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PATENT PORTFOLIOS AND FIRMS TECHNOLOGICAL CHOICES

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Patent Portfolios and Firms Technological Choices*

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Abstract

In many industrial sectors, firms amass large patents portfolios to reinforce their bargaining position vis a vis competitors. In a context where patents have a pure strategic nature, we discuss how the presence and the effectiveness of a patent system affect firms technological decisions. Specifically, we present a two-stage game where firms first choose whether to *agglomerate* (i.e. develop technologies for the same technological territory) or to *separate* (i.e. develop technologies for different territories) and then they take their patenting decisions. We show that strong patents may distort technological choices yielding to firms to follow inefficient technological trajectories in an attempt to reduce competitors' patenting activity. While an increase in the strength of patent rights − i.e. the extent to which patents can be used to extract value − undoubtedly distorts firms choices, the impact of a larger scope − the degree to which patent protection carries out in the adjacent ares as well − is ambiguous. We also discuss how such distortions change when one firm is prevented from obtaining its optimal number of patents and when firms patenting activities generate additional market value.

Keywords: patent portfolios, patent strength and scope, technological territory, strategic patenting.
JEL classifiers: D43, L13, 034

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1 Introduction

A large part of the literature on patents is devoted to discuss the role of intellectual property rights in spurring firms R&D activities. Nevertheless, patents may have an effect not only on the amount of research but also on the type and nature of firms R&D efforts. In particular, a question that has been largely ignored in the literature refers to the role that patents may play in determining firms technological trajectories. This issue has been raised by Moser (2005); using historical data on the inventions presented during two World's Fairs in the second half of the nineteenth century, she shows that there were clear differences in the innovation trajectories followed by firms in countries that had and that had not adopted a patent system. Moving from this evidence, Moser (2005) concludes by saying that "patents help to determine the direction of technical change", suggesting that the presence, and the effectiveness, of a patent system can have an impact not only on the extent of innovation but also on its direction. This observation motivates our analysis.

The role that the patent system may have in determining the direction of firms technological trajectories appears to be even more critical today. During the last years, several industrial sectors have experienced a surge in patenting activities (see Khan et al., 2011); this phenomenon is very relevant in the information and communication technologies (ICTs) where complex technologies are often protected by a large number of overlapping patents owned by different inventors (so-called patent thickets).[1] Hall et al. (2013) have shown that this surge in patenting activities and the associated fragmentation of patent rights has had an influence on firms behaviour, in particular on entrants; the authors find that the presence of patent thickets may represent a strong impediment to SMEs even to the point of inducing them not to enter the technological territories where patent fragmentation prevails.

The evidence in Hall et al. (2013) is particularly relevant for our scopes as it refers to industrial sectors where patents have acquired a strong strategic value. As pointed out in an early work on the US semiconductor industry by Hall and Ziedonis (2001), companies in high-tech sectors use patents as bargaining chips in negotiations; amassing large patents portfolios reinforces their bargaining position *vis a vis* competitors, thus improving their chances to strike better deals during licensing and cross-licensing negotiations. Piling up sizable patent portfolios may be beneficial either for defensive or for offensive reasons. In the former case, firms use patent portfolios as a safeguard against the possibility of rival firms taking legal action for patent infringement (Ziedonis, 2004); in the latter, firms may want to use them aggressively against competitors (Walsh et al., 2016; Torrisi et al., 2016).

The high degree of fragmentation of patent ownership protecting complex technologies has

[1] See Shapiro (2001); for a recent survey of the economic literature on the controversial role of patents in ICTs see Comino et al. (2019).

naturally led firms in high-tech industries to use patents strategically. The evidence found by Hall et al. (2013) reveals that large and small high-tech firms react to patent fragmentation differently, with the latter that are less able to cope with the “cost of complexity”, namely the costs associated with the uncertainty over freedom to operate, the lack of transparency, the search of relevant prior art and the costs associated with legal actions (EPO, 2017).[2]

Our intention is to combine these two evidences regarding the role of patents: their effect in determining firms technological trajectories and their strategic nature. More specifically, the aim of this paper is to analyse theoretically how the presence and the effectiveness of a patent system affect firms technological decisions in a context where patents represent the channel through which firms compete to appropriate market value. We do so through a two-stage game; in the first stage, firms choose their technological trajectory − they select in which technological area to develop their technologies - and in the second stage firms take their patenting decisions and develop their patent portfolios. Technological areas overlap to a certain degree, meaning that technologies and patents developed for one area can be (at least partially) used in the other area as well. Portfolios' size and strength influence firms ability to appropriate profits generated in the areas they operate. In this framework, we show that strong patent protection may distort the direction of R&D activities inducing firms to inefficiently concentrate on the same technological area in some cases or to excessively diversify their R&D projects in others. Specifically, firms may refrain from choosing the research trajectory which is optimal from the industrial point of view in order to induce the competitor to patent less intensively. In our analysis, while an increase in the strength of patent rights − i.e. the extent to which patents can be used to extract value − undoubtedly distorts firms choices, a larger scope − the degree to which patent protection carries out in the other area as well − can either increase or decrease efficiency. Our analysis highlights that, on top of the classical deadweight loss associated with the monopolistic position that they grant, patents can be the source of another potential inefficiency related to the distortion they may cause on firms technological choices. Different contributions in the empirical literature have shown a high degree of heterogeneity in how effective patents are considered by firms (Cohen et al., 2005; Graham et al., 2009). This fact suggests that a strengthening of patent protection may distort technological choices favoring sectors and firms for which patents represent a more effective legal tool. Interestingly, our analysis shows that also in a symmetric context − with patents affecting firms and sectors evenly −, patents may alter firms technological choices.

Our model shares some modeling assumptions with von Graevenitz et al. (2013). The

[2]On similar lines, Cockburn et al. (2010) find a significant effect of fragmentation on innovative activities for firms that need to in-license the technology they use. For these firms the presence of patent thickets markedly reduce the share of revenues they are able to collect from sales of new products.

authors of this study present a model in which firms choose the technological area (technological opportunity in their wording) where to perform their R&D activities and how many patent applications to file. A central difference with our framework is that in von Graevenitz et al. (2013) the main focus rests on determining firms patenting activity − and specifically the influence of complexity in explaining the increase in patenting observed in the data.

The surge in patenting that has taken place during the recent years and the increased importance of patent portfolios is the the main motivation behind Choi and Gerlach (2017). The authors develop a theoretical model of patent portfolios in which two firms compete to develop a new product; each firm owns a patent portfolio of a given size and strength and when one firm successfully develops a product there is a chance that it infringes on some of the patents the other firm holds. A central assumption of the theory developed by Choi and Gerlach (2017) is that the size and therefore the strength of firms portfolios are exogenous. In the reality, as discussed above, firms tend to accumulate large patent portfolios to increase their bargaining position *vis* a *vis* competitors. In other words, the size of firms portfolios, and consequently their strength, is endogenously determined by firms. This is where our paper contributes to the analysis proposed in Choi and Gerlach (2017).

Our paper also aims at contributing to the growing theoretical literature studying the distortions arising in the choice of the direction of firms R&D efforts. Bryan and Lemus (2017) show that two types of inefficiencies may arise in this context. On the one hand, firms may excessively invest in 'easy to obtain' yet less valuable research projects. On the other hand, differences in the degree of appropriability of returns may induce firms to invest too often in R&D projects which are more difficult to develop but for which appropriation is a lesser problem. Interestingly, Bryan and Lemus (2017) also show that, in general, traditional policies used to stimulate R&D − patents and prizes − fail to correct inefficiencies in the direction of innovation activities. More recently, Chen et al. (2018) look at how a tightening of patentability standards affects the choice between a safe and a risky R&D project. The authors show that there are two countervailing effects in place. On the one hand, the probability that the risky process leads to a patentable innovation reduces and this fact lowers the incentives to select it. On the other hand, however, tighter standards reduce the chances of a new patentable innovation to emerge; this lowers the probability of the patent owner being replaced and increases the expected returns generated by the risky project.

The analyses presented by Cardon and Sasaki (1998) and Dasgupta and Maskin (1987) suggest reasons why firms may end up selecting similar research projects. The rationales for this technological clustering are, however, different from the one we focus on in this paper. Cardon and Sasaki (1998) present a multi-agent search model in which firms choose their technological path and then compete in the product market. Technological clustering emerges because of the presence of strategic complementarities of different nature. For instance, in the two-firm-two-path case, clustering may occur because it serves as a form of ex-ante collusion: only one firm obtains the patent and enjoys monopoly profits during the initial period. In Dasgupta and Maskin (1987) firms choose the degree of correlation of their R&D projects. The authors show the emergence of excessive correlation in the projects that are selected. Lowering correlation would generate a positive externality increasing the likelihood of success in the case of failure of the rival's project. This socially desirable effect is however not internalized by firms and leads to an "undue similarity of project characteristics".

The paper is organized as follows. In Section 2, we present the set-up of the model; Section 3 develops the analysis and derives the main results while in Section 4 we present some extensions to the baseline model. Finally, Section 5 concludes. The proofs of the results are presented in the mathematical Appendix.

2 The model

We consider two firms, A and B, operating in an economy made of two adjacent markets, 1 and 2, each of which corresponds to a given technological territory; by technological territory we mean the set of technologies aimed at manufacturing products for a given market. The two territories overlap and this means that, to a given extent, technologies belonging to a territory can be used to realize products for the other market. We often refer to the degree of overlap with the term technological proximity. For complex products, technological proximity can be rather large; for example, one can think to mobile devices (tablets, smartphones) and personal computers. Clearly, several hardware and software technologies incorporated into a tablet can also be embedded into a personal computer, and viceversa.

The two firms are competing in the two markets and we normalize to zero their current profits. With the aim of boosting their businesses, the two firms conduct R&D activities aimed at developing and patenting new technologies. We model the interaction between A and B as a two stage game; in the first stage they simultaneously choose in which technological territory to direct their R&D efforts. For the sake of simplicity, R&D costs are normalized to zero; the only expenses of developing technologies within a given territory are related to the opportunity cost of not investing in the other one. When firms choose the same territory, we say that *agglomeration* occurs; when they go for different territories, we say that *separation* occurs. After choosing the territory where to develop technologies, in the second stage of the game, the two firms decide how many patent applications to file. Firms then use the portfolio of patents they obtain at this stage to appropriate the value generated in the two areas; in line with the spirit of our analysis, in our model patents are the only way through which firms appropriate the value generated by their technological decisions.

The degree of technological proximity between the two areas is indexed by the parameter $\beta \in [0,1]$; in the extreme case of $\beta = 1$, the two areas perfectly overlap: all the technologies developed for one area can be perfectly employed for producing the products in the other area too. At the same time, patents obtained in one area have full strength in the other area as well.

Firms decisions are crucially driven by the value their technological choices generate in the two areas. In case of separation, firms develop technologies for different areas; we indicate with $\Phi(\beta)$ the value generated in each of the two areas. This value is positively affected by β: $\Phi'(\beta) > 0$. With this assumption we want to capture the fact that technologies developed for one area can be employed, and generate value, in the other area as well; as spillovers from one area to the other are larger the more the two areas overlap, then the larger β the more firms R&D activities in the two territories reinforce each other with the effect of generating larger values in the two territories.

In case of technological agglomeration, both firms develop technologies the same area; without loss of generality, let us assume that in this case, firms choose area 1. Agglomeration has two effects on the values generated in the two territories: on the one hand, as there are no firms active in area 2, the value generated in this area is minimal. The two firms can still use the technologies they develop for area 1 to create value in area 2, but having not developed specific technologies for this area, the value they can generate is necessarily limited. We refer to this value as $\Psi(\beta)$ smaller than $\Phi(\beta)$; also in this case it is natural to assume that $\Psi'(\beta) > 0$, as the technologies developed for area 1 are better able to generate value in area 2 too the larger the degree of technological proximity between the two areas. The second consequence of agglomeration is related to the value generated in the chosen area, area 1 in this case. We assume that when firms agglomerate, the value they generate in area 1 is maximum, provided that all technologies are specifically developed for this area. Formally, we denote with $\widehat{\Phi}$ larger than $\Phi(\beta)$ this value; clearly, since with agglomeration there are no technologies specifically developed for area 2, there is no spillover effect at work and industrial profits in area 1 do not depend on the technological overlap between the two areas.

Putting everything together, the values generated in the two areas with agglomeration and separation are ranked as follows: $\widehat{\Phi} > \Phi(\beta) > \Psi(\beta)$ for $\beta \in [0,1[$, while $\widehat{\Phi} \geq \Phi(1) \geq \Psi(1)$; the following table summarizes the values of the two technological areas in the two scenarios.

	Area 1	Area 2	Total industrial profits
Separation	$\Phi(\beta)$	$\Phi(\beta)$	$2\Phi(\beta)$
Agglomeration	$\widehat{\Phi}$	$\Psi(\beta)$	$\widehat{\Phi}+\Psi(\beta)$

Table 1: values of technological areas

The third column of Table 1 reports the overall industrial profits that are collected in the two areas; from these values it follows that agglomeration is desirable from the social point of view when $\widehat{\Phi}+\Psi(\beta)\geq 2\Phi(\beta)$ while separation is preferred otherwise. The following lemma summarizes this observation:

Lemma 1 *Agglomeration is socially desirable when $\Phi(\beta)\leq\Phi_1$, where $\Phi_1=\left(\widehat{\Phi}+\Psi(\beta)\right)/2$, while separation is desirable otherwise.*

Patents. After developing technologies, firms take their patenting decisions; formally, we denote with n_i the number of technologies patented by firm $i=A,B$. In our analysis, patents are crucial as they are the instrument through which firms can appropriate the value generated in each market.

We do not model explicitly how this occurs. Our basic idea is that the stronger the patent portfolio of a firm vis a vis that of the rival, the higher the firm bargaining power and the larger the share of the market value the firm is able to appropriate. The effectiveness of a portfolio in appropriating value depends on its size $-\ n_i\ -$ and on the strength of the patents in the technological area. We indicate with $\sigma>0$ the average *strength* of each patent in the market for which the technology has been developed; hence, σn_i indicates the strength of firm i's patent portfolio in the relevant technological area. With $\alpha(\beta)$ we denote the degree to which patents developed for an area can be used to appropriate value in the adjacent area; we can interpret $\alpha(\beta)$ as the *scope* of patent i.e. the degree to which patent rights carry out in the adjacent market. Formally, the strength of firm i's portfolio in the adjacent territory is $\alpha(\beta)\sigma n_i$ with $\alpha(\beta)\in[0,1]$ and $\alpha'(\beta)>0$. This latter assumption implies that patent scope increases the greater the technological overlap between two territories.

Both σ and $\alpha(\beta)$ can be interpreted as measures of the protection conferred to the patent holder by the legal system; clearly, the larger σ and $\alpha(\beta)$, the stronger the protection.

In the next section, we determine and discuss the equilibrium in firms technological and patenting choices; in this baseline model, we assume that patents only affect the ability of firms to appropriate profits while they do not impact on the overall industrial profits shown in Table 1. We will remove this assumption in one of the extensions developed in Section 4.

3 Analysis and results

We model a two stage simultaneous moves game. In the first stage, firms choose the technological territory and in the second stage they decide how many patents to file. The cost of patenting n technologies is $n^2/2$; this cost incorporates any possible expense the firm incurs to protect its technologies via patents (i.e. cost of filing applications, renewal fees, etc.). There are two alternative first stage equilibria: agglomeration, whereby both firms choose the same area, either area 1 or 2, and separation whereby firms select different territories. Given the symmetry of our model, without loss of generality, in what follows we assume that firm A chooses area 1 and then we focus on the technological choice of firm B. Therefore, the two relevant sub-games to consider are:

1) firm A has chosen technological territory 1 while B has chosen territory 2 (technological separation);

2) both firms have chosen technological territory 1 (technological agglomeration).

We start with the analysis of the first sub-game.

3.1 Firm B has chosen area 2 (technological separation)

We model firms profits using a very simple reduced-form representation where the share of industrial profits each firm is able to appropriate depends on the strength of its patent portfolio relative that of the competitor. Hence, in technological territory $\kappa = 1, 2$, firm i enjoys a share of value which is larger the stronger its portfolio and the weaker the portfolio of the rival.

As shown in Table 1, in the case of technological separation, industrial profits in each of the two areas are $\Phi(\beta)$. Firm A's patent portfolio is more effective in area 1, where its strength is $n_a\sigma$, than in area 2, where strength is $\alpha(\beta)n_a\sigma$; the opposite is true for B's portfolio. We assume that the share of area 1 profits which firm A appropriates equals $(1+n_a\sigma-\alpha(\beta)n_b\sigma)/2$, a function which is increasing in the strength of A's patent portfolio, decreasing in that of the competitor and equal to 1/2 if the two portfolios have the same strength. The share of the value $\Phi(\beta)$ generated in area 1 going to firm B is $(1+\alpha(\beta)n_b\sigma - n_a\sigma)/2$. Similarly, the shares of the value generated in area 2 going to firm A and B are assumed to be $(1+\alpha(\beta)n_a\sigma - n_b\sigma)/2$ and $(1+n_b\sigma-\alpha(\beta)n_a\sigma)/2$, respectively. Hence, firm i's profits, $i = A, B$, when firms separate are:

$$\pi_i^{1,2}(n_i, n_j) = \frac{\Phi(\beta)}{2}\left(1 + n_i\sigma - \alpha(\beta)n_j\sigma\right) + \frac{\Phi(\beta)}{2}\left(1 + \alpha(\beta)n_i\sigma - n_j\sigma\right) - \frac{n_i^2}{2},$$

where superscripts $1,2$ remind us firms technological choices. From the first order condition it is possible to determine the optimal number of patents filed by each firm:

$$n^{1,2} = \frac{\sigma}{2}\,\Phi(\beta)\,(1+\alpha(\beta))\,. \tag{1}$$

Notice that firms patenting activity increases *i*) the larger the value generated in the two areas, $\Phi(\beta)$, *ii*) the stronger the protection guaranteed by patents (the larger σ and $\alpha(\beta)$), and *iii*) the greater the technological overlap between the two areas: $dn^{1,2}/d\beta > 0$. The reason for *i*) is obvious: for a given overlap, the larger the value, the greater firms incentives to patent to appropriate such value; as *ii*) is concerned, a larger strength and scope make portfolios more effective in appropriating profits, and this induces firms to patent more aggressively. Finally *iii*) is a combination of the previous two effects: as a matter of fact, when β increases not only $\alpha(\beta)$ goes up, a fact that by *ii*) stimulates firms' patenting, but also $\Phi(\beta)$ gets larger, due to a stronger spillover effects across areas.

Plugging expressions $n^{1,2}$ into $\pi_i^{1,2}(n_i, n_j)$ we obtain the profits the two firms earn in the case of technological separation:

$$\pi^{1,2} = \Phi(\beta) - \frac{1}{8}\Phi(\beta)^2\sigma^2(1+\alpha(\beta))^2.$$

From this expression, it is immediate to prove that the following remark holds:

Remark 1 *In case of separation, firms profits decrease with patent strength, σ, and scope, $\alpha(\beta)$.*

We have just observed that an increase in σ and $\alpha(\beta)$ induces firms to patent more aggressively, thus increasing their costs. At the same time industrial profits, which do not change with the number of patents, are equally split between the two firms. This explains the remark. In this setting, investing in patents represents a waste of resources and the model resembles a typical prisoners' dilemma game, with firms investing in costly patents but that would be better off by coordinating not to apply for any of them.

The effect of β on firms profits is instead uncertain. On the one side, firms patent more aggressively the more the two areas overlap; this, as discussed above, reduces firms profits. On the other hand, though, β positively affects the value firms generate in the two areas; this additional effect clearly pushes firms profits up. Which effect dominates is unclear and the effect of β on firms profits remains undetermined.

3.2 Firm B has chosen area 1 (technological agglomeration)

When both firms develop technologies for area 1, industrial profits are $\widehat{\Phi}$ and $\alpha(\beta)$ in area 1 and 2, respectively; the strength of firms portfolios is higher in area 1 than in area 2, $n_i\sigma$ and $\alpha(\beta)n_i\sigma$ respectively. Therefore, firms profits are

$$\pi_i^{1,1}(n_i, n_j) = \frac{\widehat{\Phi}}{2}\left(1 + n_i\sigma - n_j\sigma\right) + \frac{\Psi(\beta)}{2}\left(1 + \alpha(\beta)n_i\sigma - \alpha(\beta)n_j\sigma\right) - \frac{n_i^2}{2}, \quad i = A, B.$$

Solving firms first order conditions, it is immediate to obtain the number of patents firms invest in with agglomeration:

$$n^{1,1} = \frac{\sigma}{2}\left(\widehat{\Phi} + \alpha(\beta)\Psi(\beta)\right). \tag{2}$$

As before $n^{1,1}$ increases with σ, $\alpha(\beta)$ and β. Substituting expression (2) into $\pi_i^{1,1}(n_i, n_j)$ we can determine the level of profits the two firms obtain when they both select technology area 1:

$$\pi^{1,1} = \frac{1}{2}\left(\widehat{\Phi} + \Psi(\beta)\right) - \frac{1}{8}\sigma^2\left(\widehat{\Phi} + \alpha(\beta)\Psi(\beta)\right)^2. \tag{3}$$

Remark 2 *In case of agglomeration, firms profits decrease with patent strength, σ, and scope, $\alpha(\beta)$.*

For the same reason as before, an increase in σ and $\alpha(\beta)$ reduces firms profits. The effect of a greater technological proximity is uncertain as it impacts negatively on profits through the effect it has on firms patenting, but it impacts positively on industrial profits via the effect it has on $\Psi(\beta)$.

Before moving to the first stage of the game, it is useful to compare the number of patents firms hold when they agglomerate with that they hold in the case of separation, $n^{1,1}$ and $n^{1,2}$. This comparison is interesting since it highlights how the technological choice of a firm impacts on the patenting incentives of the competitor. This strategic effect will play a role in B's technological decision, as we will highlight below.

Remark 3 *Let us define $\Phi_2 = \left(\widehat{\Phi} + \alpha(\beta)\Psi(\beta)\right)/\left(1 + \alpha(\beta)\right)$, with $\Phi_2 > \Phi_1 > \Psi(\beta)$; comparing $n^{1,1}$ and $n^{1,2}$ it follows that:*

i) $n^{1,1} > n^{1,2}$ iff $\Phi(\beta) < \Phi_2$;

ii) $|n^{1,1} - n^{1,2}|$ increases with σ.

The interpretation of the remark is intuitive. Patenting is proportional to industrial profits. Hence, firms patent more intensively in the case of agglomeration when $\Phi(\beta)$ is small compared to $\widehat{\Phi} + \Psi(\beta)$; formally, this occurs for $\Phi(\beta) < \Phi_2$. For larger values of $\Phi(\beta)$, instead, patenting is greater under separation. Notice, however, that since the effectiveness of patents to appropriate profits in the adjacent market is reduced and proportional to $\alpha(\beta)$, it does not follow that patenting is more intense when firms technological choices are efficient. As a matter of fact, the threshold Φ_2 is larger than the threshold Φ_1 shown in Lemma 1. This implies that for $\Phi(\beta) \in (\Phi_1, \Phi_2)$ separation is efficient but firms patent more in the case of agglomeration. According to part *ii)* of Remark 3 the absolute value of the difference in patenting increases with σ. This fact follows from Remarks 1 and 2 $-$ patenting increases with σ $-$ and suggests that the strategic effect of the technological choice on the the incentives to patent of the competitor gets stronger the larger σ.

3.3 Technology choice of firm B

We are now in the position to determine the technological choice of firm B. Comparing $\pi^{1,2}$ with $\pi^{1,1}$ the following proposition holds:

Proposition 1 *The technological choice of firm B is:*

i) when $\Phi(\beta) < \Phi_1$, firm B agglomerates for $\sigma < \overline{\sigma}$ and separates otherwise;

ii) when $\Phi_1 \leq \Phi(\beta) < \Phi_2$, firm B separates;

iii) when $\Phi(\beta) \geq \Phi_2$, firm B separates for $\sigma < \overline{\sigma}$ and agglomerates otherwise,

where $\overline{\sigma} = 2\sqrt{\frac{2\,\Phi(\beta) - \widehat{\Phi} - \Psi(\beta)}{(1+2\alpha(\beta)+\alpha(\beta)^2)\Phi(\beta)^2 - \left(\widehat{\Phi} + \alpha(\beta)\Psi(\beta)\right)^2}}$.

When taking its technological decision, firm B balances tow effects. On the one hand, it aims at maximizing industrial profits. This fact favors agglomeration for $\Phi(\beta) < \Phi_1$ and separation otherwise. We label this as the "direct effect" of firm B's decision. On the other hand, firm B is also in competition with firm A and its decision may be based also on strategic motives. From our previous discussion, we have already noted that B's decision is going to affect the amount of patents piled up by firm A. In particular, from Remark 3 we know that if $\Phi(\beta) < \Phi_2$, firm A patents more aggressively when B agglomerates; as a consequence, firm B might, strategically, be induced to separate in order to reduce A's amount of patents and, consequently, to limit its bargaining power. When $\Phi(\beta) > \Phi_2$, the opposite occurs, with firm B willing to agglomerate to reduce A's patenting incentives. This is what we label as the "strategic effect" of firm B's decision. Remark 3 also shows that the strategic effect becomes stronger as σ increases.

The combination of the direct and the strategic effects determines B's decision; specifically, three scenarios can be highlighted, depending on parameters' values. The first scenario occurs when $\Phi(\beta) < \Phi_1$; in this region, the direct and the strategic effects move in opposite direction: the direct effect calls for agglomeration while the strategic effect for separation. As long as patents are not too strong, the direct effect dominates and firm B chooses agglomeration; when σ is large enough, the strategic effect dominates and firms separate. The opposite occurs when $\Phi(\beta) > \Phi_2$; in this case, the direct effect calls for separation while the strategic effect for agglomeration. Again, only if σ is not too large, the direct effect dominates and separation occurs. Finally, an intermediate scenario may emerge when $\Phi_1 < \Phi(\beta) < \Phi_2$: in this region, both effects call for separation and, irrespectively on σ, firm B opts for separating into technological area 2.

From Proposition 1 we can highlight the cases in which firm B takes an inefficient $-$ from the industrial point of view $-$ technological decision. This is shown in the following Corollary:

Corollary 1 *When $\sigma \geq \overline{\sigma}$: a) there is inefficient separation if $\Phi(\beta) < \Phi_1$ and b) there is inefficient agglomeration if $\Phi(\beta) > \Phi_2$.*

Interestingly, Corollary 1 shows that strong patent rights may distort firms technological choices, leading either to excessive separation or to excessive agglomeration. The Corollary also reveals that an increase in patents strength, σ undoubtedly harms social efficiency.

In order to provide a visual representation of the equilibrium, in Figure 1 we draw the function $\Omega_b = \pi^b_{1,2} - \pi^b_{1,1}$; when $\Omega_b > 0$ firm B chooses to separate while, when $\Omega_b < 0$, B prefers technology area 1 and agglomeration occurs. The plot has been drawn for $\widehat{\Phi} = 4$, $\Psi(\beta) = 1$ and $\alpha(\beta) = 0.5$; these values imply that $\Phi_1 = 2.5$ and $\Phi_2 = 3$. In the figure, Ω_b has been drawn for three different levels of $\Phi(\beta)$, one for each region highlighted in Proposition 1: *i*) $\Phi(\beta) = 1.8$, *ii*) $\Phi(\beta) = 2.7$ and *iii*) $\Phi(\beta) = 3.5$.

Figure 1 shows very neatly the distortions associated with a stronger patent strength. When $\Phi(\beta) = 1.8 < \Phi_1$, for large values of σ separation occurs despite agglomeration being efficient. For greater values of $\Phi(\beta)$ - $\Phi(\beta) = 3.5 > \Phi_2$ - there is inefficient agglomeration when the patent strength is large. Finally, for intermediate values of $\Phi(\beta)$ - $\Phi_1 < \Phi(\beta) = 2.7 < \Phi_3$ - separation is efficient and it is also the equilibrium outcome.

3.4 Comparative statics

In order to understand better the characteristics of the equilibrium, it is interesting to present some comparative statics analyzes. As highlighted by the discussion of Proposition 1, B's technological choice depends on *i*) which, between separation and agglomeration, is the most efficient choice, and *ii*) the strategic effect of its choice on A's patenting.

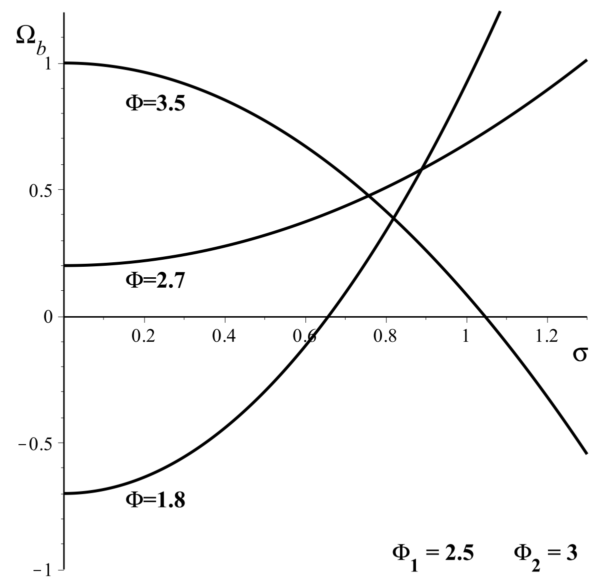

Figure 1: the choice of firm B

In the following sections we analyze how the equilibrium, and the associated distortions, change with the scope of patent protection and the degree of overlap between the two technological areas. In both cases, the results will depend on how the change in the parameter of interest impacts on *i*) and *ii*).

3.4.1 The effect of a change in the patent scope

The previous analyzes highlights the effect of patent strength, σ, on firm B's choice and on the associated distortions. Nonetheless, σ is only one dimension of patent protection; as discussed above, the protection granted to the patent holder is affected also by the scope of patents, intended as the possibility of the patent holder to use its portfolio to appropriate value in the adjacent area too. Patent scope naturally depends on the degree of overlap between the two areas, but to a certain extent is also a policy variable measuring how much the legal regime favors patent protection; formally, a legal regime κ is more pro-patent than regime τ if for a given overlap β, patents protecting technologies intended for area i are also effective in area j: $\alpha_\kappa(\beta) > \alpha_\tau(\beta)$, for any β.

Patent scope does not impact on industrial profits and therefore it does not alter the relative efficiency of separation *vs* agglomeration. This implies that the effect of a change in α on B's technological decision is entirely driven by its impact on the strategic effect on A's patenting.

Remarks 1 and 2 show that both $n^{1,1}$ and $n^{1,2}$ increase with patent scope which implies that B's profits reduce with $\alpha(\beta)$ both under agglomeration and under separation. With separation, the increase in A's patenting reduces the profits of B in proportion to $\Phi(\beta)$, the industrial profits that are generated in the two areas. Similarly, with technological agglomeration, the reduction in B's profits due to A's more intensive patenting is proportional to $\widehat{\Phi}$ and $\Psi(\beta)$, the industrial profits generated in the two areas in the case of technological agglomeration. Hence, as shown in Proposition 4 below, an increase in patent scope favors separation when $\Phi(\beta)$ is small relative to $\widehat{\Phi}$ and $\Psi(\beta)$, while it favors agglomeration otherwise.

Remark 4 *An increase in the patent scope stimulates separation if* $\Phi(\beta) < \Phi_3$*, while it stimulates aggregation otherwise, where* $\Phi_3 = \sqrt{\frac{\Psi(\beta)\left(\widehat{\Phi}+\alpha(\beta)\Psi(\beta)\right)}{1+\alpha(\beta)}} < \Phi_1$.

From this Remark, we can immediately evaluate the potential effects of an increase in patent scope on industrial efficiency. As agglomeration is efficient for $\Phi(\beta) \leq \Phi_1$ while separation is efficient otherwise, it follows that when $\Phi(\beta) < \Phi_3$ an increase in patent scope, by stimulating separation, reduces industrial efficiency. Also when $\Phi(\beta) > \Phi_1$, an increase in patent scope reduces industrial efficiency, although now the inefficiency is due to the fact that larger scope simulates agglomeration. Only when $\Phi_1 < \Phi(\beta) < \Phi_3$, a larger scope has a positive effect on efficiency.

3.4.2 The effect of a change in the technological proximity of the two areas

The effect of an increase in the technological proximity of the two areas is more complex to analyze. A larger β affects both the strategic effect on A's patenting as well as the relative efficiency of separation *vs.* agglomeration. The impact of a change in β on B's technological choice depends on the interplay of these two effects which may move in opposite directions. As far as the effect on A's patenting is concerned, we can borrow from our discussion on patent scope. A larger β increases the intensity of patenting of the competitor both in the case of separation and in the case of agglomeration. Similarly to our previous analysis, such an increase favors separation over agglomeration when $\Phi(\beta)$ is small, while the opposite occurs when $\Phi(\beta)$ is large. In addition to this effect, a change in β also impacts on the industrial profits;[3] they increase at a rate $2\Phi'(\beta)$ in the case of separation and at a rate $\Psi'(\beta)$ in the case of agglomeration. Hence, greater technological proximity makes separation more efficient than agglomeration when $\Phi'(\beta) > \Psi'(\beta)/2$ and this fact induces firm B to select area 2 more often; the opposite occurs when $\Phi'(\beta) < \Psi'(\beta)/2$. The following remark, summarizes the above discussion.

[3]Recall that total industrial profits with separation are $2\Phi(\beta)$ while they are $\widehat{\Phi}+\Psi(\beta)$ with agglomeration.

Remark 5 *A greater technological proximity favors separation when:*

i) *a larger β increases the efficiency of separation realative to agglomeration (i.e. $\Phi'(\beta) > \Psi'(\beta)/2$);*

ii) *the strategic effect caused by the increase in A's patenting is stronger under agglomeration (i.e. $\Phi(\beta)$ is small).*

4 Extensions

4.1 Patents affecting industrial profits

In the analysis presented so far we have assumed that the only role of patents is to determine how firms share the (exogenous) industrial profits. The underlying assumption was that patents *per se* do not generate value for the firms. In this section, we extend our analysis to the case where patents generate additional value for the patent holder.

One way to incorporate this issue in our model is to assume that patents generate profits which are proportional to their strength and to the industrial profits. Formally, in the case of technological agglomeration firm i's, $i, j = a, b$, profits become:

$$\pi_i^{1,1}(n_i, n_j) = \frac{\widehat{\Phi}}{2}\left(1 + n_i\sigma - \alpha(\beta)n_j\sigma\right) + \frac{\Psi(\beta)}{2}\left(1 + \alpha(\beta)n_i\sigma - n_j\sigma\right) + \left(\widehat{\Phi} + \alpha(\beta)\Psi(\beta)\right)gn_i\sigma - \frac{n_i^2}{2}. \tag{4}$$

Patents generate additional profits $\widehat{\Phi}gn_i\sigma$ in area 1 where they are fully effective and $\alpha(\beta)\Psi(\beta)gn_i\sigma$ in area 2 where they are less than fully effective, with $g \geq 0$. Similarly, in the case of technological separation, profits are:

$$\pi_i^{1,2}(n_i, n_j) = \frac{\Phi(\beta)}{2}\left(1 + n_i\sigma - n_j\sigma\right) + \frac{\alpha(\beta)}{2}\left(1 + \alpha(\beta)n_i\sigma - \alpha(\beta)n_j\sigma\right) + \Phi(\beta)\left(1 + \alpha(\beta)\right)gn_i\sigma - \frac{n_i^2}{2}, \tag{5}$$

where $\Phi(\beta)gn_i\sigma$ are the additional profits in the technological area of firm i while $\Phi(\beta)\alpha(\beta)gn_i\sigma$ indicate the additional profits collected in the adjacent area.

From firms first order conditions it follows that the optimal numbers of patents with separation and with agglomeration are:

$$n^{1,2} = \frac{\sigma}{2}(1 + 2g)\left(\widehat{\Phi} + \alpha(\beta)\Psi(\beta)\right), \tag{6}$$

and

$$n^{1,1} = \frac{\sigma}{2}(1 + 2g)\Phi(\beta)\left(1 + \alpha(\beta)\right). \tag{7}$$

Plugging expressions (6) and (7) into firm B's equilibrium profits when it separates and when it agglomerates, it follows that:

Proposition 2 *An increase in g favors agglomeration if $\Phi < \Phi_2$, while it favors separation otherwise.*

This proposition has a natural interpretation once recalled that when $\Phi(\beta) < \Phi_2$ firms patent more intensively under agglomeration than under separation, while the opposite occurs when $\Phi(\beta) > \Phi_2$. As the profits firms are able to generate is proportional to the number of patents obtained by each firm, then firms are more willing to take the technological decisions that are conducive to more intense patenting; therefore, as when $\Phi(\beta) < \Phi_2$ firms patent more with agglomeration, then they will be willing to agglomerate even more the greater the value it generates via patenting (i.e. the greater g). Obviously, the opposite occurs when $\Phi(\beta) > \Phi_2$.

4.2 Asymmetric market values

The scenario analyzed so far was characterized by symmetric market values: when firms separate, both technological areas generate the same value $\Phi(\beta)$. It is interesting to analyze an asymmetric environment, whereby one area is more valuable than the other. This analysis is useful not only to characterize the different incentives towards agglomeration/separation in an asymmetric setting, but also because it will allow us to extend the model to the case in which one firm is financially constrained and it cannot patent more than a certain amount of technologies. This additional extension which considers a practically relevant scenario will be the focus of the next section.

Let us assume that area 1 is more profitable than area 2. As mentioned, asymmetry in market values is relevant in case of separation and the easiest way to incorporate such asymmetry into the model is to assume that industrial profits are $\Phi(\beta) + \delta$ in area 1 and $\Phi(\beta) - \delta$ in area 2, where $\delta \geq 0$ indicates the degree of asymmetry. This way of modeling the asymmetry in market values has the nice property of not affecting the overall industrial profits; therefore, independently of δ, the condition for which agglomeration/separation is efficient is still given by Lemma 1.

As before, we assume that firm A chooses area 1; whether separation or agglomeration occurs depends on firm B's technological decision.

The subgame with agglomeration is the same as above and firms profits at the subgame equilibrium are as in expression (3). When firm B separates and chooses area 2, firms' profits are:

$$\pi_a^{1,2}(n_a, n_b) = \frac{\Phi(\beta) + \delta}{2}(1 + n_a\sigma - \alpha(\beta)n_b\sigma) + \frac{\Phi(\beta) - \delta}{2}(1 + \alpha(\beta)n_a\sigma - n_b\sigma) - \frac{n_a^2}{2},$$
$$\pi_b^{1,2}(n_b, n_a) = \frac{\Phi(\beta) - \delta}{2}(1 + n_b\sigma - \alpha(\beta)n_a\sigma) + \frac{\Phi(\beta) + \delta}{2}(1 + \alpha(\beta)n_b\sigma - n_a\sigma) - \frac{n_b^2}{2}.$$

From the first order conditions it is possible to determine the optimal number of patents obtained by the two firms with separation:

$$n_a^{1,2} = \frac{\sigma}{2}\left(\Phi(\beta)(1 + \alpha(\beta)) + \delta(1 - \alpha(\beta))\right) \quad \text{and} \quad n_b^{1,2} = \frac{\sigma}{2}\left(\Phi(\beta)(1 + \alpha(\beta)) - \delta(1 - \alpha(\beta))\right). \quad (8)$$

It is interesting to notice that A's number of patents increases with δ while that of firm B decreases; this amounts to say that the greater the asymmetry, the more firm A improves its bargaining position vis a vis firm B. Plugging these values back in the above expressions of $\pi_i^{1,2}(n_i, n_j)$, we obtain the firms profits in this subgame. From a comparison of the profits firm B gets with separation and those it gets with agglomeration it is possible to prove the following proposition:

Proposition 3 *The larger the asymmetry, the more firms agglomerate in the high valued area 1.*

This result has a clear interpretation. The larger δ the lower the incentives to select area 2 and build a patent portfolio which is fully effective in area 2 but less than fully effective in the more profitable area 1. Hence, asymmetry in the profitability of the two markets is another potential source of inefficiency, tilting firms technological decisions towards the most lucrative market. More specifically, as δ does not impact on the industrial profits, Proposition 3 suggests that asymmetry may induce an inefficient technological choice when separation is optimal. By contrast, when agglomeration is efficient, asymmetry may bring the market outcome out of an inefficient separation, thus reducing the distortion shown in part *a)* of Corollary 1.

4.2.1 Firm B is financially constrained

The previous section suggests an interesting extension which is worth discussing. So far we have assumed that firms could apply for any number of patents they wish. However, in some circumstances firms might have a limited patenting capacity. For instance, SMEs might lack the expertise to file patent applications, they might be unable to actually enforce patents − a fact that would reduce drastically the benefits of applying for patent protection in the first place − or they might simply be financially constrained. In this section, we study what happens to a firm when, for any reason, it is limited in the number of patents it can apply for. Specifically, we assume that, regardless of its technological decision, firm B can apply for at most $\overline{n}$ patents, a number below its optimal/desired level; formally, $\overline{n} < min\{n_b^{1,1}, n_b^{1,2}\}$.

In this setting, firms are no longer symmetric and, therefore, to characterize the equilibrium we would need to explicitly determine the technological choice of each firm. This analysis is beyond the scope of this section. What we are interested in studying here is much simpler and it essentially boils down to answering the following question: suppose that the unconstrained firm A has chosen the more lucrative area 1, does a more stringent constraint of B's ability to patent stimulate more agglomeration or more separation?

In order to discuss this issue, it is useful to reinterpret the difference between B's profits when it separates and those when it agglomerates, indicated above with Ω_b, as the incentive to separate. Clearly, when firm B cannot apply for more than $\bar{n}$ patents, this difference depends on $\bar{n}$. It is possible to prove to following:

Proposition 4 *A more stringent constraint: i) favors agglomeration if, without constraint, firm B would patent more when it separates than when it agglomerates ($n_b^{1,2} > n_b^{1,1}$), ii) favors separation if, without constraint, firm B would patent more when it separates than when it agglomerates ($n_b^{1,2} < n_b^{1,1}$).*

Proposition 4 can be easily interpreted: firm B is more likely to go for the option which is less hurted by the constraint. Hence, if unconstrained, firm B would patent more by agglomerating, when it faces the constraint the firm would be much more severely affected by the constraint if it agglomerates than if it separates; as a consequence, a more stringent constraint (smaller $\bar{n}$) is likely to induce firm B to go for this latter, less painful, option.

At this point, one may wonder whether a more stringent constraint, formally represented by a lower level of $\bar{n}$, favors the inefficiency related to patents. Discussing this issue is the aim of our last corollary.

Corollary 2 *When $\Phi(\beta) < \Phi_1$ or $\Phi(\beta) > \Phi_2 + \delta(1 - \alpha(\beta))/(1 + \alpha(\beta))$, a more stringent constraint generates more inefficiency.*

This corollary reveals that when $\Phi(\beta)$ is sufficiently small (resp. large), a more stringent constraint pushes firm B to agglomerate (resp. separate) when separation (resp. agglomeration) is socially desirable. Unless $\Phi(\beta)$ takes intermediate values, a decrease in $\bar{n}$ has the effect of generating inefficiency. Interestingly, the greater the asymmetry (formally the greater δ), the less likely it is that this occurs.[4]

[4] It is necessary to remember, however, that the game we are analysing here is very specific and based on the assumption that firm A always chooses the most profitable technological area. While this simplification does not entail any loss of generality in the symmetric game, in the presence of asymmetry, as in this case, a more complete analysis of the equilibrium would require to study also the technological choice of firm A.

5 Conclusions

In this paper we analyse theoretically how the presence and the effectiveness of a patent system affect firms technological decisions in a context where patents have a pure strategic role, that is they have the sole scope to allow firms to appropriate the market value they generate with their technological decisions. We do so through a two-stage game; in the first stage, firms choose their technological trajectory − they select in which technological area to develop their technologies - and in the second stage firms take their patenting decisions and develop their patent portfolios.

Crucially, technological areas overlap to a certain degree, meaning that technologies and patents developed for one area can be (at least partially) used in the other area as well. Portfolios' size and strength influence firms ability to appropriate profits generated in the areas they operate. We show that strong patent protection may distort the direction of R&D activities. Specifically, firms may refrain from choosing the research trajectory which is optimal from the industrial point of view in order to induce the competitor to patent less intensively. Interestingly, our analysis reveals that while an increase in the strength of patent rights − i.e. the extent to which patents can be used to extract value − undoubtedly distorts firms choices, a larger scope − the degree to which patent protection carries out in the other area as well − can either increase or decrease efficiency.

Our analysis highlights that, on top of the classical deadweight losses associated with the monopolistic positions that they grant, patents may have also an additional negative effect accruing from to the distortions they may generate in firms' technological decisions.

A. Mathematical appendix

Proof. of Remark 3. Using the expressions for $n^{1,1}$ and $n^{1,2}$ it follows that

$$n^{1,1} - n^{1,2} = \frac{\sigma}{2}\left(\widehat{\Phi} - \Phi(\beta) - (\Phi(\beta) - \Psi(\beta))\,\alpha(\beta)\right).$$

The Remark follows from simple algebraic inspection and differentiation. Note that point *iii*) holds for any acceptable parameters' value as, by construction, $\Phi(\beta) > \Psi(\beta)$. ■

Proof. of Proposition 1 and Corollary 1. Let us indicate with $\Omega_b = \pi^b_{1,2} - \pi^b_{1,1}$ the difference between the profits that firm B obtains when selecting area 2 (separation) and those it collects when it chooses area 1 (agglomeration): when $\Omega_b > 0$ firm B chooses to separate while, when $\Omega_b < 0$, B prefers area 1. Formally, using B's profits with separation and with agglomeration:

$$\Omega_b = M + H\sigma^2, \tag{9}$$

where[5]

$$H = \frac{1}{8}\left(\widehat{\Phi} + \alpha\Psi\right)^2 - \frac{1}{8}\left(1 + 2\alpha + \alpha^2\right)\Psi^2,$$

and

$$M = \Phi - \frac{1}{2}\Phi_h - \frac{1}{2}\Psi.$$

We need to determine the sign of Ω_b. Ω_b is a parabola in Φ; when $sign\{H\} = sign\{M\}$, the expression (9) does not have real roots, and $sign\{\Omega_b\} = sign\{H, M\}$. When, instead, $sign\{H\} \neq sign\{M\}$, expression (9) has real roots.

The sign of M is easily identified, as $M < 0$ for $\Phi < \Phi_1$ and $M > 0$ otherwise. Polynomial H is a symmetric concave parabola in Φ with roots:

$$\frac{\Phi + \alpha\Psi}{1+\alpha} \quad \text{and} \quad -\frac{\Phi + \alpha\Psi}{1+\alpha};$$

the latter root is negative and it can be ignored. Indicating $\Phi_2 = \frac{\Phi+\alpha\Psi}{1+\alpha}$, it follows that $H > 0$ for $\Phi < \Phi_2$ and $H > 0$ otherwise. It is also possible to check that Φ_2 is admissible as $\Phi_2 > \Psi$, where Ψ indicates the minimum level of the value generated in a technological area.

Note also that $\Phi_2 > \Phi_1$; we can therefore identify three regions: *i*) $\Phi < \Phi_1$, whereby $M < 0$ and $H > 0$, *ii*) $\Phi_1 < \Phi < \Phi_2$, whereby $M > 0$ and $H > 0$ and, *iii*) $\Phi > \Phi_2$ whereby $M > 0$ and $H < 0$.

[5]For the sake of simplicity, in the functions $\Phi(\beta)$, $\Psi(\beta)$ and $\alpha(\beta)$ we omit the argument.

In regions i) and iii), $sign\{H\} \neq sign\{M\}$, hence expression (9) has two roots: $\overline{\sigma} = \sqrt{\frac{-M}{H}}$ and $\underline{\sigma} = -\sqrt{\frac{-M}{H}}$; $\underline{\sigma} < 0$ and it can be ignored. In region i), $M < 0$ (i.e. agglomeration is desirable) and $H > 0$; in this case, Ω_b is a symmetric convex parabola in Φ, which is negative (i.e. firms agglomerate) for $\sigma < \sigma_1$ and positive otherwise. In region iii), $M > 0$ (i.e. separation is desirable) and $H < 0$; Ω_b is a symmetric concave parabola in Φ, which is positive (i.e. firms separate) for $\sigma < \sigma_1$ and negative otherwise. ■

Proof. of Remark 4. Firm B's profit difference $\Omega_b = \pi_b^{1,2} - \pi_b^{1,2}$ is given in expression (9); taking the derivative with respect to α, it follows that:[6]

$$\frac{d\Omega_b}{d\alpha} = \frac{1}{4}\left(\Psi(\widehat{\Phi} + \alpha\Psi) - \Phi^2(1+\alpha)\right)\sigma^2$$

which is positive for $\Phi(\beta) < \Phi_3$ and negative otherwise. ■

Proof. of Remark 5. Suppose that firm B separates. An increase in β affects B's profits as follows:

$$\frac{d\pi_b^{1,2}}{d\beta} = \frac{\partial\pi_b^{1,2}}{\partial\beta} + \frac{\partial\pi_b^{1,2}}{\partial n_a}\frac{dn_a^{1,2}}{d\beta} + \frac{\partial\pi_b^{1,2}}{\partial n_b}\frac{dn_b^{1,2}}{d\beta}.$$

The last term is zero by the envelope theorem. The first term equals

$$\frac{\partial\pi_b^{1,2}}{\partial\beta} = \frac{\Phi'}{2}\left(1 - n_a^{1,2}\sigma + \alpha n_b^{1,2}\sigma + 1 - \alpha n_a^{1,2}\sigma + n_b^{1,2}\sigma\right) + \frac{\Phi}{2}\left(\alpha' n_b^{1,2}\sigma - \alpha' n_a^{1,2}\sigma\right),$$

which reduces to Φ' as $n_a^{1,2} = n_b^{1,2}$ in equilibrium. Using the equilibrium expression for $n_a^{1,2}$, the second term becomes

$$\frac{\partial\pi_b^{1,2}}{\partial n_a}\frac{dn_a^{1,2}}{d\beta} = -\frac{\sigma^4}{4}\Phi(1+\alpha)\left[\Phi'(1+\alpha) + \Phi\alpha'\right].$$

Summarizing, the effect of a greater β on B's profits in case of separation equals

$$\frac{d\pi_b^{1,2}}{d\beta} = \Phi' - \frac{\sigma^4}{4}\Phi(1+\alpha)\left[\Phi'(1+\alpha) + \Phi\alpha'\right],$$

an expression which reduces with Φ.

Using similar arguments, it follows that the effect of a larger β on B's profitis in case of agglomeration equals

[6]For notational convenience, we have omitted the argument β in the functions $\Phi(\beta)$, $\Psi(\beta)$ and $\alpha(\beta)$.

$$\frac{d\pi_b^{1,1}}{d\beta} = \frac{\Psi'}{2} - \frac{\sigma^4}{4}\left(\widehat{\Phi} + \alpha\Psi\right)(\alpha'\Psi + \alpha\Psi'),$$

an expression which does not depend on Φ.

Hence, an increase in β favors separation when $\Phi' > \Psi'/2$ and for small values of Φ. ■

Proof. of Proposition 2. As before, we assume that firm A chooses area 1; whether firms agglomerate or separate depends on the decision of firm B. Using (6), the profits of firm B in case of separation are:

$$\pi_g^{1,2} = \Phi + \frac{1}{8}\,\Phi^2\sigma^2\,(2\,g-1)\,(2\,g+1)\,(\alpha\,(\beta)+1)^2\,.$$

where the subscript g indicates that we are in the scenario where patents generate value. Using (7), the profits of firm B in case of agglomeration are:

$$\pi_g^{1,1} = \frac{1}{2}\left(\widehat{\Phi} + \Psi\right) - \frac{1}{8}\sigma^2\left(1-4\,g^2\right)\left(\widehat{\Phi} + \alpha\,(\beta)\,\Psi\right)^2.$$

We can now compute the difference in firm B's profits between separation and agglomeration, $\Omega_b^g = \pi_g^{1,2} - \pi_g^{1,1}$:

$$\Omega_b^g = \Omega_b - \frac{1}{2}\left(\widehat{\Phi} + \alpha(\beta)\Psi + \alpha(\beta)\Phi + \Phi\right)\left(\widehat{\Phi} + \alpha(\beta)\Psi - \alpha(\beta)\Phi - \Phi\right)\sigma^2 g^2$$

where Ω_b is the difference in B's profits when $g = 0$, i.e. in our benchmark case. Simple differentiation reveals that $d\Omega_b^g/dg < 0$ if $\Phi(\beta) < \Phi_2$ and $d\Omega_b^g/dg > 0$ otherwise. ■

Proof. of Proposition 3. B's profits in case of agglomeration are not affected by δ; hence the impact of the asymmetry on firm B's decision is entirely driven by the effect of δ on the profits the firm obtains if it separates. Using the optimal number of patents given in expressions (8), B's profits with separation are given by:

$$\pi_b^{1,2} = \Phi(\beta) - \frac{1}{8}\left((1+\alpha(\beta))^2\,\Phi(\beta)^2 - 6\,\delta\,\left(1-\alpha(\beta)^2\right)\Phi(\beta) + \delta^2\,(1-\alpha(\beta))^2\right)\sigma^2.$$

The derivative of this expression with respect to δ is therefore:

$$-\frac{1}{4}\,(1-\alpha(\beta))\,\sigma^2\,(\delta(1-\alpha(\beta)) + 3\,\Phi(\beta)(1+\alpha(\beta)))\,,$$

which is always negative. As B's profits with separation decrease with δ while those with agglomeration are not affected by δ, the proposition easily follows. ■

Proof. of Proposition 4. Assuming that $\overline{n} < min\{n_b^{1,1}, n_b^{1,2}\}$, firm B applies exactly for $\bar{n}$ patents, regardless its technological choice. Firm A is unconstrained, hence depending on what B does, it is free to apply for its optimal number of patents, $n_a^{1,1}$ and $n_a^{1,2}$. Plugging the former into $\pi_b^{1,1}(n_a, n_b)$ and the latter into $\pi_b^{1,2}(n_a, n_b)$ and given that $n_b = \bar{n}$, it is possible to derive $\Omega_b(\bar{n})$:

$$\begin{aligned}\Omega_b(\bar{n}) = &-\frac{1}{2}\sigma\left(\widehat{\Phi} + \Psi(\beta)\alpha(\beta) - (\Phi(\beta) + \delta)\,\alpha(\beta) + \delta - \Phi(\beta)\right)\bar{n} \\ &+ \frac{1}{4}\sigma^2\left(\left(\widehat{\Phi} + \Psi(\beta)\alpha(\beta)\right)^2 - \left(\Phi(\beta)\left(1 + \alpha(\beta)\right) + \delta\left(1 - \alpha\left(\beta\right)\right)\right)^2\right) - \frac{1}{2}\Psi\left(\beta\right) - \frac{1}{2}\widehat{\Phi} + \Phi\left(\beta\right)\end{aligned}$$

Simple differentiation reveals that $d\Omega_b(\bar{n})/d\bar{n} = -\sigma\left(\widehat{\Phi} + \Psi(\beta)\alpha(\beta) - (\Phi(\beta) + \delta)\,\alpha(\beta) + \delta - \Phi(\beta)\right)/2$, which corresponds to $n_b^{1,2} - n_b^{1,1}$. ∎

Proof. of Corollary 2. From Proposition 4 we know that $d\Omega_b(\bar{n})/d\bar{n} < 0$ - a more stringent constraint stimulates separation - if $n_b^{1,2} < n_n^{1,1}$ or, equivalently, if $\Phi(\beta) < \Phi_2 + \delta(1 - \alpha(\beta))/(1 + \alpha(\beta))$. As aggregation is socially desirable if $\Phi(\beta) < \Phi_1$ and provided that $\Phi_1 < \Phi_2$, it follows that when $\Phi < \Phi_1$ a more stringent constraint stimulates separation when aggregation is desirable. Similarly, from Proposition 4 we know that $d\Omega_b/d\bar{n} > 0$ if $\Phi(\beta) > \Phi_2 + \delta(1 - \alpha(\beta))/(1 + \alpha(\beta))$. As separation is socially desirable if $\Phi(\beta) > \Phi_1$, it follows that when $\Phi(\beta) > \Phi_2 + \delta(1 - \alpha(\beta))/(1 + \alpha(\beta))$ a more stringent constraint stimulates agglomeration when separation is desirable. ∎

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