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EXCESS RETURNS IN PUBLIC-PRIVATE PARTNERSHIPS: DO GOVERNMENTS PAY TOO MUCH?

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Excess returns in Public-Private Partnerships: Do governments pay too much?

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Abstract

We study the optimal design of Public-Private Partnerships (PPPs) when there is unobservable action on the private party’s side. We show that if the private party does not have negotiating power over the project’s surplus, no inefficient delays are attributable to the moral hazard issue. However, if the private party has negotiating power, the first-best timing is not guaranteed. This time discrepancy is shown to be costly in terms of overall project efficiency. The explicit consideration of the private party’s negotiating power can explain empirical evidence showing that private parties in PPPs reap excess returns.

Keywords: public projects; public-private partnerships; moral hazard; real options; investment timing.

JEL classification: D81, D82, D86, H54

1 Introduction

The provision of public services frequently implies a contractual or market relationship between the public sector and the private sector (Quiggin, 2005). An internationally established form of such a relationship goes under the name of Public Private Partnership (PPP). A PPP is a long-term agreement between a public party and a private party regarding the delivery of a public service. The PPP procurement process can be divided into four main stages: the planning phase, the negotiation phase, the construction phase and the operation phase (Ahadzi and Bowles, 2004). During the planning phase, the public party defines the main characteristics of the project while during the negotiation phase the public party and the private party agree upon the contractual clauses. Then, during the construction and the operation phases the two parties implement the contract. In principle, the private party takes over the financing, constructing and managing of a project in return for a stream of payments that comes directly from the public party and/or indirectly from the users of the project. At the same time, the public party opts for a PPP when this proves to be the best alternative in terms of efficiency and budget management. PPPs are adopted in many sectors as, e.g., in transportation, resource management, health care and others (EPEC, 2019; Engel et al., 2014).

PPPs are a mainstream form of concession agreements but designing them remains a demanding task because of their distinguishing features (Engel et al., 2013; Dewatripont and Legros, 2005): PPPs are long-term contracts, they have varying degrees of complexity, they involve costly and irreversible investments and

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they start generating an uncertain public benefit and an uncertain cash flow as soon as the project becomes operational. Another issue that is often detected in PPPs has to do with agency conflicts that can arise between the public party and the private party. For instance, the private party often possesses a certain expertise that can be the source of information asymmetries. Similarly, there may be moral hazard issues attributed to unobservable action by the private party (see e.g., Martimort and Pouyet, 2008 and Iossa and Martimort, 2015).

Given these aspects of PPPs, there is an ongoing discussion regarding their performance. Iossa and Martimort (2015) and Martimort and Straub (2016) show that well designed PPPs may increase efficiency in public service delivery. However, they can become too costly in the presence of high levels of uncertainty and/or when the projects are highly sophisticated. Using French data Saussier and Tran (2012) find that PPPs are particularly effective in reducing cost and time overruns in public projects. However, they also find that PPPs can be too costly when the remuneration the public party pays to the private party is properly accounted for. Focusing on the cost of PPPs, Gao (2017) and Hellowell and Vecchi (2018) find that the rates of return claimed by private companies participating in PPPs are significantly higher than those that appear on corporate portfolios or other equity assets. Whiterfield (2017) reports that, while at the financial close of a PPP the average required rate of return for the private party is around 12%–15%, the PPP’s shareholders are selling their share for much more.1

Existing theoretical models associate the high rates of return observed in practice with incomplete contracting (see e.g., Dewatripont and Legros, 2005 and references therein). In this paper, we develop a model of complete contracting that allows for the presence of excess returns.

We consider a public party that is contemplating the opportunity to engage in a PPP with a private party and whose goal is the completion of a project that will start generating a public benefit and a cash flow upon delivery. We assume that there is an agency conflict between the two partners such that, while the private party can, by exerting effort, increase the probability of delivering a project of high quality in public-benefit terms, the public party cannot observe whether such effort is actually exerted or not. In order to capture some of the stylized facts of PPPs, e.g., sunk investment costs, uncertain future cash flows and temporal flexibility for the public party,2 we develop a real options model.3

Using the framework proposed by Grenadier and Wang (2005), we show that the public party can design a mechanism and eventually resolve the moral hazard issue.4 However, the mechanism proves to be sensitive to the private party’s negotiating power. The related literature shows that the private party’s negotiating power can be attributed to several reasons. For instance, private firms that are involved in PPPs are usually not well diversified (Hellowell and Vecchi, 2018) and the PPP market is not very competitive (Engel et al., 2014).5

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1Hellowell and Vecchi (2013) assess the expected returns on a sample of 77 contracts signed by health care provider organizations in the UK between 1997 and 2011. They show that the difference between the rates of return of private equity investors and the expected return rates on investments in PPP equity is 9.5%, indicating a high degree of rent extraction by private investors.

2The term temporal flexibility refers to the ability of the public party to postpone the PPP if the current conditions prove to be unfavorable.

3See Dixit and Pindyck (1994).

4Grenadier and Wang (2005) consider both hidden action and hidden information. While here we focus on the effect of hidden action on investment timing and efficiency, there is a growing literature that is instead focusing on information asymmetries, see for instance Shibata (2009) and Shibata and Nishihara (2010).

5According to Hellowell and Vecchi (2018), private firms that are involved in PPPs are usually not well diversified because their portfolios are often comprised by few projects that are concentrated in the infrastructure sector. This lack of diversification can result in construction companies being demanding when they negotiate a PPP. Another possible source of negotiating power for the agent is the degree of market competition that it is facing. Because of PPPs’ complexity, few firms have the capacity and expertise to carry them out. Therefore, a private company can use its market power to its advantage when negotiating the
If the private party does not have negotiating power, the moral hazard issue has no effect on the timing of the investment, that is, the first-best timing is guaranteed, the project’s surplus is shared between the two parties and there are no detrimental effects on the project efficiency. However, if the private party has negotiating power, the optimal investment threshold is not guaranteed, the private party obtains a higher rate of return and there is a cost in terms of project efficiency.

Our work contributes to two strands of the literature. On one hand, there is an established body of papers that discuss PPPs using concepts from mechanism design and contract theory. For instance, Martimort and Pouyet (2008), Hoppe and Schmitz (2013) and Iossa and Martimort (2012) focus on information asymmetries, Hart (2003) and Iossa and Martimort (2015) discuss task bundling, and de Bettignies and Ross (2009) analyze contract incompleteness. On the other hand, there is a growing body of papers that study PPP projects emphasizing their real-option-like characteristics see, e.g., Alonso-Conde et al. (2007), Brandao and Saraiva (2008), Martins et al. (2015) and Blank et al. (2016). Some scholars link these two literature strands see, e.g., Takashima et al. (2010), Soumare and Lai (2016), Buso et al. (2019) and Silaghi and Sarkar (2018).

What distinguishes our work from these papers is that we present a real options model that accounts for both agency conflicts and agents with various levels of negotiating power.

The rest of the paper is organized as follows. Section 2 presents the setup of the model. Section 3 describes the first-best solution and the principal-agent setting. In Section 4 we derive and discuss the optimal contracts with and without negotiating power for the agent. Finally, Section 5 concludes.

2 The model

A public authority (principal) holds the option to develop a public service that requires building a facility with sunk investment cost $I$. The principal delegates the decision to exercise this investment option to a private firm (agent) who possesses a relevant expertise. We assume that the construction of the facility can be carried out instantaneously and that the infrastructure has an infinite life.

The project, once implemented, starts generating a public-benefit flow $b$ and a cash flow $x_t$ that are observable and contractible to both parties. For example, in the case of a highway, $b$ can be interpreted as the value of the commuters’ travel-time savings and $x_t$ as the flow of profits from tolls. In the case of green public transportation the public benefit flow corresponds to the reduced carbon emissions whereas the cash flow corresponds to the fees that the users pay in order to gain access to this infrastructure. Last, in the case of social housing, $b$ can be interpreted as the value of reduced crime and better employment opportunities for the tenants, whereas $x_t$ is the monthly rent that they pay.

The public-benefit flow $b$ is assumed to be an increasing function of a random variable $\theta$ that can turn out to be equal to $\theta_1$ or $\theta_2$ where $\theta_1 > \theta_2$, $\Delta \theta = \theta_1 - \theta_2 > 0$ and $b(\theta_1) > b(\theta_2) > 0$. A draw of $\theta_1$ indicates a “high quality” facility whereas a draw of $\theta_2$ indicates a “low-quality” one.

The agent’s effort plays an important role in obtaining a high quality project and then a high public-benefit flow. When the agent exerts effort, $\theta$ obtains the value $\theta_1$ with probability $q_H \in (0, 1)$ and the value $\theta_2$ with probability $1 - q_H$. On the other hand, if the agent chooses not to exert effort, then the probability of drawing $\theta_1$ is $q_L \in (0, 1)$ whereas the probability of drawing $\theta_2$ is $1 - q_L$, where $q_H > q_L$ and

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6 For the rest of the paper we use female pronouns for the principal and male pronouns for the agent.
7 PPPs are commonly associated with long concession periods (see e.g., Song et al., 2013).
8 In PPPs the private party is in charge of designing the infrastructure (Martimort and Pouyet, 2008; Iossa and Martimort, 2015), which in turn affects the public-benefit flow (de Bettignies and Ross, 2009).
\( \Delta q = q_u - q_l > 0 \). Effort exertion is assumed to be costly for the agent who, when exerting effort, incurs a cost \( \xi > 0 \). Finally, upon the realization of \( \theta \), \( b(\theta) \) obtains its final value, \( b(\theta_1) \) or \( b(\theta_2) \), which remains fixed throughout the life of the project.  

Unlike \( b(\theta) \), the cash flow \( x_t \) is assumed to fluctuate over time. In particular, we assume that \( x_t \) evolves according to a risk-neutral geometric Brownian motion:

\[
\frac{dx_t}{x_t} = \mu dt + \sigma dz_t, \quad x_0 = x
\]  

(1)

where \( \mu \) is the risk-neutral rate of drift, \( \sigma \) is the positive constant volatility and \( dz_t \) is the increment of a standard Wiener process under the risk-adjusted measure. The risk-neutral rate of drift \( \mu \) is assumed to be smaller than the risk-free interest rate \( r \) which means that there is a positive rate-of-return-shortfall \( \delta = r - \mu > 0 \). Therefore, once installed, the project generates a rate of cash flows \( \delta x_t \), either as dividends or as liabilities to the stakeholders, which is equivalent to the payout rate that a potential investor could obtain from projects with comparable risk profiles.  

Eq. (1) is based on the hypothesis that the systematic risk of the project, i.e., the market risk, is separated from the specific risks of the project, i.e., the part of the risk that is not correlated to the market. While the former is remunerated by the market at rate \( \delta \), specific risks can be ultimately reduced to zero through an adequately diversified investment portfolio. However, when such a portfolio cannot be constructed, standard theory suggests that specific risks may require an “extra” premium (Hirshleifer, 1988).  

In PPPs the assessment of project-specific risks is the subject of negotiation between the public party and the private party during the pre-contractual phase. Since an analytical description of the pre-contractual phase is beyond the scope of this paper, we assume that an agent with negotiating power successfully claims a rate of return that is higher than the rate-of-return shortfall \( \delta \) whereas an agent without negotiating power fails to do so.  

Since the principal cannot verify whether the agent exerts or does not exert effort during the pre-contractual phase, she faces a problem of moral hazard. In order to resolve this issue, she must use an appropriate mechanism that will incentivize the agent to reveal his action. In line with Grenadier and Wang (2005), we assume that at \( t = 0 \) the principal submits a take-it-or-leave-it offer that specifies a fixed transfer \( w \geq 0 \) that the principal pays to the agent, along with the timing of the investment. Apart from \( w \), as is standard in most PPPs, the agent is also entitled to receive the cash flow generated by the project. In return, he pays the sunk investment cost \( I \) while the principal receives the flow of public benefits \( b(\theta) \).  

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9 In the case of a highway, a well-designed facility gives commuters the opportunity to save time by efficiently managing multiple toll gates. For example, in Italy, in spite of the presence of several private highway operators, a driver takes a ticket when he enters the highway and pays when he leaves. Toll payment in Greece is more complicated as the driver pays separately at every toll stop until he reaches her/his destination. The problem lies in the fact that in Greece each operator employs a different toll system (see https://www.tolls.eu/).  

10 For instance, in the case of highways the stochasticity of \( x_t \) captures the fluctuations of toll revenues attributed to changes in traffic flow overtime (Lara Galera and Sanchez Solino, 2010). Demand fluctuations will also affect the \( x_t \) when PPPs deal with green public infrastructure, social housing etc.  

Eq. (1) also consistent with situations in which the revenue is uncertain because operating costs are uncertain. For instance, under a Cobb-Douglas production technology \( b(n) = n^a \) with \( a \in (0, 1) \) and \( n \) a scalar input, the instantaneous profit maximization gives the input demand function \( n = (a/\omega)^{1/(1-a)} \), where \( \omega \) is the input price. If the uncertainty is associated with changing input prices, the profit flow would be \( x_t = N \omega_1^{a/(a-1)} \), where \( N = (1-a)/(a+a) \). By Ito’s lemma, if \( \omega_1 \) is lognormally distributed, then so is \( x_2 \) (see Bertola, 1998).  

11 Concerning systematic risks, Hellowell and Vecchi (2018) argue that, since investors in the PPP market are not well diversified, they retain many market-related risks that can be related to fluctuations in demand, inflation, currency exchange rates, availability of funds to subcontractors and so on. However, as these risks are systematic, they should already be captured in the market rate. On the other hand, concerning specific risks, Merton (1987) shows that when the markets are segmented and investors have small or concentrated portfolios, an additional premium for specific risks may be required.  

12 The fixed transfer \( w \geq 0 \) can capture various payment structures. For instance, when \( w = 0 \), we have a user-pay contract.
Contrary to \( b(\theta) \), \( x_t \) and \( I \) are, by assumption, unaffected by \( \theta \). This assumption guarantees perfect asymmetry of objectives between the principal and the agent since the agent has nothing to gain by exerting effort. However, any extension of this model that allows the agency frictions to be relaxed will lead to results that are not qualitatively different. Finally, throughout the paper, we assume that both parties are risk-neutral.

The timing of the interaction between the two parties is given in Figure 1.

<table>
<thead>
<tr>
<th>Pre-contractual Phase:</th>
<th>Contractual Phase:</th>
<th>Realization of the investment:</th>
</tr>
</thead>
<tbody>
<tr>
<td>The two parties negotiate over the agent’s rate of return. An agent with negotiating power successfully claims a rate of return larger than ( \delta ).</td>
<td>Given the outcome of the pre-contractual phase, the principal submits the contract to the agent.</td>
<td>When the investment timing specified in the contract is reached, the principal pays the transfer, the agent pays the investment cost and the investment takes place.</td>
</tr>
</tbody>
</table>

Figure 1: Timing of the interaction between the principal and the agent.

### 3 The investment problem and the first-best solution

We start by presenting the case in which the agent’s action related to his effort exertion, or lack thereof, is observable by the principal. For ease of exposition, we also assume, for now, that \( b(\theta) \) is a constant.

Again, the contract that the principal submits to the agent specifies the transfer \( w \) and the timing of the investment. Since in our set-up all the information about the future evolution of process (1) is embodied in \( x_t \), the optimal investment rule takes the form: “Invest immediately if \( x_t \) is at, or above, a critical threshold \( x_\tau \) and wait otherwise”. That is, the optimal investment time is the first time point at which the state variable \( x_t \) passes a constant threshold \( x_\tau \).

The principal needs to find the transfer \( w \geq 0 \) and the investment threshold \( x_\tau \) that maximize her expected payoff, i.e., her value of the option to invest, conditional on the agent’s participation.

Assuming that the initial value \( x \) is low enough so that it is not optimal to invest immediately (i.e., \( \tau > 0 \)), the principal solves the following maximization problem:

\[
\max_{x_\tau, w} R(x_\tau, w) = \max_{x_\tau, w} \left( \frac{x}{x_\tau} \right)^{\beta} Y(w)
\]

subject to,

\[
V(x_\tau, w) = \left( \frac{x}{x_\tau} \right)^{\beta} F(x_\tau, w) \geq 0
\]

When \( w > 0 \) instead, the contract accounts for availability payments, that is, government subsidies that are contingent on the availability of the realized project (EPEC, 2018; Engel et al., 2014). Moreover, the model can be adapted to contain a revenue sharing scheme between the government and the agent. However nothing changes in the analysis as long as the government is free to choose the combination of the two contractual tools. For revenue sharing in regulation see Sappington and Weisman (1996), Sappington (2002), and, in a real options framework, Moretto et al. (2008).

If the quality of the project is reflected in the generated cash flow, then this can be modeled as, e.g., \( y_t = m(\theta) + x_t \), with \( m(\theta) \) an increasing function of \( \theta \), instead of just \( y_t = x_t \). However, from Itô’s Lemma, \( x_t \) and \( y_t \) will have the same distribution and will differ only in the starting point \( y_0 \) instead of \( x_0 \). Therefore, the problem can be equivalently formulated as one in which the cash flow is \( x_t \) and the effective cost of exercising the option is \( I - m(\theta) \), as in Grenadier and Wang (2005).

In the standard real options model, all parties formulate optimal policies under the assumption of risk neutrality or market completeness as in Eq. (1). Introducing risk aversion would simply result in an erosion of the project’s value and an increase of the option value of waiting to invest (Hugonnier and Morellec, 2013).

Formally \( \tau = \inf(\tau > 0, \text{such that } x_t = x_\tau) \), see Dixit, Pindyck and Sodal, (1999) for more details.
where,
\[ Y(w) = E_\tau \left[ \int_\tau^\infty b(\theta) e^{-r(t-\tau)} dt \right] - w = \frac{b(\theta)}{r} - w \tag{4} \]
is the principal’s payoff at time \( \tau \) and,
\[ F(x_\tau, w) = E_\tau \left[ \int_\tau^\infty x_t e^{-r(t-\tau)} dt \right] + w - I = \frac{x_\tau}{\delta} + w - I \tag{5} \]
is the agent’s net present value (NPV) at time \( \tau \).

In the maximization problem (2)-(3) both \( Y(w) \) and \( F(x_\tau, w) \) are discounted with \( \left( \frac{x}{x_\tau} \right)^\beta \) because the optimal investment threshold lies somewhere in the future (\( \tau > 0 \), i.e., \( x_\tau > x \)) and since the terms of the contract must be specified at \( t = 0 \), future values must be properly discounted. The term \( \beta > 1 \) is the positive root of the characteristic equation \( \Psi(\zeta) = \frac{1}{2} \sigma^2 \zeta (\zeta - 1) + \mu \zeta - r = 0. \tag{16} \)

Rearranging the objective function in Eq. (2), we can write:
\[ R(x_\tau, w) = W(x_\tau) - V(x_\tau, w) \tag{6} \]
where:
\[ W(x_\tau) = \left( \frac{x}{x_\tau} \right)^\beta \left( \frac{x_\tau}{\delta} - \left( I - \frac{b(\theta)}{r} \right) \right) \tag{7} \]
represents the project’s total welfare.

The principal chooses the contract \( \{ x_\tau, w \} \) keeping in mind the agreement reached in the pre-contractual phase.

Suppose first that the agent does not have negotiating power. In this case the principal can appropriate the entire surplus generated by the project, while the agent merely expects to break even. Since \( F(x_\tau, w) \) is increasing in \( w \), the principal’s payoff is maximized when the agent’s NPV is brought down to zero, that is:
\[ F(x^{FB}, w^{FB}) = 0 \rightarrow w^{FB} = I - \frac{x^{FB}}{\delta} \tag{8} \]
where the superscript \( FB \) stands for first-best.

The investment threshold that maximizes Eq. (7) is
\[ x^{FB} = \delta' \left( I - \frac{b(\theta)}{r} \right) \tag{9} \]
where \( \delta' = (1 + \frac{1}{\beta - 1})\delta \) and \( 1 + \frac{1}{\beta - 1} > 1 \) is the standard option multiplier that captures the effect of uncertainty. Then, from Eq.(8) and Eq.(9), we obtain:
\[ w^{FB} = \frac{\beta b(\theta) - I}{\beta - 1} \tag{10} \]

In summary, the contract that the principal submits to an agent without negotiating power in the first-

\[ \text{The term } \left( \frac{x}{x_\tau} \right)^\beta \text{ corresponds to the time zero price of an Arrow-Debreu security that pays one monetary unit the first moment the threshold } x_\tau \text{ is reached. It is also known as the “expected discount factor” (see Dixit and Pindyck, 1994, pp. 315-316).} \]
best is \(\{x^{FB}, w^{FB}\} = \left\{\delta' \left(1 - \frac{b(\theta)}{r}\right), \frac{\beta b(\theta) - I}{\beta - 1}\right\}\). Given \(\{x^{FB}, w^{FB}\}\), the total welfare becomes:

\[
W(x^{FB}) = \left(\frac{x}{x^{FB}}\right)^{\frac{\beta}{\beta - 1}} \left(1 - \frac{b(\theta)}{r}\right)
\]

(11)

We can better understand the government’s offer by showing the rate of return that is implicit in the contract. The rate of return, which is also referred to as the hurdle rate or internal rate of return (IRR), is the minimum return rate that a potential investor expects to earn from an investment. A project is approved only if its expected rate of return is equal to or exceeds the IRR. The project’s IRR and NPV are the two sides of the same coin. From Eq. (8), an investor with an IRR of \(\delta\) has an NPV that is equal to zero at the time of the investment. On the other hand, an investor with an IRR higher than \(\delta\) will have a positive NPV at the time of the investment.\(^{18}\)

Now, indicated by \(h^{FB}_A\), the rate of return for the agent, expressed as the percentage of his total revenue per unit of time on the investment cost, is:

\[
h^{FB}_A = \frac{x^{FB} + \delta w^{FB}}{I} = \frac{\delta' \left(1 - \frac{b(\theta)}{r}\right) + \delta' \left(\frac{b(\theta)}{r} - \frac{I}{\beta}\right)}{I} = \delta
\]

(12)

Eq. (12) suggests that, for an agent without negotiating power, the rate of return is exactly equal to the rate-of-return shortfall \(\delta\). On the other hand, indicated by \(h^{FB}\), the rate of return of the project as a whole (i.e., the social rate of return of the project), is:\(^{19}\)

\[
h^{FB} = \frac{x^{FB}}{I} = \delta'
\]

(13)

Rearranging the characteristic equation \(\Psi(\beta)\) to read \(\frac{\delta}{\beta - 1} = \mu + \frac{1}{2} \beta \sigma^2\), the social rate of return of the project can be written as:

\[
h^{FB} = \frac{\delta}{\beta - 1} = \mu + \frac{1}{2} \beta \sigma^2
\]

(14)

This means that for every monetary unit invested, the project must pay the market premium \(\delta\) and an irreversibility premium \(\mu + \frac{1}{2} \beta \sigma^2\). The irreversibility premium has to do with the fact there is an investment cost and an opportunity cost that the project needs to pay for when the investment option is exercised. While the investment cost has to do with the mere realization of the project through the payment of \(I\), the opportunity cost is related to the foregone option to further postpone the investment decision.

Consider for instance the hypothetical case in which the agent can invest in the project privately, that is, outside of a PPP. In this case he would choose the investment trigger \(x = \arg \max \left(\frac{\mu}{\sigma^2}\right)\beta \left(\frac{\theta}{r} - I\right) = \delta' I\) which is higher than \(x^{FB} = \delta' \left(1 - \frac{b(\theta)}{r}\right)\). This would allow for a hurdle rate \(\frac{\mu}{\sigma^2} = \delta' = \delta' I\) which is equal to

\(^{17}\)For the investment threshold \(x^{FB}\) and the transfer \(w^{FB}\) to make sense, we require: \(\beta \frac{b(\theta)}{r} > I > \frac{b(\theta)}{r}\). For \(r > 0\), the initial value must be \(x < \delta' \left(1 - \frac{b(\theta)}{r}\right)\).

\(^{18}\)The IRR for an investor who is accounting for the uncertainty and irreversibility of a project is not \(\delta\), but the adjusted discount rate \(\delta' = (1 + \frac{1}{\beta - 1})\delta\) such that \(\frac{b(\theta)}{r} + \frac{x^{FB}}{\beta} - I = 0\) (Dixit, 1992, p.130; Dixit and Pindyck, 1994, p. 142). Further \(\lim_{\beta \to 1} \delta' = \delta\).

\(^{19}\)The social rate of return corresponds to the total revenue \(\left(x^{FB}\right)\) in percentage of the social cost of investment expressed as the investment cost \(I\) net of the public benefit \(\left(\frac{b(\theta)}{r}\right)\).
Last, when the agent has negotiating power, the principal solves again the problem (2)-(3) but condition (3) is slack. While the principal can still dictate the investment threshold that maximizes (7), the transfer will have to be higher than $w_{FB}$ in order to ensure participation. Therefore, despite the fact that the rate of return of the project as whole remains equal to $\delta'$, the rate of return for the agent is (slightly) higher than $\delta$.

4 The investment problem under moral hazard

Suppose now that $b(\theta)$ is a random variable as described in Section 2 and that the principal cannot verifiably observe the agent’s action. As in Grenadier and Wang (2005), the principal can design a mechanism to provide incentives to the agent to truthfully reveal his private action. While this mechanism cannot be contingent on the unobservable level of the agent’s effort, it can be contingent on the observable public benefit and cash flow.

The interaction between the principal and the agent in this case evolves as follows. Initially, that is, in the pre-contractual phase, the two parties negotiate over the agent’s rate of return. Then, at $t=0$, the principal submits to the agent a menu of contracts that commits the actions of the two parties at the time of the investment. In this case the principal submits, not one, but two contracts to the agent, one for each realization of $\theta$: $\{x_i, w_i\}, i \in \{1, 2\}$.

The principal chooses the $\{x_i, w_i\}$, $i \in \{1, 2\}$ that maximize her ex-ante investment option value conditional on the participation of an effort exerting agent. The principal’s ex-ante payoff is:

$$R(x_1, x_2, w_1, w_2) = q_H \left( \frac{x}{x_1} \right)^{\beta} Y_1 + (1 - q_H) \left( \frac{x}{x_2} \right)^{\beta} Y_2$$

where $Y_i = \frac{b(\theta_i)}{r} - w_i, i \in \{1, 2\}$.

Eq. (15) is to be maximized subject to both ex-ante and ex-post constraints.

The ex-ante constraints are

$$q_H \left( \frac{x}{x_1} \right)^{\beta} F_1 + (1 - q_H) \left( \frac{x}{x_2} \right)^{\beta} F_2 - \xi \geq q_L \left( \frac{x}{x_1} \right)^{\beta} F_1 + (1 - q_L) \left( \frac{x}{x_2} \right)^{\beta} F_2$$

$$q_H \left( \frac{x}{x_1} \right)^{\beta} F_1 + (1 - q_H) \left( \frac{x}{x_2} \right)^{\beta} F_2 - \xi \geq 0$$

where $F_i = \frac{x_i}{\delta} + w_i - I, i \in \{1, 2\}$.

Ineq. (16) is the agent’s ex-ante incentive compatibility constraint which guarantees that the agent (weakly) prefers to exert effort. In other words, Ineq. (16) ensures that there is no unobservable action from the agent’s side at the time of the investment.

Ineq. (17) is the agent’s ex-ante participation constraint of the agent which ensures that it makes sense for the agent to abide by the principal’s choice of transfers and investment triggers.

The ex-post constraints are:

$$F_1 \geq 0$$

$$F_2 \geq 0$$

---

20 Despite the fact that the rate of return remains the same ($\tilde{h} = h_{FB}$), the project is delayed when it is privately executed ($\pi > x_{FB}$) because the investor does not account for the public benefit flow $b(\theta)$.

21 The subscript $i \in \{1, 2\}$ refers to the realization of $\theta, \theta_1$ or $\theta_2$. 
and

\[ w_1 \geq 0 \quad (20) \]
\[ w_2 \geq 0 \quad (21) \]

Ineqs. (20)-(21) need to hold since, by definition, \( w_i, i \in \{1, 2\} \) are transfers from the principal to the agent. Ineqs. (18) and (19) instead complement the ex-ante constraints and provide insurance to the agent, who, is guaranteed to receive a non-negative ex-post payoff irrespective of the realization of \( \theta \).\(^{22}\)

### 4.1 Optimal contracts when the agent does not have negotiating power

Suppose that the agent fails to claim \( F_i > 0, i \in \{1, 2\} \). Section A.1 of the Appendix shows that the problem (15)-(21) can be slightly reduced since Ineqs. (17) and (18) are always slack. The principal’s problem reduces to the maximization of the objective function in Eq. (15) subject to Ineq. (16):

\[ \left( \frac{x}{x_1} \right)^\beta F_1 - \left( \frac{x}{x_2} \right)^\beta F_2 \geq \frac{\xi}{\Delta q} \quad (22) \]

and Ineqs. (19)-(21).

Solving, we obtain:

**Proposition 1** When the agent does not have negotiating power, the principal chooses:

\[ \{x_1^*, w_1^*\} = \{x_1^{FB}, w_1^{FB} + \Phi_1\} \quad (23.1) \]
\[ \{x_2^*, w_2^*\} = \{x_2^{FB}, w_2^{FB}\} \quad (23.2) \]

where \( x_i^{FB} = \delta'(I - \frac{b(\theta_i)}{\tau}), w_i^{FB} = \frac{b(\theta_i)}{\beta - 1} - I, i \in \{1, 2\} \), and \( \Phi_1 = \frac{\xi}{\Delta q} \left( \frac{x_{FB}}{\xi^\beta \Delta q} \right) > 0 \).

**Proof.** See Section A.1 of the Appendix. \( \blacksquare \)

\( x_i^{FB} \) and \( w_i^{FB} \) are reminiscent of \( x_i^{FB} \) and \( w_i^{FB} \) from Eqs. (9) and (10) and correspond to the investment thresholds and transfers chosen in the first-best when \( \theta = \theta_i, i \in \{1, 2\} \).\(^{23}\)

From Eqs. (23.1)-(23.2) we see that the principal can guarantee that the investment will take place as soon as \( x_i^{FB}, i \in \{1, 2\} \) is reached. For \( \theta = \theta_2 \), the principal does so by setting \( F_2(x_2^*, w_2^*) = 0 \). For \( \theta = \theta_1 \), she must instead pay an information premium, \( \Phi_1 > 0 \), to induce the agent to exert effort which results in \( F_1(x_1^*, w_1^*) > 0 \).

As for the condition that must hold for the principal to induce effort exertion we have (see Section A.2 of the Appendix):

\[ \Delta q A \geq \xi + \frac{qL \xi}{\Delta q} \quad (24) \]

where \( A = W(x_1^{FB}) - W(x_2^{FB}) > 0 \) and \( W(x_i^{FB}) = \left( \frac{x}{x_i^{FB}} \right)^\beta \frac{1}{\beta - 1} \left( I - \frac{b(\theta_i)}{\tau} \right) \), is the project’s total welfare in the first-best, \( i \in \{1, 2\} \).\(^{24}\)

---

\(^{22}\)Ineqs., (18) and (19) guarantee that the private firm does not have an incentive to renegotiate the contract at the time of the investment.

\(^{23}\)For the contract to make sense we require \( \beta \frac{b(\theta_i)}{\tau} > I > \frac{b(\theta_i)}{\tau} \) and \( x < \delta'(I - \frac{b(\theta_i)}{\tau}) \).

\(^{24}\)Since in the first best the NPV of the agent is set equal to zero, the total value of the project and the total welfare for the principal coincide. This is obvious from Eq. (6) since \( R(x_1^{FB}, w_1^{FB}) = W(x_1^{FB}) - V(x_1^{FB}, w_1^{FB}) \) and \( V(x_1^{FB}, w_1^{FB}) = V(x_2^{FB}, w_2^{FB}) \).
The left-hand side of Ineq. (24) indicates the expected gain related to effort exertion. This gain comes from the fact that the value of the total welfare when the project turns out to be of high quality, \( W(x^{FB}_1) \) (which is greater than the total welfare when the project turns out to be of low quality, \( W(x^{FB}_2) \)) arises with a higher probability when effort is exerted. The right-hand side of Ineq. (24) is instead the cost of inducing effort exertion, which is given by the direct effort exertion cost \( \xi \) plus the limited liability rent \( \frac{qL}{\Delta q}\xi \) (Laffont and Martimort 2002, p. 157).

Alternatively, Ineq. (24) can be written as:

\[
qH \left( \frac{x^{FB}_i}{x_i^{FB}} \right)^\beta \left( \frac{b(\theta_1)}{r} - w^*_i \right) \geq qL W(x^{FB}_1) + \Delta q W(x^{FB}_2)
\]

The right-hand side of Ineq. (25) is positive which means that \( \frac{b(\theta_1)}{r} > w^*_i \). One can easily show that also \( \frac{b(\theta_2)}{r} > w^*_2 \). Hence, when the principal chooses to induce effort exertion, the discounted flow of public benefits is strictly larger than the transfers that the principal pays, no matter the realization of \( \theta \).

As for the agent’s rate of return, we have

\[
h^*_A = \frac{x^{FB}_1 \delta w^*_1 + \delta qH}{I} = \frac{\delta^I(I - \frac{b(\theta_1)}{r}) + \delta^I(\frac{b(\theta_1)}{r} - \frac{b(\theta_2)}{r}) + \delta \Phi_1}{I}
\]

(26.1)

when \( \theta = \theta_1 \).

Also, thanks to \( \frac{b(\theta_1)}{r} > w^*_1 \), we have \( h^*_A = \delta + \delta \left( \frac{w^*_1 - \frac{b(\theta_1)}{r}}{\delta} \right) < \delta' \). Consequently, \( h^*_A \in (\delta, \delta') \). In addition, when \( \theta = \theta_2 \), we have:

\[
h^*_A = \frac{x^{FB}_2 \delta w^{FB}_2}{I} = \delta
\]

(26.2)

where the equality \( h^*_A = \delta \) is attributed to the principal’s decision to choose \( \{x^*_2, w^*_2\} \) so that \( F_2(x^*_2, w^*_2) = 0 \).

Accounting for the fact that the menu of contracts in Eq. (23) allows for two investment thresholds, one for each realization of \( \theta \), we can get an idea of the agent’s ex-ante rate of return by calculating \( h^*_A \):

\[
h^*_A = qH h^*_A + (1 - qH) h^*_A = \frac{\delta}{\text{Market Premium}} + \frac{qH \Phi_1 \delta}{\text{Moral Hazard Premium}} < \delta'
\]

(27)

Eq. (27) shows that the agent’s ex-ante rate of return \( h^*_A \) is larger than the rate of return that corresponds to the case without agency conflicts \( \left( h^{FB}_A = \delta \right) \), but it is still lower than the social rate of return \( \left( h^{FB} = \delta' \right) \).

The reasoning behind \( h^*_A \in (\delta, \delta') \) is as follows: The agent needs to be remunerated for the effort that he exerts. For this reason he gets the premium \( qH \Phi_1 \delta / I \) over the market rate \( \delta \). However, since he cannot successfully claim a rate of return higher than \( \delta \) irrespective of the realization of \( \theta \), the ex-ante rate of return will not reach \( \delta' \).

Therefore, an agent who accepts the menu of contracts in Eq. (23) expects to receive a rate of return that is lower than the rate of return that an option-holder would have accepted in order to exercise the same investment option, i.e., \( \delta' \).

Finally, since \( x^*_i = x^{FB}_i, \ i \in \{1, 2\} \), the project’s social rate of return is still equal to \( \delta' \), i.e. \( h^* = \left( \frac{x^{FB}_i}{x_i^{FB}} \right)^\beta F_i(x^{FB}_i, w^{FB}_i) = 0 \) because of \( F_i(x^{FB}_i, w^{FB}_i) = 0 \). Consequently, \( R(x^{FB}_1, w^{FB}_1) = W(x^{FB}_1) \) and \( A = R(x^{FB}_1, w^{FB}_1) - R(x^{FB}_2, w^{FB}_2) = W(x^{FB}_1) - W(x^{FB}_2), \ i \in \{1, 2\} \).
\[ q_H h_1^* + (1 - q_H) h_2^* = \delta'. \] Thus, the menu of contracts in Eq. (23) simply transfers part of the project’s return from the principal to the agent without altering the project’s risk profile.

### 4.2 Optimal contracts when the agent has negotiating power

Suppose now that the agent can successfully claim \( F_i > 0, i \in \{1, 2\} \). In this case, we need to solve the problem (15)-(21) anew, assuming that Ineqs. (18) and (19) are slack.

Solving we obtain:

**Proposition 2** When the agent has negotiating power, the principal chooses:

\[
\begin{align*}
\{ x_1^{**}, w_1^{**} \} &= \{ x_1^{FB}, w_1^{FB} + \Phi_1 + \Phi_2 \} \quad (28.1) \\
\{ x_2^{**}, w_2^{**} \} &= \left\{ x_2^{FB} + \frac{\delta' b (\theta_2)}{q_H} - x_2^{FB}, 0 \right\} \quad (28.2)
\end{align*}
\]

where \( x_1^{FB} = \delta' \left( 1 - \frac{b(\theta_1)}{r} \right), w_i^{FB} = \frac{\delta b(\theta_1)}{r} - 1, i \in \{1, 2\} \), and \( \Phi_2 \equiv \left( \frac{1 - q_H}{q_H} \right) \frac{\beta' b(\theta_2)}{r} \left( x_2^{FB} \right)^{\beta} > 0 \)

**Proof.** See Section A.3 of the Appendix. \( \square \)

Unlike the case discussed in the previous subsection here the principal faces an insurance-efficiency trade-off. In fact, she finds it optimal to distort the menu of contracts, in terms of both timing and transfers.

From Eqs. (28) we see that, unless the project turns out to be of high quality (\( \theta = \theta_1 \)), the principal chooses an investment threshold that is higher than the first-best one, that is, \( x_1^{**} = x_1^{FB} \) and \( x_2^{**} > x_2^{FB} \). Moreover, the optimal transfer is higher than the one paid in the first-best for a high quality project (\( \theta = \theta_1 \)) and lower that the one paid for a low quality project (\( \theta = \theta_2 \)), i.e., \( w_1^{**} > w_1^* \) and \( w_2^{**} = 0 \).

The inequality \( w_1^{**} > w_1^* \) is attributed to the agent’s ability to successfully claim a rate of return higher than \( \delta \). In addition, similar to \( w_1^* \), the transfer \( w_1^{**} \) contains the term \( \Phi_1 > 0 \) which accounts for the rents that must be paid for the resolution of the agency conflict. The term \( \Phi_2 > 0 \) instead measures the cost attributed to the agent’s negotiating power. The equality \( w_2^{**} = 0 \) instead, has to do with the fact that, when \( \theta = \theta_2 \), the principal finds it too costly to guarantee optimal timing so, she allows for \( x_2^{**} > x_2^{FB} \) saving at least in terms of transfer (\( w_2^{**} = 0 \)).

As for the condition that must hold for the principal to induce effort exertion we have (see Section A.4 of the Appendix):

\[
\Delta q A \geq \xi + \frac{q_L}{\Delta q} \xi + \Delta R \tag{29}
\]

where \( \Delta R > 0 \) is the difference between the principal’s ex-ante payoff \( R(x_1^*, x_2^*, w_1^*, w_2^*) \) and its equivalent for \( \{ x_1^{**}, w_1^{**} \}, i \in \{1, 2\} \). This difference measures the agency cost that has to do with the agent’s negotiating power. Ineq. (29) is clearly more demanding than Ineq. (24). This means that the principal chooses to induce effort exertion less frequently.

In addition, Ineq. (29) can be written as:

\[
q_H \left( \frac{x}{x_1^{FB}} \right)^{\beta} \frac{b(\theta_1)}{r} - w_1^{**} \geq q_L W(x_1^{FB}) + \Delta q W(x_2^{FB}) + (1 - q_H) \left[ W(x_2^{FB}) - \left( \frac{x}{x_2^{*}} \right)^{\beta} \frac{b(\theta_2)}{r} \right] \tag{30}
\]

\(^{25}\)This result is in line with evidence showing that low-quality projects may be substantially delayed (Villani et al., 2017; Ng and Loosmore, 2007).

\(^{26}\)We show that \( \Delta R > 0 \) in Section A.1 of the Appendix.

\(^{27}\)This result is standard in the literature: that is, combined with limited liability constraints, the problem of moral hazard generates an information transfer from the principal to the agent that distorts decisions concerning the optimal effort (Laffont and Martimort, 2002).
where again, \( W(x_{i}^{FB}) = \left( \frac{x_i}{x_i^{FB}} \right)^{\beta - 1} \left( I - \frac{b(\theta_i)}{r} \right), \ i \in \{1, 2\} \).

If \( q_H \) is sufficiently high, the last term on the right-hand side of Ineq. (30) is negligible and condition (30) becomes equivalent to condition (25). Thus, when the principal chooses to induce effort exertion, the discounted flow of public benefits is strictly larger than the transfers that the principal pays, no matter the realization of \( \theta \), i.e., \( \frac{b(\theta_1)}{r} > w_i^{**}, \ i \in \{1, 2\} \). However, if \( q_H \) happens to be low, Ineq. (30) does not necessarily result in \( \frac{b(\theta_1)}{r} > w_i^{**} \) but might hold even with \( \frac{b(\theta_1)}{r} < w_i^{**} \). Because, if \( q_H \) is low, the principal will gain access to a costless project \( (w_2^{**} = 0) \) generating a public-benefit flow \( b(\theta_2) > 0 \) with high probability \( (1 - q_H) \).

Regarding the agent’s rates of return, we have:

\[
\begin{align*}
\h_A^{**} & = \frac{x_1^{FB}}{I} + \delta w_i^{**} = \delta' + \frac{w_i^{**} - \frac{\beta}{\beta-1} \frac{b(\theta_1)}{r}}{\delta}, \text{ if } \theta = \theta_1 \\
\h_A^{**} & = \frac{x_2^{**}}{I} = \delta' + \frac{1 - q_H \frac{b(\theta_2)}{r}}{\delta}, \text{ if } \theta = \theta_2
\end{align*}
\]

(31.1) (31.2)

Thanks to \( x_i^{**} = x_i^*, x_2^{**} > x_2^*, w_i^{**} > w_i^* \) and \( w_2^{**} = 0 \), we have:

**Proposition 3** The rate of return of an agent with negotiating power is strictly higher than the rate of return of an agent without negotiating power, irrespective of the realization of \( \theta \), i.e.:

\[
\h_A^{**} > \h_A^{*}, \ i \in \{1, 2\}
\]

The increase from \( \h_A^{*} \) to \( \h_A^{**} \) is attributed to the agent’s ability to claim more \( (w_i^{**} > w_i^*) \) of a project that still takes place when the \( x_1^{FB} \) is reached. On the other hand, the increase from \( \h_A^{*} \) to \( \h_A^{**} \) is attributed to the further postponement of the project \( (x_2^{**} > x_2^{FB}) \) and to the principal’s decision to pay no transfer \( (w_2^{**} = 0) \) to the agent.

The social rates of return of the project are:

\[
\begin{align*}
\h_1^{**} & = \frac{x_1^{FB}}{I - \frac{b(\theta_1)}{r}} = \delta', \text{ if } \theta = \theta_1 \\
\h_2^{**} & = \frac{x_2^{**}}{I - \frac{b(\theta_2)}{r}} = \delta' + \frac{1 - q_H \frac{b(\theta_2)}{r}}{\delta'}, \text{ if } \theta = \theta_2
\end{align*}
\]

(32.1) (32.2)

Comparing the agent’s return rates and the social return rates we find that when the project happens to be of high quality \( (\theta = \theta_1) \), the principal is willing to bear a negative payoff, as long as this guarantees that the project will be delivered when the first-best investment threshold \( x_1^{FB} \) is reached. In fact, from Eqs. (31.1) and (32.1), if \( w_i^{**} - \frac{b(\theta_i)}{r} > 1 - \frac{\beta}{\beta-1} \frac{b(\theta_1)}{r} \), we have \( \h_A^{**} > \h_A^{*} = \delta' \).

On the other hand, when the project happens to be of low quality \( (\theta = \theta_2) \), the principal prefers to remunerate the agent with a high rate of return by postponing the investment. In this case the rate of return the principal requires to start the project is higher than \( \delta' \). This is reflected in \( \h_2^{**} > \delta' \).

Summarizing we get:

**Proposition 4** When the project happens to be of high quality \( (\theta = \theta_1) \), then if \( w_i^{**} \leq \frac{\beta}{\beta-1} \frac{b(\theta_1)}{r} \), we have \( \h_A^{**} \leq \h_A^{*} = \delta' \). When instead the project happens to be of low quality \( (\theta = \theta_2) \), we have \( \h_2^{**} > \h_A^{**} > \delta' \).

---

28 This result is similar to what we found for \( \{x_i^*, w_i^*\}, \ i \in \{1, 2\} \), that is, \( \frac{b(\theta_1)}{r} > w_i^*, \ i \in \{1, 2\} \).
Finally, when the agent has negotiating power, the project’s ex-ante social rate of return can be approximated by:

$$h^{**} = \frac{\delta}{\delta_{\text{Market\ Premium}}} + \frac{\mu}{2\sigma^2} + \frac{1 - q_H}{q_H} \frac{\frac{b'(\theta_2)}{b(\theta_2)}}{I - \frac{b'(\theta_2)}{b(\theta_2)}} \delta' > h^* = \delta'$$  \hspace{1cm} (33)

Ineq. (33) suggests that when a project’s completion is delegated to an agent with negotiating power, the project is required to have a higher rate of return than a similar project that is delegated to an agent without negotiating power ($h^{**} > h^*$).

The effect of this “extra” return is an undervaluation of the project’s current (time zero) total value with respect to the project’s total value with moral hazard only (see Section A.5 of the Appendix). Indeed, the presence of an agent with negotiating power comes at a cost for the project’s overall efficiency that must pay also for whatever the agent asks to accept the delegation the principal proposes.

5 Conclusion

In this paper we discuss the optimal design of a PPP explicitly assuming that there is a moral hazard issue between the public party and the private party involved in the PPP. In addition, we allow for two types of agents, one with negotiating power and one without. The originality of our contribution lies in the combination of the agency conflict and the private party’s negotiating power.

Departing from Grenadier and Wang (2005), we show that, by employing a properly designed mechanism, the public party can resolve the agency conflict. However, the mechanism that the public party chooses depends on the negotiating power of the private party, or lack thereof. If the private party does not have negotiating power, the moral hazard issue has no effect on the timing of the investment, but if the private party happens to have negotiating power, the first-best timing cannot be guaranteed. This time discrepancy is also reflected in the project’s reduced overall efficiency since the project must pay a higher return to attract the private investor.

The main message of this work is that, even if the value of the real option embedded in a PPP is explicitly accounted for, and a properly designed mechanism is used to resolve agency conflicts between the PPP partners, there is no guarantee that the first-best solution will be reached. The agent’s negotiating power is a source of inefficiency that may lead to under-investment in PPPs and limited use of incentive contracts by public administrations.
A Appendix

A.1 Proof of Proposition 1

The principal’s problem is:

\[
\max_{\{x_i, w_i\}, i \in \{1, 2\}} q_H \left( \frac{x}{x_1} \right)^\beta Y_1 + (1 - q_H) \left( \frac{x}{x_2} \right)^\beta Y_2
\]  

(A.1)

s.t.

\[
q_H \left( \frac{x}{x_1} \right)^\beta F_1 + (1 - q_H) \left( \frac{x}{x_2} \right)^\beta F_2 - \xi \geq q_L \left( \frac{x}{x_1} \right)^\beta F_1 + (1 - q_L) \left( \frac{x}{x_2} \right)^\beta F_2
\]  

(A.2)

\[
q_H \left( \frac{x}{x_1} \right)^\beta F_1 + (1 - q_H) \left( \frac{x}{x_2} \right)^\beta F_2 - \xi \geq 0
\]  

(A.3)

\[F_1 \geq 0\]  

(A.4)

\[F_2 \geq 0\]  

(A.5)

\[w_1 \geq 0\]  

(A.6)

\[w_2 \geq 0\]  

(A.7)

where \(Y_i = \frac{b_i}{r} - w_i\) and \(F_i = \frac{x_i}{\delta} + w_i - I, i \in \{1, 2\}\).

Ineq. (A.2) can be written as:

\[
\left( \frac{x}{x_1} \right)^\beta F_1 - \left( \frac{x}{x_2} \right)^\beta F_2 \geq \frac{\xi}{\Delta q}
\]  

(A.8)

From Ineq. (A.8) we obtain:

\[
\left( \frac{x}{x_1} \right)^\beta F_1 \geq \frac{\xi}{\Delta q} + \left( \frac{x}{x_2} \right)^\beta F_2 \geq \frac{\xi}{\Delta q} > 0
\]  

(A.9)

This means that Ineq. (A.4) is not binding.

From Ineq. (A.3) we have:

\[
\left( \frac{x}{x_1} \right)^\beta F_1 + \frac{1 - q_H}{q_H} \left( \frac{x}{x_2} \right)^\beta F_2 \geq \frac{\xi}{q_H}
\]  

(A.10)

From Ineq. (A.9) we have \(\left( \frac{x}{x_1} \right)^\beta F_1 \geq \frac{\xi}{\Delta q}\). Since \(\frac{\xi}{\Delta q} > \frac{\xi}{q_H}\), also \(\left( \frac{x}{x_1} \right)^\beta F_1 > \frac{\xi}{q_H}\) which means that Ineq. (A.10), and consequently Ineq. (A.3), is slack.

In summary, the reduced version of the principal’s problem is:

\[
\max_{\{x_i, w_i\}, i \in \{1, 2\}} q_H \left( \frac{x}{x_1} \right)^\beta Y_1 + (1 - q_H) \left( \frac{x}{x_2} \right)^\beta Y_2
\]  

(A.11)

subject to the following conditions:

\[
\left( \frac{x}{x_1} \right)^\beta F_1 - \left( \frac{x}{x_2} \right)^\beta F_2 \geq \frac{\xi}{\Delta q}
\]  

(A.12)

\[F_2 \geq 0\]  

(A.13)

\[w_1 \geq 0\]  

(A.14)

\[w_2 \geq 0\]  

(A.15)
The Lagrangian is:

\[ L = q_H \left( \frac{x}{x_1} \right)^{\beta} \left( \frac{b(\theta_1)}{r} - w_1 \right) + (1 - q_H) \left( \frac{x}{x_2} \right)^{\beta} \left( \frac{b(\theta_2)}{r} - w_2 \right) + \lambda_1 \left( \frac{x}{x_1} \right)^{\beta} \left( \frac{x_1}{\delta} + w_1 - 1 \right) - \left( \frac{x}{x_2} \right)^{\beta} \left( \frac{x_2}{\delta} + w_2 - 1 \right) - \frac{\xi}{\Delta q} \right] + \lambda_2 \left( \frac{x_2}{\delta} + w_2 - 1 \right) + \lambda_3 w_1 + \lambda_4 w_2 \]

where \( \lambda_1, \lambda_2, \lambda_3 \) and \( \lambda_4 \) are the four Lagrangian multipliers, one for each of the four Ineqs. (A.12)-(A.15).

The first-order condition with respect to \( w_1 \) gives:

\[ \lambda_3^{AC} = \left( q_H - \lambda_1^{AC} \right) \left( \frac{x}{x_1^{AC}} \right)^{\beta} \] \quad (A.16)

where the superscript \( AC \) stands for “agency conflict”.

Eq. (A.16) is satisfied in three possible cases:

i) \( q_H = \lambda_1^{AC} > 0 \). The positivity of \( \lambda_1^{AC} \) suggests that Ineq. (A.12) is binding. In addition, thanks to \( q_H = \lambda_1^{AC} \) we have \( \lambda_3^{AC} = 0 \) which means \( w_1^{AC} > 0 \).

ii) \( q_H > \lambda_1^{AC} > 0 \). Again, the positivity of \( \lambda_1^{AC} \) suggests that Ineq. (A.12) is binding. However, since \( q_H > \lambda_1^{AC} \) we have \( \lambda_3^{AC} > 0 \) which in turn suggests \( w_1^{AC} = 0 \). Last,

iii) \( q_H > \lambda_1^{AC} = 0 \). Since \( \lambda_1^{AC} = 0 \), Ineq. (A.12) is slack. On the other hand, since \( q_H > \lambda_1^{AC} \) we have \( \lambda_3^{AC} > 0 \) which suggests \( w_1^{AC} = 0 \).

The first-order condition with respect to \( w_2 \) gives:

\[ \lambda_2^{AC} + \lambda_4^{AC} = \left( 1 - q_H + \lambda_1^{AC} \right) \left( \frac{x}{x_2^{AC}} \right)^{\beta} \] \quad (A.17)

The right-hand side of the equality is strictly positive, so either \( \lambda_2^{AC}, \lambda_4^{AC} \), or both are positive. We have a total of three cases.

a) \( \lambda_2^{AC} = 0, \lambda_4^{AC} > 0 \). In this case Ineq. (A.13) is slack and Ineq. (A.15) is binding.

b) \( \lambda_2^{AC} > 0, \lambda_4^{AC} = 0 \). In this case Ineq. (A.13) is binding and Ineq. (A.15) is slack. Last,

c) \( \lambda_2^{AC} > 0, \lambda_4^{AC} > 0 \). In this case both inequalities are binding.

The first-order condition with respect to \( x_1 \) gives:

\[ \lambda_1^{AC} x_1^{AC} = \frac{\beta}{\beta - 1} \delta \left[ q_H \left( w_1^{AC} - \frac{b(\theta_1)}{r} \right) - \lambda_1^{AC} (w_1^{AC} - I) \right] \] \quad (A.18)

The first-order condition with respect to \( x_2 \) gives:

\[ \frac{\lambda_2^{AC}}{\delta} = \left( \frac{x}{x_2^{AC}} \right)^{\beta} \left[ \frac{(1 - q_H) \left( \frac{b(\theta_2)}{r} - w_2^{AC} \right)}{x_2^{AC}} - \lambda_1^{AC} (w_2^{AC} - I) - \frac{\lambda_1^{AC} \beta - 1}{\delta} \right] \] \quad (A.19)

Given (A.16)-(A.19) we discuss the possible solutions.
A.1.1 Case iA

In this case we have \( q_H = \lambda_1^{**} > 0 \) and \( \lambda_2^{**} = 0, \lambda_1^{*} > 0 \). In other words, Ineqs. (A.13) and (A.14) are slack whereas Ineqs. (A.12) and (A.15) are binding. From these we have

\[
\begin{align*}
  x_1^{**} &= x_1^{FB}, \\
  x_2^{**} &= x_2^{FB} + \frac{\delta}{q_H} \frac{b(\theta_2)}{r} \beta_B - 1,
\end{align*}
\]

(A.20)

(A.21)

where \( x_i^{FB} = \delta' \left( I - \frac{b(\theta_1)}{r} \right), i \in \{1, 2\} \).

From the binding (A.12) we obtain:

\[
\begin{align*}
  w_1^{***} &= \left( \frac{1}{q_H} \frac{b(\theta_2)}{r} - w_2^{FB} \right) \left( x_2^{FB} \right)^{\beta_B} + \frac{\xi}{\Delta q} \left( \frac{x}{x_1^{FB}} \right)^{-\beta_B} + w_1^{FB} \\
  &= \frac{I + \beta b(\theta_2)}{\beta - 1} \left( x_2^{FB} \right)^{\beta} + \frac{\xi}{\Delta q} \left( \frac{x}{x_1^{FB}} \right)^{-\beta} + w_1^{FB}
\end{align*}
\]

(A.22)

(A.23)

where \( w_i^{FB} = \frac{\beta b(\theta_1)}{\beta - 1} - 1, i \in \{1, 2\} \).

Finally, from \( \lambda_2^{**} > 0 \) we have \( w_2^{**} = 0 \).

The inequality \( \beta b(\theta_2) > I > \frac{b(\theta_1)}{r} \) guarantees \( x_i^{**} > 0, w_i^{FB} > 0, i \in \{1, 2\} \). From Eq. (A.23) we see that \( w_i^{**} > 0 \), so Ineq. (A.14) is indeed slack. At the same time, the solution \( \{x_i^{**}, w_i^{**}\}, i \in \{1, 2\} \) satisfies the always slack Ineqs. (A.3) and (A.4) as well as Ineq. (A.13) which is the slack in this case.

A.1.2 Case ib

In this case we have \( q_H = \lambda_1^{*} > 0, \lambda_2^{*} > 0 \) and \( \lambda_1^{*} = 0 \). In other words, Ineqs. (A.14) and (A.15) are slack whereas Ineqs. (A.12) and (A.13) are binding. From these we obtain:

\[
\begin{align*}
  x_1^{*} &= x_1^{FB} \\
  x_2^{*} &= x_2^{FB} \\
  w_1^{*} &= w_1^{FB} + \frac{\xi}{\Delta q} \left( \frac{x}{x_1^{FB}} \right)^{-\beta} \quad \text{(A.24)} \\
  w_2^{*} &= w_2^{FB} \quad \text{(A.25)}
\end{align*}
\]

(A.26)

(A.27)

where again \( x_i^{FB} = \delta' \left( I - \frac{b(\theta_1)}{r} \right) \) and \( w_i^{FB} = \frac{\beta b(\theta_1)}{\beta - 1} - 1, i \in \{1, 2\} \).

As before, the inequality \( \beta b(\theta_2) > I > \frac{b(\theta_1)}{r} \) guarantees the positivity of \( w_i^{*} \) and that \( x_i^{FB}, i \in \{1, 2\} \). One can also easily check that the solution \( \{x_i^{*}, w_i^{*}\}, i \in \{1, 2\} \) satisfies the always slack Ineqs. (A.3) and (A.4).

A.1.3 Case ic

In this case we have \( q_H = \lambda_1^{ic} > 0 \) and \( \lambda_2^{ic} > 0, \lambda_1^{ic} > 0 \). In other words, only Ineq. (A.14) is slack. From Eq. (A.18) we have \( x_i^{ic} = x_i^{FB} \) and from the binding Ineq. (A.12) we have \( w_i^{ic} = w_i^{FB} + \frac{\xi}{\Delta q} \left( \frac{x}{x_1^{FB}} \right)^{-\beta_1} \)

where \( x_i^{FB} = \delta' \left( I - \frac{b(\theta_1)}{r} \right) \) and \( w_i^{FB} = \frac{\beta b(\theta_1)}{\beta - 1} - 1 \). At the same time, from the binding Ineq. (A.15) we have \( w_2^{ic} = 0 \). Since \( w_2^{ic} = 0 \), we expect \( x_2^{ic} = x \) which is a contradiction since, by assumption, \( x_2^{ic} > x \).
A.1.4 Cases iia, iib and iic

In all these cases we have \( q_H > \lambda_1 \) and \( \gamma_H = 0 \) so \( w_1 = 0 \). Given this, from the first-order condition (A.18) we obtain:

\[
x_1^i = \frac{\beta}{\beta - 1} \delta (I - \frac{q_H b(\theta_1)}{\lambda_1})
\]  
(A.28)

The always slack Ineq. (A.4), \( \frac{x_1^i}{\delta} + w_1 = I > 0 \), is satisfied only if \( I > \frac{\beta q_H b(\theta_1)}{\lambda_1} \). Since \( q_H > \lambda_1 \) and \( b(\theta_1) > b(\theta_2) \), the inequality \( I > \frac{\beta q_H b(\theta_1)}{\lambda_1} \) is stronger than the inequality \( I > \frac{b(\theta_2)}{\gamma} \), although this result contradicts the condition \( \beta_{\frac{b(\theta_2)}{\gamma}} > I > \frac{b(\theta_1)}{\gamma} \).

A.1.5 Cases iia, iib and iic

In all these cases we have \( q_H > \lambda_1 \). Plugging \( \lambda_1 = 0 \) into Eq. (A.18) we obtain \( \frac{b(\theta_1)}{\gamma} = w_1^i \). Thanks to \( q_H > \lambda_1 \) we have \( \lambda_1 > 0 \), which suggests that \( w_1 = 0 \). Consequentely, Cases iia, iib and iic result in \( b(\theta_1) = 0 \) which is a contradiction since, by assumption, \( b(\theta_1) > 0 \).

A.1.6 The optimal solution

In summary, the principal must choose between \( \{ x_1^*, w_1^* \} \) and \( \{ x_i^*, w_i^* \}, i \in \{1, 2\} \). If the principal chooses the former, she obtains \( q_H (\frac{x}{x_1^*})^\beta (\frac{b(\theta_1)}{\gamma} - w_1^*) + (1 - q_H) (\frac{x}{x_2^*})^\beta (\frac{b(\theta_2)}{\gamma} - w_2^*) \). A similar expression results if she chooses the latter. Referring to the difference between the two as \( \Delta R \), we have:

\[
\Delta R = \frac{b(\theta_2)}{\gamma} (\frac{x}{x_1^*})^\beta (\frac{b(\theta_1)}{\gamma} - w_1^*) + (1 - q_H) (\frac{x}{x_1^*})^\beta (\frac{b(\theta_2)}{\gamma} - w_2^*) > 0
\]  
(A.29)

The positivity of \( \Delta R \) suggests that the principal opts for \( \{ x_1^*, w_1^* \}, i \in \{1, 2\} \).

As for the agent, when \( \{ x_i^*, w_i^* \}, i \in \{1, 2\} \) is the submitted menu of contracts, the ex-ante option value is \( q_H (\frac{x}{x_1^*})^\beta F_1 (x_1^*, w_1^*) + (1 - q_H) (\frac{x}{x_2^*})^\beta F_2 (x_2^*, w_2^*) - \xi \). We have a similar expression when \( \{ x_i^*, w_i^* \}, i \in \{1, 2\} \) is chosen. Referring to the difference between the two as \( \Delta V \), we have:

\[
\Delta V = q_H (\frac{x}{x_1^*})^\beta (w_i^* - w_i^*) + (1 - q_H) (\frac{x}{x_2^*})^\beta F_2 (x_2^*, w_2^*)
\]  
(A.30)

Since \( w_1^* < w_1^* \) and \( F_2 (x_2^*, w_2^*) > 0 \) we have \( \Delta V < 0 \) so the agent prefers \( \{ x_i^*, w_i^* \} \) to \( \{ x_i^*, w_i^* \}, i \in \{1, 2\} \).

A.2 Effort exertion when the agent does not have negotiating power

When the principal is delegating the investment decision to an agent without negotiating power and effort exertion is not induced, she solves the following problem.

\[
\max_{y, \alpha, \gamma} q_L \left( \frac{x}{x_1} \right)^\beta Y_1 + (1 - q_L) \left( \frac{x}{x_2} \right)^\beta Y_2
\]  
(A.31)
subject to:

\[
q_L \left( \frac{x}{x_1} \right)^\beta F_1 + (1-q_L) \left( \frac{x}{x_2} \right)^\beta F_2 \geq 0 \tag{A.32}
\]

\[
F_1 \geq 0 \tag{A.33}
\]

\[
F_2 \geq 0 \tag{A.34}
\]

\[
w_1 \geq 0 \tag{A.35}
\]

\[
w_2 \geq 0 \tag{A.36}
\]

where \( Y_i = \frac{b(x_i)}{r} - w_i \) and \( F_i = \frac{x_i^*}{\delta} + w_i - I, \ i \in \{1, 2\} \).

Since the principal can solve the problem by choosing \( F_i = 0, i \in \{1, 2\} \) and since the objective function in (A.31) is decreasing in \( x_i, w_i, i \in \{1, 2\} \), she chooses the transfers and the investment triggers so that the ex-ante participation constraint is binding. However, this means that the two ex-post participation constraints will also be binding. From this we obtain the solution \( \{\dot{x}_i, \dot{w}_i\} = \{x_i^{FB}, w_i^{FB}\}, i \in \{1, 2\} \).

In this case, effort is exerted when \( q_H \left( \frac{x}{x_1^{FB}} \right)^\beta \left( \frac{b(x_1)}{r} - w_1^* \right) + (1-q_H) \left( \frac{x}{x_2^{FB}} \right)^\beta \left( \frac{b(x_2)}{r} - w_2^* \right) \) is not smaller than \( q_L \left( \frac{x}{x_1^{FB}} \right)^\beta \left( \frac{b(x_1)}{r} - \dot{w}_1 \right) + (1-q_L) \left( \frac{x}{x_2^{FB}} \right)^\beta \left( \frac{b(x_2)}{r} - \dot{w}_2 \right) \). Comparing the two we obtain:

\[
\left( \frac{x^{FB}}{x^{FB}_1} \right)^\beta \left( \frac{b(x_1)}{r} - w_1^{FB} \right) - \left( \frac{x^{FB}}{x^{FB}_2} \right)^\beta \left( \frac{b(x_2)}{r} - w_2^{FB} \right) \geq \frac{q_H \xi}{\Delta q} \tag{A.37}
\]

Alternatively, this can be written as

\[
\Delta q A \geq q_H \frac{\xi}{\Delta q} \tag{A.38}
\]

where

\[
A = \left( \frac{x}{x_1^{FB}} \right)^\beta \left( \frac{b(x_1)}{r} - w_1^{FB} \right) - \left( \frac{x}{x_2^{FB}} \right)^\beta \left( \frac{b(x_2)}{r} - w_2^{FB} \right) = R(x_1^{FB}, w_1^{FB}) - R(x_2^{FB}, w_2^{FB}) = W(x_1^{FB}) - W(x_2^{FB}), \ R(x_1^{FB}, w_1^{FB}) = \left( \frac{x}{x_1^{FB}} \right)^\beta \left( \frac{b(x_1)}{r} - w_1^{FB} \right) \text{ and } W(x_1^{FB}) = \left( \frac{x}{x_1^{FB}} \right)^\beta \left( \frac{b(x_1)}{r} + \frac{x_1^{FB}}{\delta} - I \right). \ i \in \{1, 2\}.
\]

The equality \( R(x_1^{FB}, w_1^{FB}) - R(x_2^{FB}, w_2^{FB}) = W(x_1^{FB}) - W(x_2^{FB}) \) is attributed to the fact that \( F(x_1^{FB}, w_1^{FB}) = 0, i \in \{1, 2\} \).

### A.3 Proof of Proposition 2

When Ineqs. (A.4)-(A.5) are slack, the problem (A.1)-(A.7) has a unique solution: \( \{x_i^{*}, w_i^{*}\}, i \in \{1, 2\} \).

### A.4 Effort exertion when the agent has negotiating power

When the principal is delegating the exercise of the investment option to an agent with negotiating power, she cannot solve the problem (A.31)-(A.36) choosing \( F_i = 0, i \in \{1, 2\} \) since in this case, (A.32)-(A.34) are slack. In this case the principal’s problem is

\[
\max_{\{x_i, w_i\}, i \in \{1, 2\}} q_L \left( \frac{x}{x_1} \right)^\beta Y_1 + (1-q_L) \left( \frac{x}{x_2} \right)^\beta Y_2 \tag{A.39}
\]
subject to:

\[ w_1 \geq 0 \quad (A.40) \]
\[ w_2 \geq 0 \quad (A.41) \]

or, equivalently,

\[
\max_{\{x_i, w_i\}, i \in \{1, 2\}} q_L W_1 + (1 - q_L) W_2 - \left[ q_L \left( \frac{x}{x_1} \right)^\beta F_1 + (1 - q_L) \left( \frac{x}{x_2} \right)^\beta F_2 \right]
\]

subject to:

\[ w_1 \geq 0 \quad (A.43) \]
\[ w_2 \geq 0 \quad (A.44) \]

where \( W_i = \left( \frac{x}{x_i} \right)^\beta \left( \frac{b(\theta_i)}{r} + \frac{x_i}{2} - I \right), i \in \{1, 2\}. \)

The principal can dictate the investment threshold that maximizes the total value of the project \( W_i \), i.e., \( x_i^{FB}, i \in \{1, 2\} \). However, the transfer will have to be higher than \( w_i^{FB}, i \in \{1, 2\} \) so that the agent can benefit from a project with positive net present value at the time of the investment \( (F_i > 0, i \in \{1, 2\}) \).

Therefore, the contracts that the principal chooses when she is delegating the exercise option to an agent with negotiating power and she chooses not to induce effort exertion are \( \{\hat{x}_i, \hat{w}_i\} = \{x_i^{FB}, w_i^{FB} + \bar{\varepsilon}^+\}, i \in \{1, 2\} \) where \( \bar{\varepsilon}^+ > 0 \) is arbitrarily small.

In this case, effort exertion is induced when \( q_F \left( \frac{x}{x_1} \right)^\beta \left( \frac{b(\theta_1)}{r} - \hat{w}_1^{**} \right) \) and \( (1 - q_F) \left( \frac{x}{x_2} \right)^\beta \left( \frac{b(\theta_2)}{r} - \hat{w}_2^{**} \right) \) is not smaller than \( q_L \left( \frac{x}{x_1} \right)^\beta \left( \frac{b(\theta_1)}{r} - \hat{w}_1 \right) \) and \( (1 - q_L) \left( \frac{x}{x_2} \right)^\beta \left( \frac{b(\theta_2)}{r} - \hat{w}_2 \right) \). Comparing the two we obtain:

\[
\Delta R = \Delta qA - q_R \frac{\xi}{\Delta q} + \Gamma \bar{\varepsilon}^+
\]

where \( \Delta R \) was derived in subsection A.1.6, \( A \) was derived in subsection A.2 and \( \Gamma \equiv q_L \left( \frac{x}{x_1} \right)^\beta + (1 - q_L) \left( \frac{x}{x_2} \right)^\beta > 0 \). Letting \( \bar{\varepsilon}^+ \rightarrow 0 \), the term \( \Gamma \bar{\varepsilon}^+ \) becomes negligible and Ineq. (A.45) can be written as:

\[
\Delta qA \geq q_R \frac{\xi}{\Delta q} + \Delta R
\]

A.5 Welfare analysis

The ex-ante total value of the project when \( \{x_i^*, w_i^*\}, i \in \{1, 2\} \) is chosen is given by the following expression:

\[
q_F \left( \frac{x}{x_1} \right)^\beta Y_1 (w_1^*) + (1 - q_F) \left( \frac{x}{x_2} \right)^\beta Y_2 (w_2^*) + q_F \left( \frac{x}{x_1} \right)^\beta F_1 (x_1^*, w_1^*) + (1 - q_F) \left( \frac{x}{x_2} \right)^\beta F_2 (x_2^*, w_2^*) - \xi
\]

We have a similar expression when \( \{x_i^{**}, w_i^{**}\}, i \in \{1, 2\} \) is chosen.

Expression (A.47) can be written as:

\[
q_F \left( \frac{x}{x_1} \right)^\beta [Y_1 (w_1^*) + F_1 (x_1^*, w_1^*)] + (1 - q_F) \left( \frac{x}{x_2} \right)^\beta [Y_2 (w_2^*) + F_2 (x_2^*, w_2^*)] - \xi
\]
or as:

\[ q_H \left( \frac{x}{x_1^*} \right)^\beta \left( \frac{b(\theta_1)}{r} + \frac{x_1^*}{\delta} - I \right) + (1 - q_H) \left( \frac{x}{x_2^*} \right)^\beta \left( \frac{b(\theta_2)}{r} + \frac{x_2^*}{\delta} - I \right) - \xi \]  

(A.49)

Taking the difference between (A.49) and its equivalent for \( \{x_i^{**}, w_i^{**}\}, i \in \{1, 2\} \) and keeping in mind that \( x_1^* = x_1^{**} \), we obtain:

\[ (1 - q_H) \left[ \left( \frac{x}{x_2^*} \right)^\beta \left( \frac{b(\theta_2)}{r} + \frac{x_2^*}{\delta} - I \right) - \left( \frac{x}{x_2^{**}} \right)^\beta \left( \frac{b(\theta_2)}{r} + \frac{x_2^{**}}{\delta} - I \right) \right] \]  

(A.50)

Now, since \( x_2^* = x_2^{FB} < x_2^{**} \) and \( x_2^{FB} = \arg \max \left[ \left( \frac{r}{b} \right)^\beta \left( \frac{b(\theta_2)}{r} + \frac{y}{\beta} - I \right) \right] \), this expression is positive. This means that, for the economy as a whole, using \( \{x_i^*, w_i^*\} \) is always preferred to using \( \{x_i^{**}, w_i^{**}\}, i \in \{1, 2\} \).
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