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UNCERTAINTY AND MONETARY POLICY DURING THE GREAT RECESSION

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Uncertainty and Monetary Policy During the Great Recession*

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Abstract

We employ a nonlinear VAR framework and a state-of-the-art identification strategy to document the large response of real activity to a financial uncertainty shock during and in the aftermath of the great recession. We replicate this evidence with an estimated DSGE framework featuring a concept of uncertainty comparable to that in our VAR. We then use the estimated framework to quantify the output loss due to the large uncertainty shock that materialized in 2008Q3. We find such a shock to be able to explain about 60% of the output loss in the 2008-2014 period. The same estimated model unveils the role successfully played by the Federal Reserve in limiting the output loss that would otherwise have occurred had monetary policy been conducted as in normal times. Finally, we show that the rule estimated during the great recession is able to deliver an economic outcome closer to the flexible price one than the rule describing the Federal Reserve's conduct in normal times.

Keywords: Uncertainty shock, nonlinear IVAR, nonlinear DSGE framework, minimum-distance estimation, great recession.

JEL codes: C22, E32, E52.

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1 Introduction

Financial uncertainty has been identified as one of the drivers of the US business cycle (Bloom (2009), Leduc and Liu (2016), Basu and Bundick (2017), Ludvigson, Ma, and Ng (2019)). Notably, the two highest realizations of the VIX (a popular proxy of financial uncertainty) materialized in correspondence with two of the largest drops in real activity occurred in the last two centuries, i.e., the great recession and the Covid-19 recession.[1] Such dramatic drops in real activity called for immediate and massive interventions by the Federal Reserve to sustain the business cycle. The synchronous occurrence of record large jumps in financial uncertainty, recessions of the magnitude of the 2007-09 one and the one that began in 2020, and unprecedented monetary policy moves naturally calls for the investigation of the output loss due to financial uncertainty shocks during these extreme events on the one hand, and on the stabilizing role played by systematic monetary policy on the other hand. Given the current data availability, we focus on the great recession episode and ask two questions. First, were financial uncertainty shocks relevant contributors to the US great recession? Second, how important was monetary policy in alleviating the business cycle costs due to uncertainty shocks?

This paper addresses these questions by proceeding in three steps. First, working with a nonlinear VAR estimated with post-WWII US data, we document the response of real activity to the large uncertainty shock occurred in 2008Q4 in correspondence with Lehman Brothers' bankruptcy. Our nonlinear VAR identifies the business cycle impact of exogenous changes in uncertainty thanks to the information carried by selected events occurred during the post-WWII period. In particular, we follow Ludvigson, Ma, and Ng (2019) and focus on events characterized by bursts in financial uncertainty that are likely to be informative on the realizations of financial uncertainty shocks. This narrative identification strategy, recently put forth by Ludvigson, Ma, and Ng (2019) and Antolín-Díaz and Rubio-Ramírez (2019), enables us to avoid imposing questionable zero restrictions on the uncertainty-business cycle contemporaneous relationship.[2] We

[1] The VIX reached its historical record level of 82.69 on March 16, 2020. The second highest value ever recorded by the VIX is 80.06, which occurred on October 27, 2008. These extreme realizations are associated with spectacular drops in output during the Great Recession (-3.92% in terms of y-o-y real GDP growth in 2009Q2) and the Covid-19 pandemic (-9.03% in 2020Q2). For a paper exploiting the information associated with natural disasters (a different type of extreme events) to estimate the macroeconomic impact of uncertainty shocks, see Baker, Bloom, and Terry (2020).

[2] We follow Ludvigson et al.'s (2019) approach (vs. Antolín-Diaz and Rubio-Ramírez's 2019) for two reasons. First, Ludvigson, Ma, and Ng (2019) are concerned with the identification of financial uncertainty shocks, as we are. Second, Ludvigson, Ma, and Ng (2019) pursue a frequentist approach. This enables us to consider as "data" the impulse responses produced with their identification strategy,

find that nonlinearities are present, statistically relevant, and quantitatively important. In particular, with respect to "normal times" (whose dynamics are captured via a linear VAR), we document a peak response of output 50% larger during the great recession (conditional on a same-size shock), and a peak monetary policy response twice as large (a cut of the federal funds rate of about 100 basis points in normal times vs. 200 basis points during the great recession). Our results are robust to a variety of perturbations of the baseline nonlinear VAR, which also include controlling for the role played by first-moment financial shocks during the great recession.[3]

The second step of our analysis estimates a version of the Basu and Bundick (2017) model with the Bayesian minimum-distance direct inference approach developed by Christiano, Trabandt, and Walentin (2010), which we extend to the case of a nonlinear, third-order approximated DSGE model. This is a limited information method - alternative to the GMM approach developed by Andreasen, Fernández-Villaverde, and Rubio-Ramírez (2018) - that treats as "data" the impulse responses produced with our nonlinear VAR. This approach makes the estimation of a large set of structural parameters easy to implement. To our knowledge, the Bayesian estimation of a third-order approximated DSGE model by impulse response function matching is a novel contribution to the literature *per se*.[4] The presence in the DSGE framework of a theoretical concept of financial uncertainty in line with the proxy we use in our empirical analysis makes Basu and Bundick's model particularly suited to our purposes, because it enables us to match the dynamics of financial uncertainty in the data with its theoretical counterpart.[5] This part of the analysis shows that our DSGE framework goes a long way

something we need to do to estimate our DSGE model, as explained in Section 4. Differently, the approach by Antolín-Díaz and Rubio-Ramírez (2019) is Bayesian. Hence, their impulse responses are related to posterior densities, which naturally merge information coming from the data and the prior densities used to estimate their framework.

[3]Lhuissier and Tripier (2016), Alessandri and Mumtaz (2019) and Caggiano, Castelnuovo, Delrio, and Kima (2020) show that uncertainty shocks exert larger business cycle effects in presence of financial stress. Nonlinear structural DSGE frameworks such as the ones proposed by Gilchrist, Sim, and Zakrajšek (2014), Arellano, Bai, and Kehoe (2019), Alfaro, Bloom, and Lin (2019), and Fernández-Villaverde and Guerron-Quintana (2020) provide economic intuitions on the role of financial frictions in magnifying the real effects of uncertainty shocks. While all these DSGE models assume fully flexible prices (for an exception dealing with a calibrated model, see Bonciani and van Roye (2016)), our estimated model deals with sticky prices and focuses on the role played by monetary policy in dampening the real effects of uncertainty shocks working through channels such as households' precautionary savings and precautionary labor supply and firms' upward pricing bias.

[4]The codes to implement this procedure are available at the following address: https://sites.google.com/site/giovannipellegrinopg/home.

[5]As in Basu and Bundick (2017), we conceptually focus on the identification and modeling of ex-ante, implied financial volatility shocks and their macroeconomic effects. While the majority of the contributions in this uncertainty literature deal with implied volatility, it is important to acknowledge

in replicating our empirical facts, therefore providing us with an empirically credible microfounded framework to perform counterfactual analysis, which is what we do in the final step of our investigation. Conditional on our estimated framework, we find that an uncertainty shock as large as the one that hit the US economy in 2008Q4 might be responsible of about 60% of the output lost by the US economy during and in the aftermath of the great recession.[6]

The third step of our analysis assesses the role played by the US monetary policy in tackling the negative effects on real activity of the uncertainty shock occurred during the acceleration of the global financial crisis in correspondence with Lehman Brothers' bankruptcy. First, we show that the monetary policy response engineered by the Federal Reserve during the great recession (as interpreted by our estimated framework) successfully limited the output cost generated by the spike in financial uncertainty in 2008Q4. A comparison between the economic outcome implied by the policy rule estimated with great recession data and that estimated in normal times (i.e., conditional on the impulse responses produced with a linear VAR) reveals that the stronger policy response to output growth fluctuations during the great recession possibly halved the uncertainty shock-induced output loss, and shortened the duration of the recession. We then use our estimated DSGE framework to simulate the economic outcome implied by an unfeasible optimal simple rule tracking the evolution of the real natural interest rate, a policy conduct that would imply an allocation of resources as under flexible prices. We find that, both in terms of implied dynamics after an uncertainty shock and in terms of macroeconomic volatilities, the rule estimated under the great recession brings the economy closer to such allocation of resources. Our results offer support to prompt and aggressive policy interventions as those implemented by the Federal Reserve during the great recession.[7]

The paper develops as follows. Section 2 discusses the related literature. Section 3 presents our non-linear VAR model, the identification strategy we use, and the empirical results. Section 4 describes the DSGE model and the estimation approach, and

that also realized volatility can be a driver of the business cycle. Berger, Dew-Becker, and Giglio (2020) find that innovations in realized stock market volatility are robustly followed by contractions, while shocks to forward-looking uncertainty have no significant effect on the economy.

[6] In a short companion paper, we show that this figure can be severely underestimated if the empirical facts used to estimate the nonlinear DSGE framework at hand are generated via a standard linear VAR. For details, see Caggiano, Castelnuovo, and Pellegrino (2020).

[7] The DSGE model we work with does not explicitly feature unconventional policy interventions (namely, quantitative easing). Following Wu and Zhang (2019), Mouabbi and Sahuc (2019), and Sims and Wu (2020), we interpret a negative interest rate in presence of the zero lower bound as a close substitute for unconventional policies.

it presents the estimation results. Section 5 documents the business cycle effect of the uncertainty shock that occurred in 2008Q4; it compares the outcome of differently aggressive output-stabilizing policies; it uses the estimated DSGE framework as a laboratory to compare the macroeconomic performance implied by different policy rules. Section 6 concludes.

2 Related literature

Our focus on financial uncertainty is due to the recent paper by Ludvigson, Ma, and Ng (2019), who find that shocks to expected financial market volatility are relevant drivers of the US business cycle (for similar results, see Angelini, Bacchiocchi, Caggiano, and Fanelli (2019)).[8] We borrow their identification strategy to isolate exogenous changes in financial uncertainty and quantify their effects on the business cycle. There are three fundamental differences between our paper and theirs. First, we use a nonlinear framework to focus on the dynamic response of the US economy to the large uncertainty shock occurred in 2008Q4. Second, we identify financial uncertainty shocks by appealing to a larger set of narrative restrictions with respect to theirs. In particular, we merge their dates with those exploited by Bloom (2009) for the identification of large jumps in financial uncertainty, which are also assumed to be informative on the occurrence of financial uncertainty shocks. Third, we interpret our responses by taking a version of Basu and Bundick's (2017) microfounded DSGE model to the data, which is then used to assess the role played by systematic monetary policy in contrasting the recessionary effects of the uncertainty shock materialized in 2008Q3 in correspondence with Lehman Brothers' bankruptcy. Differently, Ludvigson, Ma, and Ng (2019) focus on the identification of the real effects of uncertainty shocks in a VAR-only context.

Methodologically, we use a nonlinear Interacted VAR (IVAR) model to establish novel facts regarding the different impact of financial uncertainty shocks on a battery of real activity indicators. In computing our impulse responses, we follow Pellegrino (2017, 2021), Caggiano, Castelnuovo, and Pellegrino (2017), and Amendola, Serio, Fragetta,

[8] Larsen (2021) conducts a study on Norwegian data and finds financial uncertainty to be a driver of the Norwegian business cycle. Similarly to our findings, he also documents a negative response of real activity to a financial uncertainty shock. Interestingly, Larsen (2021) shows that not all types of uncertainty are alike. For instance, uncertainty surrounding future mergers and acquisitions is found to be positively correlated with the Norwegian business cycle. We reiterate that our focus on financial uncertainty is justified by: i) the conceptual adherence to the measure of uncertainty we deal with in our DSGE framework; ii) the empirical literature that points to financial uncertainty as a driver of the US business cycle.

and Melina (2020) and allow the elements composing the interaction term in the nonlinear VAR - in our case, uncertainty and real activity - to endogenously evolve after an uncertainty shock. We do so to minimize the bias in our estimated responses that could otherwise emerge if uncertainty were not allowed to be endogenous and, above all, the business cycle were not allowed to react to shocks in uncertainty. Our IVAR-related findings, which point to more severe consequences of uncertainty shocks for output, investment, consumption, and hours during the great recession compared to normal times, echo those by Caggiano, Castelnuovo, and Groshenny (2014) on unemployment, and those obtained with indicators correlated with the business cycle like financial stress (Alessandri and Mumtaz (2019)). Differently from these contributions, which analyze a generic recession, our focus is on the great recession, and our identification strategy relies on narrative sign restrictions.

As anticipated above, we estimate a version of Basu and Bundick's (2017) framework with the impulse-response matching approach popularized by Christiano, Trabandt, and Walentin (2010). With respect to Basu and Bundick (2017), our stylized facts - that are specifically related to the great recession - are obtained with a nonlinear VAR framework, which we use to show that the response of real activity to an uncertainty shock is economically and significantly larger during the great recession than in normal times. Bretscher, Hsu, and Tamoni (2018) also investigate the role of uncertainty shocks during the great recession. They find that a large degree of risk aversion is needed to replicate the real effects of uncertainty shocks found in the data. Our analysis differs in at least three respects. First, our main focus is on the role played by systematic monetary policy in stabilizing the business cycle after the spike in uncertainty in 2007-09. Second, we establish the empirical facts on the response of the US business cycle to an uncertainty shock during the great recession with a nonlinear VAR where uncertainty shocks are identified using a state-of-the-art narrative sign restrictions approach. Third, we take our DSGE framework to the data by matching the nonlinear impulse responses of our VAR, which enables us to focus on the dynamics of the great recession. Finally, from a methodological standpoint our analysis deals with a one-off shock and the dynamic response of real activity it triggers. For a contribution dealing with sequences of uncertainty shocks, see Diercks, Hsu, and Tamoni (2020).

Methodologically, the closest approach to ours is probably the one by Ruge-Murcia (2014). He estimates a small-scale third-order approximated DSGE model with an impulse-response matching procedure based on a class of nonlinear VARs as auxiliary models for the purpose of indirect inference via a classical minimum distance estimator.

In doing so, he imposes the perturbation solution of the nonlinear DSGE model on the nonlinear VAR framework to approximate as closely as possible the DSGE-related policy functions. His approach, which is extremely neat, becomes unfortunately difficult to implement when one works with models with several states. Our novel estimation strategy easily accommodates large state spaces.

3 The real effects of uncertainty shocks: Empirical evidence

3.1 Nonlinear empirical methodology

Reduced-form nonlinear VAR. We represent the US macroeconomic environment with an IVAR, which augments a standard linear VAR model with interaction terms to determine how the effects of a shock to a variable depend on the level of another conditioning variable. We focus on a parsimonious IVAR to maximize the available degrees of freedom while capturing the nonlinearity of interest.

Our IVAR is the following:

$$\mathbf{Y}_t = \boldsymbol{\alpha} + \sum_{j=1}^{l} \mathbf{A}_j \mathbf{Y}_{t-j} + \left[\sum_{j=1}^{l} \mathbf{c}_j \ln VXO_{t-j} \times \Delta \ln GDP_{t-j} \right] + \boldsymbol{\eta}_t, \quad \boldsymbol{\eta}_t \sim d(0, \boldsymbol{\Omega}) \quad (1)$$

where $\mathbf{Y}_t$ is the $(n \times 1)$ vector of the endogenous variables, $\boldsymbol{\alpha}$ is the $(n \times 1)$ vector of constant terms, $\mathbf{A}_j$ are $(n \times n)$ matrices of coefficients, and $\boldsymbol{\eta}_t$ is the $(n \times 1)$ vector of error terms whose variance-covariance matrix is $\boldsymbol{\Omega}$, and $d(\cdot)$ is the distribution of the residuals. The interaction term in brackets makes an otherwise standard linear VAR a non-linear IVAR model. For each lag j, such interaction term includes a $(n \times 1)$ vector of coefficients c_j, a measure of uncertainty $\ln VXO_t$, and an indicator of the business cycle $\Delta \ln GDP_{t-j} \equiv \ln GDP_{t-j} - \ln GDP_{t-j-1}$, which is the quarter-on-quarter growth rate of real GDP. The interaction term $\ln VXO_{t-j} \times \Delta \ln GDP_{t-j}$ enables us to capture the potentially state-contingent effects of a shock to $\ln VXO_{t-j}$ (i.e., an uncertainty shock) conditional on the state of the business cycle, which is proxied by the growth rate of real GDP. Given the focus of our study, we will refer to the responses produced with the nonlinear framework calibrated with initial conditions at time $t-1 =$ 2008Q3, $t-2 =$ 2008Q2, etc. as "great recession" moments. Differently, the responses produced with the nested linear VAR characterized by $\mathbf{c}_j = 0$ per each $j = 1, ..., l$ will be referred to as average responses, or responses in "normal times".

Alternatives to IVAR frameworks - such as, e.g., regime switching frameworks or smooth transition VARs - are available to capture the nonlinear effects of macroeconomic shocks (for a recent survey, see Teräsvirta (2018)). We prefer to employ the IVAR framework (1) for three reasons. First, it resembles the approximated nonlinear policy functions of the DSGE framework we work with.[9] Second, it allows uncertainty shocks to have different effects over time because of the changing business cycle stance, which is key to isolate the impact of uncertainty during a specific recession. Third, it does not feature nuisance parameters, which are often difficult to estimate in nonlinear frameworks.[10] Finally, our IVAR does not technically allow us to deal with heterogeneities in impulse responses due to differently signed shocks or differently sized shocks. Here it is worth noticing that: i) our study aims at understanding the effects of an increase in uncertainty; ii) we want to impose the same size of the shock whose effect we simulate in the two regimes (great recession vs. normal times) to focus on the transmission mechanism and, in particular, on the role played by the systematic component of monetary policy.

Data. We model the vector $\mathbf{Y}_t = [\ln VXO, \ln GDP, \ln C, \ln I, \ln H, \ln P, SR]'$, where VXO denotes the stock market S&P 100 implied volatility index, GDP per capita GDP, C per capita consumption, I per capita investment, H per capita hours worked, P the price level, and SR the policy shadow rate. The variables in this vector are those used by Basu and Bundick (2017) in their linear VAR analysis.[11] We estimate our IVAR model with four lags over the 1962Q3-2017Q4 sample. The end of the sample is relatively similar to that of Basu and Bundick (2017). We do not include observations related to the COVID-19 period to avoid distorting our VAR impulse responses (Lenza and Primiceri (2020)). Given that the VXO is unavailable before 1986, we follow Bloom (2009) and splice it with the within-month volatility of S&P500 daily returns, which has displayed an extremely high correlation with the VXO since 1986. The sample includes the zero lower bound period experienced by the Federal Reserve during the period 2008Q4-2015Q4. We then work with the shadow rate constructed by Wu and

[9] Nonlinear policy functions feature different, higher order interaction terms. Our IVAR focuses on just one of the many interaction terms one could work with. We focus on the term featuring uncertainty and the real GDP growth because we are interested in isolating the impact of uncertainty shocks during the 2008-2009 downturn. Simulations conducted with higher order terms, and reported in our Appendix, deliver even stronger empirical results in favor of such nonlinear effects.

[10] Notice that IVARs featuring interactions terms resemble approximated Smooth Transition VAR frameworks (Teräsvirta, Tjøstheim, and Granger (2010)).

[11] Basu and Bundick's (2017) VAR also features the presence of money. Adding money implies no changes in our empirical results. The definition and construction of the variables common to our investigations is exactly the same as in Basu and Bundick (2017).

Xia (2016) to account for the effects of unconventional policy responses to financial uncertainty shocks.

A standard likelihood-ratio test favors our IVAR specification against the Basu and Bundick's (2017) linear VAR model (which is nested in our IVAR model in case of the overall exclusion of the interaction terms from model (1)). In particular, the LR test suggests a value for the test statistic $\chi_{28} = 61.99$, which allows us to reject the null hypothesis of linearity at any conventional statistical level in favor of the alternative of our I-VAR model (p-value $<< 0.01$).

Identification. We move from the reduced-form IVAR in (1) to the structural one as follows. First, we assume that the system of contemporaneous relationships mapping reduced form residuals $\boldsymbol{\eta}_t$ and structural shocks $\mathbf{e}_t$ can be described as

$$\boldsymbol{\eta}_t = \boldsymbol{B}\mathbf{e}_t, \quad \mathbf{e}_t \sim d(0, \boldsymbol{I_n}) \tag{2}$$

where $\boldsymbol{B}$ is a matrix featuring n^2 elements. Given that the reduced form covariance matrix $\boldsymbol{\Omega}$ features only $n(n+1)/2$ restrictions, further restrictions have to be imposed to identify the effects of the structural shocks $\mathbf{e}_t$ on the endogenous variables $\mathbf{Y}_t$. Without such further restrictions, infinitely many solutions satisfy the covariance restrictions $\boldsymbol{\Omega} = \boldsymbol{B}\boldsymbol{B}'$. We collect these uncountably many solutions into the set $\mathcal{B} = \{\boldsymbol{B} = \boldsymbol{P}\boldsymbol{Q} : \boldsymbol{Q} \in \mathcal{O}_n, diag(\boldsymbol{B}) \geqslant 0, \boldsymbol{\Omega} = \boldsymbol{B}\boldsymbol{B}'\}$, where $\mathcal{O}_n$ is the set of $(n \times n)$ orthonormal matrices (i.e., $\boldsymbol{Q}\boldsymbol{Q}' = \boldsymbol{I_n}$), $\boldsymbol{P}$ is the unique lower-triangular Cholesky factor with non-negative diagonal elements, i.e., $\boldsymbol{\Omega} = \boldsymbol{P}\boldsymbol{P}'$.

The set $\mathcal{B}$ is constructed by implementing the algorithm proposed by Rubio-Ramírez, Waggoner, and Zha (2010). First, we initialize the algorithm by setting $\boldsymbol{B} = \boldsymbol{P}$. Then, we rotate $\boldsymbol{B}$ by randomly drawing one million matrices $\boldsymbol{Q}$. Each rotation is performed by drawing a $(n \times n)$ matrix $\boldsymbol{M}$ from a $\mathcal{N}(\mathbf{0}, \boldsymbol{I_n})$ density. Then, $\boldsymbol{Q}$ is taken to be the orthonormal matrix in the $\mathbf{QR}$ decomposition of $\boldsymbol{M}$. Given that $\boldsymbol{B} = \boldsymbol{P}\boldsymbol{Q}$ and $\boldsymbol{Q}\boldsymbol{Q}' = \boldsymbol{I_n}$, the covariance restrictions $\boldsymbol{\Omega} = \boldsymbol{B}\boldsymbol{B}'$ are satisfied. Let $\mathbf{e}_t(\boldsymbol{B}) = \boldsymbol{B}^{-1}\boldsymbol{\eta}_t$ be the shocks implied by $\boldsymbol{B} \in \mathcal{B}$ for a given $\boldsymbol{\eta}_t$. Then, one million different $\boldsymbol{B}$ imply one million unconstrained $\mathbf{e}_t(\boldsymbol{B}) = \boldsymbol{B}^{-1}\boldsymbol{\eta}_t$, $t = 1, ..., T$.

While the set $\mathcal{B}$ contains infinitely many solutions mathematically coherent with equations (1)-(2), not all these solutions are equally credible from an economic standpoint. Following Ludvigson, Ma, and Ng (2019), we impose shock-based restrictions to select the economically interesting shocks. In particular, we impose restrictions directly on the shocks $\mathbf{e}_t(\boldsymbol{B})$ to work out the set of admissible solutions $\overline{\mathcal{B}}$ that can be considered as economically sensible. We identify uncertainty shocks by working with two types of restrictions, i.e., event constraints and external variable constraints.

Event constraints. Event constraints are justified by large jumps in financial uncertainty which have a clear interpretation from an historical standpoint. Figure 1 plots the financial uncertainty measure used in this study and identifies the events we work with. In our estimation sample, the two largest peaks occur in 1987Q4 (Black Monday in October 1987) and in 2008Q4 (acceleration of the financial crisis after the collapse of Lehman Brothers). For a financial uncertainty shock to be credible, we require it to be larger than the 75th percentile of the empirical distribution of the realizations of financial uncertainty shocks $\mathbf{e}_{FUt}(\boldsymbol{B})$ in 1987Q4 and 2008Q4.[12] Other two peaks we target are the ones in 1979Q4 and 2011Q3, which correspond to the beginning of the Volcker experiment (targeting of non-borrowed reserves) and to the debt-ceiling crisis, respectively. We require the realizations of our identified uncertainty shocks to be larger than the median value of the empirical density of the uncertainty shocks $\mathbf{e}_{FUt}(\boldsymbol{B})$ in these two dates. These four restrictions are those imposed by Ludvigson, Ma, and Ng (2019) for the identification of their financial uncertainty shocks. In an attempt to sharpen our VAR's ability to correctly identify financial uncertainty shocks, we then add further constraints. In particular, we consider all events identified by Bloom (2009) as possibly related to exogenous variations in financial uncertainty.[13] These events include, among others, the assassination of JFK, two OPEC crisis, two Gulf wars, 9/11, the Asian crisis, and the LTCM default. Bloom's (2009) sample ends in June 2008. When checking peaks in financial uncertainty in more recent times, we identify one in 2016Q1. Several uncertainty-triggering events occurred right before or during this quarter, e.g., the first increase of the federal funds rate which ended the zero lower bound phase after seven years; fears about China's economic fragility; the Central Bank of Japan going negative with the policy rate; and the announcement in February 2016 by British Prime Minister David Cameron of the Brexit referendum in June that year. For all these events (Bloom's plus those related to 2016Q1), we impose that our identified shocks must be larger than the median value of the empirical density of the uncertainty shocks $\mathbf{e}_{FUt}(\boldsymbol{B})$. Table 1 reports all the event constraints we work with.

External variable constraints. We further narrow down the set of models surviving

[12] This paper focuses on financial uncertainty. Ludvigson, Ma, and Ng (2019) jointly deal with financial and macroeconomic uncertainty, and require either one or the other (or both) to be large during the great recession. Interestingly, they find financial uncertainty shocks to be largely prevailing in correspondence to the spike in uncertainty in late 2008. A related paper that emphasizes the role of financial uncertainty as a driver of the business cycle during the great recession is Angelini, Bacchiocchi, Caggiano, and Fanelli (2019)

[13] Bloom (2009) reports the list of these events in Table A.1, page 676.

the selection conditional on the event constraints described above by imposing external variable constraints. Again following Ludvigson, Ma, and Ng (2019), we impose two such constraints. We impose that the correlation between $\mathbf{e}_{FUt}(\boldsymbol{B})$ and the aggregate stock market returns (growth rate of the real price of gold) to be below (above) the median of its empirical density. The rationale for these constraints is the negative correlation between financial volatility and stock market returns typically predicted by macro-finance models, and the role of gold as a safe asset investors go to when financial uncertainty is high.[14] These two constraints are also indicated in Table 1.[15]

Generalized impulse responses. The interaction term of our IVAR is treated as an endogenous object. We compute GIRFs à la Koop et al. (1996) to account for both the endogenous response of the growth rate of per capita GDP, i.e., our conditioning variable, to the uncertainty shock and the feedback this reaction can imply on the dynamics of the economy. Theoretically, the GIRF at horizon h of the vector $\mathbf{Y}_t$ to a shock of size δ computed conditional on an initial history $\boldsymbol{\varpi}_{t-1} = \{\mathbf{Y}_{t-1}, ..., \mathbf{Y}_{t-l}\}$ is given by the following difference of conditional expectations:

$$GIRF_{Y,t}(h, \delta_t, \varpi_{t-1}) = \mathbb{E}\left[\mathbf{Y}_{t+h} \mid \delta, \varpi_{t-1}\right] - \mathbb{E}\left[\mathbf{Y}_{t+h} \mid \varpi_{t-1}\right].$$

In our analysis, we are interested in recovering the response of $\mathbf{Y}_t$ to an uncertainty shock conditional on a specific initial history $\boldsymbol{\varpi}_{t-1} = \{\mathbf{Y}_{t-1}, ..., \mathbf{Y}_{t-4}\}$, where $t-1 = 2008Q3$, the initial history that corresponds to the quarter before the remarkable uncertainty spike in 2008Q4 (see Figure 1). Hence, the IVAR GIRFs $\widehat{\boldsymbol{\psi}^i}$ for the great recession are computed by iterating forward the system starting from the initial condition $\boldsymbol{\varpi}_{2008Q3}$. Our Appendix describes the algorithm used to compute the GIRFs. As regards the size of the shock δ, we impose a 4.4 standard deviation shock, which is the median size of the uncertainty shock in $t = 2008Q4$ among all retained shocks series.

[14] As stressed by Ludvigson et al. (2019), the external variables used here are not required to be valid exogenous instruments. Hence, this identification approach is conceptually different with respect to the one used in the proxy-SVAR literature. For a contribution in this latter direction, see Carriero, Mumtaz, Theodoridis, and Theophilopoulou (2015), Piffer and Podstawski (2018), and Alessandri, Gazzani, and Vicondoa (2020).

[15] Conditional on all the constraints we impose, the number of accepted draws is about 0.2% for both the linear VAR and the IVAR. More precisely, out of one million, we retain 2,116 draws for the linear VAR, and 2,168 for the IVAR.

3.2 Empirical results

Figure 2 plots the generalized impulse responses computed with our IVAR approach for the great recession. To appreciate the role played by nonlinearities in this analysis, the same Figure plots the corresponding responses generated by the nested linear VAR framework. In plotting the dynamic response of the US economy to an uncertainty shock, we focus on the median target model à la Fry and Pagan (2011).[16] The focus on the median target model is justified by two reasons. First, when searching for a reference to quantify the effects of macroeconomic shocks, median impulse responses are often taken as a reference. Second, in the second part of the paper, we will estimate our DSGE framework with an impulse response functions matching approach that focuses on the points estimates (as opposed to the identified sets) of the impulse responses produced with our VAR.[17]

Looking at Figure 2, a few facts stand out. An uncertainty shock induces a generalized drop in our real activity indicators during the great recession. The largest fall is in investment. The fall is persistent, with all four real activity indicators taking 20 quarters (or more, in the case of consumption) to go back to their trends. Prices also drop in a significant manner. The response of the federal funds rate is large and significant, with a peak drop of about 200 basis points after 2 years. All these responses are significant according to their 68% confidence bands.[18] Last but not least, nonlinearities matter. The responses of the four real activity indicators during the great recession are substantially larger than those suggested by the encompassed linear VAR. Moreover, prices do not fall according to the linear framework. This is true despite of the close similarity between the response of uncertainty according to the linear vs. nonlinear

[16] Following Fry and Pagan (2011), the median target impulse responses plotted in Figure 2 are those implied by the (median target) model belonging to the set of admissible solutions common to the two regimes we work with, i.e., normal times and great recession. This model is identified by searching for the model-implied impulse responses that are the closest, according to a quadratic loss function, to the responses computed by taking the median values across all models $\overline{\mathcal{B}}$ per each single variable and horizon. For details on the algorithm to identify the median target model, see Fry and Pagan (2011). The plot with all response belonging to the identified set is reported in our Appendix.

[17] Ludvigson, Ma, and Ng (2019) consider (for instructive purposes) a different selection criterion, i.e., the "max G" solution, which selects the model that maximizes the inequality constraints pertaining to the external variable constraints. We prefer to pursue the median target model because it involves all the constraints we impose to set-identify the macroeconomic responses we deal with.

[18] Our bootstrapped confidence bands are based over 1,000 realizations for the impulse responses, which are used to compute the bootstrapped estimate of the standard errors of the impulse response functions. As in Altig, Christiano, Eichenbaum, and Lindé (2011), the 68% confidence bands are constructed by considering the median target point estimates of the impulse response $\pm$ the bootstrapped estimate of the standard errors.

VAR. Table 2 reports the peak response of our business cycle indicators during the great recession. Notably, it is about 50% larger than the average response produced with the linear VAR. The same indication comes from consumption, whose peak reaction is 32% larger in great recession, and even more so for investment and hours, whose peak responses during the great recession are two and a half and two times larger than average, respectively.

Are the above-commented responses statistically larger according to our nonlinear VAR with respect to those predicted by the linear framework? Figure 3 shows the outcome of the bootstrapped test for the differences, along with the 68% confidence bands.[19] As evident from the figure, the responses of output, investment, and hours produced with our nonlinear VAR are significantly larger in the great recession. Turning to consumption, the mass of the distribution of the retained models hints to a larger response in the great recession. Finally, also the response of the price level and the nominal interest rate is found to be significantly different between the great recession and normal times.[20]

Our results are driven by an identification strategy - explained in the previous Section - that hinges upon events some of which are not directly related to financial markets and financial volatility. The interpretation one has to give to those events is an instrumental one, i.e., those events are such to generate financial volatility not caused by movements in the business cycle. Obviously, this interpretation can be challenged when dates such as the OPEC-related ones, or the Black Monday, or the one on the appointment of Paul Volcker as the chairman of the Federal Reserve are considered to achieve identification. The risk is that of confounding financial uncertainty shocks with,

[19] For each variable, the figure is based on the distribution constructed by considering 1,000 differences between responses in the linear model and responses obtained from the IVAR for the great recession. Such responses are generated from 1,000 samples obtained via the standard residual-based bootstrap around the median target responses. For each sample, we estimate the IVAR and nested linear VAR, compute the corresponding GIRFs and IRFs, and take their difference. The 68% confidence bands are constructed by considering the point estimate of the impulse responses $\pm$ the bootstrapped estimate of the standard errors. The construction of the test statistic takes into account the correlation between the estimated impulse responses. Our Appendix shows that the difference in the responses holds true also when model uncertainty is accounted for. We also find evidence in favor of a stronger response of real activity during the great recession when considering the maximum and minimum peak responses of real GDP to an uncertainty shock.

[20] Obviously, the great recession was characterized by a combination of first-moment financial shocks and uncertainty shocks (Stock and Watson (2012)). Our Appendix documents an exercise in which we model the BAA-AAA spread along with the other variables of our VAR, and we implement an event-based approach to separately identify first and second-moment financial disturbances. The impulse responses obtained with this expanded vector of variables are pretty close to the ones documented here.

respectively, oil shocks, first-moment financial shocks, and monetary policy shocks. Reassuringly, the correlations between our uncertainty shocks and proxies for the monetary and first-moment financial shocks - respectively, the Romer and Romer (2004) monetary policy shocks updated by Miranda-Agrippino and Rey (2020) and the estimates by Gilchrist and Zakrajšek (2012) of first-moment credit supply shocks turn out to be not significant at a 10% level.[21] Differently, the correlation between our uncertainty shocks and the oil supply shocks by Baumeister and Hamilton (2019) is 0.20, and it is precisely estimated (p-value: 0.03). However, an exercise (reported in Appendix) run by i) dropping the oil-related dates from our set of constraints, and ii) requiring the identified uncertainty shocks implied by our retained models to be not significantly correlated with the oil supply shocks proxy by Baumeister and Hamilton (2019) confirms the robustness of our findings.

Overall, these results point to an economically and significantly strong response of real activity to an uncertainty shock in an extreme event like the great recession. To interpret this fact, and above all to quantify the impact of the systematic policy response to the macroeconomic situation materialized during the great recession, we now use a nonlinear structural DSGE model.

4 Uncertainty-driven contractions: A structural interpretation

4.1 DSGE model: Description and estimation

Description. The Basu and Bundick (2017) framework extends an otherwise standard medium-scale New Keynesian model to consider an ex-ante second moment shock in the preference shock process, which has got a direct influence on a well-defined ex-ante financial volatility concept within the model. As stressed in the Introduction, this is the reason why we prefer to work with this model with respect to other models that have successfully captured the business cycle effects of other types of uncertainty shocks, e.g., shocks to the volatility of the world interest rate (Fernández-Villaverde, Guerrón-Quintana, Rubio-Ramírez, and Uribe (2011)), fiscal policy (Born and Pfeifer

[21] The correlations (p-values) between our uncertainty shocks (conditional on the median target model) and the proxies indicated in the text are: with excess bond premium shocks 0.19 (0.20); with monetary policy shocks -0.09 (0.19). Significance assessed by running an OLS regression involving our uncertainty shocks (left hand side) vs. a constant and one proxy at a time (right hand side). We consider White heteroskedasticity-consistent standard errors to control for heteroskedasticity.

(2014), Fernández-Villaverde, Guerrón-Quintana, Kuester, and Rubio-Ramírez (2015)), technology (Bloom (2009), Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry (2018)), and monetary policy (Mumtaz and Zanetti (2013)). We briefly describe the Basu and Bundick (2017) model here, focusing on the parts that are crucial for our study. We refer the reader to Basu and Bundick's (2017) paper for further details.

Households work, consume, and invest in equity shares and one-period risk-free bonds. They are all similar, and feature Epstein-Zin preferences over streams of consumption and leisure, formalized as follows:

$$V_t = \left[\left(1 - \beta)(a_t \tilde{C}_t^{\eta} (1 - N_{t,})^{(1-\eta)} \right)^{(1-\sigma)/\theta_V} + \beta ((E_t V_{t+1})^{1-\sigma})^{1/\theta_V} \right]^{\theta_V/(1-\sigma)}$$

where $\tilde{C}_t = C_t - H_t$, C_t is consumption, $H_t = bC_{t-1}$ captures external habit formation in consumption related to the level of aggregate consumption lagged one period, N_t is hours worked, β is the discount factor, σ is a parameter directly influencing the degree of risk aversion, ψ is the intertemporal elasticity of substitution, $\theta_V \equiv (1-\sigma)/(1-\psi^{-1})^{-1}$ captures households' preferences for the resolution of uncertainty, η weights consumption and labor in households' happiness function, and a_t is a stochastic shifter influencing the relevance of today's realizations of consumption and labor vs. those expected to occur during the next period.[22]

The stochastic process followed by this preference shock is:

$$\begin{aligned} a_t &= (1-\rho_a)a + \rho_a a_{t-1} + \sigma_{t-1}^a \varepsilon_t^a \\ \sigma_t^a &= (1-\rho_{\sigma^a})\sigma^a + \rho_{\sigma^a}\sigma_{t-1}^a + \sigma^{\sigma^a}\varepsilon_t^{\sigma^a} \end{aligned}$$

where ε_t^a is the first-moment preference shock, and $\varepsilon_t^{\sigma^a}$ is a second-moment uncertainty shock to the preference process which loads the law of motion regulating the evolution of the time-varying second moment σ_t^a relative to the distribution of ε_t^a. With respect to the framework in Basu and Bundick (2017), we add (external) habit formation in consumption to capture the hump-shaped response of consumption in the data (for another contribution jointly modeling Epstein-Zin preferences and habits in consumption, see Andreasen, Fernández-Villaverde, and Rubio-Ramírez (2018)).

[22] de Groot, Richter, and Throckmorton (2018) show that households' preferences in Basu and Bundick's (2017) paper imply an asymptote in the responses to an uncertainty shock with unit intertemporal elasticity of substitution. Our paper employs the set of preferences proposed by Basu and Bundick (2018), which do not imply any asymptote.

Intermediate goods-producing firms operate in a monopolistically competitive environment, rent labor from households, and pay wages. They own capital and choose its utilization rate, issue equity shares and one-period riskless bonds, and invest in physical capital to maximize the discounted stream of their profits. In doing so, they face quadratic costs of adjusting nominal prices à la Rotemberg (1982), capital adjustment costs à la Jermann (1998), and capital utilization costs influencing the capital depreciation rate.[23] All intermediate firms have the same Cobb-Douglas production function, and are subject to a fixed cost of production and stationary technology shocks. Intermediate goods are packed by a representative final goods producer operating in a perfectly competitive market. The model is closed by assuming that the central bank follows a standard Taylor rule, which reads as follows:

$$r_t = r + \rho_\pi(\pi_t - \pi) + \rho_y \Delta y_t \tag{3}$$

where $r_t = \ln(R_t)$, $\pi_t = \ln(\Pi_t)$, $\Delta y_t = \ln(Y_t/Y_{t-1})$, R_t is the gross nominal interest rate, Π_t is gross inflation, π is the net inflation target, and Y_t is output. Hence, monetary policymakers are assumed to systematically respond to changes in inflation and the growth rate of output.

In this framework, an uncertainty shock propagates to the economy mainly via precautionary savings and precautionary labor supply.[24] The former effect reduces current consumption in response to an increase in uncertainty, while the latter increases labor supply, which drives real wages and firms' marginal costs down. Given that prices are sticky, the price markup increases. Output, which is demand-driven in this model, falls due to the drop in consumption, and labor demand contracts driving hours down. Given the lower return on capital, investment falls too. Hence, in equilibrium, an increase in uncertainty causes a drop in all four real activity indicators, i.e., output, consumption, investment, and hours, which is what we observe in the data.

[23] Oh (2020) shows that the Rotemberg pricing scheme has got implications on the response of inflation to an uncertainty shock that are not equivalent to those implied by Calvo. We prefer to model price rigidities à la Rotemberg because, as shown in Oh (2020), the implications of this pricing scheme are more consistent with the empirical evidence on the fall in inflation in response to an uncertainty shock.

[24] Given that adjustment costs are convex, this model does not imply a "wait-and-see" effect after an uncertainty shock. The reason is that, to solve the model, we use perturbation methods which require policy functions to be differentiable, a feature which is not possessed by threshold policy functions arising in presence of real option effects. Still, investment potentially matters for the propagation of uncertainty shocks through the two channels explained in Bianchi, Kung, and Tirskikh (2019), i.e., an investment risk premium channel, which depends on the covariance between the pricing kernel and the return on investment, and a investment adjustment channel, which arises because of rigidities which prevent firms to immediately adjust investment to the desired level.

As anticipated above, the model features a well-defined implied financial volatility index. This is because intermediate firms issue equity shares on top of one-period riskless bonds.[25] Each equity share has a price P_t^E and pays dividends D_t^E, implying a one-period return $R_{t+1}^E = \left(P_{t+1}^E + D_{t+1}^E\right)/P_t^E$. The model-implied financial uncertainty index V_t^M is computed as the annualized expected volatility of equity returns, i.e., $V_t^M = 100\sqrt{4 \cdot VAR_t\left(R_{t+1}^E\right)}$, where $VAR_t\left(R_{t+1}^E\right)$ is the quarterly conditional variance of the return on equity R_{t+1}^E. Equity returns are endogenous in the model, which makes the ex-ante volatility V_t^M endogenous too. However, in this model V_t^M is almost entirely driven by second-moment preference shocks for a variety of plausible calibrations. This enables us to treat the uncertainty shock as a financial uncertainty shock proxied by V_t^M, and to sensibly match our VAR impulse responses to a financial uncertainty shock with those of the DSGE framework we aim at estimating.[26]

We work with a third-order approximation of the nonlinear DSGE model, which we solve via perturbation techniques (Schmitt-Grohe and Uribe (2004)). The third order approximation of agents' decision rules features an independent role for uncertainty, whose independent effect on the equilibrium values of the endogenous variables of the framework can therefore be studied (Andreasen (2012)). Perturbation represents an accurate and fast way to find a solution also working with frameworks featuring recursive preferences (Caldara, Fernández-Villaverde, Rubio-Ramírez, and Yao (2012)).

Estimation. We estimate the model described above via the impulse response function-matching approach popularized by Christiano, Trabandt, and Walentin (2011). This is a limited information Bayesian approach, which allows to write an approximation of the likelihood of the VAR impulse responses (that are treated as "data" here) as a function of the parameters of the DSGE model one aims at estimating (Kim (2002)). Being Bayesian, this approach enables us to impose economically sensible prior densities on the structural parameters while asking the data (i.e., our IVAR impulse responses) to shape the posterior density of the estimated model. With respect to Christiano, Trabandt, and Walentin (2011), who focus on a linearized DSGE framework and a linear

[25] Basu and Bundick (2017) assume that firms finance a share ν of their capital stock each period with one-period riskless bonds. Given that the Modigliani-Miller theorem holds in their model, leverage does neither influence firms' value nor firms' optimal decisions. Firms' leverage only influences the first two unconditional moments of financial-related quantities (e.g., the average level and unconditional volatility of the model-implied VXO and the equity premium), but it does not influences impulse responses to an uncertainty shock.

[26] A Monte Carlo simulation documented in our Appendix shows that the identification strategy based on the Narrative Sign Restrictions we work with enables our VAR to recover the "true" responses produced by the DSGE model.

VAR as auxiliary model, we estimate a nonlinear DSGE framework approximated at a third order with moments produced with an Interacted VAR. To our knowledge, this is the first application of Christiano et al.'s (2011) Bayesian estimation strategy to a nonlinear DSGE framework. Technical details on this estimation approach are offered in our Appendix.[27]

We estimate 7 structural parameters, i.e. $\zeta^i = \left[\rho_{\sigma^a}, \sigma, b, \phi_K, \phi_P, \rho_\pi, \rho_y\right]$. These parameters are the persistence of the second moment preference shock ρ_{σ^a}, the household risk aversion parameter σ, the consumption habit formation parameter b, the parameter regulating investment adjustment costs ϕ_K, the parameter regulating price adjustment costs ϕ_P, and the parameters of the Taylor rule ρ_π, ρ_y. Our priors are reported in the third column of Table 3. We calibrate the prior means with the values in Basu and Bundick's (2017) analysis, and we use diffuse priors. For the habit formation parameter and the parameters of the Taylor rule, we use the priors employed by Christiano, Trabandt, and Walentin (2011). The remaining parameters of the model are calibrated as in Basu and Bundick (2018). We discuss the calibration of these parameters in our Appendix.

It is important to stress that, consistently with our VAR analysis that features the Wu and Xia's (2015) shadow rate, our DSGE model does not impose the zero lower bound constraint on the policy rate. We do so because we interpret a possibly negative rate according to the model as a shadow rate, i.e., a short-term interest rate capturing unconventional policies whose effect is comparable to the one of a counterfactual policy leading the rate to the negative territory (for a similar approach and interpretation, see Wu and Zhang (2019) and Mouabbi and Sahuc (2019)). The underlying assumption behind this approach is that the impact of unconventional policies implemented during the great recession to circumvent the zero lower bound issue was comparable to the one of conventional policies before the great recession. Empirical support to this assumption is provided by Swanson (2020).[28]

[27] It is important to reiterate that this approach, which requires the VAR impulse responses to be interpretable as data, calls for a frequentist VAR analysis (as opposed to a Bayesian one).

[28] It is worth noticing that the federal funds rate was 1.94% in 2008Q3. Hence, according to the policy rate response predicted by our IVAR (median target estimate), the Federal Reserve had enough room to intervene with a conventional policy move to tackle the real effects of the 2008Q4 uncertainty shock.

4.2 DSGE model: Results

Our DSGE model-based estimated responses are reported in Figure 4, along with the VAR-based bootstrapped confidence bands. The model captures remarkably well the great recession facts documented with our nonlinear VAR. The DSGE impulse responses mostly lie within the 68% confidence bands of the IVAR impulse responses. As far as real activity is concerned, the model performs extremely well for output, consumption, and investment, while it goes short for hours, although it suggests a prolonged recession as in the data. Possible explanations are: i) the assumption of homogeneous workers, which misses to take into account differences the relatively faster exit from the labor market by unskilled workers during recessions (for a discussion, see Basu and Bundick (2017); for a paper dealing with skilled and unskilled workers and the responses of hours worked to an uncertainty shock, see Belianska (2020)); ii) the role played by the precautionary labor supply channel, which leads to an increase in labor supply under uncertainty and dampens the magnitude of the drop in hours worked in equilibrium (Bianchi, Kung, and Tirskikh (2019)); iii) the absence of search frictions, which can magnify the real effects of uncertainty shocks (Leduc and Liu (2016)), above all if combined with an occasionally binding constraint on downward wage adjustment (Cacciatore and Ravenna (2020)). While leaving the modification of the labor market framework in this model to future research, we note that the analysis on the role played by the Federal Reserve in tackling the recessionary effects of uncertainty shocks we carry out in Section 5 will focus on the responses of output, which the model captures well.

Turning to the nominal side, the model is able to capture the response of prices during the great recession, while it underestimates the response of the policy rate. Still, it clearly captures the persistence of the policy easing implemented by the Federal Reserve during and in the aftermath of the great recession.

Table 3 collects the estimated parameters of the DSGE model. We focus our attention on the implied estimate of relative risk aversion and on the parameters of the Taylor rule. As explained by Swanson (2012), the coefficient of relative risk aversion in this type of models is affected by the labor market structure as well as households' preference. Building on Swanson (2012), Swanson (2018) works out the expression for the coefficient of relative risk aversion conditional on endogenous labor supply, habits in consumption, and generalized recursive preferences (which include Epstein-Zin preferences). Following Swanson (2018), our estimated parameters imply a coefficient of relative risk aversion equal to 145 (see Table 3).[29] This value is larger than that calibrated (75) or estimated (110) by Rudebusch and Swanson (2012). One obvious reason for this discrepancy is that our paper aims at matching impulse responses during an extreme event, i.e., the great recession.[30] Cohn, Engelmann, Fehr, and Maréchal (2015) provide experimental evidence suggesting that financial market professionals are more risk averse during a financial bust than a boom. Guiso, Sapienza, and Zingales (2017) propose experimental evidence in favor of a fear model in which agents experience higher risk aversion in periods of crisis. Schildberg-Horisch (2018) surveys the literature on risk aversion and finds that for negative economic shocks such as the 2007-09 financial crisis, the evidence consistently points to an increase in risk aversion. A somewhat related finding is that by Cox, Greenwald, and Ludvigson (2020), who find evidence pointing to a jump in risk aversion as a relevant driver of the stock market during the first months of the COVID-19 pandemic. More in general, Cochrane (2017) points to countercyclical risk aversion as a feature macro-finance models should possess to match the data. Finally, Barillas, Hansen, and Sargent (2009) employ a max-min expected utility theory approach to show that models with high risk aversion in which rational agents are endowed with the knowledge of the true underlying structure of the economy can be reinterpreted as frameworks in which risk aversion is low but households have doubts about the model specification. Our model does not embed any doubts about the underlying economy by households. Therefore, it is likely to understate the true quantity of risk faced by households in the data, which is the reason why it requires high levels or risk aversion to match the VAR facts.[31]

[29] The formula for the RRA in our extension of the Basu and Bundick (2017) model with habits takes the following form (see our Appendix for the full derivation):

$$RRA = \left(\frac{\eta}{\eta + (1-\eta)(1-b)}\right) \cdot \left(\frac{1}{\psi}\frac{\left(1 + \frac{(1-\eta)}{\eta}(1-b)\right)}{(1-b)\left(1 + \frac{(1-\eta)}{\eta}\right)} + \left(\sigma - \frac{1}{\psi}\right)\left(\frac{\eta}{(1-b)} + 1 - \eta\right)\right)$$

[30] Our Appendix also documents the irrelevance of initial conditions for our DSGE results. Cacciatore and Ravenna (2020) prove that pruning completely eliminates state dependence in the propagation of uncertainty shocks from third-order approximated solutions. Hence, the unpruned solution of the model may in principle generate state-dependent dynamics. However, an IVAR estimated with data simulated from the unpruned approximated solution of our estimated model turns out to deliver impulse responses that are quantitatively insensitive to variations in the initial conditions.

[31] Andreasen, Fernández-Villaverde, and Rubio-Ramírez (2018) show that, in a model featuring a portfolio allocation problem related to short- and long-term bonds plus a systematic response of the central bank to the term spread, uncertainty shocks to households' preferences generate moments consistent with the data even in presence of moderate values of risk aversion. The moments studied by Andreasen, Fernández-Villaverde, and Rubio-Ramírez (2018) are, however, unconditional moments,

Turning to the Taylor rule parameters, the systematic response to inflation is estimated to be lower than that typically found in the literature (see, e.g., Clarida, Galí, and Gertler (2000)). The reason is that, for uncertainty shocks to generate a large response of real activity, a countercyclical price markup has to materialize. A too strong monetary policy response to inflation would prevent the increase of the price markup, forcing other parameters of the model (e.g., risk aversion) to adjust to enable the DSGE framework to match the empirical facts. At the same time, the imposition of a unique equilibrium under rational expectations pushes the optimizer toward delivering a policy response to inflation larger than one. Table 3 reports the estimates of the Taylor rule parameters obtained by matching the impulse responses of the linear VAR nested in our nonlinear IVAR, where the former are interpreted as "normal times" dynamics.[32] Quite interestingly, the policy response of inflation turns out to be basically the same as the one during the great recession. Differently, the policy response to output growth is estimated to be larger during the great recession. This evidence captures the rapid and massive interventions by the Federal Reserve in response of the dramatic drop in real activity occurred in 2008-09. This intuitive interpretation is supported by policy statements on the importance of contrasting the negative pressures on real activity in that period.[33]

4.3 Monetary policy and output loss during the great recession

Output loss due to the 2008Q4 uncertainty shock. What is the contribution of the large uncertainty shock materialized in 2008Q4 to the output loss recorded during and in the aftermath of the great recession? We address this question by contrasting the response of output predicted by our estimated framework with the CBO output gap, which we take as a proxy for detrended output.[34] We consider the period 2008Q4-

i.e., they are not state-specific.

[32] When taking the model to normal times data, all estimated parameters are re-optimized to ensure that adjustments in the Taylor rule parameters do not proxy for other possible changes in the structure of the economy (e.g., a lower degree of risk aversion). Given our research question, our focus is on the changes in the values of the policy rule parameters.

[33] See, for instance, the minutes of the Federal Open Market Committee meeting held on October 28-29, 2008, where all FOMC members *"[...] judged that a significant easing in policy at this time was appropriate to foster moderate economic growth and to reduce the downside risks to economic activity."* After that meeting, the federal funds rate target was cut by 50 basis points.

[34] The DSGE framework we work with does not feature any random walk or deterministically trending process. Hence, a comparison with actual data requires detrending or filtering actual output. Following Basu and Bundick (2017), we focus on the estimate provided by the Congressional Budget Office (CBO), which is based on a production function approach and on sectoral and aggregate data. For a detailed description on the estimation of the potential level of output by the CBO, see Shackleton (2018).

2014Q4, which is the period during which the output gap (normalized to zero in 2008Q3) recorded consecutive negative values before going back to a plus sign in 2015Q1.

Figure 5 plots contrasts the impulse response of output predicted by our model with the data. The response of output to an uncertainty shock follows a path pretty similar to that of output in the data, although the peak realization of the output gap occurs in 2009Q2, two quarters earlier than that predicted by the model. Then, both the model-implied output and actual output gradually go back to their trends, displaying a similar persistence. In terms of cumulative output over the 2008Q4-2014Q4 period, the model attributes a share of the actual output loss as large as 60% to the uncertainty shock materialized in 2008Q4.

Role of the switch to a more aggressive output stabilization. Equipped with the DSGE model estimated using the great recession-specific impulse responses, we now turn to the analysis of the role played by monetary policy in the propagation of the 2008Q4 uncertainty shocks. Table 3 documents a more aggressive systematic response to output growth during the great recession (the parameter attached to output in the DSGE policy rule is 0.28 for the great recession, compared with 0.20 in normal times). A natural question is to what extent such a more aggressive response of the Fed worked in favor of mitigating the depth of the great recession. This question parallels the one asked by Christiano (2003) on the role monetary policy could have played in limiting the depth of the great depression. In particular, while the depth of the great depression has been attributed to a monetary policy response not accommodative enough (Friedman and Schwartz (1963)), the avoidance of a deeper great recession has been credited to the timely and aggressive response by monetary authorities.[35] To understand the role played by the systematic monetary policy in place during the great recession, we perform a counterfactual exercise in which we replace the policy response to output growth estimated with great recession data with the value of the same parameter obtained in normal times, i.e., we replace $\rho_y^{GR} = 0.28$ with $\rho_y^{linear} = 0.20$.[36] We then generate the corresponding GIRF to a 4.4 standard deviation uncertainty shock.

Figure 6 presents the results. The counterfactual fall in output would have been roughly doubled in 2008Q4 and the recession would have been longer, lasting until the second half of 2010. Hence, according to our estimated framework, the Fed played a

[35]For instance, Jason Furman, Chairman of the Council of Economic Advisers, gave a speech at the Macroeconomic Advisers' 25th Annual Washington Policy Seminar on Sept. 9 2015 titled "It Could Have Happened Here: The Policy Response That Helped Prevent a Second Great Depression".

[36]It is useful to recall that the estimated policy response to inflation either with great recession data or with normal times data is the same.

significant role in mitigating the depth of the great recession.

Distance from scenario under optimal (unfeasible) rule. As explained by Basu and Bundick (2017), the rule that mimics the optimal policy conduct in this framework is the one that postulates a systematic response of monetary policymakers to fluctuations in the natural interest rate and the output gap, i.e.,

$$r_t = r_t^n + \pi + \rho_\pi(\pi_t - \pi) + \rho_x x_t \tag{4}$$

where r_t^n is the "natural" real interest rate from the equivalent flexible-price economy, and x_t is the output gap, i.e., the difference between the equilibrium level of output under sticky prices and that under flexible prices. This rule is, in fact, unfeasible in the real world due to the lack of perfect knowledge of latent processes such as the real natural interest rate or the model-consistent output gap. However, it is of interest to understand how different a picture we get when contrasting the macroeconomic outcome under the estimated simple rules as opposed to the one under the optimal rule.

Figure 7 jointly plots the impulse responses generated by our model under the two estimated rules and the optimal one.[37] Let us analyze the macroeconomic response under the optimal rule first. Evidently, the response of the federal funds rate tracks more closely that of the natural interest rate. As a consequence, the drop in consumption is dramatically attenuated. As far as output, investment, and hours are concerned, we observe a larger on impact reaction, but also a much more rapid return of these real activity indicators to their trend values, with the temporary effects of uncertainty shocks fully absorbed within two years after the shock. The crucial point here is the absence of a countercyclical markup. The "optimal" policy (4) prevents the equilibrium markup to exceed the desired one, therefore aligning the demand for output, investment, and hours to the one optimally supplied by firms and consumers in a flexible price scenario in spite of the presence of sticky prices.[38] Consumption decreases under all policy

[37] To minimize deviations with respect to the estimate rule (3), we calibrate $\rho_\pi = 1.05$, and $\rho_x = 0.28$ (where, for the latter parameter, we use our estimate of the response to the output growth to calibrate the response to the output gap). Given that the interest rate path under the optimal rule is fully determined by the evolution of the real natural interest rate, the calibration of the policy response to inflation and the growth rate does not play any role in determining the macroeconomic outcome under such a rule.

[38] The switch in sign in the response of output, investment, and hours is due to precautionary labor supply, which is due to the increase in the marginal utility of consumption due to precautionary savings channel that becomes active after an unceratainy shock. The increase in labor supply drives real wages downward, therefore reducing firms' marginal costs, an effect that contrasts firms' upward pricing bias in presence of uncertainty and stabilizes the price markup. Given the level of technology and capital, the larger amount of hours worked in equilibrium increases output and the returns from capital, which

rules as a consequence of agents precautionary savings. The minimum contraction of consumption is the one associated to the optimal policy rule, followed by the rule estimated with great recession data, which was more aggressive in stabilizing output, something that implies, in our model, a more effective stabilization of consumption as well. Interestingly, an outcome associated with the optimal policy is the lower on-impact hike of financial volatility, an endogenous object in our model. This result mimics the empirical evidence on the impact of expansionary monetary policy shocks on measures of uncertainty (Bekaert, Hoerova, and Lo Duca (2013), Mumtaz and Theodoridis (2019), and Pellegrino (2021)).

Turning to the comparison between the performance implied by our estimated rules and the one we would have observed if the (unfeasible) optimal rule had been implemented, the impulse responses in Figure 7 point to macroeconomic dynamics under the estimated great recession rule closer to the responses implied by the optimal rule than the dynamics implied by the rule estimated with normal times data. Moreover, Table 4 documents the lower business cycle volatility (computed both as model-implied moments and as stochastic volatility) experienced under the three rules at play.[39] The path is clear, i.e., for all business cycle indicators, the volatility turns out to be lower (and clearly so for output, investment, and hours) under the great recession rule.

5 Conclusion

This paper documents the large output costs due to financial uncertainty during the great recession and assesses to what extent the prompt and strong monetary policy intervention implemented by the Federal Reserve limited such costs. We employ a nonlinear VAR and a state-of-the-art identification strategy to estimate the causal effect going from the large jump in financial uncertainty that occurred in 2008Q4 to the US business cycle. We find evidence in favor of co-movement of output, consumption, investment, and hours, with a generalized drop in real activity substantially larger than the one predicted by a linear VAR that does not account for the nonlinear transmission of uncertainty shocks to the economy during an extreme event such as the great recession. The second part of the paper exploits the VAR evidence to estimate a nonlinear DSGE

justifies the positive response of investment. For further details on the flexible price allocation in this model, see Basu and Bundick (2017).

[39] The computation of the stochastic volatility controls for the fact that, given the stochastic volatility process of the preference shock in our model, also the endogenous variables in our DSGE framework feature a time-varying second moment.

framework that features a time-varying financial volatility concept comparable to the one modeled with our VAR. We show that such a framework replicates remarkably well our great recession facts. A simulation conducted with our estimated framework points to a contribution to the output loss during and after the great recession by the large uncertainty shock that hit the US economy in the fourth quarter of 2008 as large as 60%. A counterfactual simulation imposing monetary policy as in normal times (that features a weaker response to output growth than the one we estimate with great recession data) to the great recession scenario point to a dramatically larger output loss that the economy would have born. Finally, a comparison between the macroeconomic performance implied by our estimated rules and the one induced by the optimal (but unfeasible) simple rule suggests that the rule in place during the great recession performs better than the one relatively more concerned with inflation consistent with normal times data. Our findings support the switch to an aggressive, output stabilization-focused monetary policy during extreme events characterized by large uncertainty shocks.

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Event constraints			
t	Event	Source	Constraint on $e_{FU.t}$
1963Q4	Assassination of JFK	B	$e_{FU.t} \geqslant p(\mathbf{e}_{FUt}(\boldsymbol{B}), 50th)$
1966Q3	Vietnam buildup	B	$e_{FU.t} \geqslant p(\mathbf{e}_{FUt}(\boldsymbol{B}), 50th)$
1970Q2	Cambodia and Kent state	B	$e_{FU.t} \geqslant p(\mathbf{e}_{FUt}(\boldsymbol{B}), 50th)$
1973Q4	OPEC I, Arab-Israeli War	B	$e_{FU.t} \geqslant p(\mathbf{e}_{FUt}(\boldsymbol{B}), 50th)$
1974Q3	Franklin National	B	$e_{FU.t} \geqslant p(\mathbf{e}_{FUt}(\boldsymbol{B}), 50th)$
1978Q4	OPEC II	B	$e_{FU.t} \geqslant p(\mathbf{e}_{FUt}(\boldsymbol{B}), 50th)$
1979Q4	Volcker experiment	B, LMN	$e_{FU.t} \geqslant p(\mathbf{e}_{FUt}(\boldsymbol{B}), 50th)$
1980Q1	Afghanistan, Iran hostages	B	$e_{FU.t} \geqslant p(\mathbf{e}_{FUt}(\boldsymbol{B}), 50th)$
1982Q4	Monetary policy turning point	B	$e_{FU.t} \geqslant p(\mathbf{e}_{FUt}(\boldsymbol{B}), 50th)$
1987Q4	Black Monday	B, LMN	$e_{FU.t} \geqslant p(\mathbf{e}_{FUt}(\boldsymbol{B}), 75th)$
1990Q4	Gulf War I	B	$e_{FU.t} \geqslant p(\mathbf{e}_{FUt}(\boldsymbol{B}), 50th)$
1997Q4	Asian crisis	B	$e_{FU.t} \geqslant p(\mathbf{e}_{FUt}(\boldsymbol{B}), 50th)$
1998Q3	Russian, LTCM default	B	$e_{FU.t} \geqslant p(\mathbf{e}_{FUt}(\boldsymbol{B}), 50th)$
2001Q3	9/11	B	$e_{FU.t} \geqslant p(\mathbf{e}_{FUt}(\boldsymbol{B}), 50th)$
2002Q3	Worldcom, Enron	B	$e_{FU.t} \geqslant p(\mathbf{e}_{FUt}(\boldsymbol{B}), 50th)$
2003Q1	Iraq invasion	B	$e_{FU.t} \geqslant p(\mathbf{e}_{FUt}(\boldsymbol{B}), 50th)$
2008Q4	Great recession	B, LMN	$e_{FU.t} \geqslant p(\mathbf{e}_{FUt}(\boldsymbol{B}), 75th)$
2011Q3	Debt ceiling crisis	LMN	$e_{FU.t} \geqslant p(\mathbf{e}_{FUt}(\boldsymbol{B}), 50th)$
2016Q1	End of the US ZLB in the US, China, Japanese neg. rate, Brexit refer. ann.	This paper	$e_{FU.t} \geqslant p(\mathbf{e}_{FUt}(\boldsymbol{B}), 50th)$

External variable constraints		
External variable S_t	Source	Constraint on $\rho(e_{FUt}, S_t)$
Stock market return	LMN	$\rho(e_{FUt}, S_t) \leqslant p(\rho(e_{FUt}, S_t), 50th)$
Real price of gold (log difference)	LMN	$\rho(e_{FUt}, S_t) \geqslant p(\rho(e_{FUt}, S_t), 50th)$

Table 1: **Event and external variable constraints.** Constraints imposed to identify financial uncertainty shocks. Sources: B = Bloom (2009); LMN = Ludvigson et al. (2019). p(X,Zth) refers to the Zth percentile of the empirical density of the variable X.

	Output	Consumpt.	Investment	Hours
Baseline IVAR and VAR				
Peak response: Linear	-1.54%	-1.03%	-3.34%	-1.90%
Peak response: Great Recession	-2.32%	-1.36%	-8.32%	-3.75%
Ratio GR/Linear	1.50	1.32	2.49	1.97

Table 2: **Peak responses.** Peak responses to a one standard deviation uncertainty shock estimated with linear VAR and nonlinear IVAR for the great recession.

		Priors	Great Recession
Parameter	Interpretation	D(mean, std)	Mode, std
ρ_{σ^a}	Unc. shock, persistence	B(0.77, 0.10)	**0.65,** 0.03
b	Habit formation parameter	B(0.75, 0.15)	**0.66,** 0.04
ϕ_K	Investment adjustment costs	G(3.92, 2)	**3.21,** 0.60
ϕ_P	Price adjustment costs	G(240, 40)	**282.10,** 33.54
ρ_π	Taylor rule parameter, inflation	IG(1.5, 0.25)	**1.05,** 0.01
ρ_y	Taylor rule parameter, output growth	G(0.2, 0.15)	**0.28,** 0.05
σ	Risk aversion (fixed labor supply, no habits)	G(100, 60)	**533.04,** 59.16
RRA	Risk aversion (endogenous labor supply, habits)		**144.96**
	Normal times Taylor rule parameters		
ρ_π	Taylor rule parameter, inflation	IG(1.5, 0.25)	**1.05,** 0.01
ρ_y	Taylor rule parameter, output growth	G(0.2, 0.15)	**0.20,** 0.04

Table 3: **DSGE model: Estimated parameters.** Values estimated conditional on the IVAR impulse responses to an uncertainty shock in 2008Q4. Standard deviations estimated conditional on a Laplace approximation of the posterior density. Risk aversion in the model (RRA) computed by considering endogenous labor supply and habits as in Swanson (2018). Normal times Taylor rule parameter estimated by matching the average impulse responses of the linear VAR encompassed in the nonlinear IVAR framework.

Implied standard deviations	Output	Cons.	Invest.	Hours
Normal times Taylor rule	2.91	1.18	8.27	2.40
Great Recession Taylor rule	2.47	1.16	7.46	2.15
Optimal policy rule	1.23	0.68	6.60	1.72

Stochastic standard deviations	Output	Cons.	Invest.	Hours
Normal times Taylor rule	0.63	0.28	1.85	0.43
Great Recession Taylor rule	0.52	0.26	1.62	0.35
Optimal policy rule	0.24	0.13	1.29	0.28

Table 4: **Simulated standard deviations under different rules.** Implied standard deviations computed on simulated time series over a sample featuring 2-million observations. Stochastic standard deviations computed by considering, conditional on the above sample, the standard deviation of the time series of standard deviations computed with observations in a 5-year rolling window.

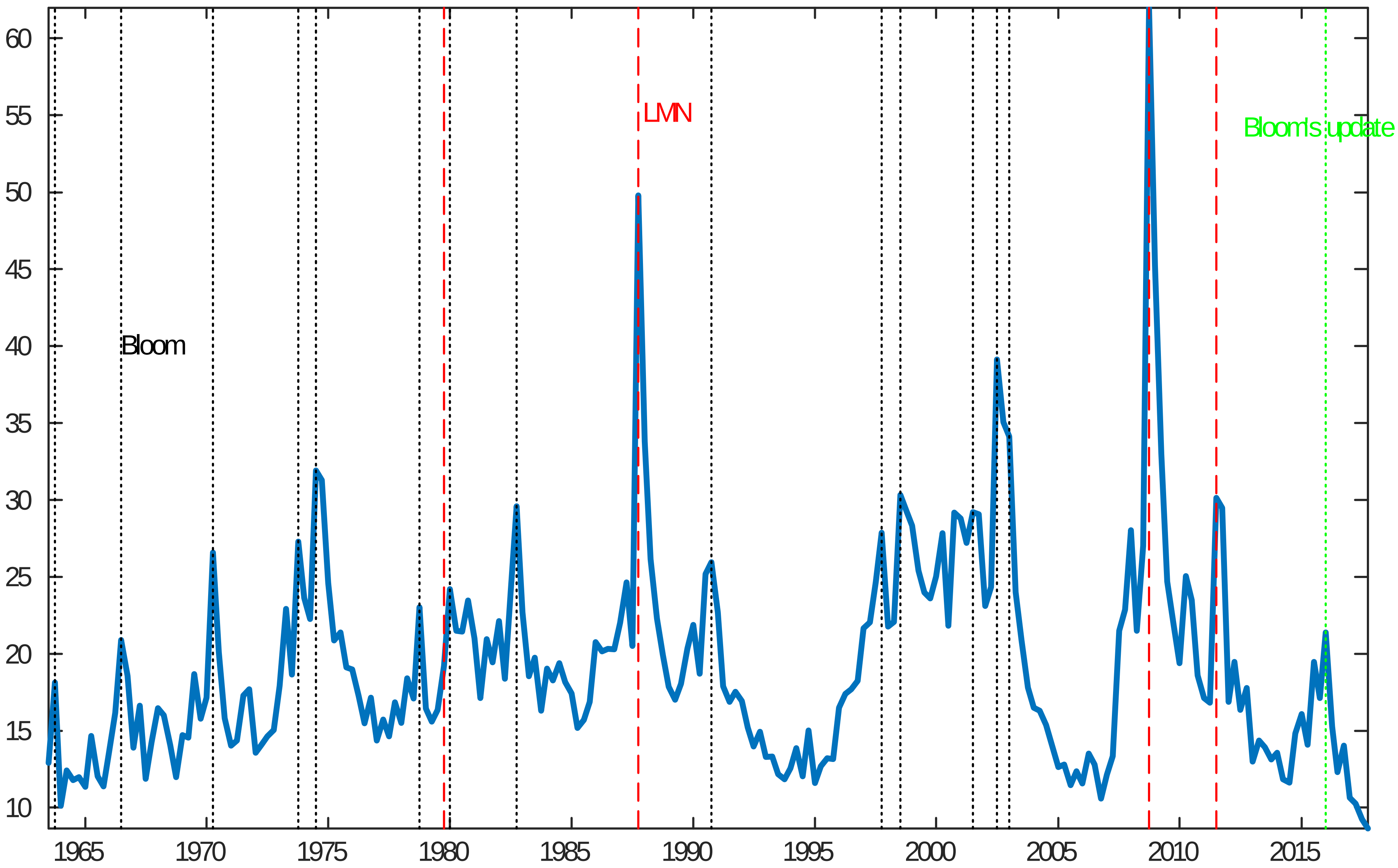

Figure 1: **Financial uncertainty: Dates exploited for identification.** Vertical lines identify the events used to identify financial uncertainty shocks. The four red lines refer to the events selected by Ludvigson, Ma and Ng (2019). The black lines identify the dates selected by Bloom (2009). The green line refers to the 2016 increase not covered in Bloom (2009). Further details are reported in Table 1.

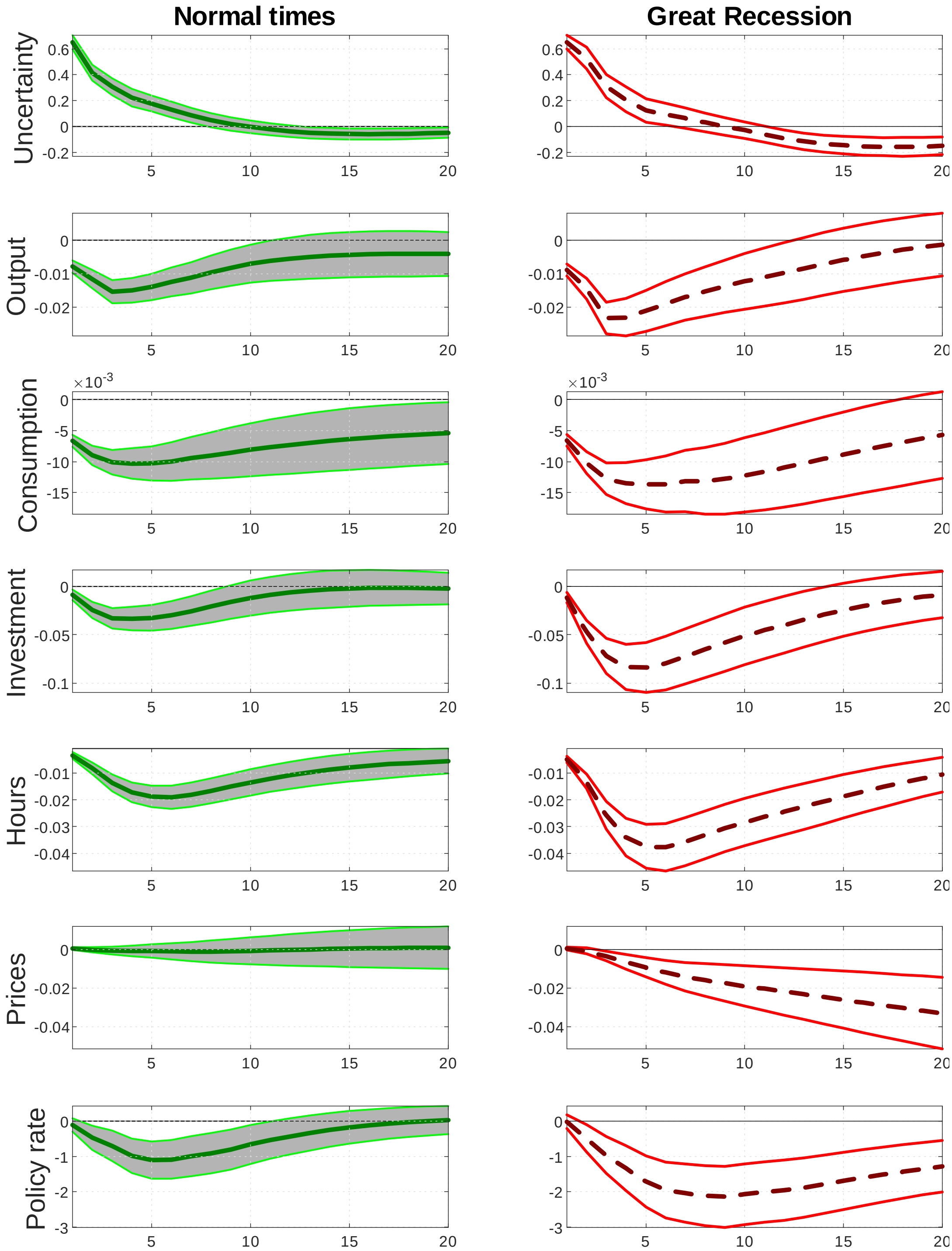

Figure 2: **Impulse responses: linear vs. great recession.** Impulse responses to a 4.4-standard deviation uncertainty shock. Solid green (dashed red) lines: Point estimates of the response of our VAR variables in normal times (during the great recession) conditional on the median target model à la Fry and Pagan (2011). Gray area (area identified by solid red lines): 68% bootstrapped confidence interval surrounding the median-target model.

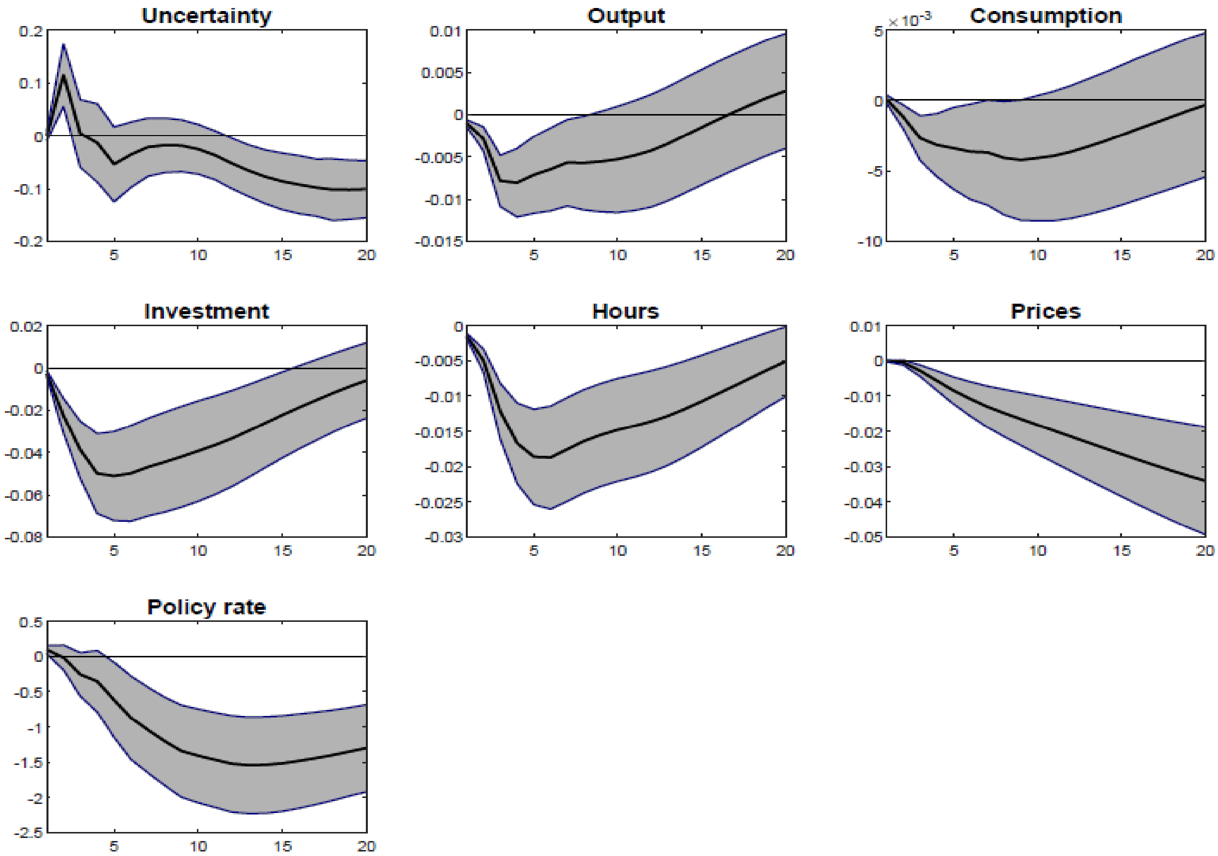

Figure 3: **Bootstrapped test for the difference of median target responses.** Grey bands: 68% confidence bands.

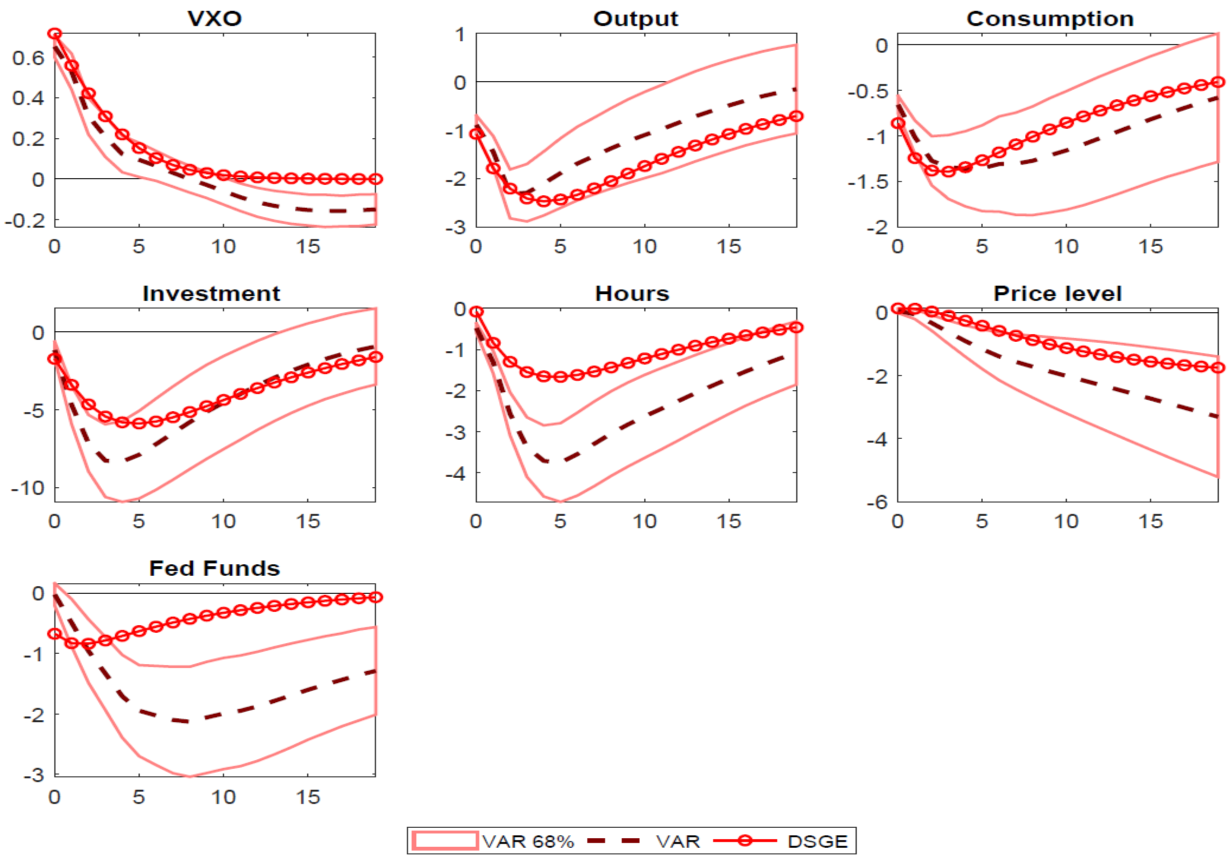

Figure 4: **VAR and DSGE impulses responses to an uncertainty shock during the great recession.** Dashed red lines: IVAR impulse reponses for the great recession with 68% confidence bands. Solid red lines with circles: Responses of the DSGE model estimated with nonlinear VAR moments. VAR estimated with four lags.

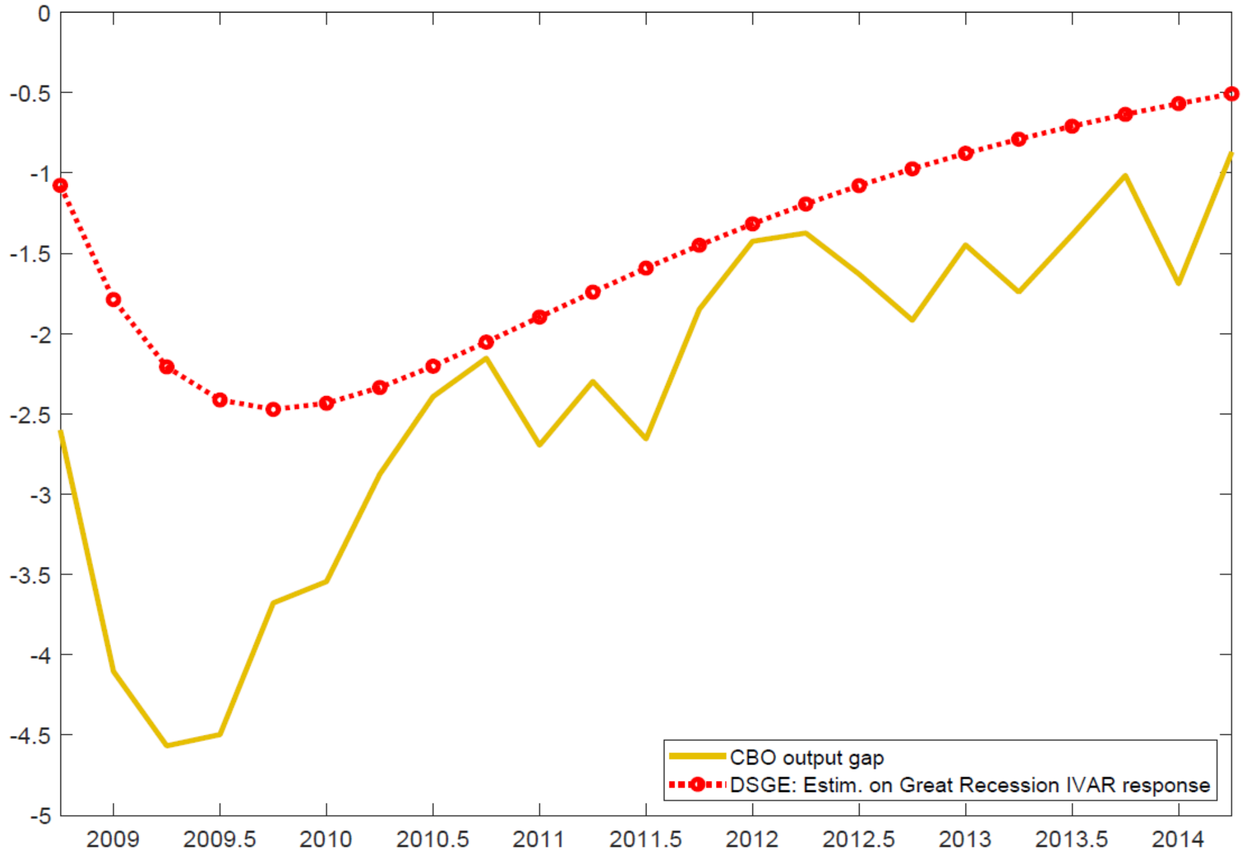

Figure 5: **Contribution of uncertainty shocks to the output loss during the great recession.** Red line with circles: Response of output to a 4.4 standard-deviation uncertainty shock according to our estimated DSGE framework. Yellow solid line: Output gap estimated by the Congressional Budget Office. Output gap normalized to zero in 2008Q3.

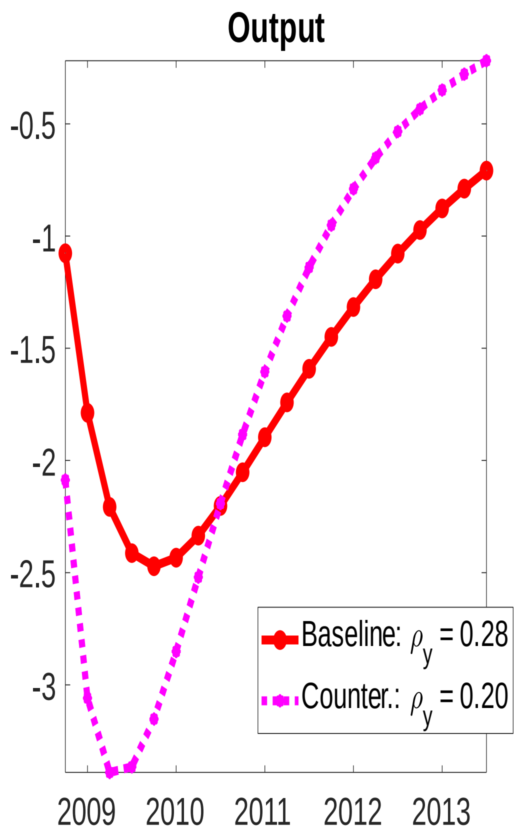

Figure 6: **Counterfactual experiment on the role of monetary policy for the propagation of the 2008Q4 uncertainty shocks and the depth of the great recession.** Baseline: GIRFs conditional on the parameters estimated with the great recession impulse responses. Counterfactual experiment conducted by replacing the Taylor rule parameter $\rho_y^{GR} = 0.28$ with the "normal times" value $\rho_y^{linear} = 0.20$.

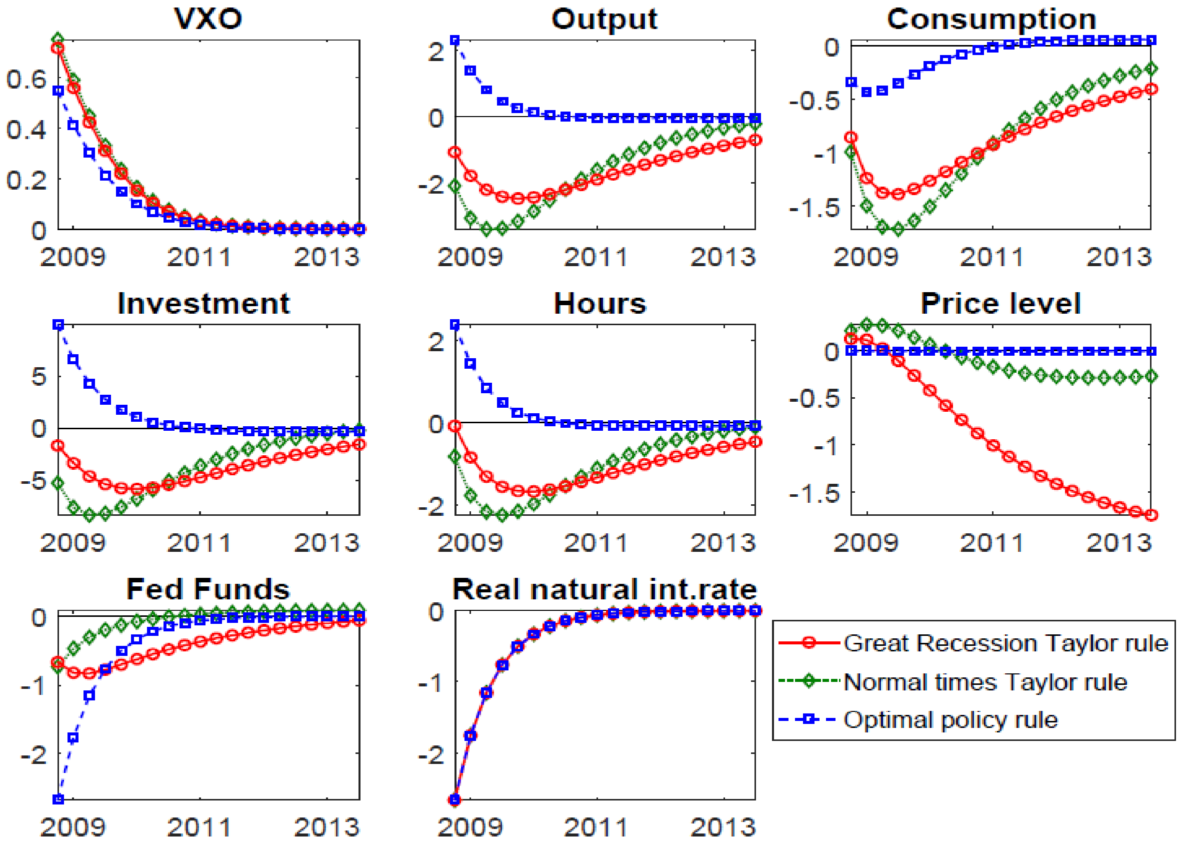

Figure 7: **DSGE impulse responses conditional on estimated vs. optimal policy rule.** Red solid line with circles: GIRFs during the great recession. Green dotted lines with diamonds: GIRFs conditional on the economy estimated with great recession data but with the policy rule estimated with normal times data. Optimal policy: GIRFs implied by the policy rule that features a systematic response of the policy rate to movements in the real natural interest rate.

Appendix of the paper "Uncertainty and Monetary Policy During the Great Recession", by Giovanni Pellegrino, Efrem Castelnuovo, and Giovanni Caggiano

This Appendix contains additional material with respect to the contents of our paper. In particular:

- Section A offers details on the way we compute the generalized impulse responses (GIRFs) with our nonlinear VAR;
- Section B documents additional results related to our baseline nonlinear VAR analysis. In particular, it documents: a) the identified set of the impulse response functions of our baseline VARs; b) a difference test that accounts for model uncertainty; c) an alternative difference test accounting for sampling uncertainty around GDP peak responses.
- Sections C offers several extensions and robustness checks to our baseline results. In particular, our results are robust to: a) adding extra interaction terms to our baseline nonlinear VAR framework; b) controlling for a proxy of credit spread, which is meant to capture first-moment financial shocks; c) excluding the dates referring to oil-related events from our baseline set of restrictions.
- Section D shows that our event-based approach for the identification of uncertainty shocks works well if the data generating process is the Basu and Bundick (2017) model;
- Section E derives the formula we use in the paper to compute the value of the relative risk aversion in the estimated DSGE framework, which depends (as also explained in the text of the paper) on the structure of the economy because of the presence of habits in consumption and endogenous labor supply;
- Section F offers details on the Bayesian IRFs matching econometric strategy used in the paper to estimate the DSGE framework in a state-dependent fashion;
- Section G discusses the calibration of the set of structural parameters of the DSGE model we work which we do not estimate;
- Section H shows that initial conditions do not materially affect the generalized impulse responses computed with our DSGE framework.

A: Computation of the Generalized Impulse Response Functions

The algorithm for the computation of the Generalized Impulse Response Functions follows the steps suggested by Koop, Pesaran, and Potter (1996), and it is designed to simulate the effects of an orthogonal structural shock as in Kilian and Vigfusson (2011). The idea is to compute the empirical counterpart of the theoretical $GIRF_{\mathbf{Y}}(h, \delta, \boldsymbol{\omega}_{t-1})$ of the vector of endogenous variables $\mathbf{y}_t$, h periods ahead, for a given initial condition $\boldsymbol{\omega}_{t-1} = \{\mathbf{Y}_{t-1}, ..., \mathbf{Y}_{t-l}\}$, where l is the number of VAR lags, and δ is the structural shock hitting at time t. Following Koop, Pesaran, and Potter (1996), such GIRF can be expressed as follows:

$$GIRF_{\mathbf{Y}}(h, \delta, \boldsymbol{\omega}_{t-1}) = \mathbb{E}[\mathbf{Y}_{t+h} \,| \delta, \boldsymbol{\omega}_{t-1}] - \mathbb{E}[\mathbf{Y}_{t+h} \,| \boldsymbol{\omega}_{t-1}]$$

where $\mathbb{E}[\cdot]$ is the expectation operator, and $h = 0, 1, ..., H$ indicates the horizons from 0 to H for which the computation of the GIRF is performed.

In our case, $\boldsymbol{\omega}_{t-1}$ corresponds to our "Great Recession" initial condition, i.e., the initial condition corresponding to the uncertainty spike occurred in $t = 2008Q4$ or:

$$\boldsymbol{\omega}_{t-1} = \boldsymbol{\omega}_{2008Q3} = \{\mathbf{Y}_{2008Q3}, ..., \mathbf{Y}_{2008Q3-l+1}\}.$$

Notice that, given that uncertainty and GDP are modeled in the VAR, such set includes the values of the interaction terms $(\ln VXO \times \Delta \ln GDP)_{t-j}$, $j = 1, ..., l$.

Given our IVAR model (formalized in the paper, see eq. (1)), we compute our GIRFs as follows:

1. use the initial condition $\boldsymbol{\omega}_{t-1} = \boldsymbol{\omega}_{2008Q3}$. Pick a matrix $\mathbf{B}$ among the set of retained matrices $\overline{\mathcal{B}}$ that satisfy our identifying narrative sign restrictions (see identification in Section 2 of the paper);

2. conditional on $\boldsymbol{\omega}_{t-1}$, $\mathbf{B}$ and the structure of the model (1), we simulate the path $[\mathbf{Y}_{t+h} \,| \boldsymbol{\omega}_{t-1}]^r$, $h = [0, 1, ..., 19]$ (which is, realizations up to 20-step ahead) by loading our VAR with a sequence of randomly extracted (with repetition) residuals $\widetilde{\boldsymbol{\eta}}^r_{t+h} \sim d(0, \boldsymbol{\Omega})$, $h = 0, 1, ..., H$, where $\boldsymbol{\Omega}$ is the VCV matrix of the IVAR residuals, $d(\cdot)$ is the empirical distribution of the residuals, and r indicates the particular sequence of residuals extracted;

3. conditional on $\boldsymbol{\omega}_{t-1}$, $\mathbf{B}$ and the structure of the model (1), we simulate the path $[\mathbf{Y}_{t+h} \,| \delta, \boldsymbol{\omega}_{t-1}]^r$, $h = [0, 1, ..., 19]$ by loading our VAR with a perturbation of the

randomly extracted residuals $\widetilde{\mathbf{u}}^r_{t+h} \sim d(0, \mathbf{\Omega})$ obtained in step 2. In particular, we use the decomposition $\mathbf{\Omega} = \boldsymbol{B}\boldsymbol{B}'$, where $\boldsymbol{B}$ is the picked admissible solution. Hence, we recover the orthogonalized elements (shocks) $\widetilde{\mathbf{e}}^r_t = \boldsymbol{B}^{-1}\widetilde{\boldsymbol{\eta}}^r_t$. We then add a quantity $\delta > 0$ to the $\widetilde{e^r}_{unc,t}$, where $\widetilde{e^r}_{unc,t}$ is the scalar stochastic element loading the uncertainty equation in the VAR. This enable us to obtain $\widetilde{\mathbf{e}}^r_t$, which is the vector of perturbed orthogonalized elements embedding $\widetilde{e^r}_{unc,t}$. We then move from perturbed shocks to perturbed residuals as follows: $\widetilde{\boldsymbol{\eta}}^r_t = \boldsymbol{B}\widetilde{\mathbf{e}}^r_t$. These are the perturbed residuals that we use to simulate $[\mathbf{Y}_{t+h} \,|\delta, \boldsymbol{\omega}_{t-1}]^r$;

4. we compute the difference between paths for each simulated variable at each simulated horizon $[\mathbf{Y}_{t+h} \,|\delta, \boldsymbol{\omega}_{t-1}]^r - [\mathbf{Y}_{t+h} \,|\boldsymbol{\omega}_{t-1}]^r$, $h = [0, 1, ..., 19]$;

5. we repeat steps 2-4 a number of times equal to $R = 500$. We then store the horizon-wise average realization across repetitions r. In doing so, we obtain a consistent estimate of the GIRF given the matrix $\mathbf{B}$, $\widehat{GIRF^{\mathbf{B}}}_{\mathbf{Y}}(h, \delta_t, \boldsymbol{\omega}_{t-1}) = \widehat{\mathbb{E}}[\mathbf{Y}_{t+h} \,|\delta, \boldsymbol{\omega}_{t-1}] - \widehat{\mathbb{E}}[\mathbf{Y}_{t+h} \,|\boldsymbol{\omega}_{t-1}]$, $h = [0, 1, ..., 19]$;

6. we repeat steps 1-5 for each given matrix $\boldsymbol{B}$ among the set of retained matrices $\overline{\mathcal{B}}$. The set of all the GIRFs for each possible $\mathbf{B} \in \overline{\mathcal{B}}$ determines our identified set. If a given matrix $\mathbf{B}$ leads to an explosive response (namely if this is explosive for most of the R sequences of residuals $\widetilde{\boldsymbol{\eta}}^r_{t+h}$, in the sense that the response of the shocked variable diverges instead than reverting to zero), then such initial condition is discarded.[1] In order to plot a summary GIRF out of this set we use the Median Target (MT) response proposed by Fry and Pagan (2011), i.e., the GIRF corresponding to the $\mathbf{B}$ model whose implied impulse responses to an uncertainty shock are the closest to the median responses computed across all retained models;

7. confidence bands surrounding the MT GIRFs estimates obtained in step 6 are computed via a bootstrap procedure. In particular, we simulate $S = 1,000$ samples of size equivalent to the one of actual data. Then, per each simulated dataset, we: i) estimate our nonlinear VAR model; ii) implement step 5.[2] In implementing

[1] This never happens for our responses estimated on actual data. We verified that it happens quite rarely as regards our bootstrapped responses.

[2] Per each simulated set we also estimate the linear VAR specification nested in the IVAR model and compute the corresponding linear response to the same shock size, so that to be consistent with what we do on the actual data. The bootstrap used is similar to the one used by Christiano, Eichenbaum, and Evans (1999) (see their footnote 23). The code discards the explosive artificial draws to be sure that exactly 1,000 draws are used. In our simulations, this happens a negligible fraction of times.

this procedure the initial conditions and VCV matrix used for our computations now depend on the particular dataset s used, i.e., $\boldsymbol{\omega}_{t-1}^{s}$ and $\boldsymbol{\Omega}_{t}^{s}$.[3] Hence, rather than using the $\mathbf{B}$ which corresponds to the MT response, we use the rotation $\mathbf{Q}$ which corresponds to the MT response, i.e., we use $\mathbf{B}^{s} = P^{s}\mathbf{Q}$ with P^{s} being the unique lower-triangular Cholesky factor associated to $\boldsymbol{\Omega}_{t}^{s}$, i.e., $\boldsymbol{\Omega}_{t}^{s} = P^{s}P^{s\prime}$. 68% confidence bands are constructed by considering the point estimates of the impulse responses ± 0.995 times the bootstrapped standard errors.

We use a shock size δ equal to the median size of the uncertainty shock in $t = 2008Q4$ among all retained shocks series.

B: Extra results on the baseline IVAR analysis

Identified set

Figure 2 in the main text shows the median target responses computed with our IVAR approach for the great recession and the median target responses obtained with the nested linear VAR together with their bootstrapped bands. Figure A1 instead reports the identified set of impulse responses along with the median target impulse response both for normal times and for the great recession.[4] A few facts stand out. First, there is evidence of a negative response of all real activity indicators to an uncertainty shock according to both models. Looking at the identified set, real activity indicators go down on impact after an uncertainty shock according to the large majority of retained models. Second, this evidence is stronger for the great recession case. The responses during the great recession are substantially larger than those in normal times. This is true despite of the close similarity between the response of uncertainty in the two states we consider. This latter evidence points to a different transmission mechanism at work in normal times vs. during an extreme event as the great recession. Three, the response of real activity indicators is more persistent during the great recession. Fourth, the response of the policy rate is negative and persistent according to both models, while that of the price level is negative during the great recession, and negligible in the linear case.

Below we show that the two identified sets are different from a statistical standpoint.

[3] To maximize comparability between the initial condition $\boldsymbol{\omega}_{t-1}^{s}$ and the Great Recession one in the actual sample, in the simulated dataset we pick the quarter t with the biggest uncertainty spike.

[4] The number of accepted draws is about 0.2% for both the linear VAR and the IVAR. More precisely, out of one million, we retain 2,116 draws for the linear VAR, and 2,168 for the IVAR. Following Fry and Pagan (2011), the median target (MT) response is produced by considering the unique retained model whose implied impulse responses are the closest to the median responses (across models) over the horizon we consider.

Difference accounting for model uncertainty

Figure 3 in the main text shows the outcome of a test for the difference of median target responses that only accounts for estimation (or sampling) uncertainty by means of the bootstrap at point 7. Figure A2 instead shows a test for the difference of state-dependent responses that focuses on model uncertainty, i.e., on the uncertainty related to all the responses in the identified set. The differences are constructed as follows. We start by considering the same set of rotations for both the linear and the interacted VARs. Among all retained draws for each model, we consider only those that are common to the two VARs. This leaves us with 77% of common retained draws. We then construct the difference among the responses belonging to the set of common retained draws and plot their distribution. Figure A2 shows that all differences remain significant, even when looking at the 99% percentile of the empirical distribution.

A test accounting both for estimation and for model uncertainty is not proposed here. Such a test would be extremely demanding from a computational standpoint, given that our VAR model is a nonlinear one and the computation of the GIRFs is time-consuming. A test of this sort is proposed by Ludvigson, Ma, and Ng (2019), who - however - focus on a linear framework and, therefore, can compute impulse responses pretty quickly given that such responses are independent from initial conditions and do not require averaging out the outcome of different simulations accounting for different initial conditions.

Difference around alternative responses than the MT responses

Figure 3 in the paper shows the outcome of a bootstrap test for the difference of our median target responses between normal times and the great recession. Specifically, we find that the responses of output, investment, consumption, and hours worked to an uncertainty shock as produced with our nonlinear VAR for the great recession are significantly larger than the same responses produced with a (nested) linear VAR once accounting for sampling uncertainty. Figure A3 shows that we find the same result if, rather than constructing the bootstrap test for the difference of responses around MT responses, we construct it around alternative target responses. In particular, we select two alternative target responses from the identified set in Figure A1: the response relative to the biggest GDP peak reaction, and that relative to the smallest GDP peak reaction. As Figure A3 documents, the response of real activity is still significantly bigger during the great recession than normal times for the two alternative targets considered.[5] The reaction of consumption is only borderline significant when targeting to the smallest GDP peak response, with the mass of the distribution that however hints to a larger response in the great recession. Overall the evidence in Figure A3 suggests that our baseline test for the difference of MT responses is representative for the spectrum of real activity responses in the identified set.

C: Extensions and robustness checks for the IVAR analysis

Parsimonious (baseline) vs. extended IVAR

The IVAR model employed in the paper is a parsimonious version of a more sophisticated IVAR which we estimated to check the robustness of our results. Thinking of the third-order approximation of the DSGE model we work with, it is natural to extend our baseline IVAR framework to add extra interaction terms involving quadratic terms as follows:

$$\mathbf{Y}_t = \boldsymbol{\alpha} + \sum_{j=1}^{L} \mathbf{A}_j \mathbf{Y}_{t-j} + \begin{bmatrix} \sum_{j=1}^{L} \mathbf{c}_j \ln VXO_{t-j} \times \Delta \ln GDP_{t-j} \\ + \sum_{j=1}^{L} \mathbf{c}_j (\ln VXO_{t-j})^2 \times \Delta \ln GDP_{t-j} \\ + \sum_{j=1}^{L} \mathbf{c}_j \ln VXO_{t-j} \times (\Delta \ln GDP_{t-j})^2 \end{bmatrix} + \mathbf{u}_t$$

Cubic terms $((\ln VXO_{t-j})^3, (\Delta \ln GDP_{t-j})^3)$ are omitted to minimize the likelihood of explosiveness.

Figure A4 contrasts the impulse responses obtained with our baseline model with those produced with the enriched framework. If anything, the reactions produced by this framework speak even more clearly in favor of nonlinearities in the data.

The role of first moment shocks

The Basu and Bundick (2017) model features frictionless financial markets. As such, it acknowledges no role to first moment financial shocks as drivers of the business cycle. Consistently with Basu and Bundick's (2017) theoretical framework, our baseline VAR specification(s) does not feature any measure of financial frictions. However, as discussed by Stock and Watson (2012), the great recessions was likely caused by a combination of first-moment financial shocks and uncertainty shocks. Hence, one may

[5] Our bootstrapped confidence bands are based over 1,000 realizations for the impulse responses, which are used to compute the bootstrapped estimate of the standard errors of the impulse response functions. As in Altig, Christiano, Eichenbaum, and Lindé (2011), the 68% confidence bands are constructed by considering the target point estimates of the impulse response ± the bootstrapped estimate of the standard errors. More details on the bootstrap we perform are available in Section A on this Appendix.

wonder if our finding on the larger business cycle effects caused by uncertainty shocks during the great recession is in fact an artifact due to having left out of the picture the role of first moment financial shocks. To address this issue, we augment our baseline vector in the IVAR specification with a measure of spread, which is meant to capture frictions in financial markets. Our model of endogenous variables is then given by: $\mathbf{Y}_t = [SPREAD, \ln VXO, \ln GDP, \ln C, \ln I, \ln H, \ln P, FFR]'$, where $SPREAD$ is the difference between the BAA yield and the AAA one, VXO denotes the stock market S&P 100 implied volatility index, GDP per capita GDP, C per capita consumption, I per capita investment, H per capita hours worked, P the price level, and FFR the federal funds rate. To jointly identify first and second moment (uncertainty) financial shocks we adopt the same methodology of Section 2. The narrative sign restrictions approach has the clear advantage of not imposing any timing restrictions on the spread-uncertainty contemporaneous relationship. This implies that, conditional on our identification strategy to separate first and second-moment financial shocks, the results we obtain are not driven by questionable zero restrictions.

The challenge at this point is to disentangle spread and uncertainty shocks, which are typically assumed to have similar effects on macro variables. To separate the two shocks, we impose the following restrictions on top of our baseline ones. First, we use two event-based identifying restrictions. First, recall that our uncertainty shock in 1987Q4 (the quarter related to the Black Monday) has to be greater than or equal to the 75th percentile of the distribution of the shocks conditional on that quarter. In other words, the uncertainty shock must be "sufficiently large". Differently, we impose that our first moment financial shock in 1987Q4 has to be smaller than or equal to the median. This requirement is supported by the evidence provided by Gilchrist and Zakrajšek (2012), whose measure of financial frictions - the excess bond premium (EBP) calculated as the fraction of a microfounded credit spread index not explained by the underlying fundamentals of bond issuers - has a negative spike in October 1987. Figure A5 plots our proxy of uncertainty, the VXO, along with three commonly used measures of financial frictions: the Baa-Aaa spread, the excess bond premium estimated by Gilchrist and Zakrajšek (2012), and the National Financial Conditions Index produced by the Chicago Fed. While all indicators show a large spike in the great recession, in October 1987 only the VXO experienced a large increase, while all other indicators displayed value below their average.

The second event-based identifying restriction we impose to separate first and second-moment financial shocks is that our first-moment financial shock in 2008Q4 be greater than or equal to the median shock. This requirement is similar to that imposed for the identification of our financial uncertainty shock. The 2008Q4-related restriction is meant to make sure that we just retain models pointing to large financial shocks (both first and second-moment financial shocks) during the great recession.

Second, we impose a further external variable restriction. We require that the identified first-moment financial shocks must be positively correlated with the Gilchrist and Zakrajšek's (2012) EBP shocks.[6] Third, we use ratio restrictions following the approach proposed by Furlanetto, Ravazzolo, and Sarferaz (2019). They identify financial and uncertainty shocks using a sign restrictions approach which features, among others, restrictions on ratios of the impulse responses of proxies for first and second-moment financial indicators. Following their approach, we impose that a financial uncertainty (spread) shock generates an on-impact response of the financial uncertainty-spread ratio bigger (smaller) than one.[7] We also impose that the spread shock and the uncertainty shock have a positive on-impact effect on the uncertainty and spread proxies, respectively, to be sure that the ratios that we constrain are also positive.

Figure A6 reports the median target GIRFs to an uncertainty shock for the great recession scenario based on the IVAR model, as well as the impulse responses for the linear case, together with their bands.[8] Figure A7 reports the difference between the linear and the nonlinear case, along with one standard deviations confidence bands. Two results stand out. First, the recessionary impact on all real activity indicators is larger, and statistically significant, in the great recession. Second, the peak responses are overall in line with those of the baseline scenario (documented in Figures 2 and 3 in the paper). Hence, our results are robust to controlling for a measure of financial frictions in our VAR. Finally, Table A1 documents the similarity between some moments implied by our baseline IVAR and the same moments produced with the IVAR enriched with financial frictions presented in this Section.

[6] We replicated Gilchrist and Zakrajšek's (2012) Cholesky-VAR(2) model and worked with their identified EBP-shock series.

[7] To implement this identification strategy, we standardized the financial spread to impose the same standard deviation of our uncertainty proxy.

[8] Out of one million draws, we retain 84 draws for the linear VAR, and 83 for the IVAR, and compute the median-target responses over these retained draws.We see the small number of retained draws as a good sign. This is in line with Uhlig's (2017) Principle 7: *"When a lot of draws are rejected, the identification is sharp. Good!"*.

The role of oil supply shocks

The analysis in the paper revealed the risk of confounding financial uncertainty shocks with oil shocks, something which would undermine our identification strategy. In particular, the correlation between our identified uncertainty shocks (conditional on the median target model) and the oil supply shocks by Baumeister and Hamilton (2019) is 0.20, and it is precisely estimated (p-value: 0.03).[9,10] We here conduct an exercise where, with respect to the baseline analysis, we do not restrict the shocks in correspondence of Bloom's uncertainty spikes dates related to oil-related events, i.e., 1973Q4 (OPEC I, Arab-Israeli War) and 1978Q4 (OPEC II). We also impose that each retained rotation $\mathbf{Q}$ should also imply an insignificant correlation coefficient (at the 5% significance level) between the rotation-implied uncertainty shocks and the externally provided oil supply shocks proxy. Figure A8 reports the median target GIRFs to an uncertainty shock for the great recession scenario based on the IVAR model together with the impulse responses for the linear case and their bootstrap confidence bands. Figure A9 documents the difference between the linear and the nonlinear case, along with one standard deviation confidence bands. The figures confirm the robustness of our findings to this robustness check and hence suggest that our findings are not driven by the confusion of financial uncertainty shocks and oil shocks. In particular, the correlation between the uncertainty shocks identified in this robustness check (conditional on the median target model) and the oil supply shocks by Baumeister and Hamilton (2019) is now 0.12 and turns out to be not significant at a 10% level.

D: Narrative Sign Restrictions and DSGE framework

This Section shows that the narrative sign restrictions (NSR) approach proposed in the paper is able to recover the true impulse responses to an uncertainty shock conditional on the Basu and Bundick (2017) model being the data generating process.

The Basu and Bundick (2017) model features an endogenous measure of financial uncertainty, a model-consistent VXO, which responds to three shocks, i.e., a first-moment technology shock, a first-moment preference shock, and a second-moment preference

[9] The oil supply shocks by Baumeister and Hamilton (2019) are online available at the website https://sites.google.com/site/cjsbaumeister/BH2_supply_shocks.xlsx?attredirects=0&d=1 . This series is available only from 1975Q2.

[10] Significance assessed by running an OLS regression involving our uncertainty shocks (left hand side) vs. a constant and the oil supply shocks proxy (right hand side). We consider White heteroskedasticity-consistent standard errors to control for heteroskedasticity.

shock, this last one being the uncertainty shock. The question is whether it is possible to identify uncertainty shocks only by observing the VXO, as we do in the data. To address this question, we simulate a sample of 2,500 observations with the Basu and Bundick (2017) model conditional on the estimates we obtained with the facts established by the linear VAR.[11] We then estimate a linear VAR and produce impulse responses to an uncertainty shock identified via our NSR restrictions.[12] In particular, consistently with what Bloom (2009) does to identify the dates we use in our baseline analysis, we select the dates with the biggest spikes in the HP-filtered (model-consistent) VXO.[13] Similarly to our baseline analysis, we require the realizations of our identified uncertainty shocks to be larger than the median value of the empirical density of the uncertainty shocks in the selected dates.[14] We focus on a population analysis and on a linear VAR to make sure that our result is not driven by any small-sample issue or fancy nonlinear reduced-form framework.

Figure A10 documents the performance of the NSR-VAR in replicating the DSGE-model consistent impulse responses. The ability of the VAR to correctly capture the responses of the DSGE model is unquestionable. This is good news not only for our VAR identification strategy, but also for the estimation of our DSGE framework. Indeed, the results in this Section imply that it makes sense to use a direct inference approach to estimate our DSGE framework, as opposed to a (much more computationally cumbersome) indirect inference approach, which would require the simulation of pseudo-data and the estimation of VAR impulse responses identified with NSR per each draw of the values of the structural parameters of the DSGE framework from its posterior density.

[11] Even if we employ a DSGE model with three shocks to simulate data which we use to estimate a seven variable-VAR model, no stochastic singularity issue arises in this exercise. The reason is that our data generating process is a nonlinear framework, hence perfect collinearity among the simulated series we use to estimate our VAR is not present even if the number of shocks is lower than the number of "observables" generated via those shocks.

[12] Although the word "narrative" loses its meaning for an exercise based on data simulated from a model, the proposed exercise resembles the identification strategy we use in our baseline analysis where uncertainty shocks are identified using information related to the VXO biggest spikes.

[13] We select the dates corresponding to the biggest 2% among VXO spikes. This selection seems appropriate because it guarantees that: i) enough responses are retained; ii) the selected dates are informative enough to identify the uncertainty shocks.

[14] Similarly to our baseline analysis and to Ludvigson, Ma, and Ng (2019), we impose that the correlation between the series of identified uncertainty shocks and (model-implied) stock market returns be smaller than the median value of the empirical density of the correlation coefficients for all draws.

E: Relative Risk Aversion for the Basu and Bundick (2017) model extended with external habits

This Section derives the expression for the Relative Risk Aversion (RRA) coefficient in the version of the Basu and Bundick (2017) model extended with external habits and which features (as the original model) endogenous labor supply.

Equivalence with Rudebusch and Swanson's (2012) notation

It is first useful to clarify that the value function that we use, which is:

$$V_t = \left[\left(1 - \beta)(a_t \tilde{C}_t^{\eta}(1 - N_{t,})^{(1-\eta)} \right)^{(1-\sigma)/\theta_V} + \beta((E_t V_{t+1})^{1-\sigma})^{1/\theta_V} \right]^{\theta_V/(1-\sigma)}$$

can be equivalently reformulated in Rudebusch and Swanson's (2012) notation as:

$$\tilde{V}_t = \tilde{U}_t(\tilde{C}_t, N_t) + \beta(E_t \tilde{V}_{t+1}^{(1-\alpha)})^{\frac{1}{(1-\alpha)}}.$$

where the $(1 - \beta)$ pre-multiplying the contemporaneous utility function in the expression above is omitted for simplicity, given its irrelevance for the computation of the RRA. It can be easily shown that the two expressions are equivalent once the following definitions are used:

$$\begin{aligned} \tilde{V} &= V^{\frac{1-\sigma}{\theta_V}} \\ \alpha &= 1 - \theta_V = 1 - \frac{1-\sigma}{1-\frac{1}{\psi}} \\ \tilde{U}_t(\tilde{C}_t, N_t) &= (a_t \tilde{C}_t^{\eta}(1 - N_t)^{1-\eta})^{\frac{1-\sigma}{\theta_V}} \end{aligned}$$

Derivation of the formula for the RRA

Swanson (2012) shows that household's labor margin has substantial effects on risk aversion. The household can absorb asset return shocks either through changes in consumption, changes in hours worked, or some combination of the two. This ability to absorb shocks along either or both margins greatly alters the household's attitudes toward risk. Following Swanson (2012) and Swanson (2018) (this latter paper extending the analysis in Swanson (2012) to - among other things - recursive preferences), we compute two measures of relative risk aversion for our model. The first measure - RRA^c - applies when there is no upper bound for labor and therefore total household wealth equals the present discounted value of consumption. The other measure - RRA^{cl} - applies when the upper bound for the household's time endowment is well-specified, meaning that total household wealth equals the present discounted value of leisure plus consumption.

Swanson (2018, equations 23 and 24) shows that, in presence of flexible labor margin and generalized recursive preferences, the expressions to compute the coefficient of steady state relative risk aversion read as follows:

$$\begin{aligned} RRA^{cl} &= \frac{-u_{11}+\lambda u_{12}}{u_1}\cdot\frac{C+w\,(1-N)}{1+w\lambda}+\alpha\frac{(C+w\,(1-N))\,u_1}{u} \\ RRA^{c} &= \frac{-u_{11}+\lambda u_{12}}{u_1}\cdot\frac{C}{1+w\lambda}+\alpha\frac{Cu_1}{u} \end{aligned}$$

where:

$$\begin{aligned} w &= -\frac{u_2}{u_1} \\ \lambda &= \frac{wu_{11}+u_{12}}{u_{22}+wu_{12}} \end{aligned}$$

and where $u_1 = \frac{\partial \tilde{U}_t}{\partial C_t}\Big|_{ss}$, $u_2 = \frac{\partial \tilde{U}_t}{\partial N_t}\Big|_{ss}$, $u_{11} = \frac{\partial^2 \tilde{U}_t}{\partial C_t^2}\Big|_{ss}$, $u_{12} = \frac{\partial^2 \tilde{U}_t}{\partial C_t \partial N_t}\Big|_{ss}$, $u_{22} = \frac{\partial^2 \tilde{U}_t}{\partial N_t^2}\Big|_{ss}$, with ss standing for steady state, and where α and $\tilde{U}_t$ (or $\tilde{U}_t(\tilde{C}_t, N_t)$) were defined earlier. Variables without time subscript indicate steady state values.

It can be easily shown that (see Andreasen et al.'s (2018) Online Appendix):

$$RRA^{cl} = \left(1+\frac{w}{C}(1-N)\right) RRA^{c}.$$

Initial computations. Without loss of generality, the derivation below is based on the following function:[15]

$$\tilde{U}(C_t, N_t) = \frac{\left((C_t - bC_{t-1})^{\eta}\,(1-Nt)^{1-\eta}\right)^{\frac{1-\sigma}{\theta_V}}}{\frac{1-\sigma}{\theta_V}} = \frac{(C_t - bC_{t-1})^{\eta\frac{1-\sigma}{\theta_V}}\,(1-N_t)^{(1-\eta)\frac{1-\sigma}{\theta_V}}}{\frac{1-\sigma}{\theta_V}}.$$

We first take the relevant derivatives and then evaluate them at the steady state. Notice that the stock of external habits (bC_{t-1}) at time t is a given for households. Hence, we have:

[15] We omit a_t from this derivation since its steady state value is 1, which implies that the impact of the preference shock on the relative risk aversion is zero.

$$
\begin{aligned}
u_{1,t} &= \eta\left(C_t - bC_{t-1}\right)^{\eta\left(\frac{1-\sigma}{\theta_V}\right)-1}\left(1-N_t\right)^{(1-\eta)\left(\frac{1-\sigma}{\theta_V}\right)}, \\
u_{11,t} &= \eta\left(\eta\left(\frac{1-\sigma}{\theta_V}\right)-1\right)\left(C_t - bC_{t-1}\right)^{\eta\left(\frac{1-\sigma}{\theta_V}\right)-2}\left(1-N_t\right)^{(1-\eta)\left(\frac{1-\sigma}{\theta_V}\right)}, \\
u_{12,t} &= -\eta\left((1-\eta)\left(\frac{1-\sigma}{\theta_V}\right)\right)\left(C_t - bC_{t-1}\right)^{\eta\left(\frac{1-\sigma}{\theta_V}\right)-1}\left(1-N_t\right)^{(1-\eta)\left(\frac{1-\sigma}{\theta_V}\right)-1}, \\
u_{2,t} &= -(1-\eta)\left(C_t - bC_{t-1}\right)^{\eta\frac{1-\sigma}{\theta_V}}\left(1-N_t\right)^{(1-\eta)\frac{1-\sigma}{\theta_V}-1}, \\
u_{22,t} &= (1-\eta)\left((1-\eta)\frac{1-\sigma}{\theta_V}-1\right)\left(C_t - bC_{t-1}\right)^{\eta\frac{1-\sigma}{\theta_V}}\left(1-N_t\right)^{(1-\eta)\frac{1-\sigma}{\theta_V}-2},
\end{aligned}
$$

In steady state, we have:

$$
\begin{aligned}
u_1 &= \eta\left((1-b)C\right)^{\eta\left(\frac{1-\sigma}{\theta_V}\right)-1}\left(1-N\right)^{(1-\eta)\left(\frac{1-\sigma}{\theta_V}\right)}, \\
u_{11} &= \eta\left(\eta\left(\frac{1-\sigma}{\theta_V}\right)-1\right)\left((1-b)C\right)^{\eta\left(\frac{1-\sigma}{\theta_V}\right)-2}\left(1-N\right)^{(1-\eta)\left(\frac{1-\sigma}{\theta_V}\right)}, \\
u_{12} &= -\eta\left((1-\eta)\left(\frac{1-\sigma}{\theta_V}\right)\right)\left((1-b)C\right)^{\eta\left(\frac{1-\sigma}{\theta_V}\right)-1}\left(1-N\right)^{(1-\eta)\left(\frac{1-\sigma}{\theta_V}\right)-1}, \\
u_2 &= -(1-\eta)\left((1-b)C\right)^{\eta\frac{1-\sigma}{\theta_V}}\left(1-N\right)^{(1-\eta)\frac{1-\sigma}{\theta_V}-1}, \\
u_{22} &= (1-\eta)\left((1-\eta)\frac{1-\sigma}{\theta_V}-1\right)\left((1-b)C\right)^{\eta\frac{1-\sigma}{\theta_V}}\left(1-N\right)^{(1-\eta)\frac{1-\sigma}{\theta_V}-2}.
\end{aligned}
$$

Consequently, we can obtain:

$$
\begin{aligned}
w &= -\frac{u_2}{u_1} = -\frac{-(1-\eta)\left((1-b)C\right)^{\eta\frac{1-\sigma}{\theta_V}}\left(1-N\right)^{(1-\eta)\frac{1-\sigma}{\theta_V}-1}}{\eta\left((1-b)C\right)^{\eta\left(\frac{1-\sigma}{\theta_V}\right)-1}\left(1-N\right)^{(1-\eta)\left(\frac{1-\sigma}{\theta_V}\right)}} = \frac{(1-\eta)(1-N)^{-1}}{\eta\left((1-b)C\right)^{-1}} \\
&= \frac{(1-\eta)}{\eta}\frac{(1-b)C}{1-N}
\end{aligned}
$$

and

$$
\begin{aligned}
\lambda &= \frac{wu_{11}+u_{12}}{u_{22}+wu_{12}} \\
&= \frac{\begin{array}{c}\left(\frac{(1-\eta)}{\eta}\frac{(1-b)C}{1-N}\right)\left(\eta\left(\eta\left(\frac{1-\sigma}{\theta_V}\right)-1\right)\left((1-b)C\right)^{\eta\left(\frac{1-\sigma}{\theta_V}\right)-2}\left(1-N\right)^{(1-\eta)\left(\frac{1-\sigma}{\theta_V}\right)}\right)+ \\ -\eta\left((1-\eta)\left(\frac{1-\sigma}{\theta_V}\right)\right)\left((1-b)C\right)^{\eta\left(\frac{1-\sigma}{\theta_V}\right)-1}\left(1-N\right)^{(1-\eta)\left(\frac{1-\sigma}{\theta_V}\right)-1}\end{array}}{\begin{array}{c}(1-\eta)\left((1-\eta)\frac{1-\sigma}{\theta_V}-1\right)\left((1-b)C\right)^{\eta\frac{1-\sigma}{\theta_V}}\left(1-N\right)^{(1-\eta)\frac{1-\sigma}{\theta_V}-2}+ \\ +\left(\frac{(1-\eta)}{\eta}\frac{(1-b)C}{1-N}\right)\left(-\eta\left((1-\eta)\left(\frac{1-\sigma}{\theta_V}\right)\right)\left((1-b)C\right)^{\eta\left(\frac{1-\sigma}{\theta_V}\right)-1}\left(1-N\right)^{(1-\eta)\left(\frac{1-\sigma}{\theta_V}\right)-1}\right)\end{array}}.
\end{aligned}
$$

Simplifying, we get:

$$\lambda = \frac{(1-N)}{(1-b)\,C}.$$

Derivation of the RRAs. We now have everything we need to derive the two expressions for the relative risk aversion. We put all the previously derived pieces in the expression:

$$RRA^{cl} = \frac{-u_{11} + \lambda u_{12}}{u_1} \cdot \frac{C + w\,(1-N)}{1 + w\lambda} + \alpha \frac{(C + w\,(1-N))\,u_1}{u}$$

This implies:

$$RRA^{cl} = \underbrace{\frac{\begin{array}{c}-\eta\left(\eta\left(\frac{1-\sigma}{\theta_V}\right)-1\right)((1-b)\,C)^{\eta\left(\frac{1-\sigma}{\theta_V}\right)-2}(1-N)^{(1-\eta)\left(\frac{1-\sigma}{\theta_V}\right)}+ \\ +\frac{(1-N)}{(1-b)C}\left(-\eta\left((1-\eta)\left(\frac{1-\sigma}{\theta_V}\right)\right)((1-b)\,C)^{\eta\left(\frac{1-\sigma}{\theta_V}\right)-1}(1-N)^{(1-\eta)\left(\frac{1-\sigma}{\theta_V}\right)-1}\right)\end{array}}{\eta\,((1-b)\,C)^{\eta\left(\frac{1-\sigma}{\theta_V}\right)-1}(1-N)^{(1-\eta)\left(\frac{1-\sigma}{\theta_V}\right)}}}_{PieceA}\cdot$$

$$\cdot\underbrace{\frac{C+\left(\frac{(1-\eta)}{\eta}\frac{(1-b)C}{1-N}\right)(1-N)}{1+\frac{(1-\eta)}{\eta}\frac{(1-b)C}{1-N}\frac{(1-N)}{(1-b)C}}}_{PieceB}$$

$$+\alpha\underbrace{\frac{\left(C+\left(\frac{(1-\eta)}{\eta}\frac{(1-b)C}{1-N}\right)(1-N)\right)\left(\eta\,((1-b)\,C)^{\eta\left(\frac{1-\sigma}{\theta_V}\right)-1}(1-N)^{(1-\eta)\left(\frac{1-\sigma}{\theta_V}\right)}\right)}{\frac{((1-b)C)^{\eta\frac{1-\sigma}{\theta_V}}(1-N)^{(1-\eta)\frac{1-\sigma}{\theta_V}}}{\frac{1-\sigma}{\theta_V}}}}_{PieceC}.$$

Simplifying each piece, we get:

$$\begin{aligned}
PieceA &= \left(1-\frac{1-\sigma}{\theta_V}\right)\frac{1}{(1-b)\,C}, \\
PieceB &= \frac{C\left(1+\frac{(1-\eta)}{\eta}(1-b)\right)}{1+\frac{(1-\eta)}{\eta}}, \\
PieceC &= \eta\left(\frac{1}{(1-b)}+\frac{(1-\eta)}{\eta}\right)\frac{1-\sigma}{\theta_V}.
\end{aligned}$$

So, putting all together, we have:

$$R^{cl} = \underbrace{\left(1-\frac{1-\sigma}{\theta_V}\right)\frac{1}{(1-b)\,C}\cdot\frac{C\left(1+\frac{(1-\eta)}{\eta}(1-b)\right)}{1+\frac{(1-\eta)}{\eta}}}_{pieceA\cdot pieceB} + \alpha\cdot\underbrace{\eta\left(\frac{1}{(1-b)}+\frac{(1-\eta)}{\eta}\right)\frac{1-\sigma}{\theta_V}}_{pieceC}$$

which, once we replace the definition of θ_V with its expression, becomes:

$$RRA^{cl} = \frac{1}{\psi} \cdot \frac{\left(1 + \frac{(1-\eta)}{\eta}(1-b)\right)}{(1-b)\left(1+\frac{(1-\eta)}{\eta}\right)} + \alpha\left(\frac{\eta}{(1-b)} + 1 - \eta\right)\left(1 - \frac{1}{\psi}\right)$$

Replacing $\alpha = \left(1 - \frac{1-\sigma}{1-\frac{1}{\psi}}\right)$, and simplifying:

$$RRA^{cl} = \frac{1}{\psi} \cdot \frac{\left(1 + \frac{(1-\eta)}{\eta}(1-b)\right)}{(1-b)\left(1+\frac{(1-\eta)}{\eta}\right)} + \left(\sigma - \frac{1}{\psi}\right)\left(\frac{\eta}{(1-b)} + 1 - \eta\right),$$

which delivers $RRA^{cl} = \sigma$ when the degree of external habits $b = 0$.

Finally:

$$\begin{aligned} RRA^{cl} &= \left(1 + \frac{w}{C}(1-N)\right) RRA^{c} \\ &= \left(\frac{\eta}{\eta + (1-\eta)(1-b)}\right) RRA^{cl} \end{aligned}$$

This implies:

$$RRA^{c} = \left(\frac{\eta}{\eta + (1-\eta)(1-b)}\right) \cdot \left(\frac{1}{\psi} \cdot \frac{\left(1 + \frac{(1-\eta)}{\eta}(1-b)\right)}{(1-b)\left(1+\frac{(1-\eta)}{\eta}\right)} + \left(\sigma - \frac{1}{\psi}\right)\left(\frac{\eta}{(1-b)} + 1 - \eta\right)\right).$$

This is exactly the expression used in the paper to compute the RRA.

F: Minimum distance estimation strategy

The Bayesian minimum distance estimator works as follows. Denote by $\widehat{\boldsymbol{\psi}}$ the vector in which we stack the (I)VAR estimated (generalized) impulse responses over a 20-quarter horizon to an uncertainty shock conditional on the great recession (i.e., the responses displayed in Figure 2, right column).[16] When the number of observations n is large (as in our case, given the long sample of data we use to estimate our IVAR), standard asymptotic theory suggests that

$$\widehat{\boldsymbol{\psi}} \overset{a}{\sim} N(\boldsymbol{\psi}(\boldsymbol{\zeta}_0), \mathbf{V}(\boldsymbol{\zeta}_0, n)) \tag{1}$$

[16] For a paper proposing information criteria to select the responses that produce consistent estimates of the true but unknown structural parameters and those that are most informative about DSGE model parameters, see Hall, Inoue, Nason, and Rossi (2012).

where $\boldsymbol{\zeta}_0$ denotes the true vector of structural parameters that we estimate, and $\boldsymbol{\psi}(\boldsymbol{\zeta})$ denotes the model-implied mapping from a vector of parameters to the analog impulse responses in $\widehat{\boldsymbol{\psi}}$.

As explained earlier, the IVAR GIRFs $\widehat{\boldsymbol{\psi}}$ for the great recession are computed by iterating forward the system starting from the initial condition $t-1 = 2008Q3$. Similarly, we compute the DSGE model-related responses for each given set of parameter values $\boldsymbol{\psi}(\boldsymbol{\zeta})$ by iterating forward the approximated solution of the DSGE model starting from the stochastic steady state.[17] Both DSGE-based and VAR-based impulse responses are interpreted as percent deviations of variables induced by an uncertainty shock, with the exception - in our case - of the interest rate response which is measured in percentage points as implied by the VAR specification.

To compute the posterior density for $\boldsymbol{\zeta}$ given $\widehat{\boldsymbol{\psi}}$ using Bayes' rule, we first need to compute the likelihood of $\widehat{\boldsymbol{\psi}}$ conditional on $\boldsymbol{\zeta}$. Given (1), the approximate likelihood of $\widehat{\boldsymbol{\psi}}$ as a function of $\boldsymbol{\zeta}$ reads as follows:

$$f(\widehat{\boldsymbol{\psi}}|\boldsymbol{\zeta}) = \left(\frac{1}{2\pi}\right)^{\frac{N}{2}} |\mathbf{V}(\boldsymbol{\zeta}_0, n)|^{-\frac{1}{2}} \times \exp\left[-\frac{1}{2}\left(\widehat{\boldsymbol{\psi}} - \boldsymbol{\psi}(\boldsymbol{\zeta})\right)' \mathbf{V}(\boldsymbol{\zeta}_0, n)^{-1} \left(\widehat{\boldsymbol{\psi}} - \boldsymbol{\psi}(\boldsymbol{\zeta})\right)\right] \quad (2)$$

where N denotes the number of elements in $\widehat{\boldsymbol{\psi}}$ and $\mathbf{V}(\boldsymbol{\zeta}_0, n)$ is treated as a fixed value.[18] We use a consistent estimator of $\mathbf{V}$. Because of small sample-related considerations, such estimator features only diagonal elements (see Christiano, Trabandt, and

[17] Following Basu and Bundick (2017), we set the value of the exogenous processes to zero and iterate forward until the model converges to its stochastic steady state. Then, we hit the model with an uncertainty shock of the same size as in the IVAR (i.e., a 4.4 standard deviation shock) and compute impulse responses as the percent deviation between the stochastic path followed by the endogenous variables and their stochastic steady state. Given that no future shocks are considered, this way of computing GIRFs does not line up with Koop, Pesaran and Potter's (1996) algorithm. We do so to avoid simulating the model several times and then integrate across all simulations, a procedure which would be very time consuming, above all when combined with the MCMC algorithm we adopt for our Bayesian estimation. Basu and Bundick (2017) show that the differences between these two ways of computing GIRFs are negligible with a framework like theirs. We also verified that our IVAR GIRFs remained unchanged when future shocks are not taken into account, something which augments the comparability between IVAR and DSGE GIRFs. Analytical expressions for GIRFs produced with nonlinear models are available in Andreasen, Fernández-Villaverde, and Rubio-Ramírez (2018).

[18] As pointed out by Christiano, Eichenbaum, and Trabandt (2016) and Bundick and Smith (2019), there are four reasons why this is only an approximate likelihood. First, standard asymptotic theory implies that, if the DSGE model is the correct data generating process with the true parameters $\boldsymbol{\zeta}_0$, $\widehat{\boldsymbol{\psi}}$ converges only asymptotically to $N(\boldsymbol{\psi}(\boldsymbol{\zeta}_0), \mathbf{V})$ as the sample size grows arbitrarily large. Second, our proxy for $\mathbf{V}$ is guaranteed to be correct only as the sample size grows arbitrarily large. Third, $\boldsymbol{\psi}$ is approximated with a nonlinear model approximated at a third order, i.e., not with the true, global nonlinear model. Fourth, differently from the linear model case, the IRFs are not a full summary of nonlinear frameworks.

Walentin (2011) and Guerron-Quintana, Inoue, and Kilian (2017)).[19] We work with a diagonal $\mathbf{V}$ with the variances of the $\widehat{\boldsymbol{\psi}^i}$ along the main diagonal.[20] This choice is widely adopted in the literature and allows one to put more weight in replicating VAR-based responses with relatively smaller confidence bands. Treating eq. (2) as the likelihood function of $\widehat{\boldsymbol{\psi}}$, it follows that the Bayesian posterior of $\boldsymbol{\zeta}$ conditional on $\widehat{\boldsymbol{\psi}}$ and $\mathbf{V}$ is:

$$f(\boldsymbol{\zeta}|\widehat{\boldsymbol{\psi}}) = \frac{f(\widehat{\boldsymbol{\psi}}|\boldsymbol{\zeta})p(\boldsymbol{\zeta})}{f(\widehat{\boldsymbol{\psi}})}, \tag{3}$$

where $p(\boldsymbol{\zeta})$ denotes the priors on $\boldsymbol{\zeta}$ and $f(\widehat{\boldsymbol{\psi}})$ is the marginal density of $\widehat{\boldsymbol{\psi}}$. As in Christiano, Trabandt, and Walentin (2011), the mode of the posterior distribution of $\boldsymbol{\zeta}$ is computed by maximizing the value of the numerator in 3 via the csminwel algorithm proposed by Chris Sims.[21] The posterior densities are estimated via Laplace approximation.

G: Model calibration

We calibrate some of the parameters of the model as in Basu and Bundick (2018), the reason being that we use a slightly modified version of their model (to which we add habits in consumption) for our analysis. Table A2 collects all the calibrated parameters. We do not estimate these parameters for several reasons. We follow a long tradition in macroeconomics and calibrate the capital's share in production α, the household discount factor β and the steady state depreciation rate δ to values that are standard in the literature. The first-order utilization parameter δ_1 and the consumption weight in the period utility function η cannot be estimated, because the first is determined endogenously by a steady state relationship (involving δ and β) and the second is fixed

[19] Guerron-Quintana, Inoue, and Kilian (2017) study the asymptotic theory for VAR-based impulse response matching estimators of the structural parameters of linearized DSGE models when the number of impulse responses exceeds the number of linear VAR model parameters. The number of impulse responses in our analysis (140) is lower than the number of estimated coefficients of the VAR (251, constants excluded). We are aware of no contributions studying the asymptotic theory for this estimator when nonlinear frameworks are employed.

[20] Denoting by $\hat{\mathbf{W}}$ the bootstrapped variance-covariance matrix of VAR-based impulse responses $\widehat{\boldsymbol{\psi}}$, i.e., $\frac{1}{M}\sum_{j=1}^{M}(\boldsymbol{\psi}_j - \bar{\boldsymbol{\psi}})(\boldsymbol{\psi}_j - \bar{\boldsymbol{\psi}})'$ (where $\boldsymbol{\psi}_j$ denotes the realization of $\widehat{\boldsymbol{\psi}}$ in the j^{th} (out of $M = 1,000$) bootstrap replication and $\bar{\boldsymbol{\psi}}$ denotes the mean of $\boldsymbol{\psi}_j$), $\mathbf{V}$ is based on the diagonal of this matrix. Notice that $\mathbf{V}$ contains the same variances used to plot the confidence intervals for the IVAR responses. This is the same approach used in Altig, Christiano, Eichenbaum, and Lindé (2011).

[21] The use of a direct inference approach to estimate the DSGE model is justified by the Monte Carlo analysis reported in Appendix D. There we show that the narrative sign restriction identification approach we use in our VAR analysis recovers the true impulse responses produced by the DSGE framework.

in order to imply a Frisch elasticity equal to 2. The steady state inflation rate Π cannot be estimated by a impulse response functions matching procedure that focuses on out-of-steady state dynamics, i.e., deviations from the (stochastic) steady state. The firm leverage parameter ν does not influence impulse responses in the absence of financial frictions and hence is not identified. As regards the parameters of the stochastic shock processes, we calibrate the volatility of the second moment preference shock σ_{σ^a} to the same value as calibrated in Basu and Bundick (2018) to match empirical moments. The parameters governing the processes of the preference and technological shocks, i.e. ρ^a, σ^a, ρ^Z and σ^Z are calibrated by borrowing values from Basu and Bundick (2018). In spite of our focus on the effects of the uncertainty shocks, we calibrate also these parameters because these stochastic processes can in principle influence (even on-impact) the response of the model-consistent VXO to an uncertainty shock. We also do not estimate the second-order utilization parameter δ_2, the elasticity of substitution between intermediate goods θ_μ, and the IES ψ to not further increase the computational burden of the estimation procedure.

H: Role of initial conditions in the nonlinear DSGE model

This Section investigates whether the initial conditions in the nonlinear DSGE model we employ play a role for the dynamics of the system after an uncertainty shock. Andreasen, Fernández-Villaverde, and Rubio-Ramírez (2018) show that the initial values of the states are potentially very important for the effects of the macroeconomic shocks they study. The computation of the GIRFs in our paper follows Basu and Bundick (2017) and do not take into account the role of initial conditions. Hence, this possible omitted factor could be behind the evidence of countercyclical risk aversion we find.[22] It is therefore important to provide a check on the relevance of initial conditions in the model we work with.

Cacciatore and Ravenna (2020) prove that pruning the third-order approximation completely eliminates state dependence in the propagation of uncertainty shocks. Hence, to check the relevance of initial conditions we switch to the unpruned third-order approximation of our model. In particular, a Monte Carlo exercise with artificial data simulated with the Basu and Bundick (2017) framework is conducted. The exercise is

[22] As explained in the main text, we compute responses in the model starting from the regime-specific stochastic steady state implied by the estimated set of parameters. As in Basu and Bundick (2017,2018), we adopt the pruned third-order approximation proposed in Andreasen, Fernández-Villaverde, and Rubio-Ramírez (2017).

conducted similarly to Section D of this Appendix, but with two differences. First, here the unpruned approximated solution is used to simulate the model. Second, on top of the linear VAR, also an IVAR model similar to the one adopted in the baseline analysis is estimated on the simulated data.

Figure A11 compares the linear VAR response and the IVAR response for a (model-consistent) very deep contraction.[23] The identified set of the linear VAR and IVAR responses lie literally on top of each other. Hence results show that the initial conditions in the DSGE model do not materially influence the computed GIRFs to an uncertainty shock, i.e., no endogenous state-dependence is generated in the DSGE model with the use of the standard workhorse solution methods.[24]

[23] We compute the IVAR response for the initial condition corresponding to the deepest contraction in the simulated sample.

[24] We were prevented to conduct a similar exercise using a forth order approximation due to large approximation errors that caused severely distorted GIRFs. In a companion paper, Andreasen, Caggiano, Castelnuovo, and Pellegrino (2020), we use an approximation around the risky steady state, rather than around the deterministic steady state, so that to both allow initial conditions to play a role for the propagation of uncertainty shocks and accurately solve nonlinear DSGE models.

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	Output	Consumpt.	Investment	Hours
Baseline IVAR and VAR				
Peak response: Linear	-1.54%	-1.03%	-3.34%	-1.90%
Peak response: Great Recession	-2.32%	-1.37%	-8.39%	-3.77%
Ratio GR/Linear	1.51	1.33	2.51	1.98
IVAR and VAR with financial spread				
Peak response: Linear	-1.20%	-0.72%	-3.03%	-1.42%
Peak response: Great Recession	-1.95%	-1.12%	-8.42%	-2.81%
Ratio GR/Linear	1.63	1.55	2.78	1.98

Table A1: **Peak responses.** Peak responses to a one standard deviation uncertainty shock estimated with linear VAR and nonlinear IVAR for the great recession

Par.	Description	Value
σ_{σ^a}	volatility of the uncertainty shock	0.004
ρ^a	persistence of the preference shock	0.98
σ^a	volatility of the preference shock	0.005
ρ^Z	persistence of the technology shock	0.35
σ^Z	volatility of the technology shock	0.019
α	capital's share in production	0.333
β	household discount factor	0.994
δ	steady state depreciation rate	0.025
δ_1	first-order utilization parameter	$1/\beta - 1 + \delta$
Π	steady state inflation rate	1.005
ν	firm leverage parameter	0.9
δ_2	second-order utilization parameter	0.0003
θ_μ	elasticity of subst. between intermediate goods	6.0
ψ	intertemporal elasticity of substitution	0.5

Table A2: **DSGE model: Calibrated parameters. Calibration borrowed from Basu and Bundick (2018).**

		Priors	Posteriors	
			Linear VAR	Great Recession
Parameter	Interpretation	D(mean, std)	Mode, std	Mode, std
ρ_{σ^a}	Unc.shock,pers.	B(0.77, 0.10)	**0.64,** 0.03	**0.65,** 0.03
b	Habit formation parameter	B(0.75, 0.15)	**0.64,** 0.06	**0.66,** 0.04
ϕ_K	Investment adjustment costs	G(3.92, 2)	**2.29,** 0.50	**3.21,** 0.60
ϕ_P	Price adjustment costs	G(240, 40)	**236.78,** 32.26	**282.10,** 33.54
ρ_π	Taylor rule parameter, inflation	IG(1.5, 0.25)	**1.05,** 0.01	**1.05,** 0.01
ρ_y	Taylor rule parameter, output growth	G(0.2, 0.15)	**0.20,** 0.04	**0.28,** 0.05
σ	Risk aversion (fixed labor supply, no habits)	G(100, 60)	**385.90,** 50.45	**533.04,** 59.16
RRA	Risk aversion (endogenous labor supply, habits)		**104.85**	**144.96**

Table A3: **DSGE model: Average evidence vs. Great recession.** Values estimated conditional on both the linear VAR impulse responses and on the IVAR impulse responses in 2008Q4. Standard deviations estimated conditional on a Laplace approximation of the posterior density. Risk aversion in the model (RRA) computed by considering endogenous labor supply and habits as in Swanson (2018).

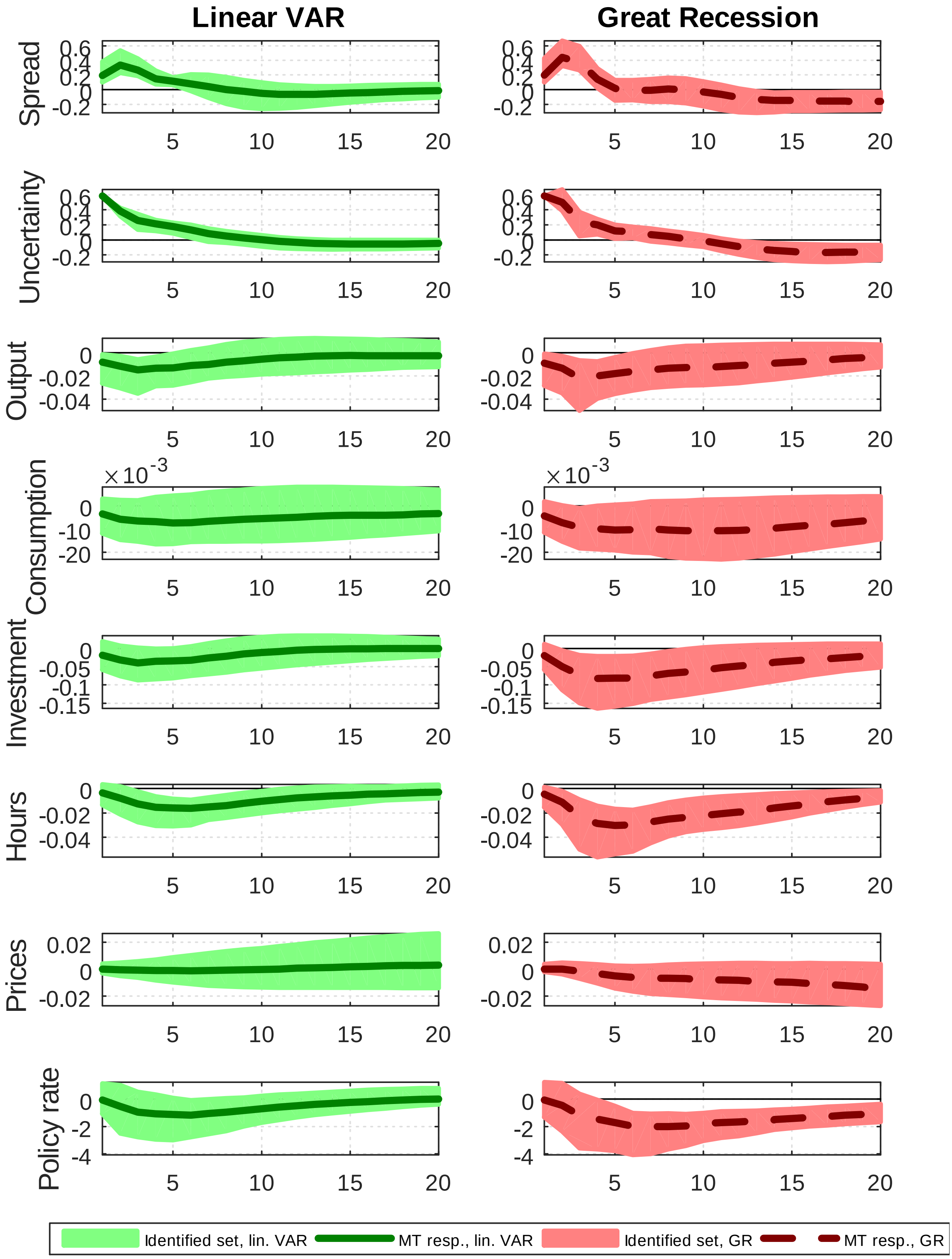

Figure A1: **IVAR impulse responses: identified sets (linear vs. great recession).** Impulse responses to a 4.4-standard deviation uncertainty shock. The solid green (red) areas report the identified set of responses produced with the linear (nonlinear) VAR model. The solid (dashed) lines report the median target impulse response for the linear (nonlinear) VAR. The number of retained draws, out a total of one million draws, is 2,168 for the IVAR and 2,116 for the linear VAR.

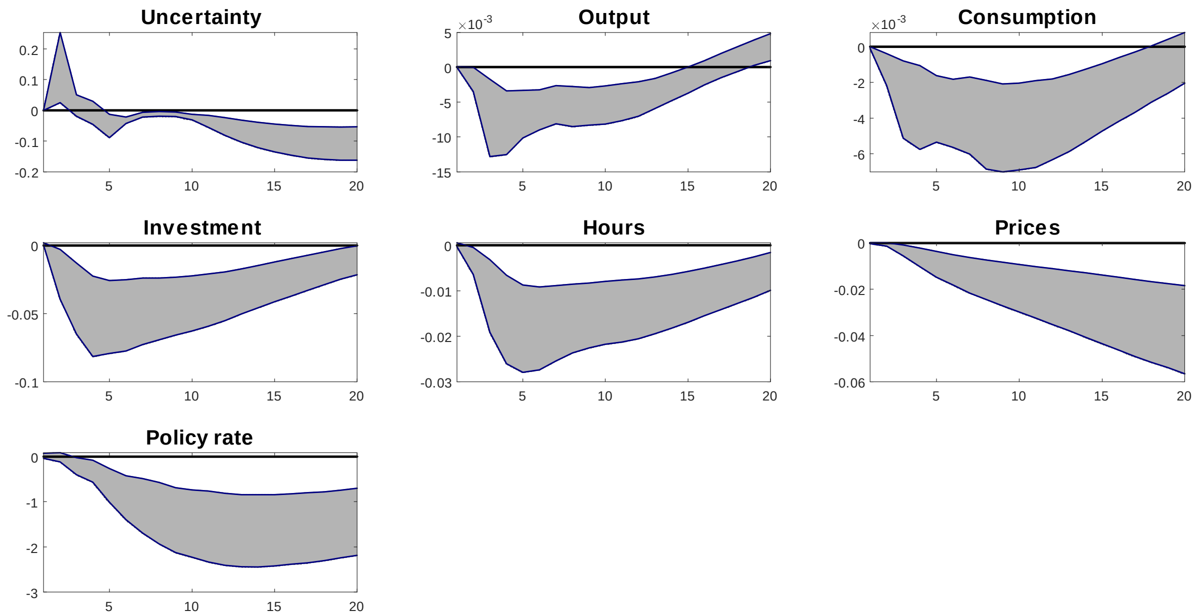

Figure A2: **Test for the difference of responses that focuses on model uncertainty.** The test takes into account the correlation between the responses. 99% confidence bands in gray.

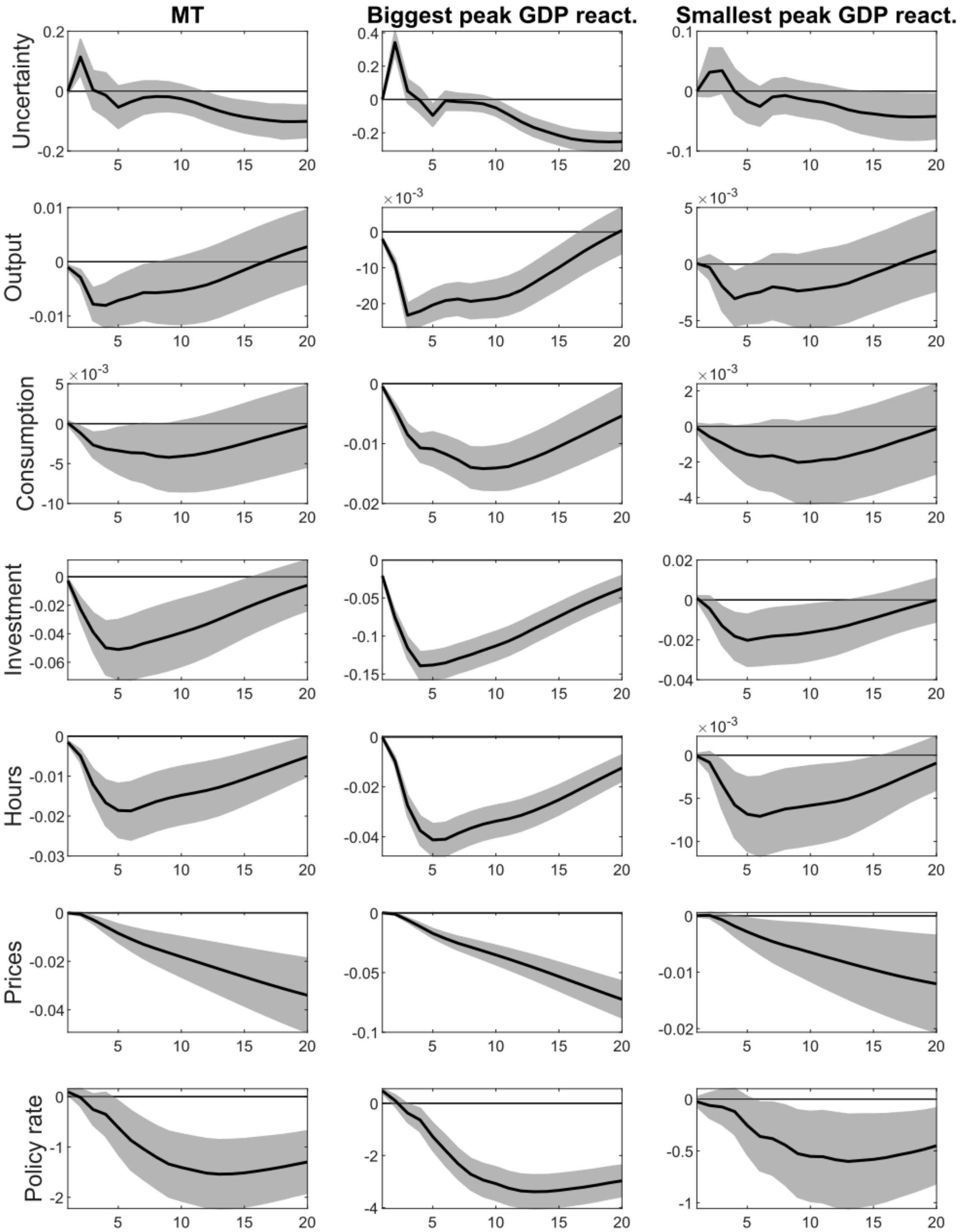

Figure A3: **Bootstrapped test for the difference of median target responses and of biggest and smallest GDP peak responses.** Grey bands: 68% confidence bands.

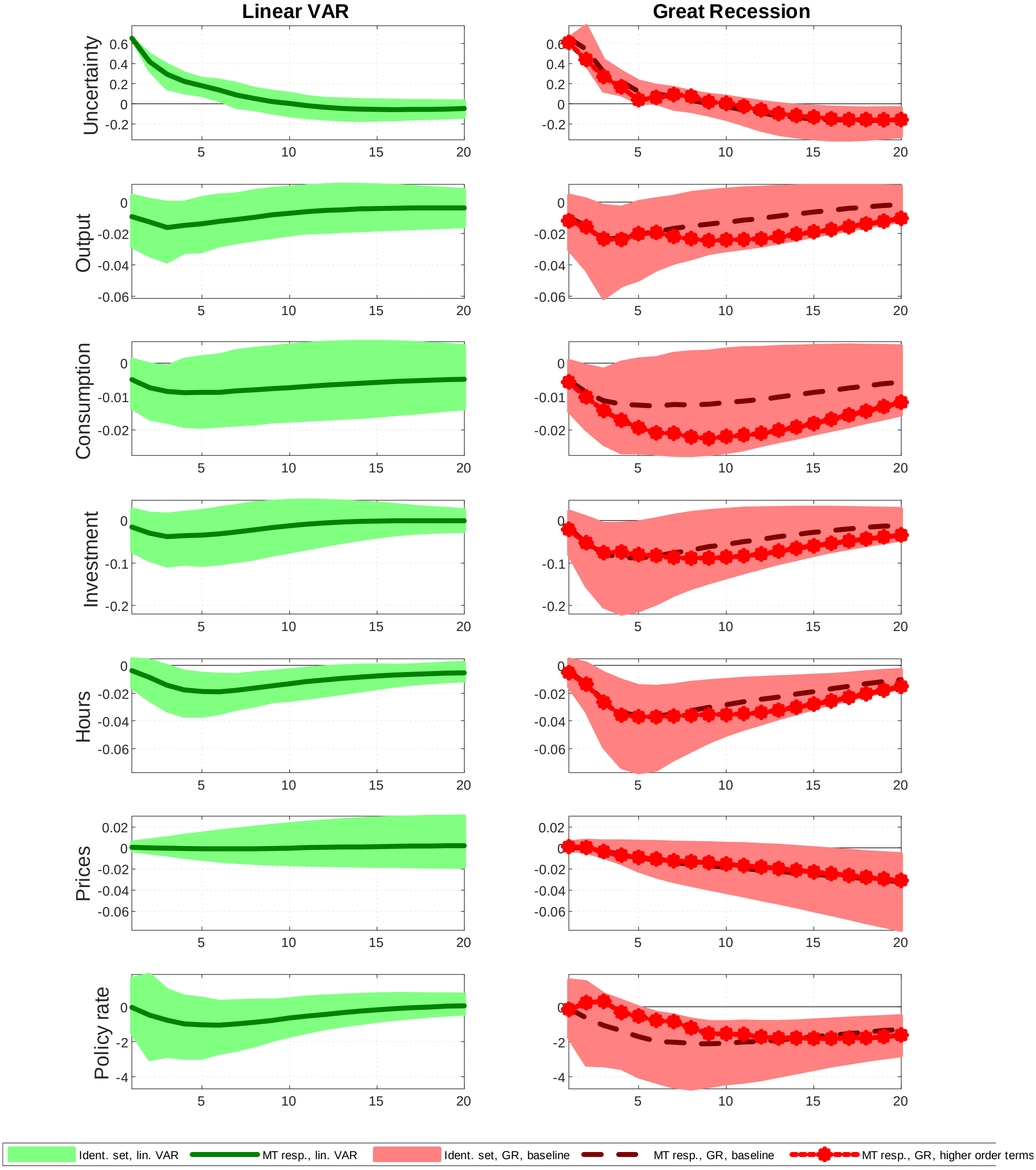

Figure A4: **IVAR impulse responses: Role of higher order terms.** Areas in the first and second columns: Identified set for impulse responses produced with the baseline, parsimonious IVAR. Solid green and dashed dark red lines in the first and second columns: Impulse responses produced with the baseline, parsimonious IVAR. Lines with red stars (second columns): Impulse responses produced with the expanded IVAR featuring extra-interaction terms.

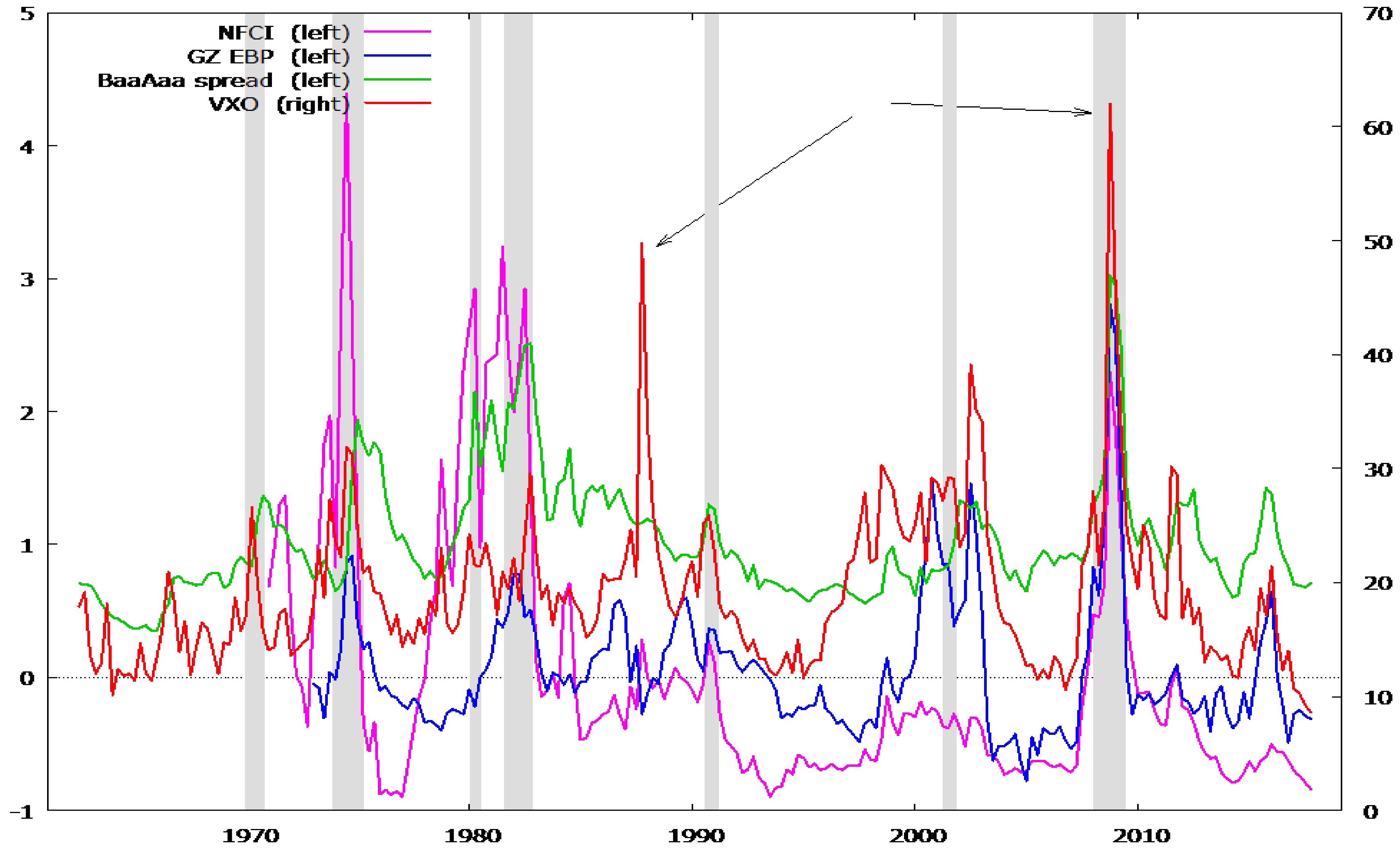

Figure A5: **VXO against common financial stress indicators.** Arrows indicate the 1987Q4 and 2008Q4 spikes in the VXO. Indicators of financial stress (related to first-moment financial shocks): NFCI = National Financial Conditions Index produced by the Federal Reserve Bank of Chicago; GZ EBP = Excess Bond Premium by Gilchrist and Zakrajï£¡ek (2012); BaaAaa spread = spread computed by subtracting the AAA yield from the BAA one.

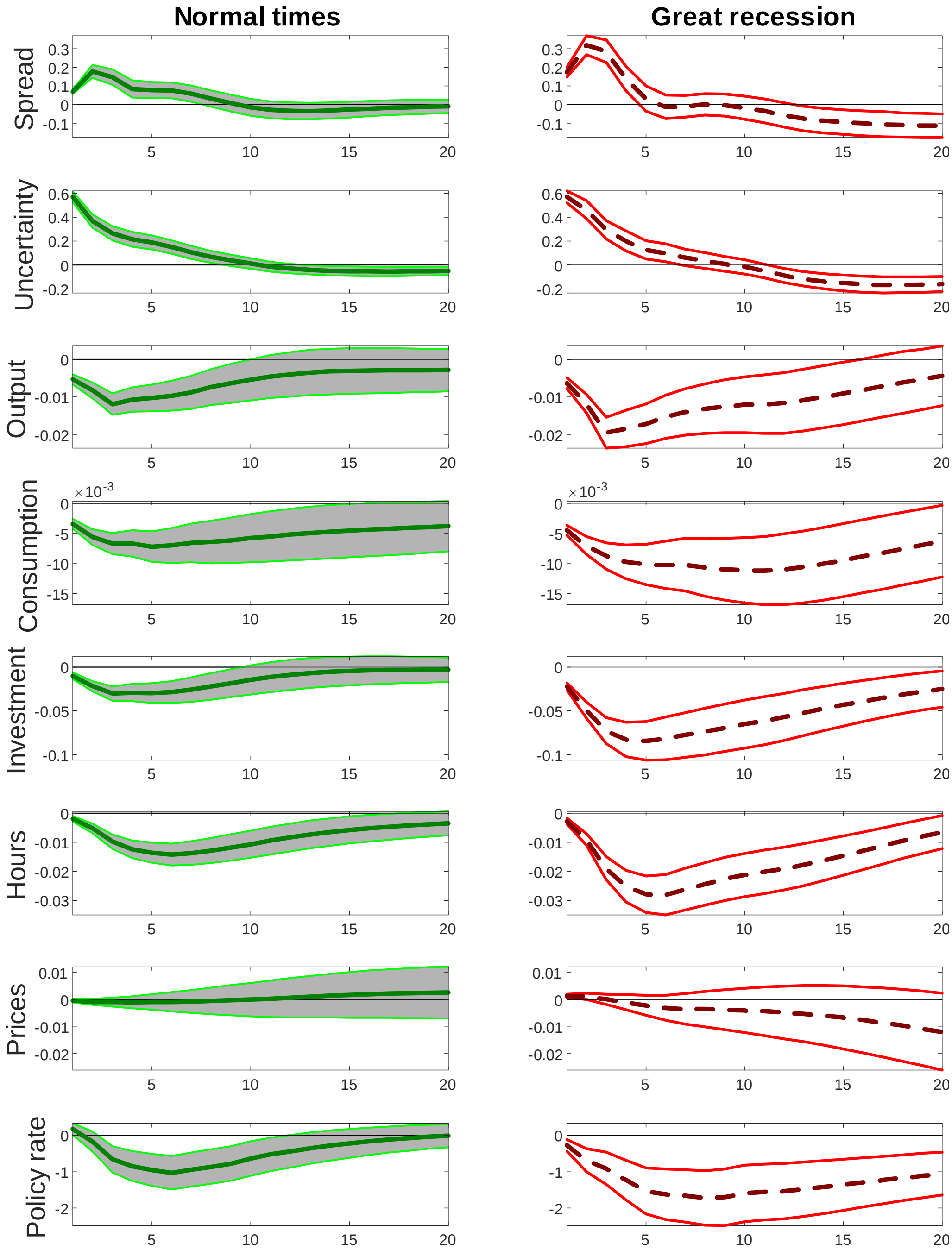

Figure A6: **Credit spread. Impulse responses: linear vs. great recession.** Impulse responses to a 4.4-standard deviation uncertainty shock. Solid green (dashed red) lines: Point estimates of the response of our VAR variables in normal times (during the great recession) conditional on the median target model à la Fry and Pagan (2011). Gray area (area identified by solid red lines): 68% bootstrapped confidence interval surrounding the median-target model.

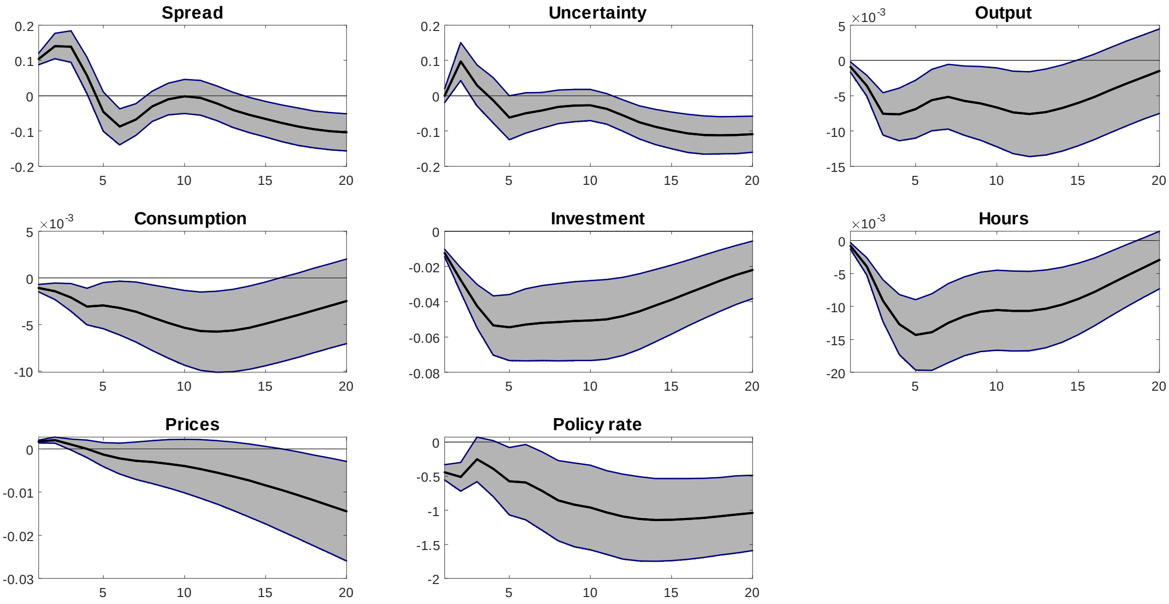

Figure A7: **Credit spread. Bootstrapped test for the difference of median target responses.** 68% confidence bands in gray.

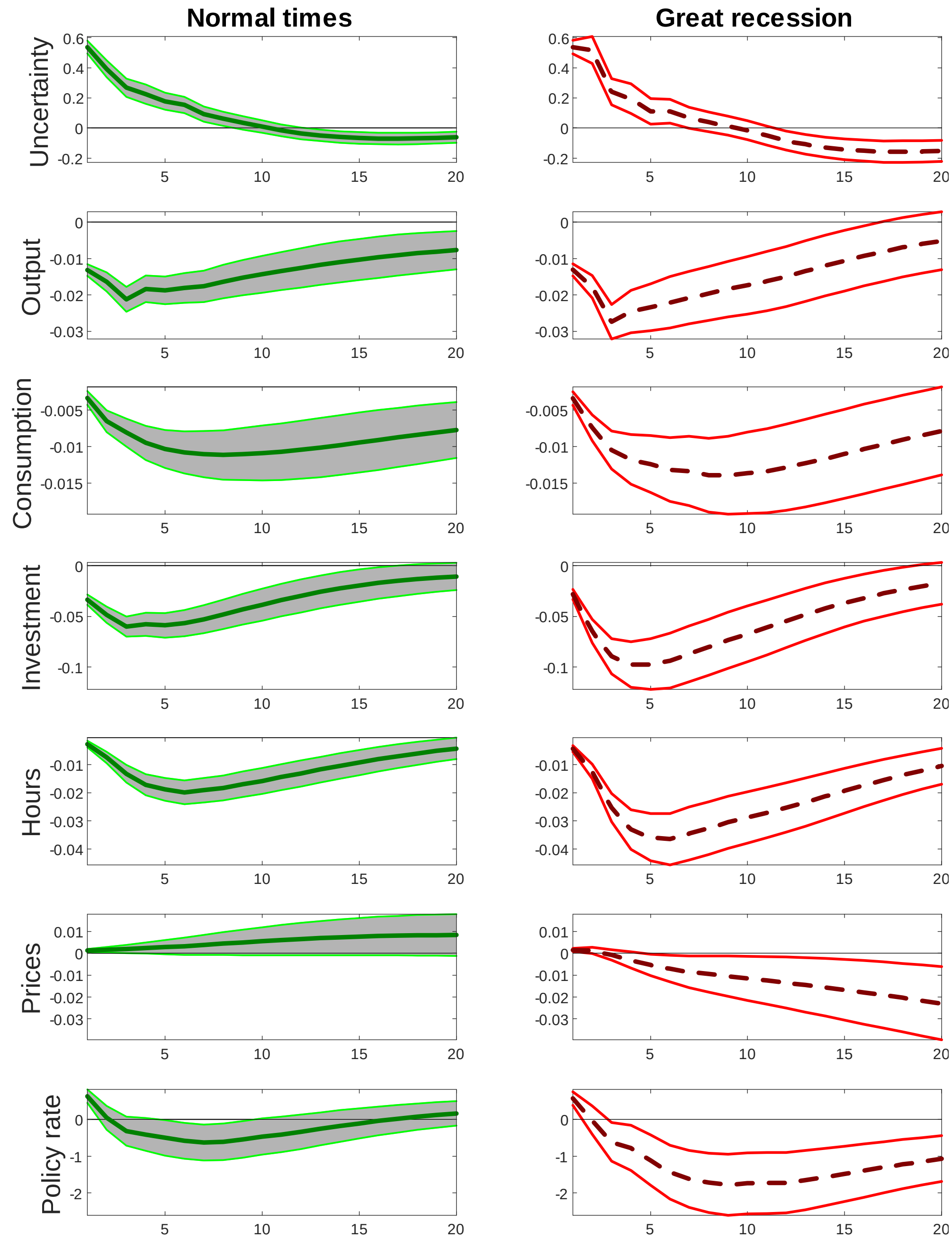

Figure A8: **No oil supply shocks dates. Impulse responses: linear vs. great recession.** Impulse responses to a 4.4-standard deviation uncertainty shock. Solid green (dashed red) lines: Point estimates of the response of our VAR variables in normal times (during the great recession) conditional on the median target model à la Fry and Pagan (2011). Gray area (area identified by solid red lines): 68% bootstrapped confidence interval surrounding the median-target model.

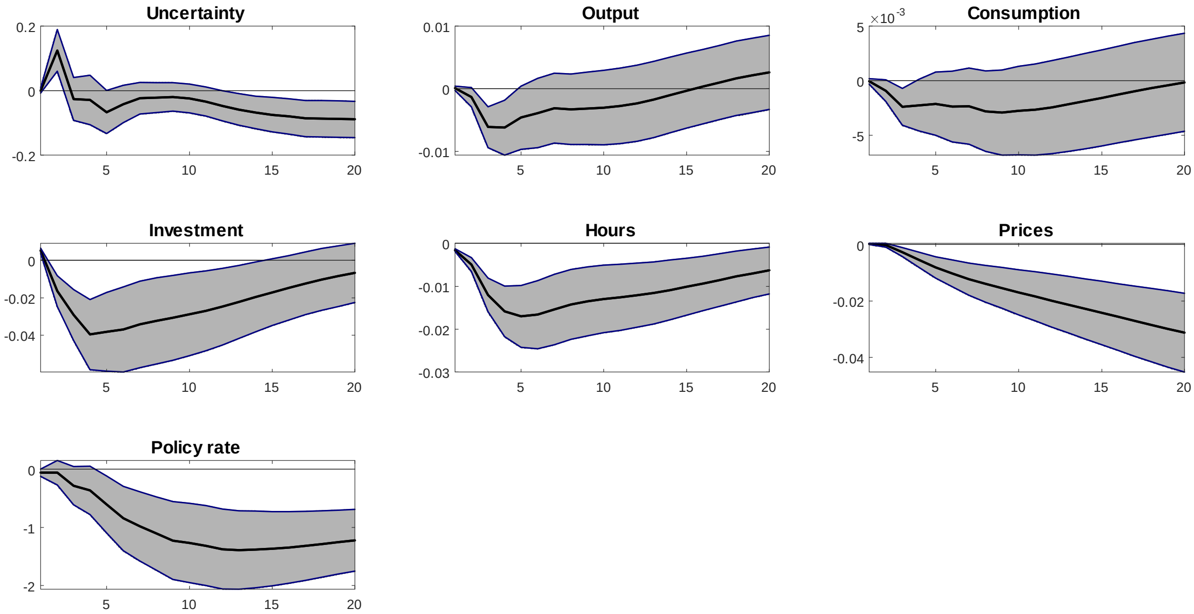

Figure A9: **No oil supply shocks dates. Bootstrapped test for the difference of median target responses.** 68% confidence bands in gray.

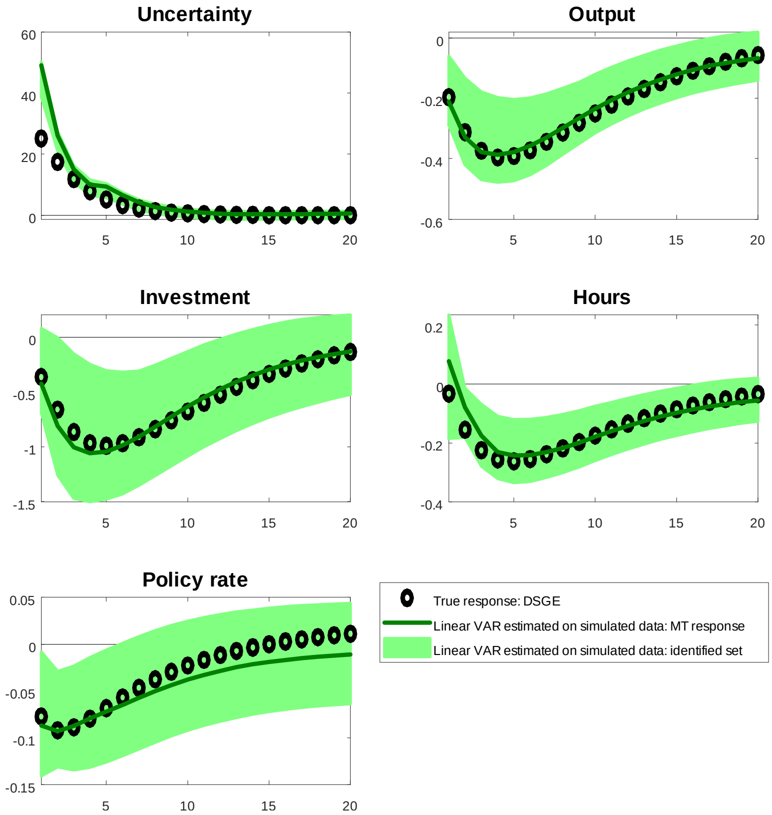

Figure A10: **Monte Carlo simulation: DSGE model vs. VAR responses to an uncertainty shock**. Calibration of the DSGE model with the estimates we obtained with the facts established by the linear VAR. Size of the simulated sample: 2,500 observations (100 of which are used as burnin). Consistently with our baseline analysis, uncertainty shocks are identified by exploiting the dates corresponding to the biggest spikes of the HP-filtered model-implied VXO, as explained in the text.

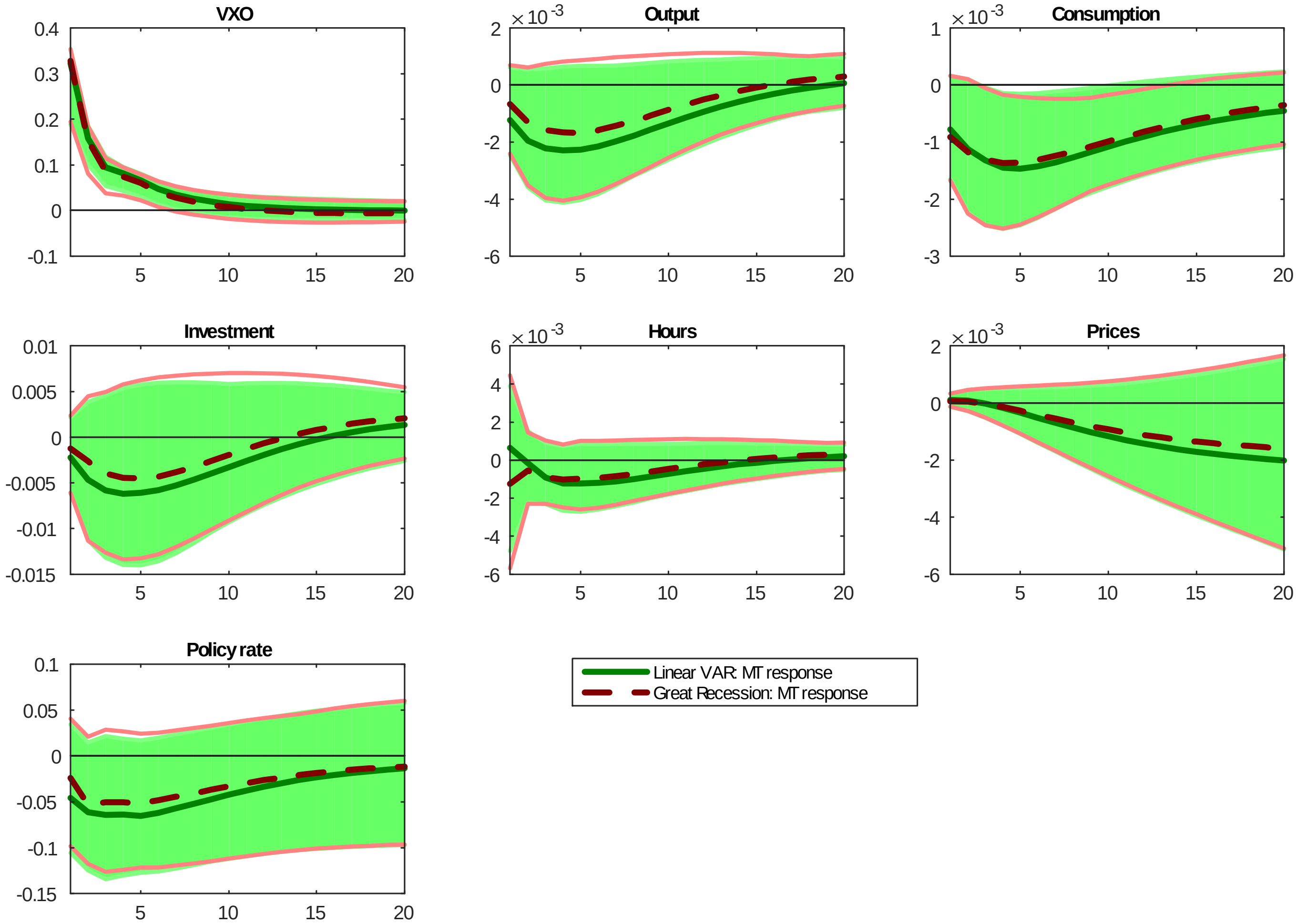

Figure A11: **Monte Carlo simulation: DSGE model vs. IVAR responses to an uncertainty shock for a model-implied very deep contraction**. Calibration of the DSGE model with the estimates we obtained with the facts established by the linear VAR. Size of the simulated sample: 2,500 observations (100 of which are used as burnin). Uncertainty shocks are identified by exploiting the information coming from the biggest spikes of the HP-filtered model-implied VXO, as explained in the text. Green areas and white area delimited by solid red lines: identified set for the linear VAR and IVAR response, respectively.