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Termination Fees and Contract Design in Public-Private Partnerships

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Abstract

We study the effects of granting an exit option that enables the private party to early terminate a PPP project if it turns out to be loss-making. In a continuous-time setting with hidden information about stochastic operating profits, we show that a revenue-maximizing government can optimally trade-off direct subsidies for capital investment against the right of opting out the PPP. In particular, the exit option, acting as a risk-sharing device, can soften agency problems and increase the value-for-money of public spending, even while taking into account the budgetary resources needed to resume the project in the event of early termination by the contractor.

Keywords: Public projects. Public-private partnerships. Adverse selection. Real options. Investment timing. Termination fees.

JEL classification: D81, D82, D86, H54

1 Introduction

Public-private partnerships (PPPs) have attracted much attention over the past decades, either because of the presumed inherent superiority of the private sector over the public in terms of operational efficiency, or due to the practical need of leveraging scarce governmental resources in the face of increasing demands for infrastructure and services. As a matter of

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fact, albeit with varying degrees of success, since the early 1990s the participation of private capital in public projects expanded worldwide (World Bank 2016). Recently, for instance, the Chinese government announced the intention to fund a package of projects worth a total of 10.6 trillion yuan using PPPs and to revise fiscal and tax policies to facilitate their implementation (The State Council The People's Republic of China 2016).

Although there is no single accepted legal definition, a PPP is generally understood as a "long-term contract between a private party and a government entity, for providing a public asset or service, in which the private party bears significant risk and management responsibility, and remuneration is linked to performance" (World Bank 2017:1). This broad definition allows for the inclusion of a wide variety of contractual arrangements, while at the same time for characterizing PPPs with respect to other forms of cooperation between the public and the private sector. For instance, compared with conventional procurement methods, a key feature of PPPs is that the private party must take a substantial proportion of risk, since contracts typically involve responsibility over several project functions and remuneration is closely tied to performance.

While tasks bundling and the direct link between rewards and performance can provide benefits both in terms of service quality for the public and value-for-money for the taxpayer (Hart 2003), the long duration of PPPs may also lead to several problems due to changing circumstances that can occur throughout the term of the contract (Saussier and de Brux 2018). For instance, "contracts [can] suffer from being signed in contexts with pervasive uncertainty over future demands and costs" (Iossa and Martimort 2015:8), and around the world there are several examples of PPPs which have encountered problems because of unrealistic demand expectations, cost inflation, changes in user preferences or changes in policies affecting the use of the facilities (Engel et al. 2014). Since public authorities retain the ultimate responsibility for service delivery (Forrer et al. 2010; Yang and Zhang 2010), governments have often been forced to undertake costly renegotiations or simply to resume operations (Guash 2014; Zhang and Xiong 2015).

Based on these evidences, scholars, as well as PPP stakeholders have often called for injecting more "flexibility" in PPP deals, with the term being used to embrace contract provisions allowing parties to rapidly react to changing circumstances during the course of the agreement (Demirel et al. 2017). One such provision might consist in letting the private party to early terminate the project if it turns out to be loss-making. For instance, although this clause is seldom expressly spelled out in PPP contracts, due to the public concern about service continuity, implicit abandonment options are often embedded in PPPs in the form of termination provisions for breach of contract.

This paper contributes to the literature on PPPs by analyzing the effects of granting, either explicitly or by implication, an exit option. Specifically, our aim is to understand whether and under which conditions the government can increase the value-for-money of public spending by blending direct support to capital expenditure with option incentives.

For this purpose, we develop a model where the private party ("the contractor") is untrusted with the responsibility for financing, building, maintaining and operating an infrastructure project of public interest, in exchange of the right to collect user charges. In order to spur investment, the government's offer consists of a capital subsidy and an exit option, whose strike price (the termination fee) is fixed at the time of contract award. Moreover, and unlike other works in the contract literature on PPPs, we propose here a general framework that enables to embrace situations where granting the contractor a delay option (rather than imposing the obligation to immediately undertake the investment) can be in the public interest. Finally, we explicitly account for the government's accountability for public service delivery, by assuming that, in the event of early termination by the contractor, the government will take over operations by affording a resumption cost in order to continue to effectively manage the project.

The model is developed in continuous time, by incorporating into a real options framework a principal-agent problem where the contractor holds private information on operating profits. In so doing, we intend to restrict our attention on situations where regulators, being unable to implement state-contingent risk-sharing contracts, have to resort on stationary incentives. The main policy implication is that a revenue-maximizing government can optimally trade-off direct subsidies for capital investment against the possibility of opting out the PPP. In particular, we show that the exit option, acting as a risk-sharing device, can soften agency problems and increase the government's payoff, even while taking into account the budgetary resources needed to resume the project in the event of early departure of the contractor. As by-products of our work, we illustrate the effects of termination fees upon the endogeneous timing of investment and the relationship between the optimal fee and the government's resumption cost.

The remainder is organised as follows. In Section 2 we position our work within the literature. In Section 3 we present the model. In Section 4 we derive the optimal mix of front-loaded and option incentives for a revenue-maximizing government, by first solving for the benchmark case of symmetric information and then turning on the case of private information on the operating profits. Section 5 concludes. The proofs are presented in the appendices.

2 Related literature

This paper can be positioned in the intersection of two literatures. The first is the literature using principal-agent frameworks to examine how contract design and incentive mechanisms can shape behaviour and PPP performance (see, e.g., Hart 2003; Martimort and Pouyet 2008; Auriol and Picard 2013; Hoppe and Schmitz 2013; Iossa and Martimort 2015). Within this literature, few studies, using a dynamic approach, have explored the effects of endogenoeus (Engel et al. 2001) or state-contingent (Danau and Vinella 2017) contract duration. The second body of literature has focused instead on the value of real options embedded into PPP contracts (Huang and Chou 2006; Alonso-Conde et al. 2007; Brandao and Saraiva 2008; Martins et al. 2015; Blank et al. 2016) without, however, providing much guidance on how to efficiently incorporate option-like incentives into contract design, namely, in the presence of asymmetric information.

Some papers have tried to bridge the gap between the two approaches by incorporating into a real option framework a contract design problem. A common feature of these models is the attention paid to the timing of service delivery which, rather than being taken as exogeneously given, is modelled as a decision variable. For instance, Takashima et al. (2010) study the interaction between a government and a private firm when they time an investment decision, while Scandizzo and Ventura (2010) consider a concession contract for developing a publicly-owned natural resource where the private party is required to pay a price to compensate the government for the loss of amenities. Broer and Zwart (2013) and Soumare and Lai (2016) depart from these works by introducing asymmetric information. In particular, Broer and Zwart (2013) examine the optimal regulation of an investment undertaken by a monopolist with private information on capital costs, while Soumare and Lai (2016) compare, within a model with hidden information, different forms of public support (loan guarantee *vs* direct investment) in PPPs. Di Corato et al. (2018), for their part, study how exit options, spurred by the lack of strong penalties for breach of contract, can affect bidding behaviour in multidimensional auctions for the provision of long-term environmental services.

Within this mixed literature, a paper close to ours is Silaghi and Sarkar (2018), who derive the timing of investment and revenue-sharing rules in a PPP setting with hidden action, by assuming the government is able to implement state-contingent contracts. However, our model differs from it in three ways. Firstly, we focus on a hidden information problem. Second, we consider time-contingent mechanisms. Third, we incorporate the accountability effect of implementing a public project by modelling the government's burden of ensuring service continuity in the event of an early departure by the contractor.

Regarding the methodology, we follow the dynamic mechanism design approach, which in turn builds on Baron and Besanko (1984), Battaglini (2005), Esö and Szentes (2007), Pavan et. al. (2014) and others. In particular, we exploit the approach developed by Kruse and Strack (2015, Forthcoming), who derive allocation mechanisms that can be implemented by relatively simple transfers that do not require continuous monitoring of the state variable. Here we extend this approach to the case where the regulator's tool-kit is expanded to include also real options.

3 The model

3.1 Set up

A government entity (e.g., a local government) intends to use a PPP to implement an infrastructure project which, besides the benefits realized by direct users, is also expected to generate valuable externalities to the community at large. One may think, as an example, of a new by-pass planned to reduce gridlocks in a heavily congested urban area and to curb local pollution by increased traffic fluidity.

The project, which has been designed in such as way as to allow for the application of user fees, requires a sunk setup cost I and then a fixed operating and maintenance cost per unit of time (say, per annum) denoted as c. To keep things as simple as possible, we suppose that construction can be instantaneously carried out, that the infrastructure has an infinite life and that the project, once implemented, will provide a perpetuity of public non-financial benefits, valued at b, above and beyond those accruing to direct users.[1]

We assume that the government, having identified a potential private partner (a firm or consortium), can commit to a take-it-or-leave-it offer including the following terms.[2]

First, the agreement, signed at $t = 0$, gives the contractor the responsibility for financing and building the infrastructure as well as the obligation for effectively operating and maintaining the facility all along the contract period which is assumed to be long enough to be approximated as infinite.

[1]In many of the applications we have in mind, public benefits may not be constant but, like cash-flows, also evolve over time. In toll roads, for instance, positive externalities (e.g., emissions abatement by increased traffic fluidity) increase with the traffic diverted from the existing roads, which in turn influences the amount of toll revenues. However, as long the process governing the project's spillovers is public information, we can generalize our model to allow for this.

[2]We assume that the agent has no outside opportunity and, thus, its reservation utility is equal to zero. For instance, this assumption can be justified on the grounds that the realization and operation of public facilities are normally subject to governmental authorization or licencing. The same reasoning applies in the case of investments involving the production of private goods (e.g., renewable energy) which require as input specific assets which are fully controlled by the government, such as publicly-owned land or state buildings.

Second, the contractor is given the right to collect user charges. For simplicity, like other works (see, e.g., Iossa and Martimort 2015) we assume that the contractor is able to extract all users' surplus.[3] However, the risks deriving from revenue (and/or input cost) fluctuations are beared by the contractor, who will only receive upon investment a fixed subsidy in the form of a one-time capital grant. By accepting this form of compensation for the external benefits to the broader community, the contractor accepts the risk that cash flows might be inadequate to repay debt or to provide an adequate return on equity.

Finally, the agreement legally binds the contractor to pay a fixed sum of money $L \geq 0$ in the event of premature termination of operations. Throughout the paper, L will be referred to as the termination (or exit) fee. We deliberately avoid terms like "penalty" or "liquidated damages", whose scope and meaning vary across different legal regimes, namely the civil law and common law systems (Di Matteo 2001; Marin Garcia 2012). For our purpose, it suffices to think of L as the strike price to be paid out if the (explicit or implicit) abandonment option is exercised. In order to ensure a credible commitment on the part of the contractor, we assume that the agreement is backed by some sort of collateral and/or third-party guarantees ensuring, in the event of early exit, the actual payment of the stipulated fee.

In the case of termination by the contractor, the government (either because of the societal relevance of the project or because of a public service obligation imposed by legislation) will take over the operations by affording a one-time (sunk) resumption cost $Z \geq 0$. For instance, Z can be seen as the effort required to acquire the information and skills needed to effectively and efficiently operate the facility (Auriol and Picard 2013), so as to achieve the same (financial and non-financial) outcome that would be achieved under private management.

The contractor's operating profits, before fixed O&M costs are deducted, are denoted by x_t and are assumed to evolve stochastically according to the following process:

$$dx_t = \sigma x_t dz_t \qquad x_0 = x \tag{1}$$

where $\sigma > 0$ is the constant instantaneous volatility, $dz_t = \varepsilon_t \sqrt{dt}$ is the increment of a standard Wiener process and ε_t are identically and independently normally distributed shocks with mean zero and unit variance.[4]

[3] This implies that the variable b must be interpreted as reflecting the (say annual) economic benefits net of those directly received by service users. For example, in the case of a toll roads, b can be taken to include travel time savings enjoyed by commuters who will continue to use existing roadway or reduced damages from air pollution experienced by the local community.

[4] The assumption of a trendless random walk allows us to focus on the pure effect of the uncertainty. However, notice that, by the Markov property of (1), our results would not be qualitatively altered by using a non-zero trend for x_t. In fact, it can be also easily shown that (1) is consistent with the case of a firm maximizing instantaneous operating profits under a Cobb-Douglas production technology $h(n) = n^\alpha$ with

While the parameter σ is public information, the realizations of the Brownian motion (1) are assumed not to be observable by the government. The initial value x, reflecting potentially different private abilities to seize opportunities coming from the project, is distributed on $[x^l, x^h]$ according to the cumulative distribution function $G(x)$, with density $g(x)$ and $g(x^l)$, $g(x^h) > 0$, which is common knowledge.[5] For instance, $G(x)$ may be viewed as the information collected by the government during the phase of project identification and planning which has been made public. The function $G(x)$ is such that $\phi(x) = \frac{1-G(x)}{g(x)x}$ is monotone and decreasing, with $g(x^l) \geq 1/x^l$. Note that this condition is strictly weaker than the familiar increasing hazard rate assumption (see, e.g., Guesnerie and Laffont 1984; Jullien 2000).

Finally, throughout the paper we shall assume that all parties are risk-neutral.[6]

3.2 The project's private value

Before focussing on the optimal government's offer, it is worth first examining the project's value to the contractor after works completion, i.e.:

$$V(x_t, x^E) \equiv E_t \left[\int_t^T e^{-r(s-t)}(x_s - c)ds - e^{-r(T-t)}L \right] \tag{2}$$

where r is the interest rate and $T = \inf\{s \geq t, x_s = x^E\}$ is the random first time the process (1) hits the threshold x^E that triggers the contractor's decision to terminate the project (the exit trigger).

Using simple algebra, Eq. (2) can be expanded as follows:

$$\begin{aligned} V(x_t, x^E) &= \frac{x_t - c}{r} - E_t \left\{ e^{-r(T-t)} \left[\int_T^\infty e^{-r(s-T)}(x_s - c)ds + L \right] \right\} \\ &= \frac{x_t - c}{r} - \left(\frac{x_t}{x^E} \right)^{\beta_2} \left(\frac{x^E - c}{r} + L \right) \end{aligned} \tag{3}$$

$\alpha \in (0,1)$ and n a scalar input. In this case the instantaneous profit maximization gives the input demand function $n = (\alpha/p)^{1/1-\alpha}$ where p is the input price. For instance, should uncertainty be associated with changing input costs, the profit flow would be: $x_t = N p_t^{\alpha/\alpha-1}$, where $N = (1-\alpha)(\alpha)^{\alpha/1-\alpha}$. By Ito's Lemma, if p_t is log-normal, then also x_t is log-normal (Dixit and Pindyck 1994).

[5] As in Arve and Zwart (2014) and Skrzypacz and Toikka (2015), this is equivalent to assuming that the contractor's private information is represented by two stochastic processes where the one representing the initial value is constant after time zero, but influences the transitions of the second one.

[6] In the standard real options approach to investment under uncertainty, agents formulate optimal policies under the assumption of risk neutrality (or market completeness). Introducing risk aversion would lead to an erosion of the project value and an increase of the option value of waiting to invest (Hugonnier and Morellec 2013).

where $\beta_2 < 0$ is the negative root of the characteristic equation $\Psi(\beta) = (\sigma^2/2)\beta(\beta - 1) - r = 0$, and $E_t(e^{-r(T-t)}) = \left(\frac{x_t}{x^E}\right)^{\beta_2}$ is the "expected discount factor" (Dixit and Pindyck 1994).

The first term on the R.H.S. of (3) measures the expected total profits if the infrastructure was operated forever, while the second term represents the option value to abandon the project. Therefore, the optimal exit trigger can be determined by minimizing the second term, leading to:

$$x^E = \frac{\beta_2}{\beta_2 - 1}(c - rL) \tag{4}$$

Eq. (4) shows the relationship between the exit trigger and the termination fee. Intuitively, other things being equal, the higher is L, the lower is x^E. For instance, if the contract provided for a "lock-in fee", i.e. $L \geqslant \frac{c}{r}$, the contractor would never find it convenient to walk away (i.e. $x^E = 0$). At the opposite extreme, if $L = 0$, then $x^E = \frac{\beta_2}{\beta_2 - 1}c > 0$. Notice that, since exit is irreversible and $0 < \frac{\beta_2}{\beta_2 - 1} < 1$, the optimal threshold for quitting the project is lower than the Marshallian one, i.e. $(c - rL)$, meaning that the contractor will find it worth to wait longer before terminating the project in the hope of recovering losses.

By substituting (4) into (3), we get:

$$V(x_t) = \frac{x_t - c}{r} + O(x_t) \quad \text{for } x_t \geq x^E \tag{5}$$

where $O(x_t) = -\left(\frac{x_t}{x^E}\right)^{\beta_2} \frac{x^E}{\beta_2 r} > 0$ represents the embedded exit-option value.

Notice that $\frac{\partial O}{\partial x_t} = -\left(\frac{x_t}{x^E}\right)^{\beta_2 - 1}/r < 0$ and $\frac{\partial O}{\partial x^E} = -\frac{1-\beta_2}{\beta_2}\left(\frac{x_t}{x^E}\right)^{\beta_2}/r > 0$, meaning, on the one hand, that the sooner the contractor will invest the lower will be the option value, and on the other, that the lower is L (and thus the higher is x^E) the higher is the option value.

4 Revenue maximization

In this section we derive a revenue-maximizing contract, namely, the combination of front-loaded and option incentives that maximizes the government's expected payoff, defined as the difference between public benefits and the budgetary resources devoted to the project. We first derive the optimal contract under symmetric information and then we study the case of private information on the operating profits.

4.1 Symmetric information

As a benchmark, we first consider the case where the initial state as well as the future realizations of the process (1) are observable by both the government and the contractor. In order to derive the government's offer we proceed as follows. We first determine the optimal stopping time (entry trigger) from the government's perspective, by taking the termination fee as given. Next, we derive the transfer function which will return the publicly desirable stopping time. Finally, we analyse the optimal choice of the termination fee.

The government's objective function to be maximized at $t = 0$ is given by:

$$R(x, x_\tau) = E_0(e^{-r\tau}) \left\{ \frac{b}{r} - [\pi + O(x_\tau)] \right\} + E_0(e^{-rT})(L - Z) \tag{6}$$

where $\frac{b}{r}$ is the total value of public benefits, $\tau = \inf\{t \geq 0, x_t = x_\tau\}$ is the stopping time, i.e. the stochastic time lapse before the process (1) hits for the first time the entry trigger x_τ, T is the random first time the process (1) hits the threshold that triggers the contractor's decision to terminate the project and $(L - Z)$ is the net revenue raised by the government when the contract is prematurely terminated.

The term in squared brackets on the RHS of (6) is the (explicit and implicit) incentive used to spur investment, which includes both the lump-sum transfer π and the option value $O(x_\tau)$, which turns out to be the government's (prospective) cost of insourcing a financially loss-making activity.

Rearranging (6) and using simple algebra, the government's expected payoff can be rewritten as follows:

$$R(x, x_\tau) = W(x, x_\tau) - F(x, x_\tau) + E_0(e^{-rT})(L - Z) \tag{7}$$

where:

$$W(x, x_\tau) \equiv E_0(e^{-r\tau}) \left(\frac{b}{r} + \frac{x_\tau - c}{r} - I \right) \tag{8}$$

represents the total welfare of all agents if the project is continuously operated by the contractor, and:

$$\begin{aligned} F(x, x_\tau) &\equiv E_0(e^{-r\tau}) \{V(x_\tau) - I + \pi\} \\ &= E_0(e^{-r\tau}) \left\{ \frac{x_\tau - c}{r} - [I - \pi - O(x_\tau)] \right\} \end{aligned} \tag{9}$$

is the project's value for the contractor, with the term in squared brackets measuring the net investment cost, given by the difference between the capital outlay (net of government's contribution) and the exit option value.

Eq. (7) implies that, for any given L and Z, the government's payoff is maximized when the private value is brought down to zero. Therefore, assuming there exists a transfer π such that $F(x, x_\tau) = 0$, the policy problem consists in finding the stopping time that maximizes (8). Denoting with $\tau^W = \inf(t \geq 0 \;/\; x_t = x_{\tau^W})$ the welfare-maximizing (first-best) stopping time, we get:

$$W(x) = \max_{x_{\tau^W}} \left(\frac{x}{x_{\tau^W}} \right)^{\beta_1} \left[\frac{b}{r} + \frac{x_{\tau^W} - c}{r} - I \right] \tag{10}$$

where $E_0(e^{-r\tau^W}) = \left(\frac{x}{x_{\tau^W}} \right)^{\beta_1}$, and $\beta_1 > 1$ is the positive root of the characteristic equation $\Psi(\beta)$.

Solving (10) leads to the first-best threshold for investing:

$$x_{\tau^W} = \frac{\beta_1}{\beta_1 - 1} (c - b + rI) \tag{11}$$

Again, the optimal trigger differs from the Marshallian one, i.e. $(c - b + rI)$, by the multiple $\frac{\beta_1}{\beta_1 - 1} > 1$ that accounts for the presence of uncertainty and irreversibility. In other words, from a public policy perspective, there is a benefit from delaying investment beyond the point where the NPV becomes positive.

By using (11), we get that, for any given termination fee, and thus, option value $O(x_{\tau^W})$, the government can always bring the private value (9) down to zero by the following transfer:

$$\begin{aligned} \pi^W &= I - O(x_{\tau^W}) - \frac{x_{\tau^W} - c}{r} \\ &= \frac{b}{r} - \frac{1}{\beta_1 - 1} \left(\frac{c - b}{r} + I \right) - O(x_{\tau^W}) \end{aligned} \tag{12}$$

Eq. (12) shows that, under uncertain operating profits ($\sigma > 0$), the optimal public transfer is lower than the conventional "Pigouvian" subsidy, i.e. $\pi^W < \frac{b}{r}$. The reason is twofold. First, since there is a social option value of delay (see Eq.11), it would not be optimal to fully internalize external benefits, since this would induce the contractor to inefficiently accelerate investment. Second, the exit option, i.e. $O(x_{\tau^W})$, reduces the net private cost of investment, which, in turn, reduces the amount of grant funding needed to spur investment. Stated differently, a revenue-maximizing government can trade-off front-load against option incentives. Obviously, both the second and the third term on the R.H.S. of (12) tend to zero as the uncertainty parameter $\sigma \to 0$.

By substituting (11) and (12) into (7), we get, for any given exit fee L, the maximum expected revenue achievable by the government:

$$R(x) = W(x) + E_0(e^{-rT})(L - Z) \tag{13}$$

Let's now turn our attention on the optimal termination fee, alias the optimal degree of exit flexibility. Recalling that $E_0(e^{-rT})(L-Z) = \left(\frac{x}{x^E}\right)^{\beta_2}(L-Z)$, maximization of Eq. (13) with respect to L leads to the following proposition.

Proposition 1 Under symmetric information the government's expected revenue is maximized by the following termination fee:

$$\begin{aligned} L^W &= Z + \frac{1}{\beta_2 - 1}\left[Z - \frac{c}{r}\right] \\ &= \frac{c}{r} + \frac{\beta_2}{\beta_2 - 1}\left[Z - \frac{c}{r}\right] \end{aligned} \tag{14}$$

with $Z < L^W < \frac{c}{r}$.

Proof: See Appendix A.

Since $\frac{\beta_2}{\beta_2-1} < 1$, the proposition implies that it would not be optimal for a revenue-maximizing government to lock-in the contractor by charging a termination fee so high that it amounts to a deterrence device against early termination (i.e., $L^W < \frac{c}{r}$).[7]

However, the exit option should be appropriately priced, so as to more than compensate the government for the direct cost of resuming operations (i.e., $L^W > Z$), the margin being related to the additional budgetary cost deriving from insourcing a financially loss-making activity. In fact, it is easy to show (see Appendix A) that the fee should be such as to allow the government to partially cover the (indirect) cost of taking responsibility for the project and that, even in the extreme case where $Z = 0$, the contractor should be charged ($L^W = -\frac{1}{\beta_2-1}\frac{c}{r} > 0$) for enjoying the right to opt out. This echoes the argument made by Scott and Triantis (2004) that when contracts include embedded options, such as the right to terminate, the amount to be paid to exercise the option may depart from the conventional compensation principle underlying the traditional common-law contract doctrine on liquidated damages.

Summarizing, the main findings are as follows. First, not surprisingly, under symmetric information revenue-maximization (rather than welfare-maximization) does not imply a distortion with respect to the economically efficient timing of investment, i.e., a government focussed on his own maximum payoff will find it optimal to achieve the same stopping timing that would be devised by a benevolent social planner. Second, however, revenue-maximization implies giving the contractor an abandonment option which turns out to lower total welfare, because the government will need to inject additional economic resources (i.e., Z) in order to resume operations. The simple intuition is that, although increasing total

[7] If the resumption cost is expected to be fairly close or even higher than fixed O&M costs (i.e. $Z \geqslant \frac{c}{r}$) then it would not be advisable to grant an exit option (i.e. $L^W = \frac{c}{r}$).

costs, the exit option allows for accelerated investment and, thus, accelerated provision of public benefits, while at the same time reducing the government's capital investment in the project.

4.2 Asymmetric information

The previous subsection assumes that both parties can perfectly observe the cash flow stream. In this case, although it is the contractor who is ultimately making the investment, the government can influence the timing of project's implementation by making an appropriate fixed payment the first time operating profits hits the desired threshold level. On the contrary, in the following we study mechanism design in a context where the contractor has private information on the initial value x as well as the future realizations of the process (1).

In this case, according to the standard direct-revelation mechanism approach, the government should offer at $t = 0$ a menu of contracts $(\tau, \pi(\tau))$ that specifies, as a function of the targeted stopping time τ, a payment $\pi(\tau)$ such that τ becomes optimal for the contractor. The contract should be incentive-compatible, i.e., whatever is his initial type $x \in [x^l, x^h]$, it should be in the contractor's best interest to report truthfully, at each time $t \geq 0$, whether or not x_t has crossed the threshold x_τ. Hence, the contractor will choose at each time $t \geq 0$ a feasible strategy as solution of the following stopping problem:

$$F(x_t) = \max_{\tau} E_t \left[e^{-r(\tau - t)} \left(\frac{x_\tau - c}{r} - I + O(x_\tau) + \pi(\tau) \right) \right] \qquad \text{for all } t \geq 0 \tag{15}$$

Unfortunately, since the space of communication strategies between the parties (i.e. the stopping times among which the contractor can choose) could be very rich, the optimal mechanism is in general hard to be implemented (Board 2007a; Pavan et al. 2014; Bergemann and Valimaki Forthcoming).

An alternative way of proceeding is that proposed by Kruse and Strack (2015, Forthcoming), who show that if the principal wishes to implement a predetermined stopping time, say τ, to which corresponds a given trigger x_τ, he can rely on a much simpler direct revelation mechanism that does not require exchange of information between the parties, with the exception of the initial value of the state variable.

Their argument can be summarized as follows. If the agent has followed up to t a truthful strategy and the future realizations of x_t are driven by independent random shocks ε_t, he will never find it optimal to stop before x_τ and there is no reason why he should change strategy in the future. Thus, working backwards, for any given x_τ a sufficient condition for a contract to be ex-ante incentive-compatible is that the maximum of (15) is attained when the agent announces the true initial value x. Moreover, by exploiting the link between reflected processes and stopping times[8], Kruse and Strack also show that the incentive-compatible transfer $\pi(\tau)$ can be simply calculated as the expected present value of all future cash flows that an agent would loose if x_t was kept below x_τ forever.

Compared with other incentive-compatible approaches, this mechanism has several attractive features. First, since the transfer is independent of the future realizations of the state variable, it can be paid even when cash flows are unobservable. Second, since the transfer only depends on the realized stopping decision, there is no need of any further transmission of information between the parties, i.e., the government only needs to know that the investment was made, rather than the reasons why it has happened.

Building on these findings, we can conclude that, for any given stopping time τ, there exists an incentive-compatible transfer $\pi(\tau)$ such that the contractor will invest the first time operating profits hit the threshold x_τ, with the latter given by the following implicit function (see Appendix B):[9]

$$\frac{\beta_1 - 1}{\beta_1} x_\tau + r\left[\pi(\tau) - \frac{x_\tau}{\beta_1}\frac{\partial O}{\partial x_\tau}\right] = c + r(I - O(x_\tau)) \tag{16}$$

Condition (16) simply says that the contractor will invest when the expected marginal cost (i.e. the R.H.S.) is equal to the expected marginal benefit (the L.H.S). This implies that the government can spur investment either directly, by lowering capital costs, or indirectly, by increasing the exit option value, i.e. by lowering the termination fee.

By substituting (16) into (15), we get, for any combination of direct and indirect incentives that lead the contractor to implement the project the first time the process (1) hits x_τ, the value to invest for a contractor of type x:

$$\begin{aligned} F(x) &= E_0(e^{-r\tau})\frac{x_\tau}{\beta_1 r}\left[1 + r\frac{\partial O}{\partial x_\tau}\right] \\ &= \left(\frac{x}{x_\tau}\right)^{\beta_1}\frac{x_\tau}{\beta_1 r}\left[1 - \left(\frac{x_\tau}{x^E}\right)^{\beta_2 - 1}\right] \quad \text{for } x^E < x < x_\tau \end{aligned} \tag{17}$$

The following Lemma allows us to apply the results of Kruse and Strack to our setting.

Lemma 1 Letting x_τ an arbitrary entry-trigger and $\tilde{x}_t$ a version of the process x_t reflected

[8]A reflected process is like a process that has the same dynamics as the original process, but is required to stay below a given barrier whenever the original process tends to exceed it. See Harrison (2013) for a formal definition of these processes.

[9]Kruse and Strack show that $\pi(\tau)$ exists even when the principal's benefits (i.e. in our framework b) are time-dependent (see footnote 1), in which case the optimal stopping time will also be time-dependent, however, in general, a closed form solution for $x_\tau(t)$ does not exist, and numerical methods are required.

at x_τ, the stopping time $\tau = \inf(t > 0 \ / \ x_t = x_\tau)$ can be implemented by the following incentive-compatible transfer:

$$\begin{aligned}\pi(\tau) &= I - O(x_\tau) - \frac{x_\tau - c}{r} + E_\tau \left\{ \int_\tau^\infty e^{-r(t-\tau)} \mid \frac{\partial O}{\partial x_t}(x_\tau) \mid d(x_t - \tilde{x}_t) \right\} \\ &= I - O(x_\tau) - \frac{x_\tau - c}{r} + \frac{x_\tau}{\beta_1 r}\left[1 - \left(\frac{x_\tau}{x^E}\right)^{\beta_2 - 1}\right]\end{aligned} \tag{18}$$

where the integral measures the reduction of cash flows, if any, in the interval $(t, t+dt)$ resulting from keeping x_t at x_τ.

Proof: See Appendix C.

Comparison between Eq. (18) and Eq. (12) shows that, as in the case of symmetric information, the public transfer should cover the difference between the investment cost and the expected present value of operating profits, net of the exit option value. However, under asymmetric information, the transfer must also compensate the contractor for the information rents, given by the option value to delay the investment beyond the government's desired threshold x_τ.

More specifically, the first line in (18) shows that, as pointed by Kruse and Strack, the information rents can be determined by calculating the expected present value of all future cash flows the contractor would loose by keeping x_t below x_τ. Notice, however, that since in our framework the contractor also benefits of an exit option, the information rents (i.e. the lost revenues) are remunerated at a lower rate than r, namely, at $\mid \frac{\partial O}{\partial x_t} \mid < 1/r$.[10] Moreover, the second line in (18) shows that the integral admits a closed-form solution and that the information rents are nothing but the private value to invest evaluated at x_τ.

Armed with these insights, let's go back to the government's revenue-maximization problem. Since any stopping time τ can be implemented by the transfer (18), by the standard mechanism design approach (Laffont and Martimort 2002) we can confine analysis on menus of ex-ante incentive compatible contracts that induce the contractor to reveal his type and such that the expected rents (17) are non-negative for any $x \in [x^l, x^h]$.

The government's problem thus reduces to choosing the entry trigger $x_\tau(x)$ that maximizes the following objective function:

[10] Formally, since the marginal incentive to delay the project is $d(x_t - \tilde{x}_t)$, in Appendix C we show that the term $\frac{\partial O}{\partial x_t} d(\tilde{x}_t - x_t) > 0$ represents the cost per unit of the distance through which x_t is reflected to keep x_t at x_τ.

$$\mathcal{R}(x, x_\tau(x)) \equiv \int_{x_l}^{x_h} R(x, x_\tau(x))\, g(x)dx \tag{19}$$
$$= \int_{x_l}^{x_h} \left[W(x, x_\tau(x)) - F(x, x_\tau(x)) + E_0(e^{-rT})(L - Z)\right]\, g(x)dx$$

In order to ensure that the contract duration is always positive and to avoid bunching, we introduce the following assumption:

Assumption 1. $\frac{x^E}{x_{\tau W}} < K \min\left[\frac{g(x^l)x^l - 1}{\beta_1 - 1},\ 1\right]$, where $K = \left(\frac{\beta_1 - 1}{\beta_1 - \beta_2}\right)^{1/1-\beta_2} < 1$.

The solution is summarized in the following proposition.

Proposition 2 Under Assumption 1, for any given termination fee:

a) the government's revenue is maximized by the stopping time $\tau^R(x) = \inf(t > 0\ /\ x_t = x_{\tau^R}(x))$, where the entry-trigger $x_{\tau^R}(x)$ is defined by the following implicit function:

$$x_{\tau^R}(x) = x_{\tau^W} + [x_{\tau^W} - r(\beta_1 - \beta_2)O(x_{\tau^R}(x))]\, \frac{\phi(x)}{1 - \phi(x)} \tag{20.1}$$

with $\frac{\partial x_{\tau R}(x)}{\partial x} < 0$;

b) the transfer that implements $\tau^R(x)$ is:

$$\pi^R(x) = \frac{b}{r} - \frac{\beta_1 - \beta_2}{\beta_1} O(x_{\tau^R}(x)) - \left(\frac{\beta_1 - 1}{\beta_1}\right) \frac{x_{\tau^R}(x) - x_{\tau^W}}{r} \tag{20.2}$$

with $\frac{\partial \pi^R(x)}{\partial x} > 0$ and $\frac{\partial \pi^R(x)}{\partial x_{\tau R}} < 0$.

Proof: See Appendix D.

Assumption 1 guarantees that the second order condition is satisfied and that the optimal trigger $x_{\tau^R}(x)$ is decreasing in $x \in [x^l, x^h]$.[11]

Eq. (20.1) shows that, except for the highest-type contractor (i.e., $x = x^h$)[12], the government has an incentive to delay the investment compared with the first-best solution, i.e. $x_{\tau^R}(x) > x_{\tau^W}$. Moreover, comparison between (20.2) and (12) shows that (except for $x = x^h$) the optimal subsidy is lower than the one paid under full information. Thus, as in the standard Principal-Agent literature, the government faces a rent-efficiency trade-off.

[11] Although it is not the main focus of our work, in Appendix D we briefly discuss the consequences of violating Assumption 1. We show that when the exit trigger x^E is relatively high (i.e. when the termination fee is too low), the optimal contract may involve a bunching interval for the most efficient types.

[12] When $x = x^h$, then $\phi(x^h) = 0$ and $x_{\tau^R}(x^h) = x_{\tau^W}$.

Eq. (20.1) also confirms the result underlined by condition (16), i.e., that the government can reduce the rents left to all inframarginal types by increasing the exit option value, and in so doing, accelerate investment. Furthermore, the time distortion (i.e., $x_{\tau^R}(x) > x_{\tau^W}$), used to squeeze information rents, allows the government to save on capital funding.

By substituting (20.1) into (19), we get that the government's expected revenue depends only on the optimal trigger $x_{\tau^R}(x)$ and equals the expected welfare when the latter is replaced by the "virtual welfare" (Myerson 1981):

$$\mathcal{R}(x, x_{\tau^R}(x)) = \int_{x_l}^{x_h} \left\{ \hat{W}(x, x_{\tau^R}(x)) + E_0(e^{-rT})(L - Z) \right\} g(x)dx \tag{21}$$

where:

$$\hat{W}(x, x_{\tau^R}(x) \equiv W(x, x_{\tau^R}(x)) - \frac{1 - G(x)}{g(x)} \frac{1}{\beta_1} F_x(x, x_{\tau^R}(x)) \tag{22}$$

Moreover, by using (20.2), we are also able to generalize the Myersonian equivalence between the expected revenue and the expected virtual surplus:

$$\begin{aligned} \hat{W}(x, x_{\tau^R}(x) = & \left[F(x, x_{\tau^R}(x)) - \frac{1 - G(x)}{g(x)} \frac{1}{\beta_1} F_x(x, x_{\tau^R}(x) \right] + \\ & - E_0(e^{-r\tau^R(x)})\beta_2 \left[\frac{1}{\beta_1} - \phi(x) \right] O(x_{\tau^R}(x)) \end{aligned} \tag{23}$$

where the first term on the R.H.S. of (23) indicates the standard virtual value, while the second term measures what the government expects to earn by increasing the option value.

Notice that the sign of the second term depends on the sign of $[\frac{1}{\beta_1} - \phi(x)]$: since $\phi'(x) < 0$ and $\beta_1 > 1$, the second term contributes to increasing the government's payoff for all values of x when $g(x^l)x^l > \beta_1$, while when $1 < g(x^l)x^l < \beta_1$ the effect is positive only if the contractor turns out to be relatively efficient (i.e., high values of x).

Before proceeding, it is worth mentioning the analogy between our findings and those obtained by Arve and Martimort (2016) using a two-period model with uncorrelated shocks. They show (Proposition 2, p. 3254) that when the agent is risk-adverse in the second period, the regulator can relax the first-period incentive-compatibility constraint by offering the agent higher profits in the second period. The income effect induced by risk aversion reduces the production distortion in the first period and thus the government does not need to distort production as much as under risk neutrality.

Here we get a qualitatively similar result, by embedding an exit option within a continuous-time model with risk-neutral agents: since the option reduces the contractor's net investment cost, and thus increase the private value, the government is able to relax the incentive-compatibility constraint in the “first period”. In other words, the granting of an exit option makes the contractor more prone to bear additional risk (i.e., to accelerate investment) and therefore the government does not need to delay the investment as much as it would occur if the project was only sustained by cash payments.

Now, let's consider the optimal termination fee. By substituting (23) in (21) and maximizing with respect to L, we get the following proposition which summarizes the properties of the optimal fees under asymmetric information.

Proposition 3 Denoting with L^{R1} the termination fee which maximizes the government's revenue when $g(x^l)x^l > \beta_1$ and with L^{R2} the termination fee which maximizes the government's revenue when $1 < g(x^l)x^l < \beta_1$, we get:

$$Z < L^{R1} < L^{R2} < L^W < \frac{c}{r}$$

Proof: See Appendix E.

The first result is that, as in the case of symmetric information, the termination fee must be such as to more than compensate the government for the resumption cost (i.e., $L^R > Z$) but, at the same time, it should not be so large as to have the effect of discouraging, under any circumstances, early exit ($L^R < \frac{c}{r}$).

The second result (i.e., $L^R < L^W$) is that the optimal fee is lower as compared with the one required under symmetric information. As already pointed out, the reason is that lower fees (i.e., higher option values) have the effect of squeezing information rents and accelerating investment, while at the same time reducing the need of public funding.

The third finding is that the optimal fee depends, as can be expected, on the project's riskiness, as well as on the distribution function of types. Specifically, $L^{R1} < L^{R2}$ is explained by the different importance “inefficient” and “efficient” contractors (low *vs* high values of x) ascribe to the exit option: while the latter, being more prone to accelerate investment, tend to attach more value to the possibility of terminating the contract, inefficient contractors have relatively little to gain from the exit option. This becomes more clear if we look more in detail at the entry trigger for a contractor of type x^l (the lowest possible efficiency level):

$$x_{\tau^R}(x^l) = x_{\tau^W} + \left[x_{\tau^W} - r(\beta_1 - \beta_2)O(x_{\tau^R}(x^l))\right] \frac{1}{g(x^l)x^l - 1} \tag{24}$$

Since $g(x^l)x^l > 1$, Eq. (24) is always positive. However, since $\beta_1 > 1$, if $g(x^l)x^l < \beta_1$, then $\frac{1}{g(x^l)x^l-1} > 1$, i.e., the entry trigger $x_{\tau^R}(x^l)$ can prove to be much higher than the first-best level x_{τ^W}. On the contrary, if $g(x^l)x^l > \beta_1$, then $\frac{1}{g(x^l)x^l-1} < 1$, i.e., the second term on the R.H.S. of (24) becomes negligible and $x_{\tau^R}(x^l) \simeq x_{\tau^W}$, meaning that the least and the most efficient type will invest more or less at the same time.

Notice that, since $\frac{\partial \beta_1}{\partial \sigma} < 0$, a higher level of uncertainty on operating profits makes less likely that $1 < g(x^l)x^l < \beta_1$. In other words, in the case of risky projects, the government will more likely be better off by lowering the termination fee. The simple intuition is that, since uncertainty increases the value of the exit option, a greater uncertainty could make also inefficient contractors prone to accept a significant reduction of capital subsidies in exchange for more flexibility in service duration.

4.3 Exit option and individual payoffs

We conclude by looking more closely at the effects of the termination fee upon the parties' expected payoffs.

As for the government, we have shown that the granting of an exit option can be an effective method for increasing the value-for-money of public spending. Specifically, the government can increase his payoff by keeping the termination fee below the level that would deter exit (i.e., $L < \frac{c}{r}$) and achieve the maximum when $L = L^R > Z$.

By contrast, the granting of an exit option reduces the contractor's expected payoff. This can be seen by totally differentiating the private value to invest (17) with respect to the amount of the termination fee:

$$\begin{aligned}\frac{\partial F(x)}{\partial L} = \frac{1}{r}\left(\frac{x}{x_{\tau^R}}\right)^{\beta_1-1}\left(\frac{1}{x_{\tau^R}}\right)\Bigg[(\beta_1-1)\left(1-\left(\frac{x_{\tau^R}}{x^E}\right)^{\beta_2-1}\right)\frac{\partial x_{\tau^R}}{\partial x^E}+ \\ -(\beta_2-1)\left(\frac{x_{\tau^R}}{x^E}\right)^{\beta_2-1}\left(\frac{\partial x_{\tau^R}}{\partial x^E}-1\right)\Bigg]\frac{\partial x^E}{\partial L} > 0\end{aligned} \tag{25}$$

This result can be interpreted as follows. The incentives used by the government to make the project privately attractive can be thought as a compound option since, on the one hand, the contractor is entitled to receive a fixed subsidy on investment (a call option) and, on the other, he is allowed to terminate the contract at a predetermined strike price (a put option). While efficient agents attach a higher value to the possibility of quitting the project, the reverse applies to less efficient agents, who attribute a greater value to the call option. This negative correlation thus reduces the variance of the project's value across different types of contractors and therefore allows the government to squeeze information rents and to get a higher payoff than just offering a single option.[13]

The following corollary comes directly from Proposition 3 together with (25).

Corollary 1 *Given the government's direct resumption cost Z, within the range $(Z, L^R]$ an increase of the termination fee results in an increase of both parties' expected payoffs.*

Basically, the corollary suggests that it would not be efficient, neither for the government nor for the contractor, to sign a contract providing for a termination fee falling below the direct public cost of resuming operations. The reason is that while low fees allow the government to accelerate investment and to save on capital expenditures, excessively low fees can prove inadequate to handle the prospective costs of insourcing a financially loss-making activity. On the other hand, while reducing the private burden of quitting the project, low fees can be accompanied by a reduction of public subsidies and, therefore, an increase in the net investment cost.

One question is whether the contract is prone to ex post adjustments of termination provisions. Although our aim is not to formally characterize a renegotiation-proof contract, it appears that owing to government's and the contractor's opposite interests there is no space for renegotiating the termination fee. In fact, while the contractor, after investing and benefitting from the public capital injection, would clearly prefer to pay a lower fee, possibly zero, the government would like to increase the resources available to offset the costs of taking over responsibility for the service.

5 Final remarks

PPPs for the provision of public infrastructures and services have gained increased interest over the past decades. While many PPPs have been success stories, others have largely failed to meet expectations. For instance, the frequency of early terminations and renegotiations has raised concern about the real benefits of PPPs over conventional procurement methods and has stimulated a debate on how to prevent or mitigate the effects of breach of contracts.

This paper contributes to this debate by arguing that the project's abandonment by the private party may not be an issue per se, as long as the risk of termination is properly accounted for at the time of contract formation. Specifically, our model shows that the granting of an exit option can enhance the value-for-money of public spending, by allowing the government to trade-off direct subsidies for capital investment against the possibility of opting out of the PPP.

[13] A similar result can be found in Board (2007a) and Dosi and Moretto (2015) where, however, rent extraction comes through the competitive bid pressure generated by a put option.

In practice, this means that, when funding capital expenditures, contracting authorities should assess the value of embedded exit options, which in turn depends on the termination fee included in the contract. As for the latter, we find that the strike price to be paid out if the option is exercised should be greater than the public cost of resuming operations, but not so high to prevent the contractor to leave an unprofitable service.

The blending of direct subsidies and option incentives in PPP contracts can prove particularly useful in the case of risky projects and asymmetric information on operating profits, when the exit option can soften agency problems and accelerate investment at a relatively lower public cost compared with lock-in contracts.

Unlike other risk-sharing provisions proposed in the academic literature on PPPs, the scheme presented in this paper, i.e. the combination of grant funding and a predetermined termination fee, does not require screening the private returns from the project. In fact, the mechanism envisaged here only requires the verification of project's implementation (rather than the circumstances that triggered the contractor's timeframe to launch the service) and, in the event of termination of service provision, the enforcement of the fee agreed upon when signing the contract.

A Proof of Proposition 1

Taking the derivative of (13) with respect to L yields:

$$\beta_2 \left(\frac{x}{x^E}\right)^{\beta_2 - 1} \left(-\frac{x}{(x^E)^2}\right) \frac{\partial x^E}{\partial L}(L - Z) + \left(\frac{x}{x^E}\right)^{\beta_2} = 0 \tag{A.1}$$

$$\left(\frac{x}{x^E}\right)^{\beta_2} \left[\beta_2 \frac{1}{(\frac{c}{r} - L)}(L - Z) + 1\right] = 0$$

$$\left(\frac{x}{x^E}\right)^{\beta_2} \left[\beta_2 \frac{r(L - Z)}{(c - rL)} + 1\right] = 0$$

form which we get:

$$L^W = \frac{c}{r} + \frac{\beta_2}{\beta_2 - 1}\left[Z - \frac{c}{r}\right] = Z + \frac{1}{\beta_2 - 1}\left[Z - \frac{c}{r}\right] \tag{A.2}$$

The second order condition is

$$\left(\frac{x}{x^E}\right)^{\beta_2} \left[\beta_2 \frac{r\left(c - rL^W\right) + r(L^W - Z)r}{(c - rL)^2}\right] = \tag{A.3}$$

$$\left(\frac{x}{x^E}\right)^{\beta_2} \left[\beta_2 \frac{r(c - rZ)}{(c - rL^W)^2}\right] < 0$$

which is always satisfied for $Z < \frac{c}{r}$.

Finally, by substituting (A.2) into (13) it is easy to prove that the government's expected payoff obtained when resuming the project is negative, i.e.:

$$\begin{aligned} R(x^E) &= \frac{x^E - c}{r} + (L^W - Z) \\ &= \left[(\frac{\beta_2}{\beta_2 - 1})^2(\frac{c}{r} - Z) - \frac{c}{r}\right] + \frac{1}{\beta_2 - 1}(Z - \frac{c}{r}) \\ &= \frac{\beta_2}{(\beta_2 - 1)^2}(\frac{c}{r} - Z) - Z < 0 \end{aligned} \tag{A.4}$$

and it remains negative even if $Z = 0$.

B Equations 16 and 17.

By Proposition 4 of Kruse and Strack (2015) and Proposition 7 of Kruse and Strack (Forthcoming), under the single crossing condition, for every transfer $\pi(\tau)$, the privately optimal investment strategy can be represented by a cut-off function $x_\tau(t)$ such that it is optimal to invest the first time that x_t hits $x_\tau(t)$. Since in our case the public benefits are time-autonomous then also the optimal threshold does not depend on time.

Let suppose that x_τ exists. Then, in the range where $x^E < x_t < x_\tau$, $F(x_t)$ satisfies the first order condition:

$$\left(\frac{x_t}{x_\tau}\right)^{\beta_1}\left\{-\beta_1\left(\frac{1}{x_\tau}\right)\left[\frac{x_\tau - c}{r} - (I - O(x_\tau)) + \pi(\tau)\right] + \left[\frac{1}{r} + \frac{\partial O(x_\tau)}{\partial x_\tau}\right]\right\} = 0 \tag{B.1}$$

while the second order condition is always satisfied:

$$\left(\frac{x_t}{x_\tau}\right)^{\beta_1}\left\{\beta_1\left(\frac{1}{x_\tau}\right)\left[\left(\frac{1}{x_\tau}\right)\left(\frac{x_\tau - c}{r} - (I - O(x_\tau)) + \pi(\tau)\right) - \frac{1}{r} - \frac{\partial O(x_\tau)}{\partial x_\tau}\right] + \frac{\partial^2 O(x_\tau)}{\partial x_\tau^2}\right\} < 0 \tag{B.2}$$

Substituting (B.1) in (15) we obtain:

$$F(x_t) = \left(\frac{x_t}{x_\tau}\right)^{\beta_1} \frac{x_\tau}{\beta_1 r}\left[1 - \left(\frac{x_\tau}{x^E}\right)^{\beta_2 - 1}\right] \quad \text{for } x^E < x_t < x_\tau \tag{B.3}$$

Now, for any transfer $\pi(\tau)$, let define the contractor's continuation value as:

$$\begin{aligned} u(x_t) &= F(x_\tau) + \pi(\tau) - F(x_t) \\ &= \frac{x_\tau}{\beta_1 r}\left[1 - \left(\frac{x_\tau}{x^E}\right)^{\beta_2 - 1}\right] + \pi(\tau) - \left(\frac{x_t}{x_\tau}\right)^{\beta_1} \frac{x_\tau}{\beta_1 r}\left[1 - \left(\frac{x_\tau}{x^E}\right)^{\beta_2 - 1}\right] \end{aligned} \tag{B.4}$$

Eq. (B.4) indicates the contractor's willingness to pay for holding the option to delay the investment beyond the current value of cash flow x_t. By definition, the continuation value is positive or equal to zero, i.e. $u(x_t) - \pi(\tau) \geq 0$. If $u(x_t) - \pi(\tau) > 0$, the contractor will continue to hold the option alive, while if $u(x_t) - \pi(\tau) = 0$ there will be no gain to go forward. When the contractor decides to stop, it will receive the transfer $\pi(\tau)$. Further, since $\beta > 1$, $u(x_t)$ is decreasing in x_t; i.e., the marginal willingness to pay for holding the option decreases as the value of cash flows increases:[14]

$$\frac{\partial u(x_t)}{\partial x_t} = -\frac{1}{r}\left(\frac{x_t}{x_\tau}\right)^{\beta-1}\left[1-\left(\frac{x_\tau}{x^E}\right)^{\beta_2-1}\right] < 0 \qquad \text{for } x^E < x_t < x_\tau \tag{B.5}$$

Since, at every point in time, the contractor is in a position to decide whether to invest or postpone the decision, if it is optimal to stop with x_t then it is also optimal to stop for $x'_t > x_t$. Thus, as the marginal incentive to keep the option alive decreases as x_t increases there exist, for any transfer $\pi(\tau)$, a level of cash flow where is optimal to stop, i.e. the contractor is indifferent between stopping and continuing when reaching x_τ.

C Proof of Lemma 1

We prove the Lemma in two steps by adapting the procedure proposed by Kruse and Strack (2015, Forthcoming) to which we refer for a rigorous derivation.

First step

For any time-autonomous stopping time τ (i.e. x_τ), the associated transfer $\pi(\tau)$ that implements it can be calculated as the expected discounted value of future cash flows the contractor would lose if the process x_t cannot exceed x_τ once that it is reached (Kruse and Strack 2015, Theorem 1; Kruse and Strack Forthcoming, Theorem 11).

For a given barrier a, a reflected process is as a process that has the same dynamics as the original process but is required to stay below a whenever the original process tends to exceed it. Defining $\tilde{x}_t$ the reflected process, it can be represented as (Harrison 2013):

$$\tilde{x}_t \equiv x_t/D_t, \quad \text{for } \tilde{x}_t \in (0, a], \tag{C.1}$$

where:

- $i)$ x_t is a geometric Brownian motion, with stochastic differential as in (1);
- $ii)$ D_t is an increasing and continuous process, with $D_0 = 1$ if $x_0 \leq a$, and $D_0 = x_0/a$ if $x_0 > a$, so that $\tilde{x}_0 = a$;
- $iii)$ D_t increases only when $\tilde{x}_t = a$.

[14] Note that Eq.(B.4) corresponds, in integral form, to the continuation value (9) in Kruse and Strack (2015), and the fact that Eq. (B.4) is decreasing in x_t follows from what they call "dynamic single crossing" condition. Equivalently, Arve and Zwart (2014) refer to Eq. (B.5) as the ex-post incentive compatible condition.

Applying Ito's lemma to (C.1), we get:

$$d\,\tilde{x}_t = \sigma \tilde{x}_t dz_t \; - d\tilde{D}_t, \quad \tilde{x}_t \in (0, a] \tag{C.2}$$

where $d\tilde{D}_t = \tilde{x}_t \frac{dD_t}{D_t}$ indicates the infinitesimally small level of "regulation" exerted to let x_t stay at a. By (C.2), until x_t hits for the first time a the two process coincide, i.e. $\tilde{x}_t = x_t$, and after that we get $\tilde{x}_t < x_t$. In particular, when $\tilde{x}_t = a$, we get $d\,\tilde{x}_t = 0$ and the rate of variation of D_t is equal to that of x_t to keep $\tilde{x}_t$ constant.

Now, defining the difference $\tilde{x}_t - x_t \equiv U_t = (D_t - 1)x_t$ as the cumulative amount of revenues lost up to t to keep the process below a, we are able to calculate the expected future values of revenues evaluated at the process reflected at a. In the specific, generalizing for any arbitrary initial value x_t, $t > 0$, we get:

$$v(\,x_t, a) = E_t \left\{ \int_t^{\infty} e^{-r(s-t)} [(\,\tilde{x}_s - c)ds + \varrho dU_s] \right\} \tag{C.3}$$

where $\varrho > 0$ is the marginal reflection cost (i.e., the value attributed to each unit of revenue) and dU_s is the reduction of revenues, if any, in the interval $(s, s + ds)$. For all $x_t < a$, the function $v(x_t; a)$ is the unique solution of the following partial differential equation:

$$\frac{1}{2}\sigma^2 x_t^2 \frac{\partial^2 v(x_t, a)}{\partial x_t^2} - rv(x_t, a) + \frac{x_t}{r} = 0 \tag{C.4}$$

with the boundary conditions:

$$\frac{\partial v(x_t, a)}{\partial x_t} \; |_{x_t = a} = \varrho \tag{C.5}$$

The general solution of (C.4) is (Harrison 2013:115):

$$v(x_t, a) = \frac{x_t - c}{r} + A x_t^{\beta_1} \tag{C.6}$$

where A is a constant to be determined. Imposing the boundary condition (C.5), it is easy to show that:

$$A = \frac{a}{\beta_1 r} \left[r\varrho - 1 \right] a^{-\beta_1} \tag{C.7}$$

which is negative if $\varrho < 1/r$. By (C.6) and (C.7) the expected discounted value of lost revenues is given by:

$$A x_t^{\beta_1} = E_t \left\{ \int_t^{\infty} e^{-r(s-t)} [\varrho dU_s] \right\} = \left(\frac{x_t}{a} \right)^{\beta_1} \frac{a}{\beta_1} \left[\varrho - \frac{1}{r} \right] \tag{C.8}$$

Setting $a = x_\tau$ and the reflection cost equals to the marginal value of the exit option evaluated at $a = x_\tau$, i.e. $\varrho = -\frac{\partial O}{\partial x}(x_\tau) = \frac{1}{r}\left(\frac{x_\tau}{x^E}\right)^{\beta_2 - 1}$, we get:

$$v(x_t, x_\tau) = \frac{x_t - c}{r} - \left(\frac{x_t}{x_\tau}\right)^{\beta_1} \frac{x_\tau}{\beta_1 r}\left[1 - \left(\frac{x_\tau}{x^E}\right)^{\beta_2 - 1}\right] \tag{C.9}$$

where we express ϱ in term of present value, i.e. the reflection cost has the dimension of the present value of the marginal cost of one unit of revenue lost forever. Notice that the second term on the R.H.S. of (C.9) is indeed the value to invest (B.3).

Second step

Finally, direct inspection of (B.4) and (C.9) shows that, provided that $v(x_t, x_\tau) < I - O(x_t)$, a transfer $\pi(t, \tau) = (I - O(x_t) - v(x_t, x_\tau)$ compensates the contractor at each time t for the loss of value due to the reflecting barrier, i.e. $u(x_t, x_\tau) - \pi(t, \tau) > 0$. When $x_t = x_\tau$, we get $u(x_\tau, x_\tau) - \pi(\tau) = 0$ and the contractor is indifferent whether to invest or postpone the decision. Thus, the transfer is:

$$\begin{aligned}\pi(\tau) &= (I - O(x_\tau)) - v(x_\tau, x_\tau) \\ &= I + \frac{c}{r} - \frac{\beta_1 - 1}{\beta_1}\frac{x_\tau}{r} - \frac{\beta_1 - \beta_2}{\beta_1} O(x_\tau)\end{aligned} \tag{C.10}$$

D Proof of Proposition 2

For every initial value x there exists a non-increasing function $\tau(x) = \inf(t > 0 \ / \ x_t = x_\tau(x))$ that maximizes (17). Further, the allocation mechanism $\tau(x)$ can be implemented by using simple transfers that only depend on the initial value x and the realized stopping time. Specifically, along with the condition (B.5), with the definition that $u(x) = u(x, x_\tau(x))$, an incentive compatible contract requires the monotonicity of the optimal trigger, i.e.:

$$\frac{dx_\tau(x)}{dx} < 0 \tag{D.1}$$

Conditions (B.5) and (D.1) are the first and the second order incentive compatibility constraints to induce the contractor to reveal its private information x. The standard approach is to ignore, for the moment, the monotonicity constraint (D.1) and solve the relaxed problem. For any choice of $x_\tau = x_\tau(\hat{x})$, applying the envelope theorem where the contractor maximizes over both the report $\hat{x}$ and the stopping time $x_\tau(\hat{x})$, we get the ex-ante value of the contractor's option to invest as:

$$
\begin{aligned}
u(x) - u(x_l) &= -\int_{x_l}^{x} \frac{1}{r}\left(\frac{y}{x_\tau}\right)^{\beta_1 - 1}\left[1 - \left(\frac{x_\tau}{x^E}\right)^{\beta_2 - 1}\right] dy \\
F(x) &= F(x_l, x_\tau(x_l)) + \int_{x_l}^{x} \frac{1}{r}\left(\frac{y}{x_\tau}\right)^{\beta_1 - 1}\left[1 - \left(\frac{x_\tau}{x^E}\right)^{\beta_2 - 1}\right] dy
\end{aligned}
\tag{D.2}
$$

where by the Revelation Principle, the optimal choice of $\hat{x}$ is x.[15] Yet, as $F(x)$ is increasing in x, it is optimal for the government to set the transfer such that the value of the lowest type is zero, i.e. $F(x_l, x_\tau(x_l)) = 0$. Substituting (D.2) in (19), the government's objective function becomes:

$$
\mathcal{R}(x, x_\tau) = \int_{x_l}^{x_h}\left[W(x, x_\tau) - \frac{1}{r}\int_{x_l}^{x}\left(\frac{y}{x_\tau}\right)^{\beta_1 - 1}\left(1 - \left(\frac{x_\tau}{x^E}\right)^{\beta_2 - 1}\right) dy + E_0(e^{-rT}(L - Z))\right] g(x)dx
\tag{D.3}
$$

Integrating by parts the second term on r.h.s. of (D.3), yields:

$$
\begin{aligned}
&\int_{x_l}^{x_h}\int_{x_l}^{x}\left(\frac{y}{x_\tau}\right)^{\beta_1 - 1}\left(1 - \left(\frac{x_\tau}{x^E}\right)^{\beta_2 - 1}\right) dy g(x)dx \\
&= \int_{x_l}^{x_h}\left(\frac{x}{x_\tau}\right)^{\beta_1 - 1}\left(1 - \left(\frac{x_\tau}{x^E}\right)^{\beta_2 - 1}\right)(1 - G(x))dx
\end{aligned}
\tag{D.4}
$$

Substituting (D.4) in (D.3), we reduce $\mathcal{R}(x, x_\tau)$ to:

$$
\begin{aligned}
\mathcal{R}(x, x_\tau) = &\int_{x_l}^{x_h}\left(\frac{x}{x_\tau}\right)^{\beta_1}\left\{\frac{x_\tau}{r}\left[1 - \left(1 - \left(\frac{x_\tau}{x^E}\right)^{\beta_2 - 1}\right)\phi(x)\right] - \left(I + \frac{c}{r} - \frac{b}{r}\right)\right\} g(x)dx \\
&+ \int_{x_l}^{x_h} E_0(e^{-rT})(L - Z)g(x)dx
\end{aligned}
\tag{D.5}
$$

where $\phi(x) = \frac{1-G(x)}{g(x)x}$, with $\phi'(x) < 0$, $\phi(x^h) = 0$ and $\phi(x^l) = \frac{1}{g(x^l)x^l} < 1$. By maximizing (C.5) with respect x_τ the first order condition is:

$$
x_{\tau W} - x_\tau(1 - \phi(x)) - \frac{(\beta_1 - \beta_2)}{(\beta_1 - 1)}\left(\frac{x_\tau}{x^E}\right)^{\beta_2 - 1} x_\tau \phi(x) = 0
\tag{D.6}
$$

from which we obtain the expression in the text with $x_\tau(x^h) = x_{\tau W}$.

Equation (D.6) may admit two solutions. Let define $f(x_\tau, x_{\tau W}) \equiv x_{\tau W} - (1 - \phi(x))x_\tau -$

[15] Eq (D.2) follows from the application of the envelope theorem. For the integral form of the envelope theorem, see Milgrom (2004).

$\left(\frac{\beta_1-\beta_2}{\beta_1-1}\right)\left(\frac{x_\tau}{x^E}\right)^{\beta_2} x^E\phi(x)$. It is easy to show that $f(x_\tau, x_{\tau^W})$ is concave in x_τ, i.e. :

$$f'(x_\tau, x_{\tau^W}) = -(1-\phi(x)) + (\beta_1-\beta_2)\left(\frac{x_\tau}{x^E}\right)^{\beta_2-1}\phi(x)$$

and

$$f''(x_\tau, x_{\tau^W}) = (\beta_1-\beta_2)(\beta_2-1)\left(\frac{x_\tau}{x^E}\right)^{\beta_2-2}\frac{1}{x^E}\phi(x) < 0$$

Moreover, since $\lim_{x_\tau\to\infty} f(x_\tau, x_{\tau^W}) = -\infty$ and $\lim_{x_\tau\to 0} f(x_\tau, x_{\tau^W}) = -\infty$, the maximum is given by:

$$(\beta_1-\beta_2)\left(\frac{x_\tau^{\max}}{x^E}\right)^{\beta_2-1}\frac{\phi(x)}{1-\phi(x)} = 1$$

Substituting $x_\tau^{\max}$ into $f(x_\tau, x_{\tau^W})$ we obtain:

$$f(x_\tau^{\max}, x_{\tau^W}) = x_{\tau^W} - (\beta_1-\beta_2)\left(\frac{x_\tau^{\max}}{x^E}\right)^{\beta_2} x^E\phi(x)\left(\frac{\beta_1}{\beta_1-1}\right)$$

Thus, if $f(x_\tau^{\max}, x_{\tau^W}) > 0$, the first order condition (D.6) admits two solutions and the optimal one satisfies $x_\tau > x_\tau^{\max}$.

Let now prove the monotonicity. Totally differentiating (D.6), we obtain:

$$\frac{dx_\tau}{dx}\left[(1-\phi(x)) - (\beta_1-\beta_2)\left(\frac{x_\tau}{x^E}\right)^{\beta_2-1}\phi(x)\right] = x_\tau\phi'(x)\left[1-\frac{(\beta_1-\beta_2)}{(\beta_1-1)}\left(\frac{x_\tau}{x^E}\right)^{\beta_2-1}\right] \quad \text{(D.7)}$$

Defining $\Omega(x_\tau) \equiv \frac{(\beta_1-\beta_2)}{(\beta_1-1)}\left(\frac{x_\tau}{x^E}\right)^{\beta_2-1}$, it is easy to show that $\frac{dx_\tau}{dx} < 0$ if:

$$\Omega(x_\tau) < \frac{1}{\beta_1-1}\frac{1-\phi(x)}{\phi(x)} = \frac{1}{\beta_1-1}\frac{\left(x-\frac{1-G(x)}{g(x)}\right)}{\frac{1-G(x)}{g(x)}} \quad \text{(D.8.1)}$$

and

$$\Omega(x_\tau) < 1 \quad \text{(D.8.2)}$$

hold simultaneously, where (D.8.1) is the second order condition of the maximization, while (D.8.2) implies $x_\tau > x_{\tau^W}$, i.e.:

$$\begin{aligned} 1-\Omega(x_\tau) &= \left[1-\frac{(\beta_1-\beta_2)}{(\beta_1-1)}\left(\frac{x_\tau}{x^E}\right)^{\beta_2-1}\right] \\ &= \left[\frac{1}{\phi(x)} - \frac{1}{x_\tau\phi(x)}\frac{\beta_1}{(\beta_1-1)}r\left(I+\frac{c}{r}-\frac{b}{r}\right)\right] \\ &= \frac{1}{\phi(x)}\frac{1}{x_\tau}\left[x_\tau - x_{\tau^W}\right] \end{aligned}$$

As $\frac{1-\phi(x)}{\phi(x)}$ is increasing in x a sufficient condition for the second order condition (D.8.1) to hold is $\Omega(x_\tau) < \frac{g(x^l)x^l - 1}{\beta_1 - 1}$. Thus, we need to distinguish two cases:

1. If $g(x^l)x^l \in (1, \beta_1]$, then (D.8.2) is satisfied if (D.8.1) is;
2. If $g(x^l)x^l > \beta_1$, then (D.8.1) is satisfied if (D.8.2) is.

Let consider the first case where $g(x^l)x^l \in (1, \beta_1]$. This implies that $\frac{g(x^l)x^l-1}{\beta_1-1} < 1$ and, as $x_\tau(x^h) = x_{\tau W}$, we can reduce (D.8.1) to:

$$\left(\frac{x^E}{x_{\tau W}}\right) < K \frac{g(x^l)x^l - 1}{\beta_1 - 1}$$

where $K = \left(\frac{\beta_1-1}{\beta_1-\beta_2}\right)^{1/1-\beta_2} < 1$. For the second case we get $\frac{g(x^l)x^l-1}{\beta_1-1} > 1$, and condition (D.8.2) is satisfied by simply setting $\left(\frac{x^E}{x_{\tau W}}\right) < K$. Therefore, Assumptions 1 guarantees that both (D.8.1) and (D.8.2) are satisfied.

Suppose now that $\frac{x^E}{x_{\tau W}}$ is such that Assumption 1 is not satisfied. Then, for higher value of x, conditions (D.8.1) and (D.8.2) may not hold. In this case we have an interval $[x', x''] \subset [x^l, x^h]$ where a constant trigger (bunching) applies such that $x_\tau(x') = x_\tau(x'') = \bar{x}_\tau > x_\tau(x^h) = x_{\tau W}$. Hence from (D.7) we should have $\phi'(x') = \phi'(x'')$ and from (D.6):

$$-(1-\phi(x'))\bar{x}_\tau - \left(\frac{\beta_1-\beta_2}{\beta_1-1}\right)\left(\frac{\bar{x}_\tau}{x^E}\right)^{\beta_2} x^E \phi(x') = -(1-\phi(x''))\bar{x}_\tau - \left(\frac{\beta_1-\beta_2}{\beta_1-1}\right)\left(\frac{\bar{x}_\tau}{x^E}\right)^{\beta_2} x^E \phi(x'') \tag{D.9}$$

$$(\phi(x') - \phi(x''))\left[1 - \left(\frac{\beta_1-\beta_2}{\beta_1-1}\right)\left(\frac{\bar{x}_\tau}{x^E}\right)^{\beta_2-1}\right]\bar{x}_\tau = 0$$

$$(\phi(x') - \phi(x''))\left[1 - \Omega(\bar{x}_\tau)\right]\bar{x}_\tau = 0$$

This leads to a contradiction as $x' > x''$ and $1 - \Omega(\bar{x}_\tau) \neq 0$ except when $\bar{x}_\tau = x_{\tau W}$. Hence we cannot have $x'' < x^h$ and if bunching is optimal it occurs at the top of the interval. However since there is no distortion at the top, the optimal solution is $x_\tau(x^h) = x_{\tau W}$ for an interval $[\hat{x}, x^h] \subset [x^l, x^h]$ for some $\hat{x}$.

The transfer is given by Lemma 1. By substituting (D.6) into (C.10):

$$\begin{aligned}\pi(\tau) &= I + \frac{c}{r} - \frac{\beta_1-1}{\beta_1}\frac{x_\tau}{r} - \frac{\beta_1-\beta_2}{\beta_1} O(x_\tau, x^E) \\ &= \frac{b}{r} - \frac{\beta_1-\beta_2}{\beta_1} O(x_\tau, x^E) - \frac{\beta_1-1}{\beta_1}\left[\frac{x_\tau^W}{r} - (\beta_1-\beta_2)\, O(x_\tau; x^E)\right]\frac{\phi(x)}{1-\phi(x)}\end{aligned} \tag{D.10}$$

with $\pi(\tau^h) = \frac{b}{r} - \frac{\beta_1-\beta_2}{\beta_1} O(x_{\tau^W}, x^E) = \pi^W$. By the monotonicity of x_τ, it is easy to show that:

$$\begin{aligned} \frac{\partial \pi(\tau)}{\partial x} &= -\frac{\beta_1 - 1}{\beta_1}\frac{1}{r}\frac{\partial x_\tau}{\partial x} + \frac{\beta_1-\beta_2}{\beta_1}\frac{1}{r}\left(\frac{x_\tau}{x^E}\right)^{\beta_2-1}\frac{\partial x_\tau}{\partial x} \\ &= \frac{1}{\beta_1 r}\frac{\partial x_\tau}{\partial x}\frac{1}{(\beta_1-1)}\left(-1 + \frac{(\beta_1-\beta_2)}{(\beta_1-1)}\left(\frac{x_\tau}{x^E}\right)^{\beta_2-1}\right) > 0 \end{aligned} \tag{D.11}$$

Finally, taking the derivative of (D.6) with respect to x^E we get:

$$\frac{\partial x_\tau(x)}{\partial x^E} = -\frac{(\beta_2-1)\frac{(\beta_1-\beta_2)}{(\beta_1-1)}\left(\frac{x_\tau}{x^E}\right)^{\beta_2}\phi(x)}{SOC} < 0 \tag{D.12}$$

where the SOC is satisfied by Assumptions 1, and the derivative of (D.10) gives:

$$\frac{\partial \pi(\tau)}{\partial x^E} = \frac{(\beta_1-\beta_2)}{\beta_1}\frac{\partial O(x_\tau, x^E)}{\partial x^E}\left[\frac{\phi(x)}{1-\phi(x)}(\beta_1-1) - 1\right] \tag{D.13}$$

which is $\frac{\partial \pi(x)}{\partial x^E} < 0$ for all x if $g(x^l)x^l > \beta_1$, while it is positive for low value of x and negative for high value of x if $1 < g(x^l)x^l < \beta_1$.

E Proof of Proposition 3

Substituting (D.6) into (D.5) we obtain:

$$\begin{aligned} \mathcal{R}(x, x_\tau) &= \int_{x_l}^{x_h}\left(\frac{x}{x_\tau}\right)^{\beta_1}\left\{\frac{x_\tau}{r}\left[1 - \left(1 - \left(\frac{x_\tau}{x^E}\right)^{\beta_2-1}\right)\phi(x)\right] - \left(I + \frac{c}{r} - \frac{b}{r}\right)\right\} g(x)dx + \\ &+ \int_{x_l}^{x_h} E_0(e^{-rT})(L-Z)g(x)dx \\ &= \int_{x_l}^{x_h}\left\{W(x,x_\tau) - \frac{1-G(x)}{g(x)}\frac{1}{\beta_1}F_x(x,x_\tau)\right\} g(x)dx + \int_{x_l}^{x_h} E_0(e^{-rT})(L-Z)g(x)dx \end{aligned} \tag{E.1}$$

where $W(x, x_\tau) - \frac{1-G(x)}{g(x)}\frac{1}{\beta_1}F_x(x, x_\tau)$ indicates the "virtual" welfare. Further by substituting (D.10) we reduce (E.1) as:

$$\begin{aligned} &= \int_{x_l}^{x_h}\left(\frac{x}{x_\tau}\right)^{\beta_1}\left\{\frac{x_\tau}{\beta_1 r}\left[(1-\phi(x)) + \beta_2\left(\frac{x_\tau}{x^E}\right)^{\beta_2-1}\phi(x)\right]\right\} g(x)dx + \int_{x_l}^{x_h} E_0(e^{-rT})(L-Z)g(x)dx \\ &= \int_{x_l}^{x_h}\left\{(1-\phi(x))F(x,x_\tau) - E_0(e^{-r\tau})\beta_2[\frac{1}{\beta_1} - \phi(x)]O(x_\tau, x^E) + E_0(e^{-rT})(L-Z)\right\} g(x)dx \end{aligned}$$

To better disentangle the effect of L on the government's revenue, we need to determine the sign of $\frac{\partial \mathcal{R}}{\partial L}$. By calling:

$$\Psi(x, x_\tau, x^E) = (1-\phi(x))F(x, x_\tau) - E_0(e^{-r\tau})\beta_2 \left[\frac{1}{\beta_1} - \phi(x)\right] O(x_\tau, x^E), \tag{E.2}$$

the sign of $\frac{\partial \mathcal{R}}{\partial L}$ is given:

$$\frac{\partial \mathcal{R}}{\partial L} = \int_{x_l}^{x_h} \left[\frac{\partial \Psi(x, x_\tau, x^E)}{\partial x_\tau}\frac{\partial x_\tau}{\partial L} + \frac{\partial \Psi(x, x_\tau, x^E)}{\partial L} + \frac{\partial E_0(e^{-rT})(L-Z)}{\partial L}\right] g(x)dx.$$

Since x_τ is the optimum, the first term is equal to zero, thus the derivative simplifies to:

$$\begin{aligned}
\frac{\partial \mathcal{R}}{\partial L} &= \int_{x_l}^{x_h} \left[-E_0(e^{-r\tau})\beta_2 \left[\frac{1}{\beta_1} - \phi(x)\right] \frac{\partial O(x_\tau, x^E)}{\partial x^E}\frac{\partial x_E}{\partial L} + E_0(e^{-rT})\left(1 + \beta_2 \frac{r(L-Z)}{(c-rL)}\right)\right] g(x)dx. \qquad \text{(E.3)}\\
&= \int_{x_l}^{x_h} \left[E_0(e^{-r\tau})\beta_2 \left[\frac{1}{\beta_1} - \phi(x)\right] \frac{1-\beta_2}{r\beta_2}\left(\frac{x_\tau}{x^E}\right)^{\beta_2} \frac{\beta_2}{\beta_2 - 1} r + E_0(e^{-rT})\left(1 + \beta_2 \frac{r(L-Z)}{(c-rL)}\right)\right] g(x)dx\\
&= \int_{x_l}^{x_h} \left[E_0(e^{-r\tau})\beta_2 \left[\frac{1}{\beta_1} - \phi(x)\right] \left(\frac{x_\tau x}{x^E x}\right)^{\beta_2} + E_0(e^{-rT})\left(1 + \beta_2 \frac{r(L-Z)}{(c-rL)}\right)\right] g(x)dx\\
&= \int_{x_l}^{x_h} \left[E_0(e^{-rT})\beta_2 \left[\frac{1}{\beta_1} - \phi(x)\right] \left(\frac{x}{x_\tau}\right)^{\beta_1}\left(\frac{x_\tau}{x}\right)^{\beta_2} + E_0(e^{-rT})\left(1 + \beta_2 \frac{r(L-Z)}{(c-rL)}\right)\right] g(x)dx\\
&= \int_{x_l}^{x_h} E_0(e^{-rT}) \left[\left(\frac{x}{x_\tau}\right)^{\beta_1 - \beta_2} \beta_2 \left[\frac{1}{\beta_1} - \phi(x)\right] + \left(1 + \beta_2 \frac{r(L-Z)}{(c-rL)}\right)\right] g(x)dx.
\end{aligned}$$

The sign of $\frac{\partial \mathcal{R}}{\partial L}$ is hard to determine. However, this expression is continuous in L with $\frac{\partial \mathcal{R}}{\partial L} \to 0$ as $L \to \frac{c}{r}$, and positive if $L = 0$. As $\mid \left(\frac{x}{x_\tau}\right)^{\beta_1-\beta_2}\left[\frac{1}{\beta_1} - \phi(x)\right] \mid < 1$ and $\phi'(x) < 0$ the sufficient condition to have $\frac{\partial \mathcal{R}}{\partial L} > 0$ is $\frac{1}{\beta_1} + \frac{1}{\beta_2} - \frac{rZ}{c} < 0$ which is always satisfied. Thus, there exists a value L^R in the compact region $[0, \frac{c}{r}]$ that maximizes (E.3).

We now claim that $L^R > Z$. To prove this let consider first the case where $\left(\frac{x}{x_\tau}\right)^{\beta_1-\beta_2}\left[\frac{1}{\beta_1} - \phi(x)\right] = 0$ for all x. In this case the exit-fee that makes $\frac{\partial \mathcal{R}}{\partial L} = 0$ is L^W as given in (14). Now let consider the opposite case where $\left(\frac{x}{x_\tau}\right)^{\beta_1-\beta_2}\left[\frac{1}{\beta_1} - \phi(x)\right] = \frac{1}{\beta_1} < 1$ for all x. Then, $\frac{\partial \mathcal{R}}{\partial L} = 0$ when $L' = Z + \frac{1}{\beta_1\beta_2 - 1}[Z - \frac{c}{r}]$. Where $L^W > L' > Z$. Now, if $\left[\frac{1}{\beta_1} - \phi(x)\right] > 0$ for all types, then L^{R1} lies in between (L', L^W), while if $\left[\frac{1}{\beta_1} - \phi(x)\right] < 0$ for at least some $x \in [x^l, x^h]$, L^{R2} is higher than L^{R1}, i.e.:

$$Z < L' < L^{R1} < L^{R2} < L^{W}$$

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