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INCUMBENT AND ENTRANT BIDDING IN SCORING RULE AUCTIONS: A STUDY ON ITALIAN CANTEEN SERVICES

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Incumbent and entrant bidding in scoring rule auctions.

A study on Italian canteen services*

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Abstract

We empirically investigate incumbents' and entrants' bids on an original dataset of 192 scoring rule auctions for canteen services in Italy. Our findings show that winning rebates are lower (i.e., prices paid by the public buyer are higher) when the contract is awarded to the incumbent supplier. This result is not explained by the observable characteristics of the auction and service awarded. We then develop a simple theoretical model that shows that such a result is consistent with a setting in which the buyer distorts the scoring function to increase the probability that the incumbent wins the auction at the cost of a higher purchasing price.

JEL codes: D44, D47, H57, L88.

Keywords: Scoring Rule Auctions, Procurement, Incumbent and Entrant, Auction design.

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1 Introduction

In the procurement of complex works or services, i.e., when suppliers have to meet "quality" specifications,[1] scoring rule auctions (SRAs) are often suggested as enhanced mechanisms for the task. SRAs are multidimensional auctions, where bids are competitively evaluated using a linear function that weights both the price and (levels of) quality dimensions. The winner is the bidder who, according to this function, obtains the highest score. Following the directions provided by the EU Directive 2014/24/EU, SRAs have been increasingly adopted in European countries; on the other side of the ocean, SRAs have been widely used, for example, to award highway construction projects in California. As highlighted by Lewis and Bajari (2011), SRAs weighting price and time to completion of the work have succeeded in increasing the provision of quality compared to First Price Auctions (FPAs) adopted in the same setting to award similar projects.

SRAs differ significantly from conventional procurement auctions because, in designing them, the buyer has discretion in - and tools for - defining the quality to be procured. Such discretion operates *ex ante* in the selection of the weight(s) for price and quality(ies) included in the linear function used to evaluate the bids: the buyer can strategically choose which element is assigned the greater weight in the score. Furthermore, this discretion operates *ex post* in the assessment of the quality component of each offered bid: the buyer can adopt a subjective valuation, and bidders cannot be certain about the score that they will achieve given the level of quality supplied (Prabal Goswami and Wettstein, 2016; Burguet, 2015; Huang, 2016). Buyers could have different reasons to implement such distortions. For instance, a risk-averse buyer might aim "to favorite" an incumbent supplier, i.e., the supplier that has previously provided the service simply to continue ongoing efficient outsourcing. On the other hand, the prospect of "exchanges" with a predetermined supplier, which increases the public buyer's utility, would also provide an incentive to manipulate the awarding mechanism[2].

In this paper, we investigate the incumbents' and entrants' bids in SRAs, specifically taking into consideration the public buyer's *ex ante* discretion in the SRA design. We do this by exploiting a small and original dataset of 192 scoring rule auctions for canteen services in Italy, awarded in the period between 2009 and 2013. We first provide descriptive empirical evidence of a positive correlation between the price paid by the public buyer and the awarding of the contract to the incumbent supplier. Then, running an econometric model, we show that this correlation does not relate

[1] Elements of quality could be the technical characteristics of the procured item, relevant delivery date, delivery conditions, etc.

[2] Note that there could be a very fine line between favoritism and corruption in the buyer's act to distort an SRA. Burguet and Che (2004) consider an SRA with two bidders and assume that both bidders are dishonest, i.e., along with quality and price, they offer a bribe. The buyer manipulates - *ex post* - the evaluation of the bid's quality in favor of the bidder submitting the larger bribe. See Wolfstetter and Lengwiler (2006) for a survey of corruption in procurement auctions.

to differences in the service and buyer characteristics, in the levels of competition - even allowing for endogenous entry and different (weaker or stronger) incumbents - or in the overall importance given to quality in the scoring function. We argue that our findings can be explained considering the buyer's *ex ante* discretion at work in the SRA design. To provide theoretical support for this explanation, we sketch a simple model in which we show that the buyer, by strategically choosing the weights of quality in the multidimensional SRA, can increase the probability of the incumbent winning. This form of buyer favoritism toward the incumbent results in a final, higher price to be paid.

We are not the first to investigate incumbent and entrant asymmetry in auctions. De Silva, Dunne and Kosmopoulous (2003) empirically document differences in the bidding patterns and winning bids between entrants and incumbents in the first price-sealed bid auctions for road construction contracts in Oklahoma. They find that entrants bid more aggressively than incumbents and win auctions with significantly lower bids than incumbents do;[3] moreover, they provide some empirical evidence about bidding distributions in asymmetric auctions.[4] To the best of our knowledge, we are the first to investigate the case of incumbent and entrant bidding behavior in multidimensional SRA. The seminal theoretical analysis by Che (1993) shows that in the case that both the quality and bidder type are single-dimensional, quality is enforceable by court, and the scoring rule is quasilinear; the most efficient firm (incumbent or entrant) will always win, regardless of the weight assigned to quality in the scoring function. Conversely, when more than one quality is included in the SRA and private information becomes multidimensional, it is no longer possible to rank firms according to their overall efficiency. With the aim of filling this gap and with a focus on incumbent and entrant asymmetry, in this paper, we develop a simple theoretical example in which both the scoring rule and private information are multidimensional. In doing so, we contribute to the existing theoretical literature on SRAs.

We add to two further strands of the literature. The first is on buyer discretion in the design of SRAs in public procurement.[5] Koning and Van de Meerendonk (2014)

[3]Hyytinen, Lundberg and Toivanen (2018) investigate the change of regime from beauty contests - where in-house suppliers went usually favored - to first-price sealed-bid and scoring auctions, using Swedish data on public procurement of cleaning services. They found that in the new regime - where favoritism went reduced - procurement costs did not change. They explain this result because bids by entrants were less aggressive than in the older regime (i.e., entrants adjusted for the lower/null favoritism in the new regime).

[4]In this respect, De Silva et al. (2003) empirically add to the theoretical analysis by Maskin and Riley (2000).

[5]In private exchanges, buyer discretion (i.e., auctioneer discretion) was recently investigated in English auctions for wholesale used cars; see Lacetera, Larsen, Pope and Sydnor (2016) and Tadelis and Zettelmeyer (2015). In a very different setting, Garicano, Palacios-Huerta and Prendergast (2005) offer empirical evidence of how professional soccer referees favor home teams to satisfy the crowds in the stadium. In their setting, referees have discretion over the addition of extra time at the end of a soccer game to compensate for lost time due to unusual stoppages. They find

empirically show that the higher the quality component in the scoring rule is, the higher the price paid.[6] Moreover, the more verifiable such quality is, the better the enforcement of the contract (Bajari and Tadelis, 2001). Branco (1997) investigates the properties of the optimal mechanisms when bidder types are single-dimensional and correlated; Asker and Cantillon (2008) show that the multidimensionality of suppliers' private information can be reduced to a single dimension (i.e., their "pseudotype"). To contextualize our empirical findings, we develop a model that adopts Asker and Cantillon's pseudotype and allows us to investigate how a public bidder can manipulate the weights of the SRA components to favor a predetermined bidder.

The second strand of the literature that we contribute to refers to the design of empirical tests to detect competitiveness and collusion in auctions. Conley and Decarolis (2015) present two statistical tests to detect coordinated entry and bidding choice. They run these tests on a dataset of average bid auctions[7] adopted for awarding public works in Turin, a town in northwestern Italy. In their setting, collusion was detected by the judge of the local court of law. In contrast, we do not have any external assessment about which auction, if any, was not competitive. Hence, we develop a mechanism to find noncompetitive behavior by investigating the features and outcomes of each auction and by exploiting the related information on the incumbent firm. Our approach is in line with those of Bajari and Ye (2003) and Aryal and Gabrielli (2013), who designed a test to disentangle collusion and competition when collusion is not directly observed. Both works used nonparametric techniques based on the FPA estimation of Guerre, Perrigne and Vuong (2000) to conduct a statistical test to detect collusion.[8] We add a new test, specifically designed for SRAs, which detects the buyer's potential favoritism toward the incumbent bidder and can be used by the regulatory authorities in charge of monitoring procurement auctions.

The rest of the paper is organized as follows. Section 2 illustrates the descriptive statistics of our dataset and some preliminary results. Section 3 implements a novel empirical strategy to investigate entrant and incumbent bidding in SRAs in detail. Section 4 presents a simple model of buyer favoritism toward the incumbent in designing an SRA. Section 5 discusses the empirical results along with our theoretical predictions, and it draws conclusions and policy implications.

that referees systematically favor home teams by shortening close games when the home team is ahead and lengthening close games when the home team is behind. In a similar vein, in our setting, public buyers, having discretion over the weights of the SRA components, could favor the incumbent suppliers by manipulating the SRA's design and ending with higher prices.

[6] In a field experiment, Decarolis, Pacini and Spagnolo (2016) empirically show that including past performances in the scoring function improves SRA performance.

[7] In the Italian framework, an average bid auction works as follows: the first 10% of the highest and lowest discounts over the reserve price is eliminated. Then the average among all remaining discounts is computed (A1). A second average (A2) is calculated among all bids strictly above A1. The winning discount is the highest discount strictly lower than A2. See Albano, Bianchi and Spagnolo (2006) and Decarolis (2009) for further details.

[8] Notice that, in our study, different from the literature, a form of collusion would be between the buyer and the incumbent bidder (and not among bidders).

2 Institutional setting and descriptive analysis

We built a small and original database of 192 public procurement contracts for canteen services awarded with sealed-bid scoring rule auctions (SRAs, henceforth) between 2009 and 2013 in Italy. This market has an HHI of 0.4, highlighting that it is moderately concentrated.[9]

The awarded contracts in our dataset last from 3 to 5 years and have a reserve price - i.e., the maximum price the public buyer is willing to pay - higher than €150,000.[10] Our cross-sectional dataset includes information on the public buyers managing such auctions, i.e., their name and whether they are *elected bodies*, *semiautonomous bodies*, or *administrative bodies*.[11]

The group of public buyers who belong to an *elected body* award 78% of the auctions in our dataset and are mostly municipalities. These buyers are locally elected every 4 or 5 years; thus, the canteens they outsource, i.e., canteens for schools located in the municipal area, are politically sensitive services. The group of public buyers belonging to a *semiautonomous body* award 7% of auctions and largely consists of public hospitals. Their governance is in between that of elected and administrative bodies, as their internal management consists of public career managers, while their executive management is appointed by the locally elected president of the region. Canteens for hospitals are typically for internal staff and patients. The group of public buyers who belong to an *administrative body* award 15% of the auctions and consist - for example - of firefighters or local branches of the Italian Tax Agency; these bodies are run by civil servants, and their canteens are usually for internal staff only.

Whatever the group these public buyers belong to, they all have discretion in designing the outsource for the canteen's service and, for SRAs, they are free to choose the weights of price and quality in the scoring function. Our database records the weights chosen in each SRA for quality and price: on average, the quality's weight adds up to 60 points over 100.[12] We also recorded whether there was a requirement for urgency in the awarding of the service.

Moreover, for each auction in our database, we have information about the identity of the winner and if he/she was the canteen's service provider in the period

[9] Specifically, in our dataset, we record 78 different winners; 4 big players won 44% of the 192 auctions, and 45 smaller players won one auction each.

[10] Using data provided by EU_TED on Italian public procurement auctions for canteen services, we can observe that the total sector value in 2015 (the most recent year available) was approximately 100 million Euro; the average participation in the corresponding auctions was 3.2 bidders.

[11] This classification follows that of Bandiera, Prat and Valletti (2009). Using Italian public procurement data, they exploit the presence of a central procuring agency to study the determinants of price differences for the same object by different public buyers of a different nature.

[12] According to Italian law, public buyers could also make decisions about firms' entry in the auction, i.e., whether to allow for free entry or to run a preliminary screening. However, this latter discretionary power is limited by law and depends on the contract's value and whether or not there is an urgent requirement.

immediately before the recorded auction took place (i.e., if she/he is the *incumbent supplier*).[13] We also observe the winning bid-to-reserve-price ratio (i.e., the winning rebate); the ratios of the maximum and the minimum bid to reserve price; and the number of participants.

We also collected data on the geographic characteristics of the area where the service should be provided and the local Purchasing Parity Power (PPP) index;[14] we used the latter as a proxy for the geographical differences in the costs of raw materials and services. To control for the electoral cycle's relevance, we then gathered information on the time lasting between the year in which the service was awarded and the next electoral year. We defined this variable as year-to-elections and included it in the empirical analysis.[15] Finally, in the case of an elected public buyer, we observed the population of its constituency.

Table 1 shows the descriptive statistics of our dataset.

[Table 1 about here]

By disentangling if the winner in the auction is the incumbent - i.e., the supplier that has previously provided the service - or an entrant, Table 2 shows the auction outcomes in terms of the winning rebate and average number of bidders, controlling also for the reserve price.

[Table 2 about here]

Our descriptive statistics in *Table 2* show that the incumbent wins 56% of the auctions in our database. In such auctions, competition and the winning rebate are lower - i.e., there is a lower number of bidders, and the public buyer pays a higher awarding price than in auctions where the incumbent does not win. In particular, the mean number of bidders in auctions where the incumbent has won is 2.1 and where she/he has not won is 3.5, and the mean winning rebates are 2.44% and 6.70%, respectively. These differences in means are statistically significant at the 99% confidence level.[16]

A two-sample Kolmogorov–Smirnov (K-S, henceforth) test of the equality of distributions confirms that both the winning rebate and bidder participation values are distributed differently in the two subsamples. Surprisingly, the K-S test does not find any difference between the two subsamples in the distribution of the reserve price, in

[13]For a subset of auctions, we also observe if the incumbent has participated but has not won the contract. See Section 3.2 for additional details.

[14]The data come from Table A2.1, Column 7, in Cannari and Iuzzolino (2009). The PPP index includes food, clothing, furniture, services and energy costs. It excludes house prices. Note that the auctions in our dataset were awarded starting from 2009.

[15]Electoral year has been considered at the national level for all the administrative bodies; at the regional level for hospitals since health is managed at the regional level in Italy; and at the local level for municipalities.

[16]The average number of bidders in the overall dataset is equal to 2.68: under symmetry, each bidder wins with a probability of 0.37.

the weight of quality in the scoring function, in the distribution of the public buyer's type, in the year the contract was awarded, in the electoral cycle, or - using NUTS groups of region codes from Eurostat - in the geographical location of the public buyer. Thus, in our dataset, considering both the group of auctions in which the incumbent has won and the that in which an entrant has won, we observe that the characteristics of the service awarded and of the SRA are identically distributed in the two groups, although the auction outcomes differ strongly.

In summary, the descriptive statistics in our database (Tables 1 and 2) show the following evidence: i) the incumbent supplier is the winner in 56% of auctions; ii) the competition is lower and the price paid by the public buyer is higher when the winner is the incumbent supplier; and iii) the other characteristics of the SRAs and the service awarded do not change.

3 Empirical analysis

In this section, we run an econometric analysis to investigate the evidence of entrants' and incumbents' bids shown in Section 2. We proceed by implementing the following empirical strategy. We begin by separating our dataset into two subsamples: the first includes all the auctions in which entrants have won, i.e., the entrants' winner subsample (EWS), and the second includes the (remaining) auctions in which incumbents were awarded the contracts, i.e., the incumbents' winner subsample (IWS). The EWS contains 84 auctions, and the IWS contains 108 auctions. We then run an econometric model on the EWS and construct two tests as a result. Finally, we apply these tests to the entire sample, with the aim of finding which auction fails to be predicted by our econometric model.

Specifically, on the EWS, we run the following parametric estimate of the winning rebate p_{wi} for each auction i:

$$p_{wi} = \alpha_1 + \beta_{11} N_i + \beta_{12} q_i + \beta_{13} X_i \tag{1}$$

and we estimate the difference between the maximum and the minimum rebate offered by bidders, Δ_{pi}, according to the following:

$$\Delta_{pi} = \alpha_2 + \beta_{21} N_i + \beta_{22} q_i + \beta_{23} X_i \tag{2}$$

where N is the number of bidders, $q \in [0, 100]$ is the weight of quality within the SRA, X is a vector of the auction characteristics that include the NUTS groups of region codes, the buyer type, the population of the municipality if the public buyer is an elected body, the log-reserve price,[17] the number of years until the next election and whether or not subcontracting was used.

[17] In empirical procurement literature, the log-reserve price has been used both as a measure of the dimension of the awarded contract and as a normalization factor for the auction's outcome. For example, on Italian data, Bucciol, Chillemi and Palazzi (2013) investigate cost overruns, defined

In the empirical procurement literature on first/second/average price auctions, the winning rebate - i.e., the ratio of the winning price to the reserve price - is used as a measure of competitiveness (see, for example, Coviello and Gagliarducci, 2010). Considering an SRA, where competition is on both the price and quality components, the higher the weight of quality is, the less important the price component in the bid. To take into account the relevance of price when quality has a positive weight, we include in our estimation the distance between the minimum and maximum rebate submitted by all bidders in the same auction, Δ_{pi}. To make the interpretation for Δ_{pi} clear, consider the following example: in an *SRA,* where $q_i = 0$ (thus corresponding to a first price auction), competition will only be on the price side, and depending on bidders' heterogeneity, Δ_{pi} has a positive value. In contrast, in an SRA, where $q_i = 100$, the price component of all bids will be equal to the reserve price, and the difference between the highest and lowest price discount will be zero. Thus, we expect to find a significant effect of q on Δ_{pi}.

The estimation results of the (1) on p_{wi} and of the (2) on Δ_{pi} are presented in *Table* 3 and *Table 4*, respectively.

In Columns 1a and 1b, *Table 3* and *Table 4*, we report results from a standard OLS model. In Columns 2a and 2b, we consider that the awarding mechanism may be endogenous with respect to the buyer's type and the dimension of the contract: for this reason, we use a two-stage least squares (2sls) approach, where q is instrumented using the buyer's type, the reserve price, and whether or not there was a requirement for urgency in the awarding of the service. The estimation for (2) considers only auctions with at least two participants.

[Table 3 about here]
[Table 4 about here]

Our results on the EWS show that the weight assigned to quality in SRAs has a strong impact on the winning rebate: the higher this weight is, the lower the competition on the price component. This result is also confirmed by looking at the negative and significant effect of quality on Δ_{pi}, i.e., on the difference between the minimum and maximum rebate over the reserve price in each auction.

As we could expect, the number of bidders is significant and has a positive effect on the winning rebate: greater competition increases the winning rebate, i.e., it reduces the buyer's purchasing price. A higher number of bidders also produces more heterogeneity across bids, i.e., Δ_{pi} increases.

We can observe that the electoral cycle also has an important influence on both p_{wi} and Δ_{pi}: the lower the years remaining to the next election are, the lower the price paid and the higher the distance between the minimum and the maximum

as the difference between the final execution price and the auction winning price: as a dependent variable, they adopt cost overrun normalized by the reserve price, and they regress it on the log-reserve price.

rebate. This is consistent with the idea of competitive auctions in periods close to election time.[18] Consider, for example, a municipality that has to manage an SRA to award the canteen service for local schools. In this setting, a mayor close to election time will be as efficient as possible in managing such a procurement process. In so doing, he/she would show to be a capable administrator and save money to be spent on gaining consensus with the aim of being re-elected or increasing support for a candidate from the same political party.

Finally, to explain fixed-effect geographical differences, in Columns 1b and 2b, we replace NUTS dummies with the local PPP Index. We obtain a significant effect, meaning that at least part of the geographical variation observed is due to the different costs of raw materials. Southern Italy has a significantly lower cost of living, being - for example - 75% of that of northwestern Italy; this difference is reflected - in the EWS - in the positive coefficients of the NUTS dummy variable *South* and in the negative and significant sign of the PPP index's coefficient.

The results presented in *Table 3* and *Table 4* remain significant using different errors (standard, robust, corrected for small sample and bootstrapped). We run other tests on the IV model as follows. First, we run an F-test of the joint significance of the additional instruments used for q on q: we find that the instruments are sufficiently correlated with the endogenous regressor. Second, we run a Sargan test, and we verify that the instruments are uncorrelated with the error term. Finally, we use the Durbin-Watson test to verify that q is is truly endogenous and, as such, needs to be treated with instrumental variables. We find that q is actually endogenous for the regression (1) on winning rebate, but not for the regression (2) on Δ_{pi}. Since an exogenous regressor estimated with the IV model is consistent but less efficient and because q was found to be endogenous on the reserve price and the buyer type in (1), we also use the IV model for the second regression. The results do not change significantly if OLS is used instead. Finally, when we estimate regression (1) on the IWS subsample, i.e., where incumbents were the winners, we find that q is no longer significant. Moreover, in this case, we obtained a much lower R^2 (specifically, 0.09 vs. 0.49). Similarly, running (2) on the IWS, we find that q is no longer significant. These results highlight that auctions' outcomes in the IWS are not well explained by the auction mechanism and service characteristics.

Robustness check: endogenous participation As a robustness check, we consider that N, the number of bidders in the *SRA,* can simultaneously be determined with the price decision because participation in the auction has a cost for each bidder. We first estimate a regression, where q is assumed to be exogenous and N and p_{wi} are simultaneously determined. We find that N correlates with all the other regressors. Since it is difficult to select an instrument that correlates only with N and not with

[18] Considering the political cycle, Moszoro and Spiller (2014) highlighted that the procurement process could be managed to reduce political hazards from opportunistic third parties (i.e. political opponents).

p_{wi}, and instrumental variables/other solutions are not available, we resort to the model proposed by Lewbel (2012). This approach exploits heteroskedasticity in data to construct an instrument for models with such issues. The results are presented in Columns 3a and 3b of *Table 3* and *Table 4*; they do not differ significantly from the standard OLS estimates. As was done previously, in Column 3b, we check for local (geographical) heterogeneity in the data using the local PPP index.

As a further robustness check, we estimate a three-stage least squares (3sls) model that considers the awarding mechanism to be endogenous in consideration of the size of the awarded contract and the buyer type. In contrast to N, the weight of quality q within the awarding procedure can be instrumented using this information. This is why we should treat the endogeneity that arises from q differently with respect to the simultaneity problem that arises from N.

As the first stage of the model, we estimate q using the reserve price, dummies for the buyer's type and dummies for whether or not the service was urgent. Then, the predicted values of q are used in the second and third stages to estimate the model proposed by Lewbel (2012). Specifically, in the second stage, we construct an instrument to estimate the number of bidders, N, and finally, in the third stage we estimate p_{wi} and Δ_{pi}, having corrected for the endogeneity of q and for the simultaneity problem of N. The stages are designed as follows:

$$\begin{array}{llll} 1 & & q_i \sim res\_price,\, eb,\, hosp & \textit{Decision } q \\ 2. & \text{(Lewbel)} & N_i \sim \widehat{q}_i, X_i', p_{wi} & \textit{Entry decision} \\ 3. & & p_{wi} \sim \widehat{N}_i, \widehat{q}_i, X_i & \textit{Auction outcome} \\ & & \Delta_{pi} \sim \widehat{N}_i, \widehat{q}_i, X_i & \textit{Auction outcome} \end{array} \tag{3}$$

The results are presented in Columns 4a and 4b, *Table 3* and *Table 4*: they are consistent with our baseline model.

Finally, we explore whether the reserve price affects auction participation.[19] We regress, in Table 5, N over the reserve price. We do not find any significant relation between the two variables.

[Table 5 about here]

3.1 Results on the whole sample

We now estimate predictions from our IV model with geographical dummies - gained on the EWS subsample - on the entire sample, and we compare the predicted and

[19] The reserve price might affect bidders' participation in auction at least in two ways. On the one hand, since the reserve price is a proxy for the contract's size, low reserve-price canteen services can make a large number of firms able to provide the service, and thus entering the awarding auctions. On the other hand, high reserve-price canteen services might provide suppliers with larger room for bidding up the price (i.e. also less efficient suppliers can provide the service), so more bidders will enter the auction.

observed values. We also estimate confidence intervals for the difference between the maximum and the minimum rebate Δ_{pi} (both above and below the predicted value) and for the winning rebate p_{wi} (only below the predicted value).

As usual, the confidence intervals are calculated as follows:

$$CI = Xb \pm \alpha SE \tag{4}$$

where Xb is the predicted value, SE is the standard error, and α is the t-value parameter that defines the width of the confidence interval. The larger α is, the wider the confidence interval. We use both standard errors of the prediction (STDP) and standard errors of the forecast (STDF), which are equal to the standard errors of the predictions plus the error variance of the regression.[20] By construction, these latter standard errors are larger than the standard errors of the prediction. As a result, they produce larger confidence intervals, so that a lower number of observations will fail to be predicted by the model.

In Figure 1, we plot α (the t-value parameter that defines a confidence interval) on the horizontal axis and the proportion of correctly predicted values - within the IWS in red and within the EWS in blue - on the vertical axis. We do this for the winning rebate, p_{wi}, and for the difference between the minimum and maximum rebate over the reserve price in each auction, Δ_{pi}, using both STDP and STDF.

[Figure 1 about here]

Depending on the subsample used - the EWS or the IWS - we find a significant difference in the precision of the model estimating the winning rebate p_{wi} and Δ_{pi}. Regardless of the confidence interval and the standard errors used, the predictions in the EWS are systematically closer to the real values than the predictions in the IWS. Note that this is no longer true if the model is estimated on a randomly chosen subsample, that is, if each observation is randomly allocated either to the EWS or to the IWS, as will be discussed in the next section referring to robustness checks.

Finally, on the basis of these results, we move from the estimate on the EWS to that on the entire database. We use the predicted values of our models as a test: if the observation is within a given confidence interval of the predicted values, then the test is passed. Given the two estimated values (p_{wi} for Test 1 and Δ_{pi} Test 2) we end up with two tests; if both tests fail, it implies that the service and buyer characteristics as well as the auction mechanism are not able to explain the observed auction outcome.

[Table 6 about here]

[20] STDP is used for within-sample predictions. STDF is used for out-of-sample predictions to control for differences in the domains on which the models are estimated (i.e., extrapolation issues). Since we derive our predictions both within- and out-of-sample, we report both statistics.

Table 6 reports the proportion of incorrectly predicted values (from 0 to 1), by confidence interval and by some characteristics of the auction. We begin looking at the results on the STDP; STDF will be discussed at the end of this section.

With a 90% confidence interval (CI), we find that 32.8% of the auctions in our entire dataset fail to pass both Test 1 for p_{wi} and Test 2 for Δ_{pi}. Obviously, the larger the confidence interval is, the smaller the number of auctions that do not enter within that interval. Accordingly, with 95% and 98% confidence intervals, 29.2% and 26% of auctions, respectively, fail to pass both tests.

However, the proportion of incorrectly predicted auction outcomes is much higher in the group of auctions where the incumbent has won. For example, with a 95% confidence interval, only 15.5% of the auctions in which the contract has been awarded to an entrant failed both tests, but this proportion increases up to 39.8% in case of a victory for the incumbent. Thus, it is much more likely for an SRA to fail our tests if the incumbent is the winner in the auction.

We find a similar effect for the electoral cycle. Considering all three confidence intervals, 56.7% of the auctions awarded during an electoral year failed to be predicted by both Test 1 for p_{wi} and Test 2 for Δ_{pi}. This proportion decreases to 28.4% (for a 90% CI) and to 20.4% (for a 98% CI) for auctions that have not been awarded during an electoral year.

Looking at the buyer type, canteens managed by semi-autonomous body (i.e. hospitals) are more likely to fail our tests, followed by contracts awarded by an elected body. In contrast, this proportion drops to 17.2% (or 13.4%, depending on the confidence interval used) for SRAs managed by nonelected administrative bodies. Finally, we do not find any difference in the proportion of incorrectly predicted auction outcomes when we disentagle auctions by the reserve price.

These results are confirmed and are even stronger using the standard errors of the forecast. First, some observations still fall outside of the confidence interval of the predictions, even if using STDF increases the width of that interval. Second, among those observations, the proportion of incorrectly predicted values is higher when the incumbent has won. Using an 80% confidence interval, 19.4% of the auctions awarded to the incumbent supplier cannot be predicted using our model, a proportion that falls to only 2.4% if the contract has been awarded to a new entrant. Using, instead, a 95% confidence interval, all the SRAs that were not predicted by our model were awarded to the incumbent. The only difference we detect, using STDF, is for the buyer type: in this case, contracts awarded by an elected body are more likely to fail our tests, followed by hospitals and central bureaucratic administrations.

In summary, in our dataset on Italian canteen services, an SRA that has failed the tests proposed in this paper is more likely to have been awarded to the incumbent supplier, to have been managed by an elected body or a hospital and to have been awarded during an electoral year.

3.2 Competing explanation and other robustness checks

In this section, we first address a competing explanation for our empirical evidence, and then, we run further robustness checks. For the former, assume that entry costs are introduced - not necessarily of monetary value (i.e., extra time or extra effort to prepare the multidimensional bid). In this case, a firm's entry decision is not exogenous, but it depends on the similar decisions made by all other players. Moreover, we assume potential entrants to observe the incumbent's characteristics relative to the setting in which the tender for the service is run; specifically, they observe how efficient is the incumbent, i.e. weak or strong incumbent. Auctions with a "weak" incumbent are more likely to have stronger competition, which keeps cost down. In this situation, the probability of the victory of the incumbent is low. In contrast, a "strong" incumbent may deter entry, resulting in higher final prices. Moreover, a strong incumbent is more likely to have the contract awarded. This situation provides for a competing explanation of our results.

First, consider that in our dataset we have sealed bid auctions: with a simultaneous bidding process, participants do not observe ex ante the level of competition in the auction, although they may have some relevant information - for example, about the presence of a strong or weak incumbent. It is possible to nonparametrically test the hypothesis that competition is fully observed ex ante or the hypothesis that competition is totally unknown to bidders. In the first case, for auctions with one participant, the unique bidder should bid a price equal to the reserve price. However, this is not verified in 75% of our observations with one bidder. In the second case, the distribution of the winning prices should not change conditional to the number of participants. Running a Kendall's rank correlation coefficient test on our dataset leads us to reject this hypothesis. We conclude that bidders receive a noisy signal on the level of competition they will face in the auction. It turns out that the distribution of prices is different between auctions with 1 bidder, with 2 or 3 bidders and with 4 and more bidders but does not change within each subsample.

We report, in *Table 7* below, the summary statistics on the auction's outcome and reserve price conditioning for the number of bidders.[21] Differences in the winning rebate, depending on whether the contract has been awarded to the incumbent or to a new entrant, persist and are significant.

[Table 7 about here]

Then, we estimate the same model as in Section 3 but conditional to a low (i.e., 2-3 bidders) or high (4+ bidders) level of competition.[22] There is no longer a simultaneity

[21] It is important to remark here that, in a sealed-bid auction, bidders do not observe ex ante the number of competitors. As a result, even in auctions with just one bidder, this bidder could not have anticipated to be the only participant, bidding the reserve price as a result.

[22] Only 13 auctions with one bidder were awarded to a new entrant, a number too small to repeat the econometric analysis on that subsample.

problem between the number of bidders and the winning rebate, but we still have to address the endogeneity problem of the scoring function. Hence, in *Table 8* below, we report both the OLS and IV estimates for the winning rebate and for the difference between the maximum and the minimum rebate, running separate regressions for auctions in the EWS with 2-3 bidders and for auctions in the EWS with 4+ bidders.

[Table 8 about here]

The results we obtain are similar to those presented in *Table 3* and *Table 4*. Finally, we use the IV models to compare the predictions of the winning rebate p_{wi} and Δ_{pi} with the observed values both in the EWS and in the IWS, separately for different levels of competition. As was done in the previous section, if both observations fall outside the confidence interval of the predictions, then the test is not passed. We compare the proportion of incorrectly predicted values (from 0 to 1) by confidence interval and controlling whether the contract has been awarded to the incumbent or to the entrant.

[Table 9 about here]

We obtain that the unobserved difference in the price paid between auctions in which the incumbent has or has not won persists. This finding has two relevant implications. First, it provides a robustness check of our estimate using the Lewbel instruments. Second, it proves that an "endogenous entry story" cannot explain our results. In fact, if prices when the incumbent has won are higher because of a lower competition, then conditioning our estimate to the number of bidders should eliminate this price difference, which does not happen.

In a subset of our observations in which the entrant has won (39.3% of the EWS), we observe whether the incumbent had at least participated in the auction.[23] Among those observations and excluding the cases where the service was previously managed in-house by the public buyer (thus, there was no incumbent), we find that the incumbent participated in only 19.2% of the cases. Hence, in a further robustness check, we compare the auction outcome and reserve price when either the incumbent won or he/she did not participate in the auction. In doing so, we remove the possibility that differences in the winning rebate and in the auction's participation between the EWS and the IWS are driven by the presence of a weak incumbent in the EWS. Moreover, an unsuccessful favoritism toward the incumbent supplier was obviously impossible if the incumbent did not participate in the auction. As we show in *Table 10*, the unobserved difference in the winning rebate and in the number of bidders between the two subsamples persists.

[23] We observe incumbent participation if (i) there was no incumbent because the service was managed in-house by the public buyer (8.3% of the EWS) and (ii) there was just one bidder, and the identity of this bidder is different from the incumbent supplier (15.5% of the EWS); (iii) we observe the identity of all bidders for auctions with more than one participant and in which an incumbent exists (15.5% of the EWS).

[Table 10 about here]

We conclude this section by presenting a final robustness check. We repeat the analysis in Section 3 using a random subsample, i.e., we randomly allocate observations to the "EWS". In this case, we obtain that there is no difference in the ability of our model to predict winning rebates and differences between the maximum and the minimum rebate in the random EWS and in the random IWS. We report in Figure 2 the proportion of correctly predicted values - within the random EWS in blue and within the random IWS in red - as a function of the width α of the confidence interval and of the type of standard error chosen - STDP or STDF.

[Figure 2 about here]

Thus, only when we use the EWS as defined at the beginning of this section we do obtain that the empirical model better predicts the winning prices of the EWS with respect to the remaining part of the dataset, while this unobserved difference disappears when a random subsample is used.

4 A theoretical example of favoritism in SRA

To explain the empirical evidence and our econometric results, in this section, we present a theoretical example with a multidimensional SRA and bidders' quadratic cost function in which a public buyer aims to increase the incumbent's probability of victory by distorting the weights of qualities in the scoring function.

The optimal design of an SRA with multidimensional private information is a difficult problem to solve, and we are not aware of any general result such as the one by Che (1993) on the unidimensional case.[24] Nishimura (2015) studies the optimal design of scoring rule auctions with multidimensional qualities; however, since he maintains a unidimensional private information setting, the buyer's distortion in the scoring function to favor a targeted bidder cannot be investigated.[25]

With the aim of contributing to filling such a gap, we consider a setting in which a public buyer,[26] or simply a "buyer" in what follows, has to award a service by

[24] Che (1993) shows that it is optimal for the buyer to undervalue quality to reduce market power of the most efficient firm. Asker and Cantillon (2008) and Hanazono, Nakabaiashi and Tsuruoka (2015) model a setting with multidimensional qualities and multidimensional private information, but they do not discuss the optimal scoring rule.

[25] There is a straightforward intuition behind this limit: if bidders can be ranked according to their overall efficiency, the contract will be awarded to the most efficient supplier, and changes in the scoring function weights will simply affect the qualities provided. In contrast, with multidimensional private information, bidders would be ranked only according to the scoring function chosen by the buyer, and in this way, their winning probabilities could be distorted.

[26] In what follows, we will refer to the public buyer using "he" and to the bidder using "she".

choosing between two bidders, $j = (I, E)$, where I is an incumbent firm (i.e., a firm that has provided the buyer with such service in the previous period), and E is an entrant firm. We assume that the buyer is willing to distort the awarding mechanism to increase the probability of winning for I, i.e., to "favor" the incumbent. The buyer adopts such a distortion in the SRA design if this increases his utility. This could occur - on the one hand - in the case in which the buyer is risk-averse, and/or he aims to continue a positive ongoing outsourcing. Alternatively, in the case in which the buyer wants to "reward" the incumbent supplier because of a story of private exchanges between them (i.e., favoritism or corruption). Note that in this example, we do not address the aim that leads the buyer to distort the SRA.

We consider a buyer that designs an SRA to award a service described by two nonmonetary characteristics, i.e., two qualities, and the price for its provision. The buyer has the following utility function:

$$U(Q, p) = q_1 + q_2 - \alpha p + f| [t_I > t_E,] \tag{5}$$

where p is the price he has to pay to the supplier for the provision of the service of quality $Q = \{q_1, q_2\}$, and α is the relative weight of the price with respect to to overall quality Q. If supplier I wins the auction, the buyer will receive an additional utility, $f \in [0, +\infty[$. Thus, considering (5), with $f = 0$, there is no buyer favoritism toward I; while, when $f > 0$, there is buyer bias in favor of I.

The buyer adopts the following scoring rule mechanism:

$$t = a_1 q_1 + a_2 q_2 - p \quad . \tag{6}$$

Such scoring rule t weights each bid $B_j = \{q_{1j}, q_{2j}, p_j\}$, and includes a linear combination with coefficients (a_1, a_2). The bidder j with the highest score t_j wins the auction.

Each bidder j has a type $\theta_j \in [0, 1] \subset \mathbb{R}^2$; she has private information on her multidimensional quality, $(\theta_{1j}, \theta_{2j})$, i.e., θ_{1j} and θ_{2j} are *i.i.d.* according to a uniform distribution between 0 and 1.

We assume that each bidder's cost function is quadratic and separable in the qualities:

$$C_j(Q, \theta_j) = \frac{1}{\theta_{1j}} q_1^2 + \frac{1}{\theta_{2j}} q_2^2. \tag{7}$$

We also assume that the buyer knows the type of the incumbent firm, $\overline{\theta}_I = (1, 0)$, [27] but that he does not observe the type of entrant E, denoted by $\theta_E = (\theta_{1E}, \theta_{2E})$.

[27] If the entrant's type dominates the incumbent's type, i.e., if $\theta_{1E} > \theta_{1I} \cup \theta_{2E} > \theta_{2I}$, then no distortion in the weights of the scoring function can allow for a victory of the incumbent. On the other hand, if the incumbent's type dominates the entrant's type, i.e. if $\theta_{1E} < \theta_{1I} \cup \theta_{2E} < \theta_{2I}$, then the incumbent will win regardless of the scoring function chosen.

A distortion in the weights of the scoring function can alter the probability of victory of the incumbent only if any bidder dominates the other one, as is always the case when the incumbent has type $\overline{\theta}_I = (1, 0)$.

The timing of the game and agents' choices at each stage are described in *Figure 3*.

$$
\begin{array}{lccc}
 & \mathit{Choice} & & \mathit{Choice} \\
 & a_1, a_2 & & p,\ q_1, q_2 \\
 & & & \\
TIME & - \ \ ---1--- & - & ---2--\longrightarrow \\
 & & & \\
PLAYER & \text{Buyer} & & \text{Bidders}
\end{array}
\qquad \text{(Figure 3)}
$$

Moving by backward induction, we start with stage 2, where equilibrium bids can be derived following the approach of Asker and Cantillon (2008).[28] Accordingly, an SRA is equivalent to a first price auction in which bidders' private values are given by their *pseudotype*, i.e., *the maximum level of social surplus that* a *supplier can generate, given her cost function and the scoring rule chosen*[29], which is as follows:

$$k(\theta_j) = \arg\max_{q_1, q_2} \{a_1 q_1 + a_2 q_2 - C_j(q_1, q_2, \theta_j)\}$$

where $k(\theta_j)$ is the pseudotype for bidder j. Q^* is defined as the level of quality that maximizes the pseudotype. It is a weakly dominant strategy for each bidder to offer Q^* as a quality component of her bid (Asker and Cantillon, 2008). Note that, in our setting, having assumed a quadratic cost function, the pseudotype becomes a linear combination of $(\theta_{1j}, \theta_{2j})$. Hence, it is possible to derive both its distribution (by convolution) and the equilibrium scores. Then, as the residual in the scoring rule function, we can also obtain the price component of the bids.

In what follows, we characterize the optimal SRA for two cases: the case in which the buyer distorts the SRA to favor the incumbent ($f > 0$), and the case in which there is no distortion ($f = 0$).

In *stage 1*, the buyer chooses a_1 and a_2 of the scoring rule t with the aim of maximizing the following:

$$\max_{a_1, a_2} \ \Pr(I\ win) \cdot U(\overline{\theta}_I | a_1, a_2) + f \cdot \Pr(I\ win) + [1 - \Pr(I\ win)] \cdot E[U(\theta_E | a_1, a_2)] \quad (8)$$

where $U(\theta_j | a_1, a_2)$ is the buyer's utility provided by bidder j, with $j \in \{I, E\}$, conditioned to the scoring rule chosen by the buyer, and $\Pr(I\ win)$ is the probability that I wins the auction. The solution of the buyer's problem (8) yields Proposition 1.

[28] See also Hanazono, Nakabaiashi and Tsuruoka (2015) for a more general discussion of equilibrium bidding behavior in SRAs.

[29] Asker and Cantillon (2008), p. 73.

Proposition 1 *Define with* (a_1^*, a_2^*) *the optimal weights for the scoring rule* t *with* $f \geq 0$*. In the case of no favoritism,* $f = 0$*, the buyer will choose* $a_1^* = a_2^* = \frac{3}{4\alpha}$*; the bidders' quality provision will be below what could have been achieved with full information.*

In the case of favoritism, $f \geq 0$*, the buyer will distort the scoring rule* t *such that* $a_1^* \geq a_2^*$*,* $a_2^* \leq \frac{3}{4\alpha}$*, the ratio* $\varepsilon_a = \frac{a_2^*}{a_1^*}$ *is a decreasing function in* f*, and a finite solution will always exist for* a_1^**,* a_2^**. Moreover, there exist two levels of favoritism* $\overline{f}$ *and* $\overline{\overline{f}} > \overline{f}$ *such that if*

1. $f \in \left[0, \overline{f}\right]$ *then* $a_1^* \geq \frac{3}{4\alpha}$*,* $q_{1I}\left(a_1^*, a_2^*\right) \geq q_{1I}\left(\frac{3}{4\alpha}, \frac{3}{4\alpha}\right)$ *and* $p_I\left(a_1^*, a_2^*\right) \geq p_I\left(\frac{3}{4\alpha}, \frac{3}{4\alpha}\right)$*,*
2. $f \in \left[\overline{f}, \overline{\overline{f}}\right]$ *then* $a_1^* \leq \frac{3}{4\alpha}$*,* $q_{1I}\left(a_1^*, a_2^*\right) \leq q_{1I}\left(\frac{3}{4\alpha}, \frac{3}{4\alpha}\right)$ *but* $p_I\left(a_1^*, a_2^*\right) \geq p_I\left(\frac{3}{4\alpha}, \frac{3}{4\alpha}\right)$*,*
3. $f > \overline{\overline{f}}$ *then* $a_1^* \leq \frac{3}{4\alpha}$*,* $q_{1I}\left(a_1^*, a_2^*\right) \leq q_{1I}\left(\frac{3}{4\alpha}, \frac{3}{4\alpha}\right)$ *and* $p_I\left(a_1^*, a_2^*\right) \leq p_I\left(\frac{3}{4\alpha}, \frac{3}{4\alpha}\right)$*.*

Proof. See Appendix. ■

Proposition 1 indicates that, in the case of no favoritism, $f = 0$, the optimal scoring rule produces a level of quality below what could have been obtained under full information. The optimal mechanism under informational asymmetry reduces the supplier's quality provision and internalizes the informational cost of the buyer.[30]

In the case of favoritism, $f > 0$, a distortion is introduced in the *SRA*: indeed, the buyer assigns a higher weight to quality a_1, of which I is largely endorsed, and the buyer assigns less weight to the other quality, a_2, of which I is scarcely endorsed. As a result, it is more likely that supplier I wins the auction. At the same time, the higher a_1 is, the higher I's market power will be and so the higher the price component of her bid will be.

Proposition 1 states that a tradeoff exists between the quality of the service and the level of favoritism. When f is small, an increase in the probability of the victory of the incumbent is obtained by increasing weight a_1 of the quality the incumbent is more endorsed with. As a result, if the incumbent actually wins, the price paid and the quality of the service provided will be higher with respect to the case where $f = 0$. This result can well explain the unobserved differences in the winning prices between the incumbent firm and entrants, as highlighted in the previous empirical analysis.

Finally, when f is high, that is, when the only concern of the buyer is to award the contract to I, then a large increase in the probability of the victory of I is obtained by reducing both weights a_1 and a_2 in the scoring function, being such reduction far greater for a_2. In this case both the resulting price and the quality of the service will be lower with respect to the case where $f = 0$.

[30] Note that this result is in line with that of Che (1993).

We are aware that this theoretical example cannot represent the exclusive explanation of our empirical results in Section 3. Note, however, that the most natural alternative explanation – an "endogenous entry story", as described in Section 3.2 – is not supported by our empirical analysis.

5 Conclusions

In this paper, we have investigated a small and original database of 192 public procurement auctions for canteen service contracts in Italy, awarded between 2009 and 2013. These are scoring rule auctions (SRAs), i.e., they contain price and quality components and are awarded by public buyers endorsed with discretion when choosing the weights of the price and quality components in the SRA.

Our analysis proceeded in three steps. First, we presented the descriptive statistics and preliminary investigations of our database. These highlight the following:

i) in 56% of our sample, the winner is the incumbent supplier, i.e., the firm that was providing the canteen's service in the period immediately before the recorded auction takes place;

ii) the competition is lower, and the price paid by the public buyer is higher, when the winner is the incumbent supplier and when the buyer is an elected body.

Second, running an econometric analysis, we showed that neither the service or buyer characteristics or competition nor the overall weight of quality given to the scoring function can explain our findings. In particular, exploiting public buyers' heterogeneity and their choices in weights in the SRA, we provide an empirical analysis to highlight auctions in which prices fail to be predicted by the observable auction characteristics.

While the number of observations in the whole dataset that fail to be predicted by our model depend on the width of the confidence intervals and the type of standard errors used, we constantly found that auctions in which the incumbent supplier wins are significantly more likely to fail our tests. Similarly, those auctions are more likely to have been managed by an elected buyer. All these results are confirmed by different robustness checks. Moreover, further investigations lead us to reject an endogenous entry story, according to which winning rebates are lower because the number of bidders - which depends on some signal potential entrants have received - is lower.

Third, we provided a simple theoretical setting in which a public buyer can favor an incumbent bidder in an SRA with multidimensional quality, and in so doing, she/he will pay a higher price than in the absence of favoritism. Indeed, to increase the probability that the incumbent will win the SRA, the public buyer will design the SRA with a higher weight for the quality component the incumbent is endorsed with, giving the incumbent market power and prompting him/her to offer a lower winning rebate (i.e., to offer the service for a higher price).

Taken together, our results suggest that multidimensional SRAs with more than one quality component could be easily distorted by public buyers. This is a relevant point since SRAs are increasingly being adopted as public procurement awarding formats in many countries, and a buyer's bias toward predetermined suppliers could annihilate competition and its potential positive effects.[31] Furthermore, even if favoritism is not present, the scoring function should be designed with great care because it may reduce competition. As Che (1993) pointed out, a large weight assigned to quality will provide, because of informational rent, excessive market power to the most efficient firms. In this respect, the methodology developed in Section 3 could be adopted for periodical screening by a regulator in charge of monitoring the performance of SRAs for public procurement and checking for their correct implementation.

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6 Proof of Proposition 1

The model of Section 2 is solved via backward induction starting from stage 2. Algebra and mathematical details are given in the *Online Appendix*.[32]

6.1 Stage 2

In stage 2, we define the equilibrium bid $B_j = \{q_{1j}, q_{2j}, p_j\}$ for bidders $j \in \{I, E\}$. As a convention, we refer to the buyer using "he" and to each bidder using "she".

Following Asker and Cantillon (2008), consider bidder j who has won the contract with a score to fulfill t_j^W. She chooses q_{1j}, q_{2j}, p_j, given the score submitted t_j^W, to maximize her profit:

$$\begin{aligned} \max_{Q} \ \pi_j &= p_j - \sum_{i=1}^{2} \frac{1}{\theta_{ij}} q_i^2 \\ s.t. \ \ t_j^W &= \sum_{i=1}^{2} a_i q_{ij} - p_j \end{aligned} \tag{9}$$

Replace p_j in the objective function to obtain:

$$\max_{Q} \ \sum_{i=1}^{2} \left(a_i q_{ij} - \frac{1}{\theta_{ij}} q_i^2 \right) - t_j^W \tag{10}$$

An important feature here is that, in equilibrium, the optimal provision of quality q_{ij} for bidder j is independent from t_j^W. Define

$$k(\theta_j) = \max_{Q} \ \sum_{i=1}^{2} \left(a_i q_{ij} - \frac{1}{\theta_{ij}} q_i^2 \right) \tag{11}$$

as the bidder j pseudotype. Solving the pseudotype maximization problem, we obtain that once the scoring rule is fixed, in equilibrium the quality decision of bidder j depends only on the bidder's ability in that quality. The optimal decision of bidder j for quality i is:

$$q_{ij}^* = \frac{1}{2} a_i \theta_{ij} \tag{12}$$

The set of pseudotypes is an interval in R, and the density inherits the smooth property of θ_j (that is distributed according to a continuous joint density function). The maximized pseudotype becomes:

$$k(\theta_j) = \sum_{i=1}^{2} \frac{1}{4} a_i^2 \theta_{ij} \tag{13}$$

[32] The Online Appendix is available at: https://sites.google.com/site/riccardocambonima/

The use of a quadratic cost function results in a pseudotype linear in the random variables θ_1 and θ_2. Denote $\frac{1}{4}a_i^2 = c_i$ to ease notation. By convolution, the cumulative distribution function (CDF) of $k(\theta)$ is given by the following piecewise function:

$$F(k) = \begin{cases} \frac{1}{2}\frac{k^2}{c_1c_2} & \text{if } 0 \leq k \leq c_2 \text{ and } c_2 \leq c_1 & (1) \\ \frac{1}{2}\frac{2k-c_2}{c_1} & \text{if } c_2 < k \leq c_1 \text{ and } c_2 \leq c_1 & (2) \\ 1-\frac{1}{2}\frac{(c_1+c_2-z)^2}{c_1c_2} & \text{if } c_1 < k \leq c_1+c_2 \text{ and } c_2 \leq c_1 & (3) \\ \frac{1}{2}\frac{k^2}{c_1c_2} & \text{if } 0 \leq k \leq c_1 \text{ and } c_1 \leq c_2 & (4) \\ \frac{1}{2}\frac{2k-c_1}{c_2} & \text{if } c_1 < k \leq c_2 \text{ and } c_1 \leq c_2 & (5) \\ 1-\frac{1}{2}\frac{(c_1+c_2-z)^2}{c_1c_2} & \text{if } c_2 < k \leq c_1+c_2 \text{ and } c_1 \leq c_2 & (6) \end{cases} \tag{14}$$

Consider that $k \in [0, (c_1 + c_2)]$. We then apply Asker and Cantillon's (2008) Theorem 1 and Corollary 1: the equilibrium bid (Q, p) in the scoring rule is equivalent to the equilibrium bid in an equivalent first price auction (FPA) where the bidder' private valuations are given by their pseudotypes and where bidders' scores are replaced by bidders' bids. The equilibrium bid in an FPA is given by:

$$t(k) = k - \frac{1}{F^{N-1}(k)} \int_0^k F^{N-1}(z)dz \tag{15}$$

$t(k)$ always exists, can be analytically estimated, and it is a finite number[33]. To conclude the characterization of equilibrium bids of a scoring rule auction in this setting, we need to define the price component p_j of the bid $B_j = (q_{1j}, q_{2j},\ p_j)$ submitted by player j. It is obtained as the residual component of the scoring function, where both scores and quality have been replaced with the equilibrium values derived above:

$$p_j = 2k(\theta_j) - t(k(\theta_j)) \tag{16}$$

In equilibrium a 1:1 relation exists between pseudotypes, scores and prices. That is, each pseudotype $k(\theta_j)$ bids a unique score t_j and a unique price p_j. However, quality provision depends only on the bidder's specific ability to provide that quality. Hence the same pseudotype may produce different level of qualities q_{1j} and q_{2j} because different configurations of $(\theta_{1j}, \theta_{2j})$ may end up having the same pseudotype, depending on the scoring rule chosen by the buyer.

6.2 Stage 1

In stage 1, the buyer observes only bidder's I type and has to decide the optimal mechanism a_1 and a_2 to award the contract. Depending on the scoring rule chosen, the utility he will receive is equal to the utility provided by the incumbent plus the

[33]Equilibrium bids are derived in Section 1, online appendix.

expected utility provided by the entrant, with each utility weighted for the probability of victory of the related bidder. Finally, we introduce favoritism as an additive utility that the buyer receives if the incumbent wins. We denote it by f. f is a measure of how much the buyer is willing to distort the scoring rule in order to let the incumbent win. If $f = 0$ there is no favoritism, if $f > 0$ there is favoritism. Hence the maximization problem becomes:

$$U = \Pr(a\ win) \cdot U(\overline{\theta}_I | a_1, a_2) + f \cdot \Pr(a\ win) + [1 - \Pr(a\ win)] \cdot E[U(\theta_E | a_1, a_2)] \quad (17)$$

where $U(\theta_j | a_1, a_2)$ is the utility provided by bidder j, with $j \in \{I, E\}$, depending on the scoring rule chosen by the buyer, while $\Pr(a\ win)$ is the probability that the incumbent wins.

We use the following solution strategy:

1. We derive the optimal scoring rule without favoritism.

   (a) In doing so, we have to derive $\Pr(a\ win)$ - the probability that the incumbent wins - then $U(\overline{\theta}_I | a_1, a_2)$ - the utility of the incumbent -, and finally $E[U(\theta_E | a_1, a_2)]$, which is the expected utility of the entrant.

   (b) Then we solve the maximization problem and we prove that there exists only one couple a_1, a_2 which is the global maximum for the function U because there is no other local maximum, the function is continuous, and because the boundary solutions for $a_i = 0$ and $a_i = +\infty$ provide a lower utility.

2. Then, we introduce favoritism. We prove that:

   (a) if $f \geq 0$ then the optimal weights (a_1^*, a_2^*) have to be constructed such that $a_1^* \geq a_2^*$: the buyer has to assign, in the scoring rule, more importance to the quality in which the incumbent is more efficient. Moreover, the optimal weight in the scoring rule for the quality in which the incumbent is less efficient is below its level without favoritism, while the optimal weight in the scoring rule for the quality in which the incumbent is more efficient can be above or below.

   (b) With infinite favoritism, due to excessive market power given to the incumbent, the provision of both quality and price are lower than under the case without favoritism. Infinite favoritism can be interpreted as the case where the buyer is no longer concerned by the quality of the service, but only in having the preferred bidder wins the auction.

   (c) With a finite level of favoritism, the price paid by CA in case of victory of the incumbent may be above (with low values of f) or below (with high level of f) that in the case without favoritism. In particular, we prove that there exists a threshold $\overline{f}$ such that if $f < \overline{f}$, then - in case of victory of the incumbent - quality 1 provision is higher and price is also higher than in the case without favoritism. There exists also a treshold $\overline{\overline{f}}$ such that if $f \in \left[\overline{f}, \overline{\overline{f}}\right]$ then quality provision will be below the case without favoritism but the price remains above and, finally, if $f > \overline{\overline{f}}$ both quality and price will be below the case of $f = 0$.

6.2.1 Derivation of $\Pr(a\ win)$

The probability that bidder $\overline{\theta}_I$ wins the auction is equivalent to the probability that the unobserved pseudotype $k(\theta_E)$ is lower than the observed pseudotype $k\left(\overline{\theta}_I\right)$, given the scoring rule chosen:

$$\begin{aligned} \Pr(a\ win) &= \\ \Pr\left[k(\theta_E) < k\left(\overline{\theta}_I\right)\right] &= \\ \Pr(a_1^2\theta_{1E} + a_2^2\theta_{2E} &< a_1^2\overline{\theta}_{1I} + a_2^2\overline{\theta}_{2I}) = \\ \Pr(Z &< 4k\left(\overline{\theta}_I\right)) = \end{aligned} \tag{18}$$

where $Z = a_1^2\theta_{1E} + a_2^2\theta_{2E}$ is a convolution of the two random variables θ_{1E} and θ_{2E}. The cumulative density function of Z, evaluated in $4k\left(\overline{\theta}_I\right)$, depends on the optimal values of a_1 and a_2. Six cases are possible, given the relative values of a_1 with respect to a_2 and of a_1,a_2 with respect to $4k\left(\overline{\theta}_I\right)$, as described in Section 2 of the online appendix. However, given the pseudotype $k\left(\overline{\theta}_I\right) = \frac{1}{4}a_1^2$ of the incumbent, three of the cases mentioned above are impossible. In the remaining three cases we have that $\Pr(a\ win)$ is equal to:

$$\begin{array}{lll} \textit{Case} & \Pr\ (Z<4k\left(\overline{\theta}_I\right)\ ) & \\ 1. & \frac{1}{2}\frac{a_1^2}{a_2^2} & \text{Only if } a_1 = a_2 \\ 2. & \frac{1}{2}\frac{2a_1^2 - a_2^2}{a_1^2} & \text{Only if } a_1 > a_2 \\ 4. & \frac{1}{2}\frac{a_1^2}{a_2^2} & \text{Only if } a_1 < a_2 \end{array} \tag{19}$$

6.2.2 Derivation of $U(\overline{\theta}_I|a_1, a_2)$

The utility of the incumbent $U(\overline{\theta}_I|a_1, a_2)$ depends on its bid $B_I = (q_{1I}, q_{2I}, p_I)$. Quality provision depends only on its ability in the given quality and is equal to $q_{1I} = \frac{1}{2}a_1$ for the quality in which the incumbent is more efficient and to $q_{2I} = 0$ for the quality in which the incumbent is less efficient. As for the price, we replace the incumbent pseudotype - which is equal to $k_I = \frac{1}{4}a_1^2$ - and equation (15) - the equilibrium bid in an FPA with two participants - in equation (16) to obtain:

$$p_I = \frac{1}{4}a_1^2 + \frac{1}{F(k_I)}\int_0^{k_I} F(z)dz \tag{20}$$

While k_I is known by the buyer, the CDF is a piecewise function, the exact piece to be used depends on the optimal scores a_1, a_2 which can be obtained only at the end of the buyer's optimization process. As before, we have to consider six cases that comes from the piecewise distribution of $F(k)$. But only three of these are possible given the incumbent's pseudotype, which are:

$$\begin{array}{lll} \textit{Case} & p_I & \\ 1. & p_I = \frac{1}{3}a_1^2 & \text{Only if } a_1 = a_2 \\ 2. & p_I = \frac{1}{4}a_1^2 + \frac{1}{12}\left(\frac{3a_1^4 - 3a_1^2a_2^2 + a_2^4}{2a_1^2 - a_2^2}\right) & \text{Only if } a_1 > a_2 \\ 4. & p_I = \frac{1}{3}a_1^2 & \text{Only if } a_1 < a_2 \end{array} \tag{21}$$

And finally the utility provided by the incumbent is:

$$U(\overline{\theta}_I|a_1, a_2) = \frac{1}{2}a_1 - \alpha p_I \tag{22}$$

6.2.3 Derivation of $E[U(\theta_E|a_1, a_2)]$

Consider the utility of the expected entrant, in case she wins.

$$\begin{aligned} E[U(\theta_E|a_1, a_2)] &= E[q_{1E} + q_{2E} - \alpha p_E] \\ &= E\left[\frac{1}{2}a_1\theta_{1E} + \frac{1}{2}a_2\theta_{2E} - \alpha p_E\right] \\ &= \frac{1}{2}a_1 E\left[\theta_{1E}\right] + \frac{1}{2}a_2 E\left[\theta_{2E}\right] - \alpha E\left[p_E\right] \end{aligned} \tag{23}$$

Note that, even if the CDF of k is a piecewise function, its expected value is simply equal to:

$$E[k] = \frac{1}{8}\left(a_1^2 + a_2^2\right) \tag{24}$$

To derive $E[\theta_{iE}]$ consider that, in order to be part of $E[k]$, a generic pair $(\theta_{1E}, \theta_{2E})$ has to satisfy three conditions:

$$\begin{aligned} \theta_{1E} &\in [0, 1] \\ \theta_{2E} &\in [0, 1] \\ \frac{1}{4}\left(a_1^2\theta_{1E} + a_2^2\theta_{2E}\right) &= \frac{1}{8}\left(a_1^2 + a_2^2\right) \end{aligned} \tag{25}$$

The distribution of each θ_{iE} remains uniform because there is a one-to-one relation between θ_{1E} and θ_{2E}, that is, once a value θ_{1E} is fixed, then only one value of θ_{2E} (at most) will be such that the pair satisfies the three conditions above, and this interval is continuous. Hence the problem of finding the expected values of $E\left[\theta_i^W\right]$ given the expected pseudotype $E[k]$ reduces in finding the extreme values of this interval. Those are given in the table below:

$$\begin{array}{llllllll} & \underline{\theta}_1^w & \overline{\theta}_1^w & \underline{\theta}_2^w & \overline{\theta}_2^w & E\left[\theta_1^w\right] & E\left[\theta_2^w\right] & C1 \\ 1 & \frac{1}{2}\left(1-\frac{a_2^2}{a_1^2}\right) & 1 & \frac{1}{2}\left(1-\frac{a_1^2}{a_2^2}\right) & 1 & \left(\frac{3}{4}-\frac{1}{4}\frac{a_2^2}{a_1^2}\right) & \left(\frac{3}{4}-\frac{1}{4}\frac{a_1^2}{a_2^2}\right) & a_1=a_2 \\ 2 & \frac{1}{2}\left(1-\frac{a_2^2}{a_1^2}\right) & \frac{1}{2}\left(1+\frac{a_2^2}{a_1^2}\right) & 0 & 1 & \frac{1}{2} & \frac{1}{2} & a_1>a_2 \\ 3 & 0 & 1 & \frac{1}{2}\left(1-\frac{a_1^2}{a_2^2}\right) & \frac{1}{2}\left(1+\frac{a_1^2}{a_2^2}\right) & \frac{1}{2} & \frac{1}{2} & a_1<a_2 \\ 4 & 0 & \frac{1}{2}\left(1+\frac{a_2^2}{a_1^2}\right) & 0 & \frac{1}{2}\left(1+\frac{a_1^2}{a_2^2}\right) & \frac{1}{4}\left(1+\frac{a_1^2}{a_2^2}\right) & \frac{1}{4}\left(1+\frac{a_2^2}{a_1^2}\right) & (a_1=a_2) \end{array} \tag{26}$$

Case 4 is equivalent to case 1, so we can disregard it.

We have finally to derive the expected value of the price of the potential entrant. The expected price is given by:

$$E\left[p_E\right]=E\left[k\right]+\frac{1}{F(E\left[k\right])}\int_0^{E[k]}F(z)dz \tag{27}$$

Since $F(k)$ is a piecewise function, we still have to consider all six cases. But only four are possible:

$$\begin{array}{lll} (1) & \text{if} & a_1=a_2 \\ (2) & \text{if} & a_1>a_2 \\ (4) & \text{if} & a_1=a_2 \\ (5) & \text{if} & a_1<a_2 \end{array} \tag{28}$$

Cases (1) and (4) are equivalent. The expected price becomes[34]:

$$\begin{array}{ll} E\left[p_b\right]=E\left[k\right]+\frac{1}{3}E\left[k\right] & \text{if} \quad a_1=a_2 \\ E\left[p_b\right]=E\left[k\right]+\frac{1}{16}a_1^2+\frac{1}{48}a_2^2 & \text{if} \quad a_1>a_2 \\ E\left[p_b\right]=E\left[k\right]+\frac{1}{16}a_2^2+\frac{1}{48}a_1^2 & \text{if} \quad a_1<a_2 \end{array} \tag{29}$$

6.2.4 Solution of the maximization problem without favoritism[35]

To solve the maximization problem, we have to consider three cases:

$$\begin{array}{rcl} a_1 & = & a_2 \\ a_1 & \geq & a_2 \\ a_1 & \leq & a_2 \end{array} \tag{30}$$

[34] Calculated with mathematica

[35] All computations are in Section 3 of the online appendix.

We solve each of these three cases, and the solution must satisfy the above condition, to be accepted.

We solve the first case, $a_1 = a_2$, and we obtain what is, at least, a local maximum (we consider also the determinant of the Hessian matrix):

$$a_1 = a_2 = \frac{3}{4\alpha} \tag{31}$$

The utility in this case assumes value $U\left(\frac{3}{4\alpha}, \frac{3}{4\alpha}\right) = \frac{3}{16\alpha}$. We then prove that if $a_1 > a_2$, only one solution exists where both the first order conditions are equal to 0 (a necessary condition for a local maximum to exists). However, looking at the Hessian matrix, reveals that this point is a saddle point. Finally, if $a_1 < a_2$, no point exists such that both FOCs are equal to zero.

To prove that the unique local maximum we found is a global maximum, we have to prove that the function is continuous and we have to look at what happens when $a_i = 0$ and when $a_i \to +\infty$ (the boundary values).

- To prove that the function is continuous. It is for any values of a_1 and a_2, because in all the three cases, if $a_1 = a_2$, then $U = \frac{1}{2}a - \frac{1}{3}a^2\alpha$
- Check the utility if $a_i = 0$. If $a_1 = a_2 = 0$ then $U(0,0) = 0$. If $a_1 = 0$ then the maximum value U can assume is equal to $U\left(\frac{2}{3\alpha}, 0\right) = \frac{1}{6\alpha} < \frac{3}{16\alpha}$. Instead, if $a_2 = 0$ then the maximum value U can assume is given by $U\left(0, \frac{2}{3\alpha}\right) = \frac{1}{12\alpha} < \frac{3}{16\alpha}$
- Check the utility if $a_i \to +\infty$. If $a_1 = a_2 \to +\infty$ then $U \to -\infty$. If $a_1 \to +\infty$ then $U \to -\infty$ for any value of a_2. If $a_2 \to +\infty$ then $U \to -\infty$ for any value of a_1.

Hence we can conclude that, without favoritism, there exists a unique couple $a_1 = a_2 = \frac{3}{4\alpha}$ such that the function is maximized, and this is a global maximum.

Comparison with quality provision under first best (full information) case
With full information, the buyer can offer to bidders a contract which maximizes his utility subject to a zero profit condition for bidders. Replacing p in the buyer's utility function and solving the maximization problem, we obtain:

$$q_i^{FB} = \frac{1}{2\alpha}\theta_j \tag{32}$$

which is lower than quality provision under the optimal scoring rule $\left(\frac{3}{4\alpha}, \frac{3}{4\alpha}\right)$, obtainted from equation (12):

$$q_i^{*} = \frac{3}{8\alpha}\theta_j \tag{33}$$

6.3 Favoritism

We now introduce favoritism. The function the buyer maximizes becomes:

$$\max U = \ H(a_1, a_2) + f \cdot \Pr_{(a\ win)}(a_1, a_2) \tag{34}$$

where $H(a_1, a_2)$ is the continuous function studied in the previous section. It has a single maximum point, where $a_1 = a_2 = \frac{3}{4\alpha}$. We define this global maximum as $(\overline{a}_1, \overline{a}_2)$. Consider also that:

$$\begin{aligned} \lim_{\substack{a_1 \to \infty \\ a_2 \longrightarrow \infty}} H(a_1, a_2) &= -\infty \\ \lim_{a_1 \to \infty} H(a_1, a_2) &= -\infty \\ \lim_{a_2 \to \infty} H(a_1, a_2) &= -\infty \end{aligned} \tag{35}$$

And,

$$\begin{aligned} \nexists \widetilde{a}_1 &: \quad H(\widetilde{a}_1, 0) > H(\overline{a}_1, \overline{a}_2) \\ \nexists \widetilde{a}_2 &: \quad H(0, \widetilde{a}_2) > H(\overline{a}_1, \overline{a}_2) \\ H(0,0) &< \quad H(\overline{a}_1, \overline{a}_2) \end{aligned} \tag{36}$$

$H(a_1, a_2)$ is twice differentiable.

$\Pr(a\ win)$ is a continuous function globally increasing in a_1 and globally decreasing in a_2, as can be seen from first order conditions: $\frac{\partial \Pr(awin)}{\partial a_1} > 0$ and $\frac{\partial \Pr(awin)}{\partial a_2} < 0$. Moreover $\Pr(a\ win) \in [0, 1]$ and f is a positive number.

6.3.1 A maximum exists

Consider that, at least for finite values of f, a maximum exists. In fact, $U(a_1, a_2)$ has an upper bound because:

$$U(a_1, a_2) \leq H(\overline{a}_1, \overline{a}_2) + f \tag{37}$$

since $\max H(a_1, a_2) = H(\overline{a}_1, \overline{a}_2)$ and $\max \Pr(a_1, a_2) = 1$. Moreover consider that:

$$\begin{aligned} \lim_{a_1 \to \infty} H(a_1, a_2) + f \cdot \Pr(a_1, a_2) &= -\infty \\ \lim_{a_2 \to \infty} H(a_1, a_2) + f \cdot \Pr(a_1, a_2) &= -\infty \end{aligned} \tag{38}$$

Hence the function has at least one maximum and this maximum is finite.

6.3.2 Domain of the optimal weights of the scoring function

Define by (a_1^*, a_2^*) the weights of the scoring function that maximizes the buyer's utility given a level of favoritism $f \geq 0$. Then $a_1^* = a_2^* = \frac{3}{4\alpha}$ if $f = 0$. We now prove that $a_1^* > a_2^*$[36] if $f > 0$. Moreover, we prove that $a_1^* \in \left[\frac{2}{3\alpha}, \widehat{a}_1\right]$ where $\widehat{a}_1$ is a finite value such that $\widehat{a}_1 > \frac{3}{4\alpha}$ and $a_2^* \in \left[0, \frac{3}{4\alpha}\right]$. We split the proof into two parts.

[36] Or, at most, $(a_1^*, a_2^*) = (\overline{a}_1, \overline{a}_2)$

First:

$$\text{if } f > 0 \rightarrow \nexists \left(\widetilde{a}_1, \widetilde{a}_2\right) : \left(\widetilde{a}_1 < \widetilde{a}_2 \cup U\left(\widetilde{a}_1, \widetilde{a}_2\right) > U\left(\overline{a}_1, \overline{a}_2\right)\right) \tag{39}$$

Consider first that $H\left(\widetilde{a}_1, \widetilde{a}_2\right) < H\left(\overline{a}_1, \overline{a}_2\right)$. Then consider that, if $\widetilde{a}_1 < \widetilde{a}_2$, then $\Pr_{(a\ win)}\left(a_1, a_2\right) = \frac{1}{2}\frac{a_1^2}{a_2^2} < \frac{1}{2}$ while $\Pr_{(a\ win)}\left(\overline{a}_1, \overline{a}_2\right) = \frac{1}{2}$. It follows that $U\left(\widetilde{a}_1, \widetilde{a}_2\right) < U\left(\overline{a}_1, \overline{a}_2\right)$. This concludes the proof.

Second:

$$\text{if } f > 0 \rightarrow \nexists \left(\widetilde{a}_1, \widetilde{a}_2\right) : \left(\widetilde{a}_1 = \widetilde{a}_2 \cup U\left(\widetilde{a}_1, \widetilde{a}_2\right) > U\left(\overline{a}_1, \overline{a}_2\right)\right) \tag{40}$$

Consider first that $H\left(\widetilde{a}_1, \widetilde{a}_2\right) \leq H\left(\overline{a}_1, \overline{a}_2\right)$. Then consider that, if $\widetilde{a}_1 = \widetilde{a}_2$, then $f \cdot \Pr_{(a\ win)}\left(\widetilde{a}_1, \widetilde{a}_2\right) = f \cdot \Pr_{(a\ win)}\left(\overline{a}_1, \overline{a}_2\right) = \frac{1}{2}\frac{a_1^2}{a_2^2}p = \frac{p}{2}$. It follows that $U\left(\widetilde{a}_1, \widetilde{a}_2\right) \leq U\left(\overline{a}_1, \overline{a}_2\right)$. This concludes the proof.

Hence The couple $\left(a_1^*, a_2^*\right)$ that maximizes $U\left(a_1, a_2\right)$ in case $f > 0$ must be constructed such that $a_1^* > a_2^*$ or, at most, $\left(a_1^*, a_2^*\right) = \left(\overline{a}_1, \overline{a}_2\right)$. Finally, consider that a necessary condition for a maximum (which we know to exists) is that the point that maximizes U will be the one such that:

$$\begin{aligned} \nabla U\left(a_1, a_2\right) &= 0 \\ \begin{bmatrix} \frac{\partial U}{\partial a_1} \\ \frac{\partial U}{\partial a_2} \end{bmatrix} &= \begin{bmatrix} \frac{\partial H}{\partial a_1} \\ \frac{\partial H}{\partial a_2} \end{bmatrix} + f \begin{bmatrix} \frac{\partial \Pr_{(a\ win)}(a_1, a_2)}{\partial a_1} \\ \frac{\partial \Pr_{(a\ win)}(a_1, a_2)}{\partial a_2} \end{bmatrix} = 0 \end{aligned} \tag{41}$$

$$\begin{aligned} \nabla H\left(a_1, a_2\right) &= -f \cdot \nabla \Pr_{(a\ win)}\left(a_1, a_2\right) \\ \begin{bmatrix} \frac{\partial H}{\partial a_1} \\ \frac{\partial H}{\partial a_2} \end{bmatrix} &= -f \begin{bmatrix} \frac{\partial \Pr_{(a\ win)}(a_1, a_2)}{\partial a_1} \\ \frac{\partial \Pr_{(a\ win)}(a_1, a_2)}{\partial a_2} \end{bmatrix} \end{aligned} \tag{42}$$

This, with some manipulations, can be expressed as:

$$\frac{\partial H}{\partial a_2}\left(\frac{\partial \Pr_{(a\ win)}\left(a_1, a_2\right)}{\partial a_2}\right)^{-1} = \frac{\partial H}{\partial a_1}\left(\frac{\partial \Pr_{(a\ win)}\left(a_1, a_2\right)}{\partial a_1}\right)^{-1} \tag{43}$$

To define the domain of a_1, the above condition can be expressed in terms of a_1 and the ratio $\varepsilon_a = \frac{a_2}{a_1}$ which we know must be such that $\varepsilon_a \in [0, 1]$.We show in Section 4 of the online appendix that there exists for a_1 a lower bound equal to $\frac{2}{3\alpha}$ and a finite upper bound greater than $\frac{3}{4\alpha}$ (the optimal weight for a_1 without favoritism) such that if a_1 does not belong to this interval, then equation (43) has no solution. Similarly, equation (43) can be expressed in terms of a_2 and the ratio $\varepsilon_a = \frac{a_2}{a_1}$; it has solutions only if $a_2^* \in \left[0, \frac{3}{4\alpha}\right]$.

6.3.3 Infinite favoritism

Suppose $f \to \infty$. Consider the buyer's maximization problem:

$$\max U = \; H(a_1, a_2) + f \cdot \underset{(a\ win)}{\Pr}(a_1, a_2) \tag{44}$$

It is optimal to choose $a_2 = 0$. In this case in fact $\Pr_{(a\ win)}(a_1, a_2) = 1\ \forall a_1$[37]. The buyer then will choose a_1 to maximize the residual part of her utility function: $H(a_1, a_2)$. The problem becomes:

$$\max \; H(a_1, 0) \tag{45}$$

$$\max \; \frac{1}{2}a_1 - \frac{3\alpha}{8}a_1^2 \tag{46}$$

The solution is: $a_1 = \frac{2}{3\alpha}$. Since $a_1 < \frac{3\alpha}{4}$ then the quality provision will be below the case without favoritism; the price is also lower. In fact, in this case the buyer is concerned only with ensuring the preferred bidder wins the auction. To do so, he must give a very high market power to the incumbent and, as a result, quality 1 becomes very costly and not as important for the buyer.

6.3.4 Proof of Proposition 1 under $f > 0$[38]

Define by (a_1^*, a_2^*) the weights of the scoring function that maximize the buyer's utility given a level of favoritism $f \geq 0$. Using all results derived above, we are now going to prove that there exist two tresholds in the level of favoritism f - which we define as $\overline{f}$ and $\overline{\overline{f}}$ with $\overline{\overline{f}} > \overline{f}$, such that:

- if $f = 0$ then this is the benchmark case with $(a_1^*, a_2^*) = \left(\frac{3}{4\alpha}, \frac{3}{4\alpha}\right)$ already proved before.
- if $f \in \left[0, \overline{f}\right]$ then $p_I(a_1^*, a_2^*) \geq p_I\left(\frac{3}{4\alpha}, \frac{3}{4\alpha}\right)$ and quality provision is above the case without favoritism: $q_{1I}(a_1^*, a_2^*) \geq q_{1I}\left(\frac{3}{4\alpha}, \frac{3}{4\alpha}\right)$. If $f > \overline{f}$ then $q_{1I}(a_1^*, a_2^*) < q_{1I}\left(\frac{3}{4\alpha}, \frac{3}{4\alpha}\right)$.
- if $f \in \left[0, \overline{\overline{f}}\right]$ then $p_I(a_1^*, a_2^*) \geq p_I\left(\frac{3}{4\alpha}, \frac{3}{4\alpha}\right)$. If $f > \overline{\overline{f}}$ then $p_I(a_1^*, a_2^*) < p_I\left(\frac{3}{4\alpha}, \frac{3}{4\alpha}\right)$.

Favoritism and the relative importance of a_1 w.r.t. a_2 Define by (a_1^*, a_2^*) the weights of the scoring function that maximize the buyer's utility given a level of favoritism $f \geq 0$. It is convenient to use the ratio ε_a of the two weights of the scoring

[37] Recall $\Pr_{(a\ win)}(a_1, a_2) = 1 - \frac{1}{2}\frac{a_2^2}{a_1^2}$

[38] All calculations are in Section 5, online appendix.

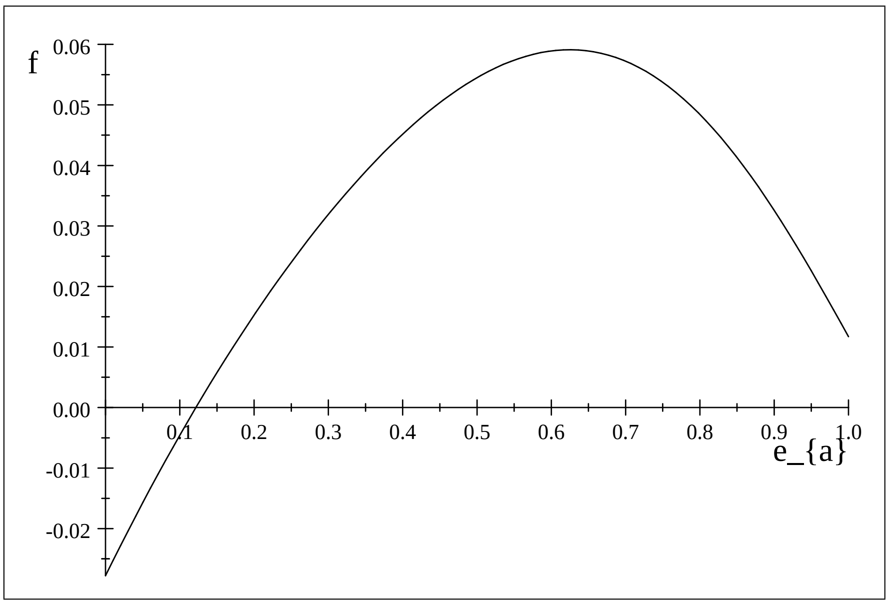
rule to get rid of a_2. To this end, define $\varepsilon_a = \frac{a_2}{a_1}$, such that $\varepsilon_a \in [0,1]$. Then equation (43) can be stated as:

$$a_1^* = \frac{1}{\alpha}\frac{24+6\left(\varepsilon_a^3-\varepsilon_a^2\right)}{11\varepsilon_a^4-15\varepsilon_a^2+36} \tag{47}$$

Using equation (47) and one of the two FOCs of the buyer's maximization problem, it is possibile to express favoritism as a function of the relative importance of a_1^* and a_2^*:

$$f = \frac{1}{\alpha}\frac{3\left(4+\varepsilon_a^2\left(\varepsilon_a-1\right)\right)\left(\varepsilon_a\left(108+\varepsilon_a\left(11\varepsilon_a^3-30\varepsilon_a-73\right)\right)-12\right)}{4\left(11\varepsilon_a^4-15\varepsilon_a^2+36\right)^2} \tag{48}$$

Plotting (48) we obtain that:



(Graph 1)

It immediately follows that there exists a level of f, which we denote as $\overline{\overline{\overline{f}}}$, such that if $f > \overline{\overline{\overline{f}}}$ then no internal solution is possible. In this case, the optimal boundary solution is for $(a_1^*, a_2^*) = \left(\frac{2}{3\alpha}, 0\right)$. In fact, the optimal weights have to be within the domain of (a_1^*, a_2^*) derived above and, of all the boundary solutions of that domain, this is the utility-maximizing one. We obtain that $\overline{\overline{\overline{f}}} \approx 0.059$[39].

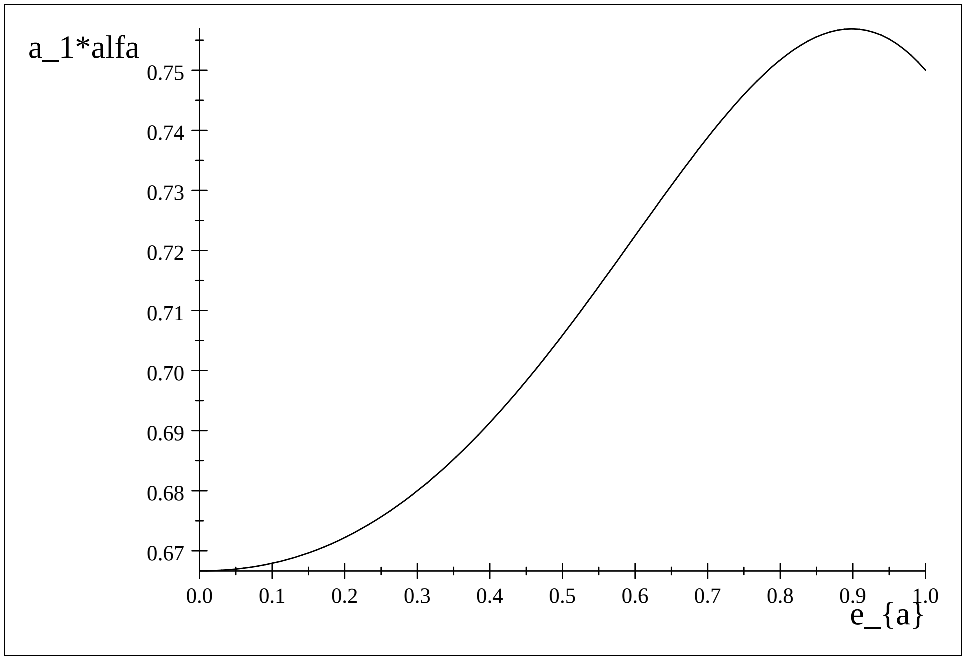
Consider the case of $\varepsilon_a^* = 1$. In this case, from (48) we obtain $f = \frac{3}{256\alpha}$. For any value of $f \in \left[\frac{3}{256\alpha}, \overline{\overline{\overline{f}}}\right]$, two local maximum (a_1^*, a_2^*) exists. For "normal" values of α, i.e. less than 5, the global maximum is given by the highest of the two solutions for ε_a for any of that level of favoritism[40]. Moreover, the domain of the global-utility-maximizing ratio $\varepsilon_a^* = \frac{a_2^*}{a_1^*}$ can be derived using numerical techniques. It results equal

[39] All numerical solutions are obtained with the software Mathematica.

[40] See Online Appendix.

to $\varepsilon_a^* \in \left[\frac{0.625}{\alpha}, 1\right]$. Hence, it finally follows that the ratio ε_a^* is monotonically decreasing in f.[41]

Quality provision under favoritism Consider that quality 1 provision of the incumbent is proportional to a_1: if $a_1 \geq \frac{3}{4\alpha}$ then the quality provision with favoritism is higher than in the case where $f = 0$; if $a_1 < \frac{3}{4\alpha}$, then the quality provision is lower. Quality 2 provision of the incumbent is always equal to zero. Plotting the solution for a_1 in equation (47), we obtain:



(Graph 2)

From graph (2) it can be seen that there exists a ratio $\bar{\varepsilon}_a$ such that if $\varepsilon_a \in [\bar{\varepsilon}_a, 1]$ then $a_1 \geq \frac{3}{4\alpha}$. Since for each level of favoritism there is an associated optimal ratio ε_a^* which is monotonically decreasing in f, then there exists a level $\overline{f}$ such that if $f \in \left[0, \overline{f}\right]$ then $\varepsilon_a \in [\bar{\varepsilon}_a, 1]$, then $a_1 \geq \frac{3}{4\alpha}$ and hence the quality provision with favoritism is above the quality provision without.

By solving the FOCs of the buyer's utility maximization problem in f and a_2 for $a_1^* = \frac{3}{4\alpha}$ it is possible to derive the maximum level of favoritism such that the quality provision is above than in the case with $f = 0$. Solutions of f are:

$$f = \left\{ \left\{ \frac{3}{256\alpha}, -\frac{1}{2816\alpha} \left(\left(\frac{94}{\sqrt[3]{1314\sqrt{6}+\sqrt{9528\,992}}} + \sqrt[3]{1314\sqrt{6} + \sqrt{9528\,992}} - 7\sqrt{6} \right)^2 - 183 \right) \right\} \right. \quad \text{if} \quad \alpha \neq 0$$

The first solution, $f = \frac{3}{256\alpha}$, yields to $a_1 = a_2 = \frac{3}{4\alpha}$ which is not acceptable, given the constraint $a_1 > a_2$. The second one, instead, which can be numerically approximated to $\overline{f} = \frac{0.050367}{\alpha}$, is that threshold.

[41] For $f \in \left[0, \frac{1}{256\alpha}\right]$ any internal solution is impossible. In this case, $(a_1^*, a_2^*) = \left(\frac{3}{4\alpha}, \frac{3}{4\alpha}\right)$ and $\varepsilon_a^* = 1$.

Price under favoritism We now prove that there exists a level $\overline{\overline{f}}$ such that, as long as $f < \overline{\overline{\overline{f}}}$, then the price paid is higher than in the case without favoritism. Consider first that, for the price with favoritism to be higher with respect to the case where $f = 0$, we need the following condition to be true:

$$p_I\left(a_1^*, a_2^*\right) > p_I\left(\frac{3}{4\alpha}, \frac{3}{4\alpha}\right) \tag{49}$$

where both prices are given by the price equations derived in (20), the first for $a_1^* > a_2^*$ and the second for $a_1 = a_2$. Then the inequality can be simplified setting $a_2 = a_1 \varepsilon_a$ to obtain:

$$a_1 > \frac{1}{\alpha}\sqrt{\frac{\frac{3}{4} - \frac{3}{8}\varepsilon_a^2}{\frac{3}{2} - \varepsilon_a^2 + \frac{1}{6}\varepsilon_a^4}} \tag{50}$$

which becomes a numerical problem using the solution for a_1 in (47). For a solution to be acceptable given the constraint $a_1^* > a_2^*$ it must be that $\varepsilon_a^* \in [0, 1]$. However, if $\varepsilon_a^* \precsim \frac{0.625}{\alpha}$ then the solution is no longer a global maximum. Given these constraints, the approximate solution we found for that inequality is $\varepsilon_a \succsim \frac{0.65}{\alpha}$, which yields $\overline{\overline{f}} \approx 0.058$. Note that $\overline{\overline{f}} < \overline{\overline{\overline{f}}}$, so there exists also a case where both price and quality are below the case of $f = 0$ but an internal solution for (a_1^*, a_2^*) still exists.