} [\alpha^{*L}(c^{*L}) B c^{l\beta_1} - (1 - \alpha^{*L}(c^{*L}))(\frac{d}{r} - \frac{c^l}{r-\gamma}) + k_3] \end{aligned} \tag{B.8}$$

Optimality requires:

$$\begin{aligned} -\frac{\beta_2}{c^{*L}} (\frac{c_t}{c^{*L}})^{\beta_2} [\frac{p-d}{r} + \alpha^{*L}(c^{*L}) A c^{*L\beta_2} - (k_1 + \frac{k_2}{2}\alpha^{*L}(c^{*L})^2)] \\ + \{(\frac{c_t}{c^{*L}})^{\beta_2} (A c^{*L\beta_2} - k_2 \alpha^{*L}(c^{*L})) - (\frac{c_t}{c^l})^{\beta_2} [B c^{l\beta_1} + (\frac{d}{r} - \frac{c^l}{r-\gamma})]\} \frac{\partial \alpha^{*L}(c^{*L})}{\partial c^{*L}} + \\ + (\frac{c_t}{c^{*L}})^{\beta_2} \alpha^{*L}(c^{*L}) A c^{*L\beta_2} \frac{\beta_2}{c^{*L}} = 0 \end{aligned}$$

which reduces to

$$\frac{p-d}{r} - (k_1 + \frac{k_2}{2}\alpha^{*L}(c^{*L})^2) = 0 \tag{B.8.1}$$

Solving for $\alpha^{*L}(c^{*L})$ yields

$$\alpha^{*L}(c^{*L}) = (2\frac{\frac{p-d}{r} - k_1}{k_2})^{1/2} = \alpha^{*U}(c^{*U}) \tag{B.8.2}$$

The investment threshold is instead given by

$$c^{*L} = \{\frac{[2k_2(\frac{p-d}{r} - k_1)]^{1/2}}{A - c^{l-\beta_2}[B c^{l\beta_1} + (\frac{d}{r} - \frac{c^l}{r-\gamma})]}\}^{1/\beta_2} \leq c^{*U}. \tag{B.8.3}$$

Note that

$$c^{*L} = \alpha^{*L}(c^{*L})^{1/\beta_2} \tilde{c}^L .$$

Hence,

$$c^{*L} \geq \tilde{c}^L \text{ for } \alpha^{*L}(c^{*L}) \leq 1$$

or

$$c^{*L} \geq \tilde{c}^L \text{ for } \frac{p-d}{r} \leq k_1 + \frac{k_2}{2}.$$

Notice that for

$$\frac{p-d}{r} > k_1 + \frac{k_2}{2} > k_1 + \frac{k_2}{2}\alpha^{*L}(c^{*L})^2$$

$O^L(c_t, c^{*L})$ is increasing in c^{*L} and Eq. (B.8.1) has no solution. This implies that the firm invests immediately, i.e., at $c_0 = c$, and sets

$$\alpha^{*L}(c^{*L}) = A c^{\beta_2} / k_2. \tag{B.8.4}$$

Scenario 1.2.B By substituting Eqs. (3), (A.3), (B.4) and (B.6) into Eq. (B.7) we have

$$O^L(c_t, c^{*L}) = \max_{c^{*L}} (\frac{c_t}{c^{*L}})^{\beta_2} [\frac{p-d}{r} + Ac^{*L\beta_2} - (k_1 + \frac{k_2}{2})] - (\frac{c_t}{c^l})^{\beta_2} (Bc^{l\beta_1} + k_3). \tag{B.9}$$

By taking the first derivative of the objective with respect to c^{*L} we have:

$$\frac{\partial O^L(c_t, c^{*L})}{\partial c^{*L}} = -\frac{\beta_2}{c^{*L}} (\frac{c_t}{c^{*L}})^{\beta_2} [\frac{p-d}{r} - (k_1 + \frac{k_2}{2})]. \tag{B.9.1}$$

This implies that

$$c^{*L} = \begin{cases} \tilde{c}^L, & \text{if } \frac{p-d}{r} > k_1 + \frac{k_2}{2} \\ d, & \text{if } \frac{p-d}{r} \leq k_1 + \frac{k_2}{2} \end{cases} \tag{B.9.2}$$

B.5.2 Scenario 2.B

For $\tilde{c}^L < d$, the firm always sets $\alpha^{*L} < 1$. The analysis of this case is identical to the one for scenario 1.1.B. Note that

$$\begin{aligned} c^{*L} &= \alpha^{*L}(c^{*L})^{1/\beta_2} \tilde{c}^L \geq d \\ &\rightarrow \\ \frac{p-d}{r} &\leq k_1 + \frac{k_2}{2}(\frac{d}{\tilde{c}^L})^{2\beta_2} < k_1 + \frac{k_2}{2}. \end{aligned}$$

In contrast, the firm invests immediately, i.e., at $c_0 = c$, and chooses

$$\alpha^{*L}(c^{*L}) = Ac^{\beta_2}/k_2 \text{ for } \frac{p-d}{r} > k_1 + \frac{k_2}{2}(\frac{d}{\tilde{c}^L})^{2\beta_2}.$$

C Appendix C: Venture capital

C.1 Optimal outsourcing share

The optimal vertical arrangement is given by:

$$\begin{aligned} \alpha^{*V} &= \arg\max[(1-\psi)F^U(c_t; \alpha) - (k_1 + \frac{k_2}{2}\alpha^2 - K^V)] \\ &= \arg\max[(1-\psi)(\frac{p-d}{r} + \alpha A c_t^{\beta_2}) - (k_1 + \frac{k_2}{2}\alpha^2 - K^V)] \end{aligned} \tag{C.1}$$

where $K^V \leq k_1 + \frac{k_2}{2}\alpha^2$. The relative FOC is:

$$(1-\psi)Ac_t^{\beta_2} - k_2\alpha = 0 \tag{C.1.1}$$

while the SOC is always satisfied. Solving for α yields:

$$\alpha^{*V}(c_t) = \begin{cases} 1 & \text{if } \quad c_t \leq \tilde{c}^V \\ [(1-\psi)A/k_2]c_t^{\beta_2} & \text{if } \quad c_t > \tilde{c}^V \end{cases} \tag{C.2}$$

where $\tilde{c}^V = [k_2/(1-\psi)A]^{1/\beta_2} < \tilde{c}^U$ for $\psi \in (0, 1)$.

C.2 Investment timing

The value function for the case where a venture capitalist is present is

$$O^V(c_t, c^{*V}) = \max_{c^{*V}}(\frac{c_t}{c^{*V}})^{\beta_2}(\psi F^U(c^{*V}, \alpha^{*V}(c^{*V})) - K^V) \tag{C.3}$$

where $T^{V*} = \inf\{t \geq 0 \mid c_t = c^{V*}\}$ is the optimal investment timing and $\alpha^{*V}(c^{*V})$ is the optimal outsourcing share at $t = T^{V*}$.

Let's solve the maximization problem allowing for the two potential scenarios, that is

$$\begin{array}{lll} \text{Scenario 1.C} & \alpha^{*V} \leq 1, & \text{if } \tilde{c}^V \geq d \rightarrow Ad^{\beta_2} \geq \frac{k_2}{1-\psi} \\ \text{Scenario 2.C} & \alpha^{*V} < 1, & \text{if } \tilde{c}^V < d \rightarrow Ad^{\beta_2} < \frac{k_2}{1-\psi} \end{array}$$

C.2.1 Scenario 1.C

At $c_0 = c$, when evaluating the investment decision and the relative optimal timing, two potential investment scenarios may arise, that is,

$$\begin{array}{lll} \text{Scenario 1.1.C} & \alpha^{*V} \leq 1, & \text{if } c \geq \tilde{c}^V \\ \text{Scenario 1.2.C} & \alpha^{*V} = 1, & \text{if } \tilde{c}^V \geq c < d \end{array}$$

Scenario 1.1.C By substituting Eqs. (3), (A.3) and (C.2) into Eq. (C.3.1) we have

$$O^V(c_t, c^{*V}) = \max_{c^{*V}}(\frac{c_t}{c^{*V}})^{\beta_2}[\frac{p-d}{r} + (1-\psi)\frac{(Ac^{*V\beta_2})^2}{k_2} - \frac{K^V}{\psi}]\psi. \tag{C.4}$$

Optimality requires:

$$-\frac{\beta_2}{c^{*V}}(\frac{c_t}{c^{*V}})^{\beta_2}[\frac{p-d}{r} - (1-\psi)\frac{(Ac^{*V\beta_2})^2}{k_2} - \frac{K^V}{\psi}]\psi = 0 \tag{C.4.1}$$

which reduces to

$$\frac{p-d}{r} - (1-\psi)\frac{(Ac^{*V\beta_2})^2}{k_2} = \frac{K^V}{\psi}. \tag{C.4.2}$$

Solving for c^{*V} yields

$$c^{*V} = \{\frac{[\frac{k_2}{1-\psi}(\frac{p-d}{r} - \frac{K^V}{\psi})]^{1/2}}{A}\}^{1/\beta_2}. \tag{C.4.3}$$

Substituting c^{*V} into Eq. (C.2) gives

$$\alpha^{*V}(c^{*V}) = [\frac{1-\psi}{k_2}(\frac{p-d}{r} - \frac{K^V}{\psi})]^{1/2}. \tag{C.4.4}$$

Note that

$$c^{*V} = \alpha^{*V}(c^{*V})\tilde{c}^V,$$

hence,

$$c^{*V} \geq \tilde{c}^V \text{ for } \alpha^{*V}(c^{*V}) \leq 1$$

or

$$c^{*V} \geq \tilde{c}^V \text{ for } \frac{p-d}{r} \leq \frac{K^V}{\psi} + \frac{k_2}{1-\psi}.$$

Notice that for

$$\frac{p-d}{r} > \frac{K^V}{\psi} + \frac{k_2}{1-\psi}$$

$O^V(c_t, c^{*V})$ is increasing in c^{*V} and Eq. (C.4.2) has no solution. This implies that the venture capitalist calls for an immediate investment, i.e., at $c_0 = c$, which in turn corresponds to

$$\alpha^{*V}(c^{*V}) = (1-\psi)Ac^{\beta_2}/k_2 \tag{C.4.5}$$

Scenario 1.2.C By substituting Eqs. (3), (A.3) and (C.2) into Eq. (C.3.1) we have

$$O^V(c_t, c^{*V}) = \max_{c^{*V}} (\frac{c_t}{c^{*V}})^{\beta_2} (\frac{p-d}{r} + Ac^{*V\beta_2} - \frac{K^V}{\psi})\psi \tag{C.5}$$

By taking the first derivative of the objective with respect to c^{*V} we have:

$$\frac{\partial O^V(c_t, c^{*V})}{\partial c^{*V}} = -\frac{\beta_2}{c^{*V}} (\frac{c_t}{c^{*V}})^{\beta_2} (\frac{p-d}{r} - \frac{K}{\psi})\psi. \tag{C.5.1}$$

This implies that

$$c^{*V} = \begin{cases} \tilde{c}^V, & \text{if } \frac{p-d}{r} > \frac{K^V}{\psi} \\ d, & \text{if } \frac{p-d}{r} \leq \frac{K^V}{\psi} \end{cases} \tag{C.5.2}$$

C.2.2 Scenario 2.C

For $\tilde{c}^V < d$, the firm always sets $\alpha^{*V} < 1$. The analysis of this case is identical to the one for scenario 1.1.C. Note that

$$\begin{aligned} c^{*V} &= \alpha^{*V}(c^{*V})^{1/\beta_2}\tilde{c}^V \quad \geq d \\ &\rightarrow \\ \frac{p-d}{r} &\leq \frac{K^V}{\psi} + \frac{k_2}{1-\psi}(\frac{d}{\tilde{c}^V})^{2\beta_2} < \frac{K^V}{\psi} + \frac{k_2}{1-\psi} \end{aligned}$$

In contrast, the venture capitalist calls for immediate investment, i.e., at $c_0 = c$, and the firm chooses

$$\alpha^{*V}(c^{*V}) = (1-\psi)Ac^{\beta_2}/k_2 \text{ for } \frac{p-d}{r} > \frac{K^V}{\psi} + \frac{k_2}{1-\psi}(\frac{d}{\tilde{c}^V})^{2\beta_2}.$$

C.2.3 Partial outsourcing

Comparative statics Studying $\alpha^{*V}(c^{*V}) < 1$ and the corresponding c^{*V} we notice that:

$$\frac{\partial \alpha^{*V}(c^{*V})}{\partial K^V} = -\frac{1}{2}\frac{\alpha^{*V}(c^{*V})}{\psi\frac{p-d}{r} - K^V} < 0 \tag{C.6}$$

$$\frac{\partial c^{*V}}{\partial K^V} = \frac{1}{\beta_2}\frac{\partial \alpha^{*V}(c^{*V})}{\partial K^V}\frac{c^{*V}}{\alpha^{*V}(c^{*V})} > 0 \tag{C.7}$$

and:

$$\frac{\partial \alpha^{*V}(c^{*V})}{\partial \psi} = -\frac{1}{2}\frac{\alpha^{*V}(c^{*V})}{\psi\frac{p-d}{r} - K^V}\frac{\psi}{1-\psi}(\frac{p-d}{r} - \frac{K^V}{\psi^2}) \tag{C.8}$$

$$\frac{\partial c^{*V}}{\partial \psi} = \frac{1}{\beta_2}\frac{\partial \alpha^{*V}(c^{*V})}{\partial \psi}\frac{c^{*V}}{\alpha^{*V}(c^{*V})} + \frac{\partial \tilde{c}^V}{\partial \psi}\frac{c^{*V}}{\tilde{c}^V} \tag{C.9}$$

$$= \frac{1}{\beta_2}\frac{c^{*V}}{1-\psi}\frac{\frac{1}{2}\psi\frac{p-d}{r} - \frac{K^V}{\psi}(\psi - \frac{1}{2})}{\psi\frac{p-d}{r} - K^V})$$

$$\frac{\partial \tilde{c}^V}{\partial \psi} = \frac{1}{\beta_2}\frac{\tilde{c}^V}{1-\psi} < 0. \tag{C.10}$$

Comparison with the benchmark Let's identify the conditions under which $c^{*V} < c^{*U}$. The inequality holds if:

$$J(\psi) = 2\psi^2(\frac{p-d}{r} - k_1) - \psi(\frac{p-d}{r} - 2k_1) - K^V > 0. \tag{C.11}$$

Note that $J(\psi)$ is a convex in ψ, $J(1) = \frac{p-d}{r} - K^V > 0$, and $J(0) = -K^V$. We remind that $\psi > \frac{rK^V}{p-d}$ for $c^{*V} > 0$. Therefore,

$$\begin{array}{lll} c^{*V} < c^{*U} & J(\psi) > 0 & \text{for } \psi \in (\widehat{\psi}, 1) \\ c^{*V} \geq c^{*U} & J(\psi) \leq 0 & \text{for } \psi \in (\frac{rK^V}{p-d}, \widehat{\psi}) \end{array}$$

where $\widehat{\psi}$ is the positive root of the equation $J(\psi) = 0$.

Let's now check the condition $\alpha^{*V} > \alpha^{*U}$. The inequality holds if:

$$Q(\psi) = \psi^2\frac{p-d}{r} + \psi(\frac{p-d}{r} - 2k_1 - K^V) + K^V < 0 \tag{C.12}$$

Note that $Q(\psi)$ is convex in ψ, $Q(1) = 2(\frac{p-d}{r} - k_1) > 0$, $Q(0) = K^V$. It is immediate to see that the inequality (C.12) never holds for

$$\frac{p-d}{r} - 2k_1 \geq K^V.$$

Hence, let's study $Q(\psi)$ in the interval

$$\frac{p-d}{r} - 2k_1 < K^V.$$

Notice that

$$Q(\frac{rK^V}{p-d}) = 2K^V(1 - \frac{r}{p-d}k_1) > 0$$

$$\left.\frac{\partial Q(\psi)}{\partial \psi}\right|_{\psi=\frac{rK^V}{p-d}} = K^V + (\frac{p-d}{r} - 2k_1).$$

It follows that the inequality (C.12) does not hold for

$$K^V \geq -(\frac{p-d}{r} - 2k_1)$$

that is, always for $\frac{p-d}{r} - 2k_1 \geq 0$. Otherwise, i.e., $K^V < -(\frac{p-d}{r} - 2k_1)$, inequality (C.12) holds in the interval $\psi \in (\psi_1, \psi_2)$ where ψ_1 and ψ_2 are the two positive roots of the equation $Q(\psi) = 0$.

D Appendix D

D.0.4 The shareholder's problem

In order to identify the optimal ψ, the shareholders maximize the following function:

$$\max_{\psi}(\frac{c_t}{c^{*V}})^{\beta_2}[(1-\psi)F^U(c^{*V};\alpha^{*V}(c^{*V}))-(k_1+\frac{k_2}{2}\alpha^{*V}(c^{*V})^2-K^V)]. \tag{D.1}$$

The following constraints must hold

$$\psi F^U(c^{*V};\alpha^{*V}(c^{*V}))\geq K^V\rightarrow\frac{p-d}{r}\geq\frac{K^V}{\psi} \tag{Constraint 1}$$

$$I(\alpha^{*V}(c^{*V}))\geq K^V\rightarrow\frac{p-d}{r}\geq\frac{K^V}{\psi}+2\frac{K^V-k_1}{1-\psi} \tag{Constraint 2}$$

$$(1-\psi)F^U(c^{*V};\alpha^{*V}(c^{*V}))\geq I(\alpha^{*V}(c^{*V}))-K^V\rightarrow\frac{p-d}{r}\geq\frac{\frac{K^V}{\psi}+2\frac{K^V-k_1}{1-\psi}}{3} \tag{Constraint 3}$$

$$\alpha^{*V}(c^{*V})\leq 1\rightarrow\frac{p-d}{r}\leq\frac{K^V}{\psi}+\frac{k_2}{1-\psi}. \tag{Constraint 4}$$

Note that Constraint 1 is met by assumption while Constraint 3 implies Constraint 2.

Plugging α^{*V} and c^{*V} into Problem (D.1) we have

$$\max_{\psi}\{\frac{1}{2}\frac{A}{(k_2)^{1/2}}c_t{}^{\beta_2}(\frac{1-\psi}{\frac{p-d}{r}-\frac{K^V}{\psi}})^{1/2}[3(1-\psi)\frac{p-d}{r}+(2-\frac{1}{\psi})K^V-2k_1]\}.$$

As can be easily seen, solving Problem (D.1) is equivalent to solve the following problem

$$\max_{\psi}G(\psi)=(\frac{1-\psi}{\frac{p-d}{r}-\frac{K^V}{\psi}})^{1/2}[3(1-\psi)\frac{p-d}{r}+(2-\frac{1}{\psi})K^V-2k_1]$$

The relative first order condition is

$$-(1/2)\frac{(\frac{1-\psi}{\frac{p-d}{r}-\frac{K^V}{\psi}})^{1/2}}{(\frac{1-\psi}{\frac{p-d}{r}-\frac{K^V}{\psi}})}[3(1-\psi)\frac{p-d}{r}+(3-\frac{1}{\psi})K^V-2k_1]\frac{(\frac{p-d}{r}-\frac{K^V}{\psi})+(1-\psi)\frac{K^V}{\psi^2}}{(\frac{p-d}{r}-\frac{K^V}{\psi})^2}+$$
$$-(\frac{1-\psi}{\frac{p-d}{r}-\frac{K^V}{\psi}})^{1/2}(3\frac{p-d}{r}-\frac{K^V}{\psi^2})=0$$

which reduces to

$$\frac{1}{1-\psi}+\frac{\frac{K^V}{\psi^2}}{\frac{p-d}{r}-\frac{K^V}{\psi}}+2\frac{3\frac{p-d}{r}-\frac{K^V}{\psi^2}}{3(1-\psi)\frac{p-d}{r}+(3-\frac{1}{\psi})K^V-2k_1}=0. \tag{D.2}$$

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