Luca Sandrini, Department of Economics and Management, University of Padova

INCENTIVES FOR LABOR-AUGMENTING INNOVATION: THE ROLE OF WAGE RATE

June 2019

Marco Fanno Working Papers - 232
Incentives for labor-augmenting innovation: The role of wage rate*

Luca Sandrini†

June 5, 2019

Abstract

This paper analyzes how the incentives to produce and to adopt labor-augmenting innovation are linked to the wage rate. I design a model of a vertically related industry, where downstream manufacturers can choose between a standard low-quality capital input or a superior one produced by an upstream innovator. High-quality capital input allows adopting firms to have a higher labor productivity. I show that there is an inverted-U shaped relationship between the wage rate and the incentive to invest in innovation based on two opposite forces: a positive cost-reducing effect and a negative output contraction effect. Finally, this paper provides some support for the introduction of a minimum wage, which is found able to increase both the investments in innovative activities and the social welfare.

JEL Code: J31, L13, O31

Key Words: labor-augmenting innovation, vertical relation, oligopoly, minimum wage

*This paper was presented at the Augustin Cournot Doctoral Days 2019, University of Strasbourg, in April 2019. I wish to thank my PhD supervisor Professor Fabio Manenti for his precious help, suggestions and comments during the development and the writing of this work. Also, special thanks go to my co-supervisor Professor Luciano Greco for his suggestions and availability. I am grateful to Dr. Levent Eraydin for his comments as discussant of the paper during the ACDD 2019. This research has been partially carried out during a visiting period of six months at the School of Business and Social Sciences at Aarhus University. I am grateful to Professor Johannes Jansen for his help during the early stage of this research. Finally, I wish to thank the participants of the XXXIII Jornadas de Economia Industrial and the XXX SIEP Conference for their useful comments on the earlier versions of this paper.

†Email address: luca.sandrini.3@phd.unipd.it. Dept. of Economics and Management, University of Padua (Italy).
1 Introduction

The relationship between technological progress and labor market conditions represents an issue that is increasingly attracting the attention of scholars and policy makers. Usually, firms adopt factor-augmenting technologies in order to replace the more expensive factors of production, making better use of cheaper and relatively more abundant ones (Dosi [1984], and Acemoglu [2002]). In particular, innovative technologies often improve the productivity of labor thus allowing adopting firms to substitute labor with capital inputs. In this paper, I argue that the degree of such substitution is influenced by the cost of labor as the higher the wage rate, the more a firm is willing to adopt such technologies. On the other hand, much more than in the past, when large and vertically integrated firms used to develop in-house the innovations they used in production, these innovative technologies are developed by upstream innovators who then sell them to downstream adopting firms. As a consequence, labor market conditions are likely to affect also the incentives to develop labor augmenting innovations via their effect on the incentive to adopt such technologies by downstream adopters. To the best of my knowledge, this paper is the first attempt to analyze the role of labor market conditions, namely the wage level, in shaping the incentives not only to adopt labor productivity enhancing technologies, but also to produce such technologies. I do so by following an industrial organization approach based on a model of a vertically related industrial sector where the innovation is produced upstream and then sold downstream to manufacturers.

Examples of labor productivity augmenting technologies abound. One may think to ICTs; developed by large upstream innovators, they allow adopting firms to significantly improve the productivity of labor. Another example are robots and cobots (i.e., collaborative robots). In particular, cobots are industrial robots designed to collaborate with humans in the production process. Differently from traditional industrial robots, cobots are flexible units that help workers perform their routinized activities. Due to their flexibility, cobots are increasingly adopted by firms where industrial robots have never been adopted before. In particular, one of the most interesting sector is logistics, where many firms have introduced cobots in order to increase the productivity of labor in piece picking tasks.

This is by no means the first paper that attempts to clarify how innovation and vertical relations affect the market performance. However, the novelty of this paper

---

1 Cabral and Vasconcelos [2011], among others, analyze the effect of matching contracts on the provision of strategic input in markets with partial vertical integration. Although this paper does not deal with the issue of vertical integration, it is built on the authors' assumption of the presence of an
is to investigate how incentives to adopt innovation may be at odds with market power for firms competing in the market, and how upstream innovators exploit the competition (that they themselves promote) between manufacturers downstream. Several studies attempt to show how innovation works, and how it affects the production process, but the vast majority adopt a macro approach (Acemoglu and Restrepo [2017], and Aghion et al. [2017], among others). Instead, in this paper I follow an industrial organization approach in order to further highlight the nexus between innovation and firms’ strategic interaction and market condition. This paper also represents a first attempt to bridge two usually disconnected literatures: the macroeconomic literature and the industrial organization one, with the aim of looking at innovation incentives both from the labor market perspective and from that of firms. In particular, I adopt a production function that is directly inspired by the literature on automation (Acemoglu and Autor[2011], Acemoglu and Restrepo[2019]), where the quality of capital input determines the amount of labor that is necessary for the production of the final good.

I model innovation in a vertically related market, in a way similar to Farrell and Shapiro [2008], where an upstream firm produces an innovative/superior technology and sells it to a series of downstream firms. However, differently from Farrell and Shapiro [2008] who focus on the effects of weak patents protecting the innovation, I analyze how labor costs affect firms’ decision to adopt the innovative technology and, more in particular, how these costs affect the innovator’s choice about how much to invest in innovation. More specifically, I show how the wage level of the downstream segment of the industry may affect positively the amount of investment of the upstream innovator and the downstream firms’ willingness to adopt the innovation.

This paper also extends the Acemoglu and Pischke’s [1999] findings on firms’ incentives to invest in labor-augmenting activities in an imperfect labor market. In particular, I apply the authors’ results to an oligopolistic framework in order to discuss how the incentives for process innovation are affected by competition. The existence of a relationship between innovation and the degree of competition has been extensively investigated in literature (Aghion et al. [2005], Vives [2008], among others).

\footnote{alternative source of the input (and an alternative competitive price), that may lead the downstream firms to refuse the provision of the input from the incumbent supplier.}

\footnote{From an industrial organization perspective, increasing in labor productivity is hardly distinguishable from automation in tasks. In fact, in both cases firms are ending up with a more capital intensive production function.}

\footnote{Moreover, I show that innovation’s profitability is not the only element that plays a role in affecting the incentive to innovate and to adopt innovation. Indeed, the conditions of the labor market represent a direct condition for the innovator to prompt or shrink the investments in innovation.}

3
However, this paper explicitly distinguishes the monopolistic advantage to produce innovation (Schumpeter [1947]) from the competitive incentive to adopt innovation (Arrow [1962]).

From the policy perspective, this paper suggests that the imposition of a minimum wage may represent an important policy in order to improve the level of investments in innovative activities, and to increase the social welfare. Interestingly, I show that under particular market conditions, industrial performance improves along with the wage level\textsuperscript{4}. Rather oddly, relatively few papers have addressed the problem of how an increase in a sector's minimum wage may affect private firms' incentive to innovate\textsuperscript{5}. Finally, this paper suggests that, in order to promote a social efficient level of innovation, the policy maker should impose a switch-off of the old technology. In fact, this would allow the innovator to entirely extract downstream rent and invest efficiently.

The rest of the paper is organized as follows: Section 2 describes the model and the timing of the game; Section 3 outlines the main results of the analysis, focusing first on the nature of the innovation and the conditions enabling its production and its adoption, then on the equilibrium outcomes of the game; a welfare analysis and different extensions to the main model are developed in Section 4 and 5, respectively; finally, Section 6 concludes.

2 The model

Assume there is an industry made of an upstream and a downstream segment. The downstream segment is populated by \( n \) identical firms (manufacturers, henceforth) which compete in the production and distribution of the final good \( q \). In order to produce, downstream firms employ labor and capital in fixed proportions, with the proportions depending on the quality of the capital input. Just to give a relevant example, the capital input can be seen as a personal computer, or, alternatively, as an industrial collaborative robot. Technological progress improves the efficiency of

\textsuperscript{4}The role of the wage on the incentives to innovate has been the subject of a vast literature, particularly focusing on the rent sharing mechanism and the unionization structure (Grout [1984], Calabuig and Gonzalez-Maestre [2003], Hauk and Wey [2004], Manasakis and Petrakis [2009], and Mukherjee and Pennings [2011]); none of these paper, however, analyzes the problem using a vertically related perspective. Moreover, these papers focus mainly on the effects of the degree of centralization of the wage setting mechanism and not on the strategic interaction among firms. Instead, this paper analyzes how the upstream innovator and the downstream firms behave and how this behavior is influenced by the wage level.

\textsuperscript{5}See, Lordan and Neumark [2018], Petrakis and Vlassis [2004], and Kleinknecht et al. [1998]
PC/cobots, namely it increases the quality of the capital input, and this allows the manufacturers to reduce the amount of labor necessary to produce a unit of the final good.

Two types of capital inputs are available to downstream firms: the current capital input (the *standard*, henceforth) and an innovative one. The former is produced competitively; it is of low quality and it is produced in accordance with the current state of the art of the technology. The latter is produced by an upstream monopolist (the *supplier*, henceforth) who owns a patent protecting the technology it uses to produce the high quality input. I normalize to zero the quality of the current capital input, while the high quality input produced by the supplier has quality $\alpha > 0$.

Formally, the technology adopted to produce the final good is:

$$q_i = \min\left\{ \frac{L}{1 - \alpha_i}; K \right\}$$

where $L$ and $K$ indicate the labor and the capital input, respectively. The parameter $\alpha_i = \{0; \alpha\}$ represents the type of the capital input chosen by firm $i$; a higher quality increases the productivity of workers and allows the adopting manufacturers to reduce the original amount of labor required to produce one unit of the final good.

The production function of both capital inputs requires one unit of labor for each unit of capital input produced, $K = L$. Competition drives the price of the standard capital input down to its marginal cost of production, that is the minimum wage paid to upstream workers, $w$. As the price of the innovative capital input is concerned, we assume that the supplier follows a two-part tariff scheme made of a per-unit price and a fixed fee. Following standard theory of optimal non linear pricing, the supplier sets the per-unit price at its marginal cost of production, that is the minimum wage $w$, and then sets the fixed fee $F$ to extract the downstream value it generates.

Downstream firms observe the prices of the two types of capital inputs and decide which one to buy. Clearly, the upstream supplier finds optimal to set the fixed fee $F$ at the highest possible level, which is the one that makes downstream firms indifferent between the standard or the new capital input. We assume that, in case of indifference, downstream firms buy the high quality input. Once taken their decision about which capital to use, downstream firms hire the desired amount of workers at the minimum wage $w$ - uniform and common to all the manufacturers regardless of the technology adopted - and compete. I assume that competition occurs à la Cournot and that market demand is linear $P(Q) = \delta - Q$, where $\delta$ measures the market size, and $Q = \sum_{i=1}^{n} q_i$ is the total output of the downstream segment, with $i = 1, 2, 3, .., n$. 

5
As mentioned, the supplier monopolistically produces a high quality capital input. The actual level of quality is chosen by the supplier, which invests an amount \( I(\alpha) = \gamma \alpha^2 \) of resources to produce a capital input of quality \( \alpha \), where \( \gamma \) is a cost parameter\(^6\).

Putting all these things together, we can now define firms’ profits functions, starting from downstream firms. In theory, there are two types of manufacturers: those who adopt the new high quality capital input, and those who, instead, keep using the standard technology. Let us indicate with \( m \) the number of firms that adopt the innovation, with \( m \leq n \). Firms’ profit functions are:

\[
\Pi_{d,i} = q_i (\delta - q_i - Q_{-i}) - q_i((2 - \alpha_i)w) - F(\alpha),
\]

with \( F(0) = 0 \). As we discuss below, the optimal fixed fee set by the upstream innovator depends on the quality of the supplied technology, hence we indicate \( F(\alpha) \).

The subscript \( d \) indicates the downstream segment of the industry.

As discussed above, the upstream supplier charges a two-part tariff, where the per unit price is set at the cost \( w \). Hence its only source of revenues comes from the fixed fee paid by the \( m \leq n \) manufacturers which adopt its high quality capital input. The objective function of the innovator therefore is given by:

\[
\Pi_u = mF - \gamma \alpha^2
\]

Where the subscript \( u \) indicates the upstream segment of the industry.

The timing of the game is as follows. At time 1, the supplier invests in R&D and decides the optimal level of \( \alpha \). At time 2, the innovator sets the price of the high quality capital input. At time 3, the manufacturers observe the prices of the two types of capital input and decide which one to pick. Then, at time 4, manufacturers hire the necessary amount of labor at price \( w \) and compete (Cournot competition).

As I am interested in the Subgame Perfect Nash Equilibrium, the game is solved by backward induction.

\(^6\)Through the paper I assume \( \gamma > \frac{n^2w^2}{(n+1)^2} \equiv \bar{\gamma} \). This assumption is sufficient for the innovation to be non drastic.
3 Results

3.1 Optimal level of quality of the new technology

Manufacturers choose their output to maximize profits. There are two types of downstream manufacturers: those who adopt the new technology (they are $m \leq n$), and those who keep producing with the standard technology (they are $m - n$). In order to simplify the problem, we impose symmetry among the firms in the same subset. So, the total output can be written as:

$$Q = m q^h + (n - m) q^s$$

where $h$ and $h$ stand for high quality and standard quality, respectively.

From the maximization of expression (1), the optimal output levels for the two types of firms are:

$$q^h = \frac{\delta - w(2 - \alpha(n - m + 1))}{n + 1}$$

$$q^s = \frac{\delta - w(2 + m \alpha)}{n + 1}$$

where $q^h$ is the output level of firms adopting the high quality capital input, and $q^s$ that of firms adopting the standard one. Using these two expressions the profits of the two types of manufacturers are:

$$\Pi^h = \frac{(\delta - w(2 - \alpha(n - m + 1)))^2}{(n + 1)^2} - F(\alpha)$$

$$\Pi^s = \frac{(\delta - w(2 + m \alpha))^2}{(n + 1)^2}$$

As mentioned earlier, the innovator can influence the manufacturers’ choice by varying the price of the high quality capital input, and in particular the fixed fee $F(\alpha)$. The higher the rent extracted, the weaker the appeal of the high quality input and, conversely, the lower the fixed fee the more downstream manufacturers will be induced to adopt the high quality input. Therefore, the optimal fixed fee set by the supplier must depend on the chosen quality level, for reasons that are straightforward. First, if $\alpha$ increases, the investment $I(\alpha)$ increases too, and the innovator must raise the price in order to recover the initial investment. Second, the higher the quality

\[7\] See the appendix for the formal details.
of the capital input the smaller the amount of labor that the manufacturers need to produce the final good. Employing less labor makes marginal costs to fall and profits to rise. This positive effect is stronger, the higher the level of quality $\alpha$ produced by the innovator.

Let us analyze the maximum fixed fee the innovator can set in order to elicit the adoption of the new technology for $m$ firms. Building on Bester and Petrakis (1993), $F$ is such that, for the $m^{th}$ manufacturer it must be indifferent whether to keep producing with the standard technology or to adopt the new one. Thus, the $m^{th}$ manufacturer would adopt the new technology if and only if the following conditions are satisfied:

$$\Pi_{d,m}^h \geq \Pi_{d,n-(m-1)}^s$$ and $$\Pi_{d,n-m}^s > \Pi_{d,m+1}^h.$$

Using the definition of firms’ profits, these two inequalities boil down to:

$$F \leq \frac{(\delta - w(2 - \alpha(n - m + 1)))^2}{(n + 1)^2} - \frac{(\delta - w(2 + (m - 1)\alpha))^2}{(n + 1)^2}$$

$$F > \frac{(\delta - w(2 - \alpha(n + 1 - (m + 1))))^2}{(n + 1)^2} - \frac{(\delta - w(2 + m\alpha))^2}{(n + 1)^2}$$

(7a)

(7b)

If condition (7a) applies, then none of the $m$ adopters has incentives to deviate and to produce with the standard technology, as they would not be able to gain any extra profits on top of what they gain with the new one. Also, when condition (7b) applies, none of the firms producing with the standard technology wants to adopt the new one, as it would guarantee less profits than the standard one. One can see that condition (7a) is necessary and sufficient for the equilibrium to be stable. Therefore, the fixed fee that induces $m$ firms to adopt the new technology is:

$$F(\alpha) = \frac{\alpha n w(2(\delta - 2w) + \alpha w(n - 2(m - 1)))}{(n + 1)^2}$$

(8)

By setting the fixed fee, the upstream firm decides how many firms $m$ adopt the innovation, given a certain quality $\alpha$. Formally, the optimal number of downstream firms maximizes the total revenues of the upstream supplier, $TR_u = m F(\alpha)$. The following corollary states under which conditions all downstream firms adopt the innovation:

**Corollary 1.** When the number of downstream firms is not too large, the innovator

---

*See the appendix for the mathematical proof.*
sets the adoption fee so that all the firms buy the new technology. Formally, the optimal number of contracts signed by the innovator is \( n \) if:

\[
n < \frac{2(\delta - w(2 - \alpha))}{3w}
\]

**Proof.** See the appendix.

Going backward, we are now in the position to solve for the optimal investment by the upstream firm. For the sake of simplicity, we solve the model assuming that the condition in Corollary 1 holds and \( m = n \). Therefore, the first stage innovator’s profits given in expression (2) can be written as:

\[
\Pi_u = n F(\alpha) - \gamma \alpha^2 \\
= \frac{\alpha n^2 w(2(\delta - 2w) - \alpha(n - 2)w)}{(n + 1)^2} - \gamma \alpha^2. \tag{9}
\]

The innovators chooses \( \alpha \) to maximize this expression. Simple differentiation reveals that the upstream supplier chooses \( \alpha = \alpha^* \), where:

\[
\alpha^* = \frac{n^2 w(\delta - 2w)}{\gamma(n + 1)^2 + (n - 2)n^2 w^2}.
\]

It is useful to rewrite \( \alpha^* \) as:

\[
\alpha^* = \frac{n(\delta - 2w)}{n + 1} \frac{n(n + 1)w}{\gamma(n + 1)^2 + (n - 2)n^2 w^2} = f(w) \bar{Q}_d(w), \tag{10}
\]

where \( \bar{Q}_d(w) = n\bar{q}_d(w) = n(\delta - 2w)/(n + 1) \) is the total output sold downstream when all the manufacturers keep producing with the standard technology. I refer to \( \bar{Q}_d \) and \( \bar{q}_d \) as “the benchmark”. From expression (10) the following proposition follows immediately:

**Proposition 1.** The optimal level of investment undertaken by the upstream innovator depends on the wage rate; \( \alpha^*(w) \) increases with \( w \) when \( \delta > \bar{\delta} \), and decreases otherwise, where \( \bar{\delta} = \frac{4\gamma(n + 1)^2 w^3}{\gamma(n + 1)^2 - (n - 2)n^2 w^2} \). Moreover, \( \alpha^*(0) = 0 \).

**Proof.** See the appendix.

This proposition represents one of the crucial results of the paper; see Figure 1 for a visual representation. Formally, it follows from simple differentiation of eq. (10):

\[
\frac{\partial \alpha^*}{\partial w} = f'_w \bar{Q}_d + f(w) \bar{Q}'_d.
\]
The effect of $w$ on the level of investment undertaken by the innovator can be easily interpreted by looking at $\alpha^*(w)$ as the interaction of two terms: the cost reducing effect of an increase in $w$, $f'_w$, which is always positive, and the output contraction effect $Q'_w$, which is always negative. More precisely, when the wage rate increases, manufacturers react by substituting labor with high quality capital input. Thus, the higher the wage level in the downstream sector, the more downstream manufacturers are induced to adopt the innovative capital input and to reduce their consumption of labour; this clearly stimulates the innovator to invest in R&D and to increase $\alpha$. On the other hand, an increase in the wage rate raises the manufacturers’ marginal costs of production, thus inducing the total output to shrink. This is what I refer to as the output contraction effect. If the total output shrinks, manufacturers employ fewer workers and the effect of the innovative input is smaller. When the market is large enough, the output contraction effect is dominated by the cost reducing effect and the amount of innovation/level of quality offered by the innovator increases with $w$.

One can see that, when $\gamma < (n-2)n^2w^2/(n+1)^2 \approx \bar{\delta}$, the positive effects always dominate the negative output contraction effect - i.e., the threshold $\bar{\delta}$ is negative. This implies that when the costs associated with the innovative activity are small enough, the cost reducing effect prompted by an increase in the wage rate is so high that it always dominates the output contraction effect. Although this case is interesting per se, the relevant scenarios occur when the dominance of one of the two effects is not so obvious. Thus, the rest of the paper focuses on the case where $\gamma > \bar{\delta}$. 

Figure 1: the optimal quality, $\alpha^*$
3.2 Price of capital input and innovator’s profits

Once determined $\alpha^*$, we can evaluate the equilibrium outcomes. Let us start from the adoption fee and the upstream innovator’s profits. By substituting expression (10) in expressions (8) and (9), we obtain:

$$F^* = \frac{n^3w^2(\delta - 2w)^2}{(n+1)^2} \frac{2\gamma(n+1)^2 + (n-2)n^2w^2}{\gamma(n+1)^2 + (n-2)n^2w^2}$$

and

$$\Pi_u^* = n F^* - \gamma \alpha^* = \frac{n^4w^2(\delta - 2w)^2}{(n+1)^2 (\gamma(n+1)^2 + (n-2)n^2w^2)}.$$

It is useful to rewrite $\Pi_u^*$ as:

$$\Pi_u^* = \frac{n w}{n+1} \frac{n^2w(\delta - 2w)}{\gamma(n+1)^2 + (n-2)n^2w^2} \frac{n(\delta - 2w)}{n+1} = g(w) \alpha^*(w) \tilde{Q}_d \quad (11)$$

From expression (11) it is immediate to obtain the following proposition:

**Proposition 2.** The innovator’s profits depend on the wage rate: $\Pi_u^*(w)$ increases with $w$ when $\delta > \tilde{\delta}$ and decreases otherwise, where $\tilde{\delta} = \frac{2w(2\gamma(n+1)^2 + (n-2)n^2w^2)}{\gamma(n+1)^2} < \delta$. Moreover, $\Pi_u^*(0) = 0$.

**Proof.** See the appendix.
Proposition 2 follows from simple differentiation of expression (11):

\[
\frac{\partial \Pi^*_u}{\partial w} = g'_w \alpha^*(w) \bar{Q}_d + \alpha'_w g(w) \bar{Q}_d + \bar{Q}'_w g(w) \alpha^*(w)
\]

The analysis follows the same logic as that used for \(\alpha^*(w)\). The effect of a wage rate increase on the innovator profits, in fact, is given by the interaction of different forces of opposite signs. First, there is a clear positive rent expansion effect, \(g'_w\). If the wage rate rises, manufacturers are more willing to adopt the innovation as it not only increases productivity but it becomes also relatively cheaper. The gains from the adoption of the innovative input is higher, the higher is the price of the input that is partially displaced. The rent expansion effect is reinforced (or mitigated) by the effect that a wage rate increase has on the quality of the innovation, \(\alpha^*(w)\). As \(\alpha'_w > 0\) (resp. \(< 0\)), the rent generated by the innovation in the downstream segment rises (resp. falls) and so do the innovator’s profits. Finally, a negative output contraction effect, \(\bar{Q}'_w\), operates as described in the previous paragraph. When the market is large enough (\(\delta > \tilde{\delta}\)), the positive effects dominate and the profits of the innovator increase with \(w\). Figure 2 shows the relationship between supplier’s profits and the wage rate. One can see that an inverted-U relationship applies also to this case. Moreover, when we compare the two thresholds, we can see that \(\bar{\delta} > \tilde{\delta}\), for any \(\gamma > \bar{\gamma}\).

### 3.3 Downstream manufacturers

Let us now turn to the downstream segment of the industry. As long as Corollary 1 applies, asymmetric equilibria are not possible: all the manufacturers either choose the new technology, or they keep producing with the old one. Moreover, one can see that when the adoption fee set by the innovator satisfies Condition (7a), then all the manufacturers choose the new technology. Thus, we can rewrite the expressions (3)-(6) as:

\[
q^h = q^* = \frac{\delta - w(2 - \alpha^*)}{n + 1}, \quad \Pi^h = \Pi^*_d = \frac{(\delta - w(2 - \alpha^*))^2}{(n + 1)^2},
\]

\[
q^s = \emptyset, \quad \Pi^s = \emptyset.
\]
By substituting eq. (9) into expressions (12) and (14), we obtain:

\[
q^* = \gamma \left( \frac{(n+1)^2 + (n-1)n^2w^2}{\gamma(n+1)^2 + (n-2)n^2w^2} \right) \frac{\delta - 2w}{n+1} = \psi \bar{q}_d
\]

\[
\Pi^*_d = \frac{\left( \gamma(n+1)^2 - n^2w^2 \right)^2}{\left( \gamma(n+1)^2 + (n-2)n^2w^2 \right)^2} \frac{(\delta - 2w)^2}{(n+1)^2} = \vartheta \bar{\Pi}_d,
\]

where \( \psi > 1 \), and \( \vartheta < 1 \), and where, as previously defined, \( \bar{q}_d \) indicates downstream firms output when they use the standard technology, and \( \bar{\Pi}_d = (\delta - 2w)^2/(n+1)^2 \) the corresponding level of profits. From equation (16), Proposition 3 follows immediately:

**Proposition 3.** The manufacturers’ optimal level of output depends on the wage rate: \( q^*(w) \) increases with \( w \) when \( \delta > \hat{\delta} \) and decreases otherwise, where \( \hat{\delta} = \frac{(n-2)(n-1)n^2w^3}{\gamma(n+1)^2} + \frac{\gamma(n+1)^2 + (2n-1)n^2w^2}{n^2w} \). Moreover, \( q^*(0) = \bar{q}_d \)

**Proof.** See the appendix.

**Corollary 2.** The manufacturers are forced into a prisoner dilemma alike situation, where the adoption of innovation is the only equilibrium, although it is not the efficient outcome: \( \Pi^*_d < \bar{\Pi}_d \).

**Proof.** Proof of Corollary 2 follows immediately from expression (17).
a wage increase has on the quality of the innovation. As a matter of fact, when the market is large enough ($\delta > \bar{\delta}$), an increase in the wage rate generates a more than proportional increase in $\alpha^*(w)$ and, via this effect, higher output levels. Interestingly, an increase in total output implies also an increase in social welfare as it raises not only the private but also the Consumers surplus. Figure 3 provides a graphical representation of the relationship between the wage rate and the per-firm output level. Also, Corollary 2 states that the manufacturers are worse off after the adoption of innovation. However, adopting the innovation is the only equilibrium as the innovator sets the price $F$ according to conditions (7a) and (7b) in order to exploit the competition in the downstream segment of the industry.

In order to be able to evaluate the impact of a minimum wage policy, it is important to correctly compare the thresholds in propositions (1) - (3). We already know that $\bar{\delta} > \tilde{\delta}$, for any $\gamma > \bar{\gamma}$, and it is easy to show that $\bar{\delta} > \tilde{\delta}$ under the same condition. Instead, if we compare $\bar{\delta}$ and $\tilde{\delta}$ we can derive the following conditions:

\[
\begin{align*}
\tilde{\delta} & \geq \bar{\delta} \quad \text{if} \quad \gamma \geq \frac{(n-1)n^2w^2}{n+1} \equiv \tilde{\gamma}, \\
\tilde{\delta} & < \bar{\delta} \quad \text{otherwise}.
\end{align*}
\]

(18a)

(18b)

with $\tilde{\gamma} > \bar{\gamma}$.

From conditions (18a) and (18b), we derive the size and the ordering of the thresholds in propositions (1) - (3). In particular, let us assume that condition (18b) holds: in this case we have that any positive effect on the level of quality implies a positive effect on the industry output; in other words, whenever the cost reducing effect dominates the output contraction effect ($\alpha_{w}^* > 0$), not only the quality of innovation benefits from an increase in the wage rate, but also the industry output increases. This is due to the fact that the low level of innovation costs ($\gammaum$) allows the innovator to be more effective in investing in $\alpha^*$ in response to an increase in the wage rate, making the new technology capable of lowering the manufacturers’ costs of production regardless of the increase in the cost of labor.

On the other side, when condition (18a) holds, it means that an increase in the wage rate following a minimum wage policy does not always imply an increase in the industry output. In other words, in this case the innovation costs ($\gamma$) are relatively large and they reduce the increase of $\alpha^*$; thus, the adoption of the new technology does not always make the manufacturers capable of recovering the negative effects of
an increase in the cost of labor.

Propositions (1) - (3) fully characterize the impact of the wage rate on market outcomes. For this reason they are important from a policy perspective; their combination reveals under which conditions a minimum wage policy might be effective. As a matter of fact, a policy maker able to identify the size of the market can anticipate the effects of the introduction of a minimum wage on the quality of the innovation, on the profits of the innovator, and on social welfare. For example, in the specific case of a sufficiently large cost of the innovation, formally $\gamma > \tilde{\gamma}$, it follows that:

Remark 1. Given the parameters of the model, an increase in the wage rate:

1. increases the optimal level of quality, the profits of the supplier and the social welfare, in region (i);
2. increases the profits of the supplier and the optimal level of quality, but decreases the social welfare, in region (ii);
3. increases the profits of the supplier, but decreases the optimal level of quality and the social welfare, in region (iii);
4. decreases the optimal level of quality, the profits of the supplier and the social welfare, in region (iv).

Figure 4: effects of an increase in $w$ when $\gamma > \tilde{\gamma}$
The proof of this remark is immediate by looking at Figure 4.

4 Welfare analysis

One may wonder whether the level of the innovation undertaken by the upstream firm is also efficient from the social point of view. It is easy to see that this is not the case. To show this, let us consider the social welfare $W$, defined, as usual, as the sum of Consumers and Private surplus. Using the above findings:

$$W = \frac{1}{2} \left( \delta - \left( \delta - n \frac{\delta - w(2 - \alpha)}{n + 1} \right) \right) \frac{\delta - w(2 - \alpha)}{n + 1} + n \left( \frac{\delta - w(2 - \alpha)}{n + 1} \right)^2 - \gamma \alpha^2$$

It is easy to see that this function is concave in $\alpha$; its maximization reveals that the socially optimal level of the innovation is:

$$\alpha^w = \frac{n(n + 2)w(\delta - 2w)}{\gamma(n + 1)^2 - n(n + 2)w^2}.$$ (19)

It is immediate to check that the upstream firm underinvests with respect to the socially optimal level: $\alpha^w > \alpha^*$. This result has a clear interpretation if one thinks to the standard theory of non-linear wholesale pricing; this theory suggests that by setting a two-part tariff wholesale price, i) an upstream firm is able to fully appropriate downstream firms’ profits and ii) this pricing scheme maximizes social welfare. In our setting, the upstream firm can only partially extract downstream firms’ profits; as a matter of fact, the presence of the standard technology puts some competitive pressure on the innovator who, therefore, cannot fully extract the surplus of downstream firms. Due to this appropriability problem, the innovator is induced to underinvest with respect to the social optimum. Paradoxically, to guarantee social optimality, a policy maker should mandate a switch-off of the old technology; in this way, the innovator regains full appropriability of downstream rents and it invests efficiently.

Remark 2. A switch-off of the old technology promotes a social efficient level of innovation.
5 Extensions

5.1 Linear Pricing

So far, we have assumed that the innovator was charging its technology according to a two-part tariff scheme. This assumption is natural when observing how innovations are usually licensed in practice. Nonetheless, one may wonder how the findings above change if this assumption is removed and it is assumed that the innovator sets a linear price $r > w$ for the high quality capital input. In this section, I follow Kamien and Tauman [2002] on patent licensing.

The upstream innovator sets the optimal price $r$ solving the following maximization problem:

$$\max_r \Pi_u = n \cdot \frac{\delta - r - w(1 - \alpha)}{n + 1} (r - w) - \gamma \alpha^2,$$

s.t. $w(1 - \alpha) + r \leq 2w$.

The result is easily derived:

$$r(\alpha) = (1 + \alpha)w.$$ (20)

Going backward to $t=0$, we can now find the optimal level of innovation. Plugging equation 20 into innovator’s profits, and maximizing it with respect to the optimal quality, we can write the innovator’s investment decision as:

$$\alpha^+ = \frac{nw(\delta - 2w)}{2\gamma(n + 1)} < \alpha^*,$$ (21)

while the upstream innovator gains:

$$\Pi_u^* = \frac{n^2w^2(\delta - 2w)^2}{4\gamma(n + 1)^2} < \Pi_u^*.$$ (22)

We can see that if the innovator charges a linear price, the equilibrium quality level is lower than under a two-part tariff scheme. This is due to the fact that the price of the innovation compensates the reduction in the labor costs following the adoption of the innovative technology. Therefore, it is not possible for the manufacturer to expand their output and increase the private surplus, which in turn implies that the innovator can extract a lower rent from the downstream segment. However, the results in proposition 1 hold with minor differences. In fact, if we rewrite expression
\[ (21) \text{ as:} \]
\[
\alpha^+ = h(w) \bar{Q}_d
\]
where, \( h'_w < f'_w \). This means that the cost reducing effect is lower under linear pricing scheme than in the scenario where the supplier sets a two-part tariff, while the output contraction effect is the same. Thus, the following proposition applies:

**Proposition 4.** *The optimal level of investment undertaken by the upstream innovator depends on the wage rate: \( \alpha^+(w) \) increases with \( w \) when \( \delta > \delta^+ \), and decreases otherwise, where \( \delta^+ = 4w \leq \bar{\delta} \). Moreover, \( \alpha^+(0) = 0 \).*

*Proof.* See the appendix.

Instead, the results in Proposition 3 cannot be replicated with a linear pricing scheme, as manufacturers do not reduce their costs of production \((w(1 - \alpha) + r = 2w)\) after the adoption of the innovative technology. Let us write down the output function of the downstream manufacturers:

\[
q^+ = \frac{\delta - r(\alpha^+) - w(1 - \alpha^+)}{n + 1}
\]

\[
q^+ = \frac{\delta - w(1 + \alpha^+) - w(1 - \alpha^+)}{n + 1} = \frac{\delta - w(1 - \alpha^+ + 1 + \alpha^+)}{n + 1}
\]

\[
q^+ = \frac{\delta - 2w}{n + 1}
\]

The optimal output of the manufacturers does not depend on the level of innovation. Therefore, any positive effect of a minimum wage policy on the quality of the innovative technology does not apply to the output level. This is so because, from an economic perspective, the wage rate policy and the pricing of innovation are strategic complement and an increase of the former generates a proportional increase of the latter. Thus, although a higher wage rate makes the innovation more convenient for the manufacturers, the rise in the innovation’s price prevents them from obtaining any cost reduction. Consequently, we can state that, under linear pricing scheme, the innovation has no effect on the Consumers surplus (the price of the final good does not change), while it increases the Private surplus by transforming part of the wage bill into profits for the upstream innovator.

### 5.2 Total wage bill and profits

In this section, I analyze the impact of the innovation on the remuneration of input factors, namely the total wage bill and profits. We already know that innovation’s
main impact is to lower the requirement of labor input in the production of the final good. Moreover, the manufacturers that adopt the innovative technology reduce the necessary amount of labor input, in order to produce one unit of final good, from 1 to \((1 - \alpha)\). However, from eq. (16), we know that when all the firms adopt the innovative technology the total output increases and so does the demand for labor. Also, since one unit of capital input is required to produce one unit of final good and the capital input is produced by labor, increasing the output means also to increase the demand for the labor necessary to produce more units of capital input. Thus, it is not immediate to understand the net effect of the innovation on the total wage bill. If the productivity increasing property of the innovation dominates the expansion of output, we say that introducing the innovative technology would harm labor remuneration. Vice versa, when the output expansion effect dominates on the productivity effect, the displaced workers are hired back to expand the production.

In formula we have that:

\[
LW = n \left( (1 - \alpha) \frac{\delta - (2 - \alpha)w}{n + 1} + \frac{\delta - (2 - \alpha)w}{n + 1} \right)
\]

\[
LW = n \frac{\delta - (2 - \alpha)w}{n + 1} (2 - \alpha)w
\]

Let us focus on the effect of innovation on the wage bill we see that \(LW'_{\alpha}\). We have that:

\[
\frac{\partial LW}{\partial \alpha} = - n w (\delta - 2(2 - \alpha)w)
\]

which is positive for \(\delta < 2w(2 - \alpha)\). If we look at the two extremes of this condition, namely \(\alpha = 0\) and \(\alpha = 1\), we can see that in the latter case, the condition cannot be satisfied, as with \(\delta < 2w\) there is no market at all. Instead, for small value of the quality of innovation, the condition is more easily satisfied. We can interpret this result as we already mention. Small innovations have relatively small effects in increasing the productivity of labor, while they do increase the production of the final good. By combining together a small displacement effect and the output expansion, we obtain a positive net effect on the total wage bill. Instead, if the displacement of workers is large due to a high increase in productivity of labor (\(\alpha\) is high), the expansion of the output is insufficient to hire the displaced workers back and the net effect on the wage bill is negative. Interestingly, this result contradicts the one in Acemoglu and Restrepo [2019], according to whom small innovations - or innovations
that generate small increase in automation - are the most harmful to labor demand.

Remark 3. When the quality of the innovation - i.e., the increase in labor productivity that the innovative technology generates - is high, then its introduction engenders a negative net effect on the total wage bill. This is so because when the innovation is of high-quality, the displacement of workers due to increase in labor productivity is higher than the increase in labor demand due to the output expansion.

Now, let us define total industrial profits as the sum of the net profits of all the active firms, namely the upstream innovator and the downstream manufacturers. Since innovator’s revenues simply consist of a transfer of private surplus from downstream manufacturers, we can write:

$$\Pi_{tot} = n \Pi_d - \gamma \alpha^2$$

$$\Pi_{tot} = n \left( \frac{\delta - w(2-\alpha)^2}{(n + 1)^2} - \gamma \alpha^2 \right)$$

Simple differentiation reveals \( \frac{\partial \Pi_{tot}}{\partial \alpha} > 0 \) if \( \delta + \frac{a(\gamma(n+1)^2 - nw^2) + 2nw^2}{nw} \). Interestingly, this threshold moves in a similar direction as for total wage bill. More in particular, with little innovations (\( \alpha \to 0 \)) the condition gets close to \( \delta > 2w \), which is always satisfied as this is the condition for a market to exist. Instead, as the innovation increases, the condition becomes stricter \( \delta > \frac{\gamma(n+1)^2}{nw} + w \) although not prohibitive.

Remark 4. Small innovations tend to have beneficial impacts on both total wage bill and profits. Large innovations are more likely to have a negative impact on the total wage bill than on the total profits.

5.3 Unionization and incentives to adopt the innovation

Finally, a consideration on unionization and its effect on incentives to adopt innovation. Hitherto, we assumed the wage rate was given and did not vary as the quality of the technology implied in the production process increased. This is not always the case, however. From a theoretical perspective, higher productivity of labor should automatically lead towards higher wage rates, and this is true for all kinds of labor (Aghion et al. [2017]). From an empirical perspective this mechanism is more puzzling, as automation may have both positive and negative effects on wages (Ace-moglu and Restrepo [2019, 2017]). Anyway, evidence says that sectors with better technological equipment are those that face higher increase in wage rate (Pianta and Tancioni [2008]).
Moreover, at least in developed countries, variations in the wage rate are determined in accordance with a bargaining process between firms and workers. Although many levels of bargaining are generally involved, many European countries have strong unions and very centralized bargaining processes.

In this section, I explicitly address the problem of how the incentives to adopt innovation vary if it directly affects the price of labor at the sector level. Before proceeding, let us just make some slight modifications to the model setup, in particular regarding the wage rate. Let us assume that the industry has a very strong union with full bargaining power. Therefore, the wage rate chosen by the union applies to all the firms in the downstream segment. Let us also assume that there are just two manufacturers and that the wage rates in the upstream and downstream segments are \( w_u \) and \( w_d \), respectively. Moreover:

\[
\begin{align*}
  w_u &= 1; \quad w_d(x) = \frac{1}{1 - x}
\end{align*}
\]

where \( x = \alpha \) if at least one manufacturer adopts the new technology, and \( x = 0 \) if the two manufacturers produce with the backstop technology. In other words, the introduction of a new technology in the downstream segment of the industry alters the cost of labor for all the firms, regardless of their actual technological equipment. Thus, we can think of the innovation as to a means to increase the rival’s costs of production (Salop and Sheffman [1983], Williamson [1968]). Obviously, this is a simplification that does not take into consideration any bargaining process, as the union has full bargaining power, and it therefore represents an unlikely situation. Theoretically, a more realistic assumption would be that the wage level increases to some level \( w(x) = \frac{1}{1 - x} \), with \( x = \frac{m}{n} \alpha \), representing a gradual increase in the wage rate following the rate of adoption of the new technology, \( \frac{m}{n} \). However, I believe this is a useful simplification to understand how the incentives to adopt innovation are affected by variations in the wage rate.

Let us define \( q_0 \) as the output of each manufacturer when none of them adopts the innovative technology, and \( q_2 \) as the output of each manufacturer when both adopt the innovation. Instead, when just one of the manufacturers adopts the innovative technology, we define \( q_i^h \) as the output of the adopter, while \( q_i^s \) is the output of the non-adopter.

Easy computations show that

\[
q_0 = \frac{(\delta - 1) - 1}{3} \equiv \frac{\Delta}{3}
\]
\[ q_2 = \frac{(\delta - 1) - \frac{1-\alpha}{1-\alpha}}{3} = \frac{\Delta}{3} \]

\[ q_1^h = \frac{\Delta}{3} + \frac{\alpha}{3(1-\alpha)} > \frac{\Delta}{3} \]

\[ q_1^s = \frac{\Delta}{3} - \frac{2\alpha}{3(1-\alpha)} < \frac{\Delta}{3} \]

where \( \Delta = \delta - 2 \).

The first thing it is worth remarking is that the innovation does not bring any advantage to the manufacturers when they behave symmetrically \((q_0 = q_2)\). This is due to the fact that, even if the innovation increases the productivity of labor, the union is able to raise the wage rate up to the new productivity level. The net effect is, therefore, nil.

Instead, asymmetric equilibria display some interesting features. Keeping in mind that, by assumption, the introduction of the new technology by at least one firm makes the union raise the wage rate to the new potential productivity (the labor productivity that the firms would obtain by adopting the technology), we can see that innovation is now guaranteeing some advantages to the adopting manufacturer. Moreover, the adopter is now able to expand the output and gain some market share, while the non-adopting rival is forced to reduce its output. Clearly, since the manufacturer that keeps producing with the backstop technology cannot increase labor productivity, the increase in the wage rate operated by the union increases its costs of production. Consequently, the adopter, which does not gain anything in terms of costs of productions - as the new higher wage zeros out the boost in labor productivity - can nonetheless expand the output and gain more profits.

This results highlight the fact that innovation still provides the manufacturers with a unilateral incentive to adopt the innovation. The innovator, who sets the price of the innovation in order to maximize its total revenues, can exploit this strategic effect of the innovation and charges the manufacturers a price which satisfies conditions (7a) and (7b), as reported in section 3.

Briefly, since the profits of the manufacturers in each scenario are:

\[ \Pi_0 = \left( \frac{\Delta}{3} \right)^2; \quad \Pi_2 = \left( \frac{\Delta}{3} \right)^2 - F \]

\[ \Pi_1^h = \left( \frac{\Delta}{3} + \frac{\alpha}{3(1-\alpha)} \right)^2 - F \]
\[ \Pi_1^* = \left( \frac{\Delta}{3} - \frac{2\alpha}{3(1 - \alpha)} \right)^2 \]

we can easily derive the maximum adoption fee \( F \) as the one that satisfies \( \Pi_2 = \Pi_1^* \): \[
F = \frac{4\alpha}{9(1 - \alpha)} \left( \Delta - \frac{\alpha}{1 - \alpha} \right)
\]

This is the price that allows the innovator to sell the innovation to both firms, exploiting their unilateral incentive to adopt the innovation to harm their respective rival. We do not go backward to previous stages as the analytical computations become intricate. However, we can see from the analyses above that incentives to adopt the innovation exist also when the wage rate reacts to the introduction of a labor augmenting innovation. This is possible because of the strategic role of the innovation to increase the rival’s costs and, more in particular, to increase the cost of that input the rival mostly relies on. From the considerations above, we can state:

**Remark 5.** When the innovation can be used strategically to increase the rivals’ costs of production, it also generates incentives for technology adoption in the downstream segment. Moreover, strong centralized unions have ambiguous effects on incentives to adoption. On the one hand, a central union decreases the cost reducing effect of innovation, lowering the appeal of the innovative technology from the manufacturers’ perspective; on the other hand, however, by raising the wage rate after the adoption of the innovation by all the manufacturers, it generates a positive strategic effect for the adopters.

6 Discussions and conclusions

Using a model of a vertically related industry, this paper shows the relationship that links the incentives to produce (and to adopt) a labor augmenting innovation and the wage rate. This paper represents a novel attempt to understand the possible implication of a minimum wage policy on the investments in R&D and on the industry performance. The model suggests that, depending on the size of the market, a policy aimed at increasing the minimum wage paid to workers can have a beneficial impact on both the incentive to produce and adopt the innovation and on the Consumer Surplus. This model is particularly interesting when we consider technologies such as ICT’s cloud services and/or industrial cobots, where the new capital inputs are able to improve the workers’ performance and to increase the labor productivity of the firms. Interestingly, this model is able to explain why the producers of this kind
of innovations increase their market power, while at the same time they seem to reduce the market power of the adopters. The results are confirmed when the labor market is dominated by a central union which controls the wage rate. In this case, the innovation is not used as a tool to increase the efficiency of the firms, but as a tool to increase the rivals’ costs of production.

Finally, the model suggests that small innovations are those that are more likely to have a better impact on the distribution of surplus among the factors of production. This result is at odds with the findings in automation literature, where usually larger automation processes are associated to higher returns for the labor share. Unfortunately, this model is unable to explain clearly how to identify the market size thresholds properly, and consequently how a policy maker can evaluate the actual effect of an employee-friendly policy in his/her particular situation. This aspect, as well as the empirical testing of the predictions stated in the model, are left aside as subjects of other researches.

References


APPENDIX

Proof of Conditions (7a) and (7b)

Proof. Price of capital input is determined in order to make the $m^{th}$ manufacturer indifferent between the adoption of the new technology and the standard one. For the sake of simplicity, I drop the subscript $d$, unless necessary. The potential profits of the $m^{th}$ manufacturer if (s)he adopts the new technology are:

$$\Pi^h_m = \frac{(\delta - w(2 - \alpha(n - m + 1)))^2}{(n + 1)^2} - F$$

Instead, if (s)he keep the standard technology, the payoff is:

$$\Pi^s_{m-(m-1)} = \frac{(\delta - w(2 + \alpha(m - 1)))^2}{(n + 1)^2}$$

One can see that $\Pi^h_m \geq \Pi^s_{m-(m-1)}$ is Condition (7a):

$$F \leq \frac{(\delta - w(2 - \alpha(n - m + 1)))^2}{(n + 1)^2} - \frac{(\delta - w(2 + \alpha(m - 1)))^2}{(n + 1)^2} \equiv C(7a)$$

Also, the $m + 1^{st}$ manufacturer must not have any incentive to adopt the new technology. Moreover, if (s)he keeps the standard technology, the payoff is

$$\Pi^s_{n-m} = \frac{(\delta - w(2 + \alpha m))^2}{(n + 1)^2}$$

Instead, if (s)he adopts the new technology, the payoff is:

$$\Pi^h_{m+1} = \frac{(\delta - w(2 - \alpha(n + 1 - (m - 1))))^2}{(n + 1)^2} - F$$

One can see that $\Pi^h_{m+1} < \Pi^s_{n-m} \geq 0$ is Condition (7b):

$$F > \frac{(\delta - w(2 - \alpha(n + 1 - (m + 1))))^2}{(n + 1)^2} - \frac{(\delta - w(2 + m\alpha))^2}{(n + 1)^2} \equiv C(7b)$$

If Condition (7a) holds with equality, it is possible to see that $F$ satisfies also (7b)
with strict inequality:
\[
C(7a) - C(7b) = \frac{2n\alpha^2w^2}{(n+1)^2} > 0
\]
The opposite is not true. Therefore, Condition (7a) is necessary and sufficient for the equilibrium in the downstream subgame to be stable.

Proof of Corollary 1

Proof. At time 2, the innovator sets the price in order to maximize the total revenues \(mF\), given the costs of innovations that are considered as sunk. Since the price of the innovation is decreasing in \(m\), the innovator faces a trade off between quantity and magnitude.

\[
\max_m \Pi_u = mF(m, \alpha) - I(\alpha)
\]

\[
m^* = \frac{2(\delta - 2w) + w(2\alpha + n)}{4w}
\]

Therefore, the optimal number of contracts from the innovator perspective is \(m = \min\{m^*, n\}\). One can see that:

\[n < m^* \quad \text{if} \quad n < \frac{2(\delta - w(2 - \alpha))}{3w}\]

Proof of Proposition 1

Proof. Optimal quality is derived from simple differentiation of eq.(9)

\[
\frac{\partial \Pi_u}{\partial \alpha} = \frac{2n^2w(\delta - 2w)}{(n+1)^2} + \frac{2\alpha(n-2)n^2w^2}{(n+1)^2} - 2\gamma \alpha
\]

Equalizing marginal revenues and marginal costs, we obtain

\[
\alpha^* = \frac{n^2w(\delta - 2w)}{\gamma(n+1)^2 + (n-2)n^2w^2}
\]

One can see that \(\alpha^* \in [0, 1)\) if \(\gamma > \frac{n^2w^2}{(n+1)^2} \equiv \bar{\gamma} \).
Proposition 1 follows from simple differentiation of expression (10):

\[
\frac{\partial \alpha^*(w)}{\partial w} = n^2 \left( \frac{\delta \left( \gamma(n+1)^2 - (n-2)n^2w^2 \right) - 4\gamma(n+1)w}{(\gamma(n+1)^2 + (n-2)n^2w^2)^2} \right)
\]

Clearly,

\[
\frac{\partial \alpha^*(w)}{\partial w} > 0 \quad \text{if} \quad \delta > \frac{4\gamma(n+1)^2w}{\gamma(n+1)^2 - (n-2)n^2w^2} \equiv \bar{\delta}
\]

and vice versa.

\(\square\)

**Proof of Proposition 2**

**Proof.** Proposition 2 follows from simple differentiation of expression (11):

\[
\frac{\partial \Pi_n^*}{\partial w} = \frac{2n^4w(\delta - 2w) \left( \delta \gamma(n+1)^2 - 2w \left( 2\gamma(n+1)^2 + (n-2)n^2w^2 \right) \right)}{(n+1)^2 \left( \gamma(n+1)^2 + (n-2)n^2w^2 \right)^2}
\]

Clearly,

\[
\frac{\partial \Pi_n^*}{\partial w} > 0 \quad \text{if} \quad \delta > \frac{2w \left( 2\gamma(n+1)^2 + (n-2)n^2w^2 \right)}{\gamma(n+1)^2} \equiv \tilde{\delta}
\]

Moreover

\[
\bar{\delta} - \tilde{\delta} = \frac{2(n-2)n^2w^3 \left( \gamma(n+1)^2 + (n-2)n^2w^2 \right)}{\gamma(n+1)^2 \left( \gamma(n+1)^2 - (n-2)n^2w^2 \right)}
\]

which is always positive for \(\gamma > \frac{(n-2)n^2w^2}{(n+1)^2}\)

\(\square\)

**Proof of Proposition 3**

**Proof.** Proposition 3 follows from simple differentiation of expression (16):

\[
\frac{\partial q^*}{\partial w} = -\frac{2 \left( -\gamma n^2(n+1)^2w(\delta - 2nw + w) + \gamma^2(n+1)^4 + n^4 (n^2 - 3n + 2) w^4 \right)}{(n+1) \left( \gamma(n+1)^2 + (n-2)n^2w^2 \right)^2}
\]

Clearly,

\[
\frac{\partial \Pi_u^*}{\partial w} > 0 \quad \text{if} \quad \delta > \frac{(n-2)(n-1)n^2w^3}{\gamma(n+1)^2} + \frac{\gamma(n+1)^2 + n^2(2n-1)w^2}{n^2w} \equiv \tilde{\delta}
\]
Moreover

\[
\tilde{\delta} - \bar{\delta} = \frac{\left(\gamma(n+1)^2 + (n-2)n^2w^2\right)^2 \left(\gamma(n+1)^2 - (n-1)n^2w^2\right)}{\gamma n^2(n+1)^2 w \left(\gamma(n+1)^2 - (n-2)n^2w^2\right)}
\]

which is positive for \(\gamma > \frac{(n-1)n^2w^2}{(n+1)^2}\)

\[
\Box
\]

**Proof of Proposition 4**

**Proof.** Proposition 4 follows from simple differentiation of eq. (20):

\[
\frac{\partial \alpha^+}{\partial w} = \frac{n\delta - 4nw}{2\gamma(n+1)} = 0
\]

\[
\Rightarrow \delta^+ = 4w
\]

One can see that \(\delta^+ \leq \bar{\delta}\) if \(n \geq 2\).

\[
\Box
\]