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THE OPTIMALITY OF PUBLIC-PRIVATE PARTNERSHIPS UNDER FINANCIAL AND FISCAL CONSTRAINTS

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The Optimality of Public-Private Partnerships
under Financial and Fiscal Constraints∗

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Abstract

The government may delegate two sequential tasks (e.g., building and operating an infrastructure) to the same or different agents (i.e., partnership versus sequential contracts). Agents are risk-neutral but face financial constraints, whereas the government’s contractual capacity may be limited by the renegotiation-proof and fiscal constraints. By relying on history-dependent incentives, the partnership contract corrects moral hazard more effectively than sequential contracts. Thus, it is socially preferred unless bundling different tasks deteriorates the agent’s financial conditions. Our results shed new light on the role of firms’ financial and government’s fiscal conditions in driving the cost-benefit analysis of public-private partnerships.

Keywords: Sequential moral hazard, Bundling, Limited liability, Budget constraint, Memory contracts

JEL classification: D86, H11, H57, L33

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1
1 Introduction

The involvement of private companies in the design, construction, and operation of public infrastructures and services is a well-established practice (Bezançon, 2005). However, in the last thirty years, two distinctive features have characterized the evolution of new forms of public-private partnerships (PPPs) in contrast to traditional concession contracts: a greater emphasis on the “value for money” for taxpayers and a growing institutional and financial complexity.¹

Although potential efficiency gains of PPPs that may derive from enhanced management of tasks and risks have been extensively analyzed by the economic literature (e.g., Iossa and Martimort, 2015), fiscal and financial determinants of PPP investments are much less clear. Empirical analysis showed a positive correlation between stricter fiscal constraints and the choice to undertake PPPs (Hammami et al., 2006; Albalate et al., 2015) that does not find convincing explanations in the literature. Although normative economics highlights the irrelevance of financial leverage arguments in favor of PPPs (Engel et al., 2013), intuitive political economy interpretations of the link between fiscal stress and PPP investments—for example, debt-hiding and non-compliance to fiscal rules (Von Hagen and Wolff, 2006; Buti et al., 2007; Maskin and Tirole, 2008)—have not been corroborated by compelling empirical analysis.²

More recently, cursory evidence suggests the existence of transmission channels between the fluctuations of PPP investments and the volatility of financial markets, and a more complex relationship between the former and the fiscal constraints. For example, the share of PPPs of total public investments of the EU countries suddenly dropped after the 2008 financial crisis and remained low afterward, whereas the debt-to-GDP ratio soared in the

¹An important characteristic of new forms of PPPs is the assignment of different tasks of the public project to a single special purpose vehicle established by firms that act as subcontractors of the consortium itself. Such bundling agreements are implemented through different contractual arrangements, taking into account country-specific legislation (Engel et al., 2014).

²In the case of France, Buso et al. (2017) confirm the correlation between adverse conditions of local public finance and the decision of municipalities to start PPPs. However, relying on a quasi-experimental setting, they rule out any debt-hiding motive as an explanation of such behavior.
same period (see Figure 1). The financial crisis has affected PPPs by, for instance, cutting available credit. Before the crisis, the ratings of PPP project bonds were enhanced by high-rating monoline insurance companies that acted as guarantors against project risks. After the 2008 crisis, most insurers were downgraded, thus reducing the liquidity of the bond market for infrastructure projects (Burger et al., 2009; EPEC, 2009).

Figure 1: PPPs and government debt of the EU countries from 1990 to 2018

Source: Our elaboration on the data set by EPEC PPP Market Updates (http://www.eib.org/epec/), Eurostat and OECD.
Legend: Only contracts above ten million euros closed each year in one of the 27 countries of the European Union and the UK are considered. General government debt and investment expenditures are calculated based on the Eurostat and OECD data sets.

Few articles have taken a theoretical approach to investigating the role of finance in PPPs, and they focused on the monitoring technology of financial intermediaries (Iossa and Martimort, 2012, 2015). In this article, we analyze the impact of financial and fiscal constraints on incentives, relying on a standard representation of a public project as a sequential moral hazard problem, where an infrastructure is first build, and then it is operated (Engel et al., 2014). To this aim, we consider a risk-neutral principal (or government) that faces a potentially binding budget (or fiscal) constraint and delegates the implementation of two
sequential tasks (i.e., building and operation) to risk-neutral agents who face limited liability (or financial) constraints. Each task has a contractible output (e.g., infrastructure quality and operational costs) that is affected by the agent’s task-specific effort and by an exogenous shock. As usual in this literature, the government can use alternative contractual schemes: under unbundled or sequential contracts, two agents (i.e., the builder and the operator) are hired to independently implement tasks; under bundled or partnership contract, a single agent (i.e., a consortium of the building and operating firms) is hired to implement the two tasks. To better understand the role of the financial and fiscal constraints in the optimal design of contracts, we abstract from any production externality between the building and operating tasks.³

We obtain three main findings. First, abstracting from any difference in financial constraints across private firms, the government can design more effective incentive schemes under partnership than sequential contracting. This moral-hazard correction component of the welfare comparison is driven by a kind of financial externality that endogenously arises between the building and operating tasks because of the history-dependent nature of the partnership contract, which is absent in the sequential contracts.

A second result is that the welfare comparison of outcomes under the sequential and partnership contracts is also driven by the limited liability differential—that is, by the heterogeneity of financial wealth—among firms. In other terms, if the aggregate financial “pockets” of the builder and operator under the sequential contracts are “deeper” than those of the consortium of firms under the partnership contract, the government may be able to design better incentive schemes in the former case than in the latter.⁴ Of course, if such financial effect is not strong enough, moral-hazard correction prevails, and the partnership contract dominates the sequential contracts in terms of social welfare.

³We also abstract from any agency problem within the private consortium that may actually affect the structure of the optimal contract (Hoppe et al., 2013; Greco, 2015). Moreover, we consider complete contracts.

⁴This is a straightforward implication of the well-known result that, if the limited liability constraint is relaxed, the principal can reach the first-best allocation by punishing the agent in the case of bad outcomes.
In our setting, the limited liability differential is exogenous and can take both positive or negative values, depending on financial market conditions. We can interpret such conditions in the light of relevant findings from the corporate finance literature. On one side, when firms bundle within a consortium (e.g., establish a special purpose vehicle), a **coinsurance** (or **trading adjuvant**) effect may improve the rating of financial assets that the consortium issues, thus expanding the consortium’s financial wealth (i.e., loosening the limited liability constraint) in contrast to the aggregate financial wealth of individual firms (Whinston, 1990; Banal-Estanol et al., 2013; Farhi and Tirole, 2015). On the other side, the financial assets issued by the consortium may be less liquid because of a **risk-contagion** (or **insulation**) effect, which tightens the consortium’s limited liability constraint in contrast to the aggregate of individual firms (Gorton and Pennacchi, 1990; De Marzo and Duffie, 1999; Banal-Estanol et al., 2013; Farhi and Tirole, 2015). The risk-contagion effect may prevail in the presence of **high uncertainty on financial markets** (e.g., when returns are low on average, very volatile, negatively skewed, or positively correlated) or costly bankruptcy procedures and weaker creditor rights (Banal-Estanol et al., 2013).

A third result of our article is that the described findings are also retrieved when we consider that the contracting capacity of the government may be limited by the renegotiation-proof and fiscal constraints. Also, in this framework, the partnership contract is more effective at correcting moral hazard, thanks to its history-dependent nature. Moreover, we uncover that fiscal and financial constraints are intertwined. Particularly, if bundling does not involve a stricter limited liability constraint (i.e., the coinsurance prevails over the risk-contagion effect), then the fiscal constraint does not affect the capacity of the partnership contract to create more welfare than the sequential contracts. However, if the risk-contagion effect is sufficiently strong such that bundling shrinks the agent’s financial wealth, a stricter fiscal constraint may change the welfare ranking between the partnership and sequential contracts.

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5In the case of PPPs, an additional benefit may arise from the involvement of outside financiers in evaluating risks, thus reducing asymmetric information and further relaxing the limited liability constraint (Iossa and Martimort, 2015).
contracts. In other words, when agents face heterogeneous limited liability constraints, and financial uncertainty is high, the fiscal constraint may affect the cost-benefit comparison between PPPs and traditional procurement.

Our analysis of the financial and fiscal drivers of the welfare comparison between the partnership and sequential contracts (i.e., the balance between the moral hazard correction and the limited liability differential components) provides a new explanation of the apparent negative impact of high financial market volatility on PPP investments (Figure 1). Such a theoretical prediction can be exploited to construct robust empirical tests of the impact of financial and fiscal constraints on PPP investments and shed new light on this hotly debated issue.

The article is structured as follows. Section 2 discusses the links of our work with different strands of the literature on PPPs and contract theory. Section 3 presents the model setup. Section 4 analyzes the sequential and partnership contracts in a baseline setting where the government’s contracting capacity is limited only by participation, incentive and limited liability constraints of the agents. Then, Section 5 extends the analysis to consider that the government cannot commit not to renegotiate contracts and faces a fiscal constraint. Finally, Section 6 concludes.

2 Related literature

In this article we develop a model that analyzes the bundling of tasks in a context of asymmetric information and financial constraints. The optimality of bundling tasks in PPPs was studied for the first time by Hart (2003) in a context of incomplete contracting. According to this seminal study, PPPs may provide incentives for both desirable investments that improve service quality and undesirable investments that reduce costs at the expense of service quality. Starting from this analysis, the pros and cons of bundled contracts in the presence

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6Although we analyze the impact of private and public financial constraints on the decision to adopt PPPs, it is beyond the scope of this article to endogenize the financial structure of PPPs (Fay et al., 2021) or analyze the role of financial intermediaries (Iossa and Martimort, 2015).
of related tasks have been investigated through models that either include agency issues or consider the financial aspect of PPPs.

Agency problems are of two forms: adverse selection and moral hazard. Adverse selection models applied to PPPs analyze situations where, in the first stage, the private player has or can gather an informational advantage over the principal about future costs (Hoppe and Schmitz, 2013; Buso, 2019). However, a possible problem of moral hazard may arise if the private player can exert effort during the building stage that is not verifiable by the government and has a direct effect on the costs incurred during the operating stage (Martimort and Pouyet, 2008; Iossa and Martimort, 2015). Alternatively, some recent contributions to the PPP literature consider two-stage repeated moral hazard models where risk-neutral firms are protected by limited liability. Martimort and Straub (2016) develop a two-stage moral hazard model where the second-stage reward cannot depend on the first-stage outcome, and the effort level must satisfy an irreversibility constraint such that it cannot be smaller in the second stage than in the first stage. Close to our setting, Hoppe and Schmitz (2021) do not consider any irreversibility constraint and allow for history-dependent (or memory) contracts. Our contribution is characterized by important differences with respect to Hoppe and Schmitz (2021). First, we do not consider any production externality between the two stages. Second, we assume that the principal faces a budget constraint. Third, we allow the exogenous wealth featuring the limited liability constraints to differ between bundled and unbundled contracts. The latter extension helps us to highlight the crucial role of financial constraints as a driver of the choice between PPPs and traditional procurement.

As for the fiscal aspect of PPPs, Engel et al. (2013) develop a model where the private firm is risk averse and receives a combination of state-dependent user fees and subsidies as a compensation for its efforts. In a framework characterized by demand uncertainty, the authors show that the presence of a budget constraint is not a sufficient reason to opt for

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7Ohlendorf and Schmitz (2012), among others, analyze repeated moral hazard models where agents are risk neutral and protected by limited liability. However, they do not focus on the differences between bundling and unbundling.
PPPs. The intuition is that by adopting an intertemporal perspective a PPP allows the
government to postpone the disbursement of payments but does not release public funds.
In a context of multiple tasks and moral hazard, Schmitz (2013) analyzes the optimality
of bundling tasks in PPPs when the government is budget constrained and private firms
are protected by limited liability. Differently from Schmitz (2013), in our setting, tasks are
sequential and asymmetric—that is, one of them comes before the other, and they affect
the principal’s objective function in different ways. This difference which explains why, in
our extended setting, we have history-dependent contracts, and we find that, when limited
liability constraints are not looser under the sequential contracts than under the partnership
contract, PPPs are preferred to traditional procurement from the social welfare point of view
even if the principal faces a binding budget constraint.

![Figure 2: Sequential structure of the game](image)

3 The model

A public infrastructure must be built and operated. The gross social surplus generated by
the public infrastructure is $Sq$, where $q$ is the level of infrastructure quality, and $S > 0$ is
its social marginal benefit. The infrastructure quality is determined in the first phase of the
public infrastructure cycle (see Figure 2), as a random outcome of the builder’s productive
effort $e_b \in [0, 1]$. We assume that quality is high $q^h$, with probability $e_b$, and low $q^l$, with
probability $1 - e_b$. Investing in quality entails a monetary cost $kq$ (with $k < S$) and a non-monetary (or management) cost for the builder $\phi(e_b)$, where $\phi(0) = 0$ and $\phi(1) > (S - k)q^h$; moreover, $\phi'(e_b) \geq 0$, $\phi''(e_b) > 0$, and $e_b \frac{\phi'''(e_b)}{\phi''(e_b)} > -2$ for all $e_b$.

The operational costs $c$ are determined during the second, service-provision phase of the public infrastructure cycle (see Figure 2) as a random variable of the operator’s effort to cut costs $e_o \in [0, 1]$. Operation costs are low $c^l$, with probability $e_o$, and high $c^h$, with probability $1 - e_o$. The non-monetary cost of the operator is $\psi(e_o)$, where $\psi(0) = 0$ and $\psi(1) > c^h - c^l$; moreover, $\psi'(e_o) \geq 0$, $\psi''(e_o) > 0$ and $e_o \frac{\psi'''(e_o)}{\psi''(e_o)} > -2$ for all $e_o$.

We assume that the government maximizes the expected net social value of the public infrastructure $W = Sq - T$, where $T$ are the total payments to the private contractors that carry out the building and operating tasks. In designing contracts, the government may face two constraints. The first is the impossibility of committing to contractual clauses, which implies that the contract should satisfy a renegotiation-proof constraint (RPC). The second is a state-independent cap to possible government expenditures on the considered infrastructural project. We model the latter budget constraint (BC) as an upper bound to total payments to the private contractors—that is, $F \geq T$.

The government cannot directly verify the effort of its contractors during the investment and operation phases. However, it can ex post verify the level of infrastructure’s quality $q$ and operational costs $c$. We assume that the public procurement procedures are such that the government has all the bargaining power (e.g., it designs a public tender). In our analysis, we focus on two contractual schemes that the government may choose. Under the sequential contracts (i.e., so-called “traditional procurement” in the literature on PPPs), the contracting game is such that: the government proposes a take-it-or-leave-it contract to the builder, specifying a payment $t_b(q, c)$; then it offers a contract to the operator with a

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8 We abstract from possible production externalities between the building and operation tasks, which are common in the literature on PPPs. These would imply that a component of costs is determined by the quality of infrastructure, as in Hoppe and Schmitz (2021), but would not change our main findings.

9 The conditions on the third derivatives of $\phi(.)$ and $\psi(.)$ are necessary to warrant the concavity of the government’s optimization problem.
payment \( t_o(q, c) \). Under the \textit{partnership contract}, the government chooses to bundle all tasks by contracting with a single consortium of firms acting as builder and operator.\(^{10}\) The total payment to the consortium that is specified by the bundled contract is \( t(q, c) \).

Under the sequential contracts, the state-contingent monetary profit of the building firm is \( \pi_b = t_b(q, c) - kq \), and the state-contingent utility of the building firm’s management, which factors in the managerial effort, is \( u_b = \pi_b - \phi(e_b) \). Meanwhile the state-contingent monetary profit of the operating firm is \( \pi_o = t_o(q, c) - c \), and the state-contingent utility of the operating firm’s management is \( u_o = \pi_o - \phi(e_o) \). Firms have to accept the contract that the government offers (e.g., they have to participate in a public tender); hence, a feasible contract must satisfy the following \textit{participation constraint} (PC): \( E(u_b) \geq 0 \), for the builder, and \( E(u_o) \geq 0 \), for the operator, where we normalize to zero the reservation utility of firms’ management. Moreover, we assume that each firm faces a state-independent \textit{limited-liability constraint} (LLC) such that the ex post monetary profit cannot drop below the firm’s financial wealth. Particularly, \( \pi_b \geq -l_b \), and \( \pi_o \geq -l_o \), where \( l_b \) and \( l_o \) are the financial wealth of the builder and the operator, respectively.

Under the partnership contract, the state-contingent monetary profit of the consortium carrying out both the building and operating tasks is \( \pi_p = t(q, c) - kq - c \), and the state-contingent utility of the consortium’s management is \( u_p = \pi_p - \phi(e_b) - \phi(e_o) \). Also, the consortium faces a PC \( E(u_p) \geq 0 \), and a LLC \( \pi_p \geq -l_c \), where \( l_c \) is the consortium’s financial wealth.

Finally, we assume that the fiscal constraint of the government (BC) and the financial constraints of the firms (LLCs) are such that the first-best investment and operational costs can be financed in all possible states of the world and, particularly, in the state of the world \( \{q^h, c^h\} \), which involves the maximum level of investment and operational costs under both the sequential and partnership contracts—that is, \( F + \min \{l_b + l_o, l_c\} \geq kq^h + c^h \).\(^{11}\)

\(^{10}\)In our analysis, we abstract from possible agency problems within the consortium of the builder and operator. Such problems may reduce the value that the government can gain from the partnership contract (Greco, 2015).

\(^{11}\)In the real world, we may have public investment projects that are abandoned in very adverse fiscal or
4 Contracting under private financial constraints

To make our analysis more tractable, we proceed in two steps. In this section, we focus on comparing the partnership and sequential contracts that have to satisfy agents’ participation, incentives and financial constraints, whereas the government can fully commit to contracts and does not face any fiscal constraint.

4.1 The first-best solution

As a benchmark, we consider the case where the government can observe the contractors’ efforts $e_b$ and $e_o$. Thus, payments to contractors can be conditioned only on effort and must satisfy the PC, which can be written as follows:\footnote{It is worth noticing that the same first-best optimal solution can be obtained if, instead of a single PC (1), we consider two separate PCs:}

$$t_b - k[e_b q^h + (1 - e_b) q^l] - \phi(e_b) + t_o - e_o c^l - (1 - e_o)c^h - \psi(e_o) \geq 0. \tag{1}$$

The government aims at reducing the payments to contractors. Thus (1) is binding, and the maximization problem of the government is:

$$\max_{e_b, e_o} (S - k) [e_b q^h + (1 - e_b) q^l] - \phi(e_b) - e_o c^l - (1 - e_o)c^h - \psi(e_o).$$

The first-best optimal efforts, $e_b^*$ and $e_o^*$, are such that:

$$\phi'(e_b^*) = (S - k)(q^h - q^l), \tag{2}$$

$$\psi'(e_o^*) = c^h - c^l. \tag{3}$$
The first-best solution can be implemented by the government even if it cannot observe the efforts of the agents, provided that the LLCs of the agents do not bind at the second-best optimum. In this case, the government can extract the full information rent from the firms. Therefore, we have the following result:

**Proposition 1** If LLCs are not binding, the sequential and partnership contracts determine the same first-best levels of effort and social welfare.

### 4.2 Sequential contracts

In this case, the government awards two contracts—one for each phase or task of the public infrastructure cycle—to different firms, the builder and the operator.

#### 4.2.1 Implementable sequential contracts

For the characterization of implementable contracts, we proceed by backward induction. Any contract awarded by the government to the operator must satisfy the PC, ICC and LLC. As shown in Figure 2, at the operation phase, the state of the world is characterized by the realized quality of the infrastructure. Thus, the operator’s PC and ICC may, in general, depend on the realization of $q$ and can be written as follows:

$$\max_{e_o} e_o(t_o(q, c^l) - c^l) + (1 - e_o)(t_o(q, c^h) - c^h) - \psi(e_o) \geq 0. \quad (4)$$

The LLCs can be written as:

$$\pi_o(q, c^l) = t_o(q, c^l) - c^l \geq -l_o;$$

$$\pi_o(q, c^l) = t_o(q, c^h) - c^h \geq -l_o.$$

By the assumptions on the shape of $\psi(.)$, the second-order condition of the problem (4) is negative; hence, the solution is unique. Thus, following the first-order approach, the ICC
can be written as:

\[(t_o(q, c^l) - c^l) - (t_o(q, c^h) - c^h) = \psi'(e_o) \geq 0.\]  

Among the implementable sequential contracts, everything else equal, the government chooses payments involving the least fiscal burden. Thus, by the LLCs and ICC, the state-contingent, implementable payments to the operator can be written as:

1. \[t_o(q, c^h) = t_o(c^h) = c^h - l_o\]  
2. \[t_o(q, c^l) = t_o(c^l) = c^l + \psi'(e_o) - l_o.\]  

Let us remark that the implementable payments, and the operator’s effort that they induce, do not depend on \(q\), but only on \(c^l, c^h\) and on the shape of the non-monetary cost function, \(\psi(.)\).

Anticipating the effort of the operator \(e_o\) (that is induced by the operation contract awarded by the government), any implementable building contract must also satisfy the PC and ICC,

\[
\max_{e_b} e_b(e_o t_b(q^h, c^l) + (1 - e_o) t_b(q^h, c^h) - k q^h) + \\
+ (1 - e_b)(e_o t_b(q^l, c^l) + (1 - e_o) t_b(q^l, c^h) - k q^l) - \phi(e_b) \geq 0,
\]

as well as the LLCs,

\[
\pi_b(q^h, c^l) = t_b(q^h, c^l) - k q^h \geq -l_b, \\
\pi_b(q^h, c^h) = t_b(q^h, c^h) - k q^h \geq -l_b, \\
\pi_b(q^l, c^l) = t_b(q^l, c^l) - k q^l \geq -l_b, \\
\pi_b(q^l, c^h) = t_b(q^l, c^h) - k q^l \geq -l_b.
\]
As in the case of the operation contract, by the assumptions on the shape of $\phi(.)$, the second-order condition of the problem (8) is negative and, following the first-order approach, the ICC can be written as:

$$
[e_o t_b(q^h, c^l) + (1 - e_o) t_b(q^h, c^h) - kq^h) +
-(e_o t_b(q^l, c^l) + (1 - e_o) t_b(q^l, c^h) - kq^l)] = \phi'(e_b) \geq 0,
$$

Again, considering that the government aims to reduce the payments to contractors, by the LLCs and ICC, we can characterize the state-contingent, implementable payments to the builder as follows:

$$
t_b(q^l, c^l) = t_b(q^l, c^h) = t_b(q^l) = kq^l - l_b
$$

$$
e_o t_b(q^h, c^l) + (1 - e_o) t_b(q^h, c^h) = kq^h + \phi'(e_b) - l_b.
$$

It is worth noticing that the implementable payments to the builder are independent of the realized operational costs when the quality of the infrastructure is low, whereas they may also be contingent on the realization of operational costs when the quality of the infrastructure is high.

As we show in the Appendix (Lemma I.1), for sufficiently low $l_b$ and $l_o$, the PCs do not limit the set of implementable sequential contracts. The intuition is that if the financial wealth of the firm (i.e., $l_b$, for the builder, or $l_o$, for the operator) is sufficiently large, the PC is binding, and the expected information rent is equal to zero. In such a case, as argued in Proposition 1, the first-best solutions are implemented. In the following, we assume that this is never the case.
4.2.2 Optimal sequential contracts

The government maximizes the following expected social welfare function:

\[
\max_{e_b, e_o} e_b [S_q h - e_o (t_b(q^h, c^l) + t_o(c^l)) - (1 - e_o)(t_b(q^h, c^h) + t_o(c^h))] + \\
+ (1 - e_b)[S_q l - e_o (t_b(q^l, c^l) + t_o(c^l)) - (1 - e_o)(t_b(q^l, c^h) + t_o(c^h))].
\]  

Substituting the payment schedules that satisfy the ICCs and LLCs of the builder (10)-(11) and the operator (6)-(7) into (12), the government’s maximization problem can be written as:

\[
\max_{e_b, e_o} (S - k)q^l - c^h + l_b + l_o + e_o (c^h - c^l - \psi'(e_o)) + \\
+ e_b [(S - k)(q^h - q^l) - \phi'(e_b)].
\]  

By the problem (13), we obtain the optimization conditions that characterize the second-best optimal efforts under sequential contracts:

\[
\phi'(e_b^*) = (S - k)(q^h - q^l) - e_b^* \phi''(e_b^*); \quad (14)
\]

\[
\psi'(e_o^*) = c^h - c^l - e_o^* \psi''(e_o^*). \quad (15)
\]

By inspection of the first-best and second-best optimization conditions—(2)–(3) and (14)–(15), respectively, we have the following result:

**Proposition 2** Under the sequential contracts, the second-best optimal efforts of the builder and operator are strictly smaller than the first-best ones.

As usual in moral hazard problems, the introduction of (binding) LLCs increases the cost of inducing agents’ efforts, thus introducing a second-best optimal downward distortion of the efforts.
4.3 Partnership contract

In this case, the government awards a single (bundled) contract to a consortium carrying out both the building and operation tasks.

4.3.1 Implementable partnership contracts

The feasible payment functions must satisfy the PC and ICC of the consortium, which can be written as:

\[
\max_{e_b, e_h, e_l} e_b\left[ e_h(t(q^h, c^l) - c^l) + (1 - e_h)(t(q^h, c^h) - c^h) - kq^h - \psi(e_h) \right] + \\
+ (1 - e_h)[e_o(t(q^l, c^l) - c^l) + (1 - e_o)(t(q^l, c^h) - c^h) - kq^l - \psi(e_o)] + \\
- \phi(e_b) \geq 0. 
\] (16)

Similarly, the LLCs must be satisfied:

\[
t(q^h, c^l) - kq^h - c^l \geq -l_c, \quad (17)
\]
\[
t(q^h, c^h) - kq^h - c^h \geq -l_c, \quad (18)
\]
\[
t(q^l, c^l) - kq^l - c^l \geq -l_c, \quad (19)
\]
\[
t(q^l, c^h) - kq^l - c^h \geq -l_c. \quad (20)
\]

Also, in this case, we can rely on the first-order approach.\(^{13}\) Thus, the consortium’s ICC

\(^{13}\)In the considered setting, the first-order approach characterizes the optimal solutions for the consortium, given that the Hessian matrix of the second-order partial derivatives of its objective function (16) is negative definite.
is represented by the following system of optimization conditions:

\[
[e_h^b(t(q^h, c^l) - c^l) + (1 - e_h^b)(t(q^h, c^h) - c^h) - kq^h - \psi(e_h^b)] +
- [e_o^l(t(q^l, c^l) - c^l) + (1 - e_o^l)(t(q^l, c^h) - c^h) - kq^l - \psi(e_o^l)] = \\
= \phi'(e_b) \geq 0;
\]

\[ (t(q^h, c^l) - c^l) - (t(q^h, c^h) - c^h) = \psi'(e_h^b) \geq 0; \quad (21) \]

\[ (t(q^l, c^l) - c^l) - (t(q^l, c^h) - c^h) = \psi'(e_o^l) \geq 0. \quad (22) \]

These conditions imply that the contract is also robust against state-contingent deviations of the consortium after \( q \) is realized. In other terms, the system of equations (21-23) satisfies both the ex ante and ex interim consortium’s ICC.

Similar to what we obtained in the case of sequential contracts, by the characterization of feasible payments to the consortium, we show in the Appendix (Lemma I.2) that, for sufficiently low \( l_c \), the PC does not limit the set of implementable partnership contracts. Again, the intuition is that if the consortium’s financial wealth is sufficiently large, the PC is binding, the expected information rent is equal to zero, and the first-best solution can be implemented (Proposition 1 holds). In the following, we assume that this is never the case.

In the Appendix (Lemma I.3), we show that, among the LLCs, only the condition (20) binds. Thus, considering that the government aims at minimizing the payments to the consortium (other things equal), by the LLCs and ICC, we characterize the state-contingent, implementable payment functions as follows:

\[ t(q^l, c^h) = kq^l + c^h - l_c, \quad (24) \]

\[ t(q^l, c^l) = kq^l + c^l - l_c + \psi'(e_o^l), \quad (25) \]

\[ t(q^h, c^h) = kq^h + c^h - l_c + \tau(e_o^l, e_o^h, e_b), \quad (26) \]

\[ t(q^h, c^l) = kq^h + c^l - l_c + \tau(e_o^l, e_o^h, e_b) + \psi'(e_o^l), \quad (27) \]
where, as shown in the Appendix (see the proof of Lemma I.3),
\[
\tau(e^l_0, e^h_0, e_b) = e^l_0 \psi'(e^l_0) - \psi(e^l_0) - e^h_0 \psi'(e^h_0) + \psi(e^h_0) + \phi'(e_b) \geq 0.
\]

### 4.3.2 Optimal partnership contract

The optimization problem of the government is:

\[
\max_{e_b, e^h_0, e^l_0} \left[ Sq^h - c^h t(q^h, c^l) - (1 - e^h_0) t(q^h, e^h_0) \right] + (28)
\]

\[
+ (1 - e_b) \left[ Sq^l - c^l t(q^l, c^l) - (1 - e^l_0) t(q^l, e^l_0) \right].
\]

Again, substituting the payment schedules that satisfy the ICC and LLCs of the consortium (24)-(25) into (28), the government’s maximization problem can be written as:

\[
\max_{e_b, e^h_0, e^l_0} (S - k)q^l - c^l + I_c + e^l_0 (c^h - c^l - \psi'(e^l_0)) + (e^h_0 - e^l_0) (c^h - c^l) + \psi(e^l_0) - \psi(e^h_0) \]

Under the partnership contract, the second-best optimal efforts in the building phase \(e^b_b\), in the operation phase when the quality of infrastructure is high \(e^{hp}_o\), and when it is low \(e^{lp}_o\), are determined by the following optimization conditions:

\[
\phi'(e^b_b) = (S - k)(q^h - q^l) + (e^{hp}_o - e^{lp}_o)(c^h - c^l) + \psi(e^{lp}_o) - \psi(e^{hp}_o) - e^b_b \phi''(e^b_b) \quad (30)
\]

\[
\psi'(e^{hp}_o) = c^h - c^l \quad (31)
\]

\[
\psi'(e^{lp}_o) = c^h - c^l - \frac{e^{lp}_o}{1 - e^b_b} \psi''(e^{lp}_o) \quad (32)
\]

We observe that, at the optimum, the government actually exploits the possibility of writing partnership contracts with memory, given that the operation effort is different depending on the realized quality of the infrastructure.

By the optimization conditions (30)-(32), we obtain two results that help us to delve into
the analysis of the optimal partnership contract. The first result compares the second-best optimal efforts induced by the partnership contract with the first-best optimal efforts:

**Proposition 3** Under the partnership contract, the second-best optimal effort of the builder can be smaller, equal or larger than the first-best optimal effort, whereas the second-best optimal effort of the operator is equal (lower) than the first-best optimal effort when the quality of the infrastructure is high (low).

**Proof.** See the Appendix.

From Proposition 3, two important differences with respect to the sequential contracts arise. First, the partnership contract allows the government to implement the first-best optimal operation effort when the quality of infrastructure is high. Second, the second-best optimal building effort is not necessarily less than the first-best.

Building on these results, we can compare the builder’s and operator’s efforts of the partnership and sequential contracts:

**Proposition 4** The second-best optimal effort of the builder under the partnership contract is strictly larger than under the sequential contracts. The second-best optimal effort of the operator under the partnership contract, when infrastructure quality is high (low), is strictly larger (smaller) than under the sequential contracts.

**Proof.** See the Appendix.

Proposition 4 relies on the well-known result that history-dependent contracts improve the welfare of the principal in models of dynamic moral hazard (e.g., Iossa and Martimort, 2015, p. 31–32). Even though no production externality exists between the building and operating tasks, the partnership contract allows the principal to design more powerful (and less costly) incentive schemes to punish (reward), in the second stage, the perceived insufficient (good) effort of the agent in the first stage. Such a mechanism cannot be used in the
framework of sequential contracts, given that the agent of the first stage is not the same as that of the second stage.\textsuperscript{14}

4.4 Partnership vs. sequential contracts: welfare analysis

Substituting the second-best optimal efforts in the government’s objective function, we can write the maximum social welfare under the partnership contract as:

\[
W^p = (S - k)q^l - c^h + l_c + e_{o}^{lp}(c^h - c^l - \psi'(e_{o}^{lp})) + e_{b}^{p2}\phi''(e_{b}^{p})
\]

and the maximum social welfare under the sequential contracts as:

\[
W^s = (S - k)q^l - c^h + l_b + l_o + e_{o}^{s}(c^h - c^l - \psi'(e_{o}^{s})) + e_{b}^{s2}\phi''(e_{b}^{s}).
\]

Thus, the total increase (reduction) of the social welfare that is determined by the partnership contract, compared to the sequential contracts, can be written as:

\[
\Delta W = W^p - W^s = MHC + l_c - l_b - l_o. \tag{33}
\]

The expression (33) is the composition of two factors. The first component

\[
MHC = e_{o}^{lp}(c^h - c^l - \psi'(e_{o}^{lp})) - e_{o}^{s}(c^h - c^l - \psi'(e_{o}^{s})) + e_{b}^{p2}\phi''(e_{b}^{p}) - e_{b}^{s2}\phi''(e_{b}^{s})
\]

is variation of the social welfare that is driven by the enhanced capacity to control moral hazard through the partnership contract compared to the sequential contracts (i.e., moral hazard correction). The second component \(l_c - l_b - l_o\) is the variation of the social welfare

\textsuperscript{14}We may find examples of history-dependent clauses in real-world, long-term concessions. For example, airport PPPs in Sao Paulo (Brasil), Rio de Janeiro (Brasil) and Santiago de Chile include incentives to attract demand and a history-dependent mechanism for capacity expansion. If the concessionaire’s effort to attract air traffic is successful, the concession is expanded and allows the concessionaire to invest in new airport capacity. Our theoretical findings can be interpreted as a suggestion to expand similar history-dependent clauses in PPPs.
that is driven by the size of the financial wealth of the consortium under the partnership contract compared to aggregate of the building and operating firms under the sequential contracts (i.e., limited liability differential).

In our setting, \( l_c - l_b - l_o \) is exogenous, and can take positive or negative values, depending on financial market conditions. As discussed in the Introduction, the corporate finance literature provides us with an interpretation of different signs of such a component. If the coinsurance effect prevails over the risk-contagion effect when firms are bundled within a consortium, then \( l_c > l_b + l_o \). If the opposite is true, \( l_c < l_b + l_o \). The latter situation is likely to arise when the financial markets feature high volatility and low appetite for risk and, hence, low overall liquidity of risky assets, which may spread asymmetric information among traders (Banal-Estanol et al., 2013; Farhi and Tirole, 2015). Considering this interpretation of the limited liability differential, we have the following important result:

**Proposition 5** When \( l_c \geq l_b + l_o \), the partnership contract always dominates the sequential contracts in social welfare terms.

**Proof.** See the Appendix. ■

The interpretation of Proposition 5 is that, as already pointed out, the partnership contract is history dependent, which affords the principal a more powerful incentive mechanism. Therefore, the MHC component of the social welfare difference between the partnership and sequential contracts is always strictly positive. Moreover, when the volatility of financial markets is low, the LLCs are equally or less constraining under the partnership than under the sequential contracts (i.e., \( l_c \geq l_b + l_o \)). In turn, the government may transfer more risk to the agent and, thus, design higher-powered incentive contracts in the former than in the latter case, which involves a smaller loss of efficiency with respect to the first-best allocation.

In contrast, if financial markets are affected by high uncertainty, such that the risk-contagion effect prevails on coinsurance (i.e., \( l_c < l_b + l_o \)), then sequential contracts may
become socially optimal. In particular, we have the following result:

**Corollary 1** Sequential contracts dominate the partnership contract in social welfare terms if and only if:

\[-(l_c - l_b - l_o) > MHC.\]

By the proof of Proposition 5, we know that $MHC > 0$. Therefore, a necessary condition for the sequential contracts to improve the social welfare with respect to the partnership contract is that financial constraints are stricter in the case of bundled tasks than in the case of unbundled tasks—that is, $l_c < l_b + l_o$.

## 5 Limited contracting capacity of the government

In what follows, we extend the model of Section 4 to consider two types of constraints that, in the real world, limit the contracting capacity of governments. We first relax the assumption that contracts cannot be renegotiated (Section 5.1). Then, we also introduce a binding fiscal constraint (Section 5.2).

### 5.1 Renegotiation

Renegotiation may affect only the partnership contract, which includes clauses regarding both sequential tasks. Particularly, if we relax the assumption that government can perfectly commit to the initial contract, after the quality of the infrastructure is determined, the government and the consortium may find it mutually convenient to renegotiate the contractual clauses that regulate the operation task. In turn, the set of feasible contracts must also satisfy the RPC, which may reduce the efficiency of the optimal partnership contract and, at least in principle, affect the results obtained in Section 4.
5.1.1 Implementable partnership contracts

When the government cannot commit not to renegotiate contracts, the implementable partnership contracts must satisfy the PC and ICC constraints from an *ex ante* (see Section 4.3.1) as well as an *ex interim* perspective. Particularly, the ex interim PC and ICC can be written as:

\[
\max_{e^i_o} e^i_o (t(q^i, c^i) - c^i) + (1 - e^i_o)(t(q^i, c^h) - c^h) - kq^i - \psi(e^i_o) - \phi(e^p_b) \geq E(u^i_o'), \tag{34}
\]

where \(q^i\), with \(i \in \{h, l\}\), is the realization of the infrastructure quality, after the (optimal) first-period investment \(e^p_b\) has been implemented, \(e^i_o\) is the operation effort that the consortium implements in the second stage (considering the renegotiated contract), and

\[
E(u^i_o') = e^i_o (t(q^i, c^i) - c^i) + (1 - e^i_o)(t(q^i, c^h) - c^h) - kq^i - \psi(e^i_o) - \phi(e^p_b)
\]

is the net expected utility that the consortium would obtain under the full-commitment contract, with \(e^i_o\) the optimal full-commitment efforts of the consortium depending on the realization of \(q^i\), which is determined by the optimization conditions (31)-(32).

As in Section 4.3, the LLCs (17)-(20) must be satisfied.

By the first-order approach, we can substitute the ICC with the condition:

\[
(t(q^i, c^i) - c^i) - (t(q^i, c^h) - c^h) = \psi'(e^i_o) \geq 0, \tag{35}
\]

which corresponds to the condition (22) or (23) in case \(i = h\) or \(i = l\), respectively.

Considering that the government aims at minimizing the (renegotiated) payments to the consortium, by the LLCs (17)-(20) and the ex interim ICC (35), the ex interim utility of the consortium’s management in the case of renegotiation can be written as:

\[
E(u^i_o) = e^i_o \psi'(e^i_o) - \psi(e^i_o) - l_c - \phi(e^p_b), \tag{36}
\]
where $i \in \{h,l\}$. The same expression (36), with $e^{ip}_o$ instead of $e^i_o$, represents the ex interim utility of the consortium’s management when the full-commitment contract is implemented. Moreover, we remark that $\frac{\partial E(u^i_o)}{\partial e^i_o} = e^i_o \psi''(e^i_o) \geq 0$, with strict inequality when $e^i_o > 0$, which brings us to the following result:

**Lemma 1** The operation-task clauses of the full-commitment partnership contract are renegotiated if and only if $e^i_o > e^{ip}_o$.

Compared to the full-commitment case, an improvement of the ex interim utility of the consortium may be warranted only if the government is willing to renegotiate a larger operating effort, which makes renegotiation feasible. Conversely, when the condition of Lemma 1 is violated (i.e., $e^i_o \leq e^{ip}_o$), the full-commitment partnership contract is also robust against any possible renegotiation.

Therefore, in the optimization problem of the government we can substitute the ex interim PC with the RPC, which can be simply introduced as a lower bound on the level of the quality-contingent operation effort—that is, $e^i_o \geq e^{ip}_o$ for $i \in \{h,l\}$.

### 5.1.2 Optimal partnership contracts

If the ex interim PC (34) is satisfied, the optimization problem of the government that is willing to renegotiate the contractual clauses about the operational phase coincides with the optimization problem of the government about the operation task under the sequential contracts (see Section 4.2.2). However, the quality-contingent operation effort cannot be set below the full-commitment effort, because of the ex interim PC of the consortium. In other terms, the government is always willing to renegotiate $e^{ip}_o$, considering the ex interim social welfare, and implement $e^s_o$ instead. By Propositions 3 and 4, we know that $e^*_o = e^{hp}_o > e^s_o > e^{ip}_o$. Thus, by Lemma 1, the renegotiation of the full-commitment contract takes place when quality is low (given that both the social welfare and the utility of the consortium’s management may grow), but not when it is high (given that any renegotiation would hurt the consortium’s management in such a case).
It is worth noticing that also the optimal renegotiation-proof partnership contract is history dependent. Therefore, we have the following result:

**Corollary 2** Proposition 5 also holds when the government cannot commit not to renegotiate contractual clauses.

**Proof.** See the Appendix. ■

Let us focus on the intuition of Corollary 2. Proposition 5 relies on the enhanced capacity of the partnership contract to control moral hazard by incorporating a memory mechanism that increases the rent of the consortium when quality is high—above the level that is reached with sequential contracts—and reduces it when the quality is low. The latter mechanism (the punishment) cannot be implemented with a renegotiation-proof partnership contract, whereas the former can still be implemented. In turn, the welfare-improving effect of the history-dependent structure of the partnership contract is not fully destroyed by the impossibility of committing not to renegotiate, although the power of incentives on the building effort is reduced. Particularly, by the optimization condition (30), we see that the building effort is strictly lower when the partnership contract must satisfy the RPC than under full commitment.

### 5.2 Fiscal constraint

We now extend the model of the previous section, which already takes into account the RPC, to consider the BC as an additional limit to the capacity of the government to design contractual clauses. Relying on the characterization of the implementable and optimal contracts of the previous sections, we analyze the maximum, state-contingent payments from the government to contractors that may be affected by the fiscal constraint both under the sequential and partnership contracts. Then we study how a fiscal constraint that limits the maximum level of payments changes our previous results.
5.2.1 Sequential contracts

Let us first analyze the maximum fiscal burden of government payments to firms under the sequential contracts to understand in which states of the world the fiscal constraint may bind.

From the LLCs and ICCs of the builder and the operator (see Section 4.2.1), it is easy to check that any implementable payments to the builder (10)-(11) and operator (6)-(7) are such that:

\[ t_b(q^h, c^h) + t_o(c^h) \geq t_b(q^l) + t_o(c^h), \]
\[ t_b(q^h, c^l) + t_o(c^l) \geq t_b(q^l) + t_o(c^l). \]

Moreover, we observe that the government can implement different payment schedules to reach the same outcome. Particularly, when the quality is high, the feasible payments to the builder must satisfy the condition (11), which implies that either \( t_b(q^h, c^l) + t_o(c^l) \) or \( t_b(q^h, c^h) + t_o(c^h) \) may entail the largest fiscal outlays for the government. However, we can establish the following result:

**Lemma 2** The BC binds if and only if:

\[ t_b(q^h, c^h) + t_o(c^h) = t_b(q^h, c^l) + t_o(c^l). \]  

Moreover, the maximum fiscal burden is associated with payments on the left- and right-hand sides of the equation (37).

**Proof.** See the Appendix ■

We now assume that the condition (37) holds and that the BC (potentially) affects the optimal sequential contracts only in the states of the world \( \{q^h, c^h\} \) and \( \{q^h, c^l\} \) that entail
the most expensive payments. Therefore, the government maximizes the problem (12) under the BCs:

\[ F \geq t_b(q^h, c^l) + t_o(c^l) \quad \text{and} \quad F \geq t_b(q^h, c^h). \]  

(38)

However, given Lemma 2, it is easy to show that the BCs (38) boil down into a single constraint. Thus, substituting the payment schedules that satisfy the ICCs and LLCs of the builder and the operator into the government optimization problem (12) under the BC (38), the government’s maximization problem can be written as:

\[
\max_{e_b, e_o} (S - k)q^l - c^h + l_b + l_o + \\
+ e_o(c^h - c^l - \psi'(e_o)) + e_b[(S - k)(q^h - q^l) - \phi'(e_b)] + \\
+ \lambda[F + l_b + l_o - kq^h - c^h - \phi'(e_b) + e_o(c^h - c^l - \psi'(e_o))],
\]

(39)

where \( \lambda \) is the Lagrangian multiplier of the BC. By the problem (39), we obtain the optimization conditions that characterize the second-best optimal efforts under the sequential contracts with the fiscal constraint:

\[
\phi'(e_{sf}^b) = (S - k)(q^h - q^l) - (e_{sf}^b + \lambda)e_{sf}''(e_{sf}^b),
\]

(40)

\[
\psi'(e_{sf}^o) = c^h - c^l - e_{sf}^o \psi''(e_{sf}^o).
\]

(41)

We obtain interesting findings. First, the optimal operation contract is not affected by the fiscal constraint (i.e., \( e_{sf}^o = e^o \)), whereas the building contract is. Particularly, if \( \lambda > 0 \) (i.e., the BC binds), the builder’s effort is strictly smaller than in the case without the fiscal constraint: \( e_{sf}^b < e^b \). Moreover, considering that the BC is binding and \( e_{sf}^o = e^o \), the

\[15\] If the BC becomes very stringent such that also the less expensive payments become unaffordable for the government, the principal loses the capacity to provide incentives that induce the agent(s) to implement different levels of effort in different states of the world.
optimization condition (40) can also be written as follows:

\[
\phi'(e_{sf}^{sf}) = F + l_b + l_o - kq^h - c^h + (e_o^s)^2 \psi''(e_o^s),
\]

(42)

from which we see that, under a binding fiscal constraint, \(e_{sf}^{sf}\) is determined by the available fiscal and financial resources (i.e., \(F + l_b + l_o\)) and \(\frac{de_{sf}^{sf}}{dF} = \frac{de_{sf}^{sf}}{dl_b} = \frac{de_{sf}^{sf}}{dl_o} = \frac{1}{\phi''(e_{sf}^{sf})} > 0\).

The reason why only the optimal building contract is affected by the fiscal constraint is that we are considering a model with sequential moral hazard where different tasks influence the final outcome asymmetrically. Particularly, by providing costly incentives to increase the building effort in the first phase, the government faces a trade-off between the objective to foster higher social welfare and the fiscal constraint. For this reason, the optimal building effort is lower under the fiscal constraint than in the absence of it. Conversely, by providing incentives to increase the operation effort in the second phase, the government is pursuing higher social welfare but also reducing the payment to the operator, thus easing the trade-off between the objective and the fiscal constraint. To see why this is the case, consider that the payment to the operator—which covers the cost \(c_i\), for any \(i \in \{h, l\}\), and the information rent when the operation cost is \(c^l\)—is lower in the states of the world where the cost is \(c^l\) than in the states of the world where the cost is \(c^h\).

The described results make evident our contribution to the literature on dynamic moral hazard. To fully understand the role of the principal’s budget constraint in the design of optimal contracts and in the comparison between bundling and unbundling in frameworks featuring sequential moral hazard, we cannot rely only on the findings of models of repeated moral hazard. The reason is that in the latter, the efforts of the agents symmetrically influence the principal’s objective function. Therefore, the way the principal’s budget constraint distorts the second-best efforts is the same for all sequential tasks. Our model shows that the results may differ quite sensibly when we consider that sequential tasks affect the principal’s objective function asymmetrically.
5.2.2 Partnership contract

We now analyze the effect of the fiscal constraint on the optimal renegotiation-proof partnership contract. Let us first remark that the RPC does not change with respect to Section 5.1. The reason is that, as shown in the previous section, the fiscal constraint does not affect the second-best optimal effort of the operator that the government is willing to implement under the sequential contracts, which—as shown in Lemma 1—is the lower bound of admissible operation efforts for any renegotiation-proof partnership contract.

Again, we analyze the fiscal burden associated with the optimal payments (without BC) in different states of the world. By the analysis of Section 4.3.1, we know that the implementable payment schemes (24)-(25) are such that:

\[
\begin{align*}
t(q^h, c^h) &> t(q^l, c^h), \\
t(q^h, c^l) &\geq t(q^l, c^l).
\end{align*}
\]

Hence, as under the sequential contracts, both \( t(q^h, c^h) \) and/or \( t(q^h, c^l) \) may entail the maximum fiscal burden.

Therefore, the government maximizes the problem (28) under the RPC (i.e., \( e^l_o \geq e^*_o \)) and the BCs:

\[
F \geq t(q^h, c^h) \quad \text{and} \quad F \geq t(q^h, c^l).
\]

By the expressions of implementable payments under partnership contracts when the quality of the infrastructure is high (26) and (27), we see that \( t(q^h, c^h) = t(q^h, c^l) \) if and only if \( \psi'(e^h_o) = c^h - c^l \). In principle, we may have that the government, at the optimum, aims at distorting the operator’s effort in the state of the world with high infrastructure quality. Particularly, if the second-best optimal operator’s effort is \( e^h_o < e^* \) (or \( e^h_o > e^* \), then \( t(q^h, c^h) > t(q^h, c^l) \) (or \( t(q^h, c^h) < t(q^h, c^l) \)).\(^{16}\)

\(^{16}\)If we substitute the optimal efforts for the building and operating tasks that maximize the govern-
Therefore, substituting the payment schedules that satisfy the ICCs and LLCs of the builder and operator in the government optimization problem (28) under the RPC (i.e., $e_o^l \geq e_o^e$) and both BCs (43), the government’s maximization problem can be written as:

$$\begin{align*}
\max_{e_b,e_o^e, e_b^o} & \quad (S - k)q^l - c^h + l_c + e_o^l(c^h - c^l - \psi'(e_o^l)) + \\
& + e_b[(S - k)(q^h - q^l) - \phi'(e_b) + (e_o^h - e_o^l)(c^h - c^l) + \psi'(e_o^h) - \psi'(e_o^l)) + \\
& + \lambda_{hh}(F + l_c - kq^h - c^h - e_o^l\psi'(e_o^l) + \psi(e_o^l) + e_o^h\psi'(e_o^h) - \psi(e_o^h) - \phi'(e_b)) + \\
& + \lambda_{hl}[F + l_c - kq^h - c^l - e_o^l\psi'(e_o^l) + \psi(e_o^l) + (e_o^h - 1)\psi'(e_o^h) - \psi(e_o^h) - \phi'(e_b)] + \\
& + \mu(e_o^l - e_o^e),
\end{align*}$$

where $\lambda_{hh}$, $\lambda_{hl}$ and $\mu$ are the Lagrangian multipliers of the BCs (43) and of the RPC, respectively. By the problem (44), we derive the optimization conditions:

$$\begin{align*}
\phi'(e_b^{pf}) &= (S - k)(q^h - q^l) + (e_o^{hpf} - e_o^{lpf})(c^h - c^l) + \psi'(e_o^{lpf}) - \psi'(e_o^{hpf}) + \\
& - (e_b^{pf} + \lambda_{hh} + \lambda_{hl})\phi''(e_b^{pf}),
\end{align*}$$

$$\begin{align*}
\psi'(e_o^{hpf}) &= c^h - c^l + \frac{\lambda_{hh}e_o^h - \lambda_{hl}(1 - e_o^h)}{e_b^{pf}}\psi''(e_o^{hpf}),
\end{align*}$$

$$\begin{align*}
\psi'(e_o^{lpf}) &= c^h - c^l + \frac{\mu - (1 + \lambda_{hh} + \lambda_{hl})e_o^{lpf}\psi''(e_o^{lpf})}{1 - e_b^{pf}}.
\end{align*}$$

From the optimization conditions (45)-(47), we derive the following results:

**Proposition 6** When the government is constrained by the RPC and BC, the second-best optimal effort of the builder under the partnership contract is strictly larger than under the sequential contracts, provided that $l_c - l_b - l_o$ is not too negative. The second-best optimal effort of the operator under the partnership contract, when infrastructure quality is high (low), government’s objective function under the RPC (and without the BC), the maximum optimal payments from the government to the consortium are:

$$t(q^h, c^h) = kq^h + e^h - l_c + e_o^e\psi'(e_o^e) - \psi'(e_o^e) - e_o^{hp}\psi'(e_o^{hp}) + \psi(e_o^{hp}) + \phi'(e_b) = t(q^h, c^l).$$

However, this is not necessarily generally true under a binding fiscal constraint.
is equal to the first-best optimal effort (the second-best optimal effort under the sequential contracts).

**Proof.** See the Appendix. ■

The interpretation of Proposition 6 is that when the fiscal constraint limits (in a symmetric way) the maximum payments that the government can award to its agents under both the partnership and sequential contracts, still the history-dependent mechanism allows the former contractual scheme to outperform the latter in correcting moral hazard. Particularly, the optimal effort in the operation phase is the first-best effort \( e^*_o \), when the infrastructure quality is high, and the sequential-contract effort \( e^*_o \), when the infrastructure quality is low. If the financial wealth of the agents is not too much unbalanced against bundling (i.e., \( l_c - l_b - l_o \) is not too negative), then the partnership contract provides stronger incentives to increase the building effort.

Also, under the partnership contract as already observed in Section 5.2.1 for the sequential contracts, Proposition 6 allows us to prove that the BC affects only the optimal building effort, which increases the fiscal cost of an additional unit of social welfare, whereas it does not affect the optimal operational effort, which helps at increasing the social welfare while reducing the government’s payments to the consortium.\footnote{Note that the latter result may not hold under full commitment. If we solve the problem (44) without the RPC, we obtain the equivalent of the optimization condition (47), which shows that the optimal effort \( e^{lbf}_o \) is, in general, smaller when the fiscal constraint binds.} Particularly, given that the BC is binding in the states of the world \( \{q^h, c^h\} \) and \( \{q^h, c^l\} \) and given that \( e^{hpf}_o = e^*_o \) and \( e^{lpf}_o = e^*_o \), we can characterize the optimal building effort as follows:

\[
\phi'(e^{pf}_b) = F + l_c - kq^h - c^h - e^*_o \psi'(e^*_o) + \psi(e^*_o) + e^*_o \psi'(e^*_o) - \psi(e^*_o), \tag{48}
\]

from which we see that, under a binding fiscal constraint, \( e^{pf}_b \) depends on the aggregate available fiscal and financial resources (i.e., \( F + l_c \)) and \( \frac{de^{pf}_b}{dF} = \frac{de^{pf}_b}{dl_c} = \frac{1}{\phi''(e^*_o)} > 0. \)

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5.2.3 Partnership vs. sequential contracts: welfare analysis

In this section, we assess the impact of the fiscal constraint on the relative performance of the partnership and sequential contracts in terms of social welfare. We first observe that the maximum fiscal burden is associated with the optimal payments featuring a high quality of the infrastructure under both the sequential (Section 5.2.1) and partnership (Section 5.2.2) contracts. The difference between the maximum fiscal burden under the sequential and partnership contracts may be positive or negative depending on the technology and on private financial conditions.\(^{18}\) To save space, in the following we consider only the most interesting case, which is when both the sequential and partnership contracts are affected by the government’s BC.

Substituting the optimal building and operating efforts into the government’s objective function when both the RPC and the BC bind, we can write the maximum social welfare under the partnership contract as:

\[
W_{pf} = (S - k)q^l - c^l + l_c + e_o^s(c^h - c^l - \psi'(e_o^s)) + \\
+ e_p^f[(S - k)(q^h - q^l) - \phi'(e_p^f) + (e_o^s - e_o^s)(c^h - c^l) + \psi(e_o^s) - \psi(e_o^s)]
\]

and the maximum social welfare under the sequential contracts as:

\[
W_{sf} = (S - k)q^l - c^l + l_b + l_o + e_o^s(c^h - c^l - \psi'(e_o^s)) + \\
+ e_s^f[(S - k)(q^h - q^l) - \phi'(e_s^f)].
\]

Thus, using the expressions (42) and (48) to substitute for, respectively, \(\phi'(e_s^f)\) and \(\phi'(e_p^f)\), the difference between the maximum social welfare under the partnership and sequential

\(^{18}\)Considering the optimization conditions (14)–(15) and (30)–(32), the difference between the maximum payments under the sequential and partnership contracts

\[
\Delta T = e_p^f \phi''(e_p^f) - e_s^f \phi''(e_s^f) + l_c - l_b - l_o,
\]

is likely to be positive when \(l_c \geq l_b + l_o\) (a sufficient condition is that \(\phi''(e) + e\phi'''(e) \geq 0\)). However, it may be negative, particularly when \(l_c < l_b + l_o\).
contracts can be written as:

\[ \Delta W = W_{pf} - W_{sf} = (e_{pf}^b - e_{sf}^b)(S - k)(q^h - q^l) - F - l_c + kq^h + c^h - e_s^o(c^h - c^l - \psi'(e_s^o)) + (1 - e_{sf}^b)(l_c - l_b + l_o). \]  

Thus, we find the following result:

**Corollary 3** Proposition 5 also holds when the government’s contractual capacity is limited by both the RPC and the BC.

**Proof.** See the Appendix. ■

By Corollary 3, we see that also when the fiscal constraint binds, the partnership contract is still history dependent. Thus, when the financial markets feature low uncertainty such that \( l_c - l_b - l_o \geq 0 \), the partnership contract outperforms the sequential contracts in terms of social welfare. The latter findings can be interpreted as a generalization of the irrelevance of public finance constraints found by Engel et al. (2013).

However, the same expression (49) highlights, once more, that this result depends on firms’ financial conditions. When financial markets are troubled by high uncertainty, bundling different firms within a consortium may reduce their financial wealth (i.e., \( l_c - l_b - l_o < 0 \)), thus reducing the efficiency of the partnership contract with respect to the sequential contracts. A way to see this is to consider the impact of a variation of the fiscal constraint on the welfare differential between the partnership and sequential contracts:

\[ \frac{d\Delta W}{dF} = - (e_{pf}^b - e_{sf}^b) - \frac{l_c - l_b - l_o}{\phi''(e_{sf}^b)} + \left( \frac{1}{\phi''(e_{pf}^b)} - \frac{1}{\phi''(e_{sf}^b)} \right) [(S - k)(q^h - q^l) - F + kq^h + c^h - e_s^o(c^h - c^l - \psi'(e_s^o))] \]  

When the private financial conditions are such that \( l_c \geq l_b + l_o \), the expression (50) is likely
to be negative.\(^{19}\) In other words, a harder fiscal constraint tends to reinforce the preference in terms of social welfare for the partnership contract versus the sequential contracts. When the financial conditions are such that \(l_c < l_b + l_o\), the expression (50) may take different signs.

### 5.2.4 Interaction between financial and fiscal constraints

To better understand the interaction between the fiscal constraint of the government and the financial constraints of the firms and its impact on the welfare comparison between the sequential and partnership contracts, in this section, we run a numerical simulation of a simple case that is characterized by the following specification of the non-monetary costs of efforts: \(\phi(e_b) = \frac{e_b^2}{2}\) for the building effort; and \(\psi(e_o) = \frac{e_o^2}{2}\) for the operation effort.\(^{20}\)

We first consider the case in which contracts can be renegotiated, but the fiscal constraint does not bind.\(^{21}\) Substituting the optimal efforts obtained with our specification into the expression (33), we derive the condition such that the partnership and sequential contracts are equivalent in social welfare terms, which can be written as:

\[
\Delta W = \left(\frac{e_b^h - e_l^l}{4}\right)^2 \left(\frac{e_b^h - e_l^l}{4}\right)^2 + (S - k)(q^h - q^l) + l_c - l_b - l_o = 0.
\]

The condition (51) is reported as a continuous red line in the graph of Figure 3 with the operational cost differential \(c^h - c^l\) on the horizontal axis, and the limited liability differential \(l_c - l_b - l_o\) on the vertical axis.

We focus on the case in which \(l_c - l_b - l_o < 0\) in Figure 3, given that the partnership contract always dominates the sequential contracts when \(l_c - l_b - l_o \geq 0\). Particularly, in the area above the continuous red line, the partnership contract is socially optimal (i.e.,

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\(^{19}\)For example, when \(\phi'''(.) = 0\), the last term of the expression (50) disappears, and the expression is negative when \(l_c \geq l_b + l_o\).

\(^{20}\)Under the considered specification, the third derivative of \(\phi(.)\) is zero and, thus, the last term of the expression (50) disappears. However, also in such a case, the expression (50) may take different signs when \(l_c - l_b - l_o < 0\).

\(^{21}\)In other terms, \(F\) is so large that \(\lambda_{hh} = \lambda_{hd} = 0\). It is also worth remarking that our numerical results would not qualitatively change considering full-commitment contracts.
Figure 3: Optimal choice between partnership and sequential contracts

Legend: The graph is derived considering the following values for the model’s parameters: $k = 1$, $S = 2$, $q^h = 1$, $q^l = 0.15$.

$\Delta W > 0)$, whereas below the continuous red line, the sequential contracts are socially optimal (i.e., $\Delta W < 0$).

As a second step, we consider the case in which the BC is binding. Considering the expression (50) with our specification\(^{22}\), we derive the following condition:

$$
\frac{d\Delta W}{dF} = \left(\frac{c^h - c^l}{4}\right)^2 + l_c - l_b - l_o = 0. \\
(52)
$$

We report the condition (52) as a dotted blue line in the graph of Figure 3. When $F$ decreases, the government’s preference for the partnership contract may increase (above the dotted blue line) or decrease (below the dotted blue line).

Considering both conditions (51) and (52), we can identify four areas in Figure 3. In the area $A (D)$, the social welfare is higher under the partnership contract (the sequential contract) regardless of the amount of available public funds $F$. The intuition is that, in

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\(^{22}\)In our simple case, the last term of expression (50) disappears, given that $\phi'''(.) = 0$, and the formula does not depend on the value of $F$. Our main results are also obtained with more general specifications of $\phi(.)$. 

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these areas, the social welfare difference between the partnership and sequential contracts that we obtain without a binding fiscal constraint (i.e., \( F \) is large enough) grows when fiscal resources are reduced (i.e., \( F \) drops).

The most interesting results are found in areas \( B \) and \( C \). In the area \( B \), the social welfare is larger under the sequential contracts without a binding BC. However, as we see in Figure 4 (which reports \( F \) on the horizontal axis and \( \Delta W \) on the vertical axis), when \( F \) decreases, the social welfare gain of relying on the sequential contracts (i.e., \( \Delta W < 0 \)) is progressively eroded. A sufficiently strict fiscal constraint eventually flips the social welfare ranking between the two alternative contractual schemes (in our example, \( \Delta W > 0 \) for \( F < 0.3 \)). The opposite is true in the area \( C \) (see Figure 5), where the maximum social welfare is reached with the partnership contract without a binding fiscal constraint, but such a welfare gain (i.e., \( \Delta W > 0 \)) is progressively reduced when \( F \) decreases. Also in this case, the welfare ranking changes for a sufficiently strict fiscal constraint (i.e., \( \Delta W < 0 \) for \( F < 0.35 \)).

![Figure 4: From sequential to partnership contracts (area B of Figure 3)](image)

Legend: The graph is derived considering the following values for the model's parameters: \( k = 1 \), \( S = 2 \), \( q^h = 1 \), \( q^l = 0.15 \), \( c^h - c^l = 1 \), \( l_e - l_b - l_o = -0.0585 \).
Figure 5: From partnership to sequential contracts (area C of Figure 3)

Legend: The graph is derived considering the following values for the model’s parameters: \( k = 1, S = 2, q^h = 1, q^l = 0.15, c^h - c^l = 2, l_c - l_b - l_o = -0.27 \).

What lessons can we draw from this numerical exercise? From the theoretical point of view, a central role in driving the welfare ranking between the partnership and sequential contracts is played by the power of incentives of the memory contract (in contrast with history-independent contracts). The latter is proxied by the operational cost differential \( c^h - c^l \) and represents the enhanced capacity of the partnership contract to correct moral hazard compared to the sequential contracts. Looking at Figure 3, we see that, given any negative value of the limited liability differential (and the other parameters), as \( c^h - c^l \) increases, \( \Delta W \) grows and, above some value of \( c^h - c^l \), it turns from negative to positive. The interpretation is that the moral hazard correction component of the social welfare differential eventually more than compensates for the limited liability differential. A similar mechanism also operates when we consider the impact of the fiscal constraint on the government’s preference for partnership and sequential contracts. Again, for a sufficiently large power of incentives underlying the memory contract, \( \frac{d \Delta W}{df} \) grows and, above some value of \( c^h - c^l \), it flips from negative to positive. If the limited liability differential is negative but above a
given threshold (in the example of Figure 3, $l_c - l_b - l_o = -0.15$), a sufficiently large power of incentives of the memory contract widens the area of parameters in which the partnership contract is socially optimal when the fiscal constraint becomes stricter (i.e., the area $B$ of Figure 3). Conversely, when the limited liability differential is below a given threshold it also drives the marginal effect of a stricter fiscal constraint (i.e., the area $C$ of the Figure 3).

Our analysis can also be used to retrieve empirically testable predictions. Empirical works find that PPPs are more likely to be implemented by budget-constrained governments (Hammami et al., 2006; Albalate et al., 2015; Buso et al., 2017), but there are no clear theoretical explanations for this correlation. Considering that other exogenous and randomly distributed factors (e.g., a fixed cost to implement PPPs compared to traditional procurement) may also affect the choice of PPPs, our analysis can be interpreted as follows. When $l_c - l_b - l_o$ is positive or slightly negative, fiscal constraints increase the likelihood of PPP investments. Conversely, when $l_c - l_b - l_o$ is very negative, fiscal constraints decrease the likelihood of PPPs. Cursory evidence, which seems to confirm this prediction (see Figure 1), must be rigorously tested through empirical analyses that look at the combined effect of fiscal and financial conditions.

6 Conclusions

Since their introduction in the early 1990s, the evolution of PPPs, in terms of the number of projects and investment volumes, has followed an uncertain trend: increasing until the 2008 crisis and decreasing afterward. Empirical and theoretical analyses suggest some possible determinants explaining the choice of PPPs by local and central authorities, such as the nature of the public infrastructure (technology required, innovation incentives, etc.) or fiscal and institutional variables. However, none of the previous analyses can explain the uncertain trend of PPPs, and it is still debated whether PPPs are chosen for efficiency or alternative reasons (e.g., political incentives, the presence of fiscal constraints).
Departing from much of the extant theoretical literature on PPPs, which considers the benefits (costs) of bundling as related to the presence of positive (negative) production externalities between sequential tasks (e.g., Hart, 2003; Martimort and Pouyet, 2008; Iossa and Martimort, 2015), this article focuses on the financial and fiscal determinants of the social welfare differential between PPPs (or partnership contracts) and traditional procurement (or sequential contracts). Our results can be summarized as follows.

First, absent any fiscal constraint, but in the presence of private financial constraints, we show that the partnership contract allows the government to design a more powerful incentive scheme, where the operation-phase payment depends not only on the operational costs but also on the building’s quality. Such a history-dependent payment schedule affords the partnership contract with an enhanced capacity (compared to sequential contracts) to control moral hazard. However, the capacity of the partnership contract to generate a welfare gain (compared to the sequential contracts) also depends on the difference between the total financial wealth of the consortium under the partnership contract and the building and operating firms under the sequential contracts. Following the corporate finance literature, this difference is positive (negative) if the coinsurance effect—among different firms bundled within the consortium—prevails (does not prevail) over the risk contagion effect, which happens when the financial markets feature low (high) volatility.

Second, we show that the previous result is robust against the introduction of renegotiation and the fiscal constraint. In this last case, we show that the impact of the budget constraint can affect the welfare difference between partnership and sequential contracts either negatively or positively. In particular, the impact is likely to be negative when financial markets are affected by very high uncertainty.

These theoretical predictions provide interesting insights for future empirical analyses. The model suggests that the volatility of financial markets is a relevant determinant explaining the adoption of PPPs. Indeed, the impact of the fiscal constraint on the probability of implementing PPPs is positive in the presence of low volatility of financial markets and neg-
ative otherwise. More generally, we need to control for the role of private financial conditions to identify the relationship between the fiscal constraint and the choice of PPPs.

Our results also have important policy implications. Following the COVID-19 health and economic crisis, national and supranational governments have been developing important packages for infrastructure to support economic recovery. A crucial policy issue is whether they will choose PPPs or traditional procurement. The message of our article is that PPPs may help governments obtain high-quality infrastructures provided that the private sponsors of the projects perform high credit ratings. Alternatively, governments may provide public guarantees to foster private partners’ ratings (EPEC, 2009). Our analysis explains why such a policy may work, though a clear assessment of the cost of government guarantees should enter the cost-benefit analysis of PPPs.

References


I Appendix

**Lemma I.1** Any sequential contracts that satisfy the ICCs and LLCs also satisfy the PCs for sufficiently low $l_b$ and $l_o$.

**Proof.** Substituting the implementable payment functions that satisfy the ICCs and LLCs of the builder (10)–(11) in (8) and of the operator (6)–(7) into (4), the PCs can be written as

\[ e_b \phi'(e_b) - \phi(e_b) \geq l_b \]  
(A1)

for the builder and

\[ e_o \psi'(e_o) - \psi(e_o) \geq l_o \]  
(A2)

the operator. The right-hand side of (A1) and (A2) is equal to zero when $e_b = 0$ and $e_o = 0$, respectively. Moreover, $\frac{\partial}{\partial e}(e \phi'(e) - \phi(e)) = e \phi''(e) > 0$ and $\frac{\partial}{\partial e}(e \psi'(e) - \psi(e)) = e \psi''(e) > 0$ for all $e \in (0, 1)$. Thus, if $l_b$ and $l_o$ are sufficiently low, (A1) and (A2) are satisfied. ■
Lemma I.2 Any partnership contract that satisfies the ICC and LLC also satisfies the PC for sufficiently low $l_c$.

Proof. Substituting (21), (22) and (23) into the agent’s objective function, the PC can be written as:

$$t(q^l, c^h) - kq^l - c^h + e_b\phi'(e_b) - \phi(e_b) + e_o^l\psi'(e_o^l) - \psi(e_o^l) \geq 0. \quad (A3)$$

By the proof of Lemma I.1, $e_b\phi'(e_b) - \phi(e_b) \geq 0$ and $e_o^l\psi'(e_o^l) - \psi(e_o^l) \geq 0$. Thus, (20) implies (A3) if $l_c \leq e_b\phi'(e_b) - \phi(e_b) + e_o^l\psi'(e_o^l) - \psi(e_o^l)$. ■

Lemma I.3 The optimal partnership contract is such that, among the LLCs, only the condition (20) binds.

Proof. By the optimization condition (22), if the LLC (18) is satisfied, then also condition (17) is satisfied. In the same way, by the optimization condition (23), if the LLC (20) is satisfied, also condition (19) is satisfied. We now substitute the optimization conditions (22) and (23) into the condition (21) and, after some algebra, we obtain:

$$t(q^h, c^h) - kq^h - c^h = t(q^l, c^h) - kq^l - c^h + \tau(e_o^l, e_o^h, e_b),$$

where $\tau(e_o^l, e_o^h, e_b) = e_o^l\psi'(e_o^l) - \psi(e_o^l) - e_o^h\psi'(e_o^h) + \psi(e_o^h) + \phi'(e_b)$. If $\tau(e_o^l, e_o^h, e_b) \geq 0$, the LLC (18) is satisfied when the condition (20) is satisfied, and the lemma holds. Assume, by contradiction, that $\tau(e_o^l, e_o^h, e_b) < 0$. Under this assumption, we substitute the binding constraints (18), (21), (22) and (23) into (16); thus the government’s optimization program can be written as:

$$\max_{e_b, e_o^l, e_o^h} (S - k)q^l - c^h - e_o^l(c^l - c^h) - e_o^h\psi'(e_o^h) - \psi(e_o^h) + \psi(e_o^h) + \phi'(e_b) +$$

$$+ e_b[(S - k)(q^h - q^l) + (e_o^h - e_o^l)(c^h - c^l) + \psi(e_o^l) - \psi(e_o^h) - \phi'(e_b)] + l_c.$$
Proof of Proposition 4.

By the first-order conditions, we find that:

\[
\phi'(e_b^p) = (S - k)(q^h - q^l) + (e_o^h - e_o^l)(c^h - c^l) + \psi(e_o^h) - \psi(e_o^l) + (1 - e_b^p)\phi''(e_b^p)
\]

\[
\psi'(e_o^h) = c^h - c^l - \frac{h}{e_b^p} \phi''(e_o^h)
\]

\[
\psi'(e_o^l) = c^h - c^l;
\]

and, given the properties of the \( \psi \) function, we derive that \( e_o^p \geq e_o^h \). However, by the proof of Lemma I.1, \( \tau(e_o^l, e_o^h, e_b^p) < 0 \) only if \( e_o^p < e_o^h \). Hence, we have a contradiction. ■

Proof of Proposition 3. Contrasting the optimization conditions (3) and (31)-(32):

\( e_o^* = e_o^h > e_o^l \). By the optimization condition (31),

\[
(e_o^h - e_o^l)(c^h - c^l) + \psi(e_o^l) - \psi(e_o^h) = e_o^h \psi'(e_o^h) - \psi(e_o^h) - z(e_o^l),
\]

where \( z(e) \equiv e\psi'(e^h) - \psi(e) \) is such that: \( z'(e) = \psi'(e^h) - \psi'(e) \) is strictly positive (negative) for all \( e < e_o^h \) \( (e > e_o^h) \), and it is zero when \( e = e_o^h \); \( z''(e) = -\psi''(e) < 0 \); and \( z(e_o^h) = e_o^h \psi'(e_o^h) - \psi(e_o^h) \). Thus, \( e_o^h \psi'(e_o^h) - \psi(e_o^h) > z(e) \) for all \( e \neq e_o^h \), and in particular: \( e_o^h \psi'(e_o^h) - \psi(e_o^h) - z(e_o^l) > 0 \). Contrasting the optimization conditions (2) and (30), \( e_b^p \) can be larger or smaller than \( e_o^* \) whenever \( (e_o^h - e_o^l)(c^h - c^l) + \psi(e_o^l) - \psi(e_o^h) > 0 \) is larger or smaller than \( e_b^p \phi''(e_b^p) > 0 \). ■

Proof of Proposition 4. By the proof of Proposition 3, we know that \( (e_o^h - e_o^l)(c^h - c^l) + \psi(e_o^l) - \psi(e_o^h) > 0 \). Considering that, by assumption, \( e_b^p \phi''(e_b^p) > -2 \), contrasting the optimization conditions (14) and (30), \( e_b^p > e_b^* \). Similarly, given that, by assumption, \( e_o^h \phi''(e_o^l) > -2 \), by the optimization conditions (15) and (31)-(32): \( e_o^h = e_o^* > e_o^s > e_o^p \). ■
Proof of Proposition 5. The expression (33) can be written as:

\[ \Delta W = W^p - W^p(e^*_b, e^*_o) + W^p(e^*_b, e^*_o) - W^s, \]

where:

\[ W^p(e^*_b, e^*_o) = (S - k)q^l - c^h + e^*_o(c^h - c^l - \psi'(e^*_o)) + \]
\[ + e^*_b [(S - k)(q^h - q^l) + (e^*_o - e^*_b)(c^h - c^l) + \psi(e^*_o) - \psi(e^*_b) - \phi'(e^*_b)] + l_c \]

is the value of the social welfare function under the partnership contract if the consortium implements the sequential-contracts optimal efforts for the building task \( e^*_b \) and for the operation task when the infrastructure quality is low \( e^*_o \) (whereas it continues to implement \( e^*_o \) when the quality is high). Given that the social welfare function of the government reaches a maximum when the building effort is \( e^*_b \), and the operation effort in case of low infrastructure quality is \( e^*_o \), then \( W^p - W^p(e^*_b, e^*_o) \geq 0 \). Using the conditions characterizing the optimal sequential contracts (14)-(15), we can write:

\[ W^p(e^*_b, e^*_o) - W^s = e^*_b [(e^*_o - e^*_b)(c^h - c^l) + \psi(e^*_o) - \psi(e^*_b)] + l_c - l_b - l_o. \]

By the argument of the proof of Proposition 3, it is straightforward to show that \((e^*_o - e^*_b)(c^h - c^l) + \psi(e^*_o) - \psi(e^*_b) > 0 \). Therefore, by \( l_c \geq l_b + l_o \), \( W^p(e^*_b, e^*_o) - W^s > 0 \), which completes the proof. \( \blacksquare \)

Proof of Corollary 2. The proof follows as for Proposition 5. \( \blacksquare \)

Proof of Lemma 2. The maximum implementable payment is \( t_b(q^h, c^h) + t_o(c^h) \) and/or \( t_b(q^h, c^l) + t_o(c^l) \). Let us remark that the government has some degrees of freedom in reducing \( t_b(q^h, c^h) \) or \( t_b(q^h, c^l) \), provided that the condition (11) is satisfied. Therefore, any
implementable payment scheme that minimizes the maximum fiscal burden must be such that the condition (37) is satisfied. This is particularly the case when the BC binds. Substituting the condition (37) into the condition (11), we obtain the formulas of implementable payments:

$$t_b(q^h, c^h) = kq^h - l_b + \phi'(e_b) - e_o(c^h - c^l - \psi'(e_o)),$$  \hspace{1cm} (A4)

$$t_b(q^h, c^l) = kq^h - l_b + \phi'(e_b) + (1 - e_o)(c^h - c^l - \psi'(e_o)).$$  \hspace{1cm} (A5)

However, if $\phi'(e_b) < e_o(c^h - c^l - \psi'(e_o))$, then (A4) would violate the LLC. Therefore, in such a case $t_b(q^h, c^h) + t_o(c^h) = kq^h + e^h - l_b - l_o > t_b(q^h, c^l) + t_o(c^l)$ and, by the assumption that $F + l_b + l_o \geq kq^h + e^h$, the BC cannot bind. By the same argument, if $\phi'(e_b) < -(1 - e_o)(c^h - c^l - \psi'(e_o))$, then (A5) would violate the LLC, and also, in such a case, the BC cannot bind. In turn, the BC binds only if condition (37) is satisfied.

**Lemma I.4** Assume that the solutions of the problem (44) are strictly positive. Then, $\lambda_{hh} > 0$ if and only if $\lambda_{hl} > 0$. Moreover, at the optimum $e^{hpf}_o = e^*_o$.

**Proof.** Assume that, at the optimum, $\lambda_{hh} > 0$. Then, given that the BC of the government in the state of the world $\{q^h, c^h\}$ is binding,

$$c^h = F + l_c - kq^h - e^l_o \psi'(e^l_o) + \psi(e^l_o) + e^h_o \psi'(e^h_o) - \psi(e^h_o) - \phi'(e_b).$$  \hspace{1cm} (A6)

Assume, by contradiction, that the BC of the government in the state of the world $\{q^h, c^l\}$ is slack (i.e., $\lambda_{hl} = 0$). Substituting the expression (A6) into the latter, we have that, at the optimum, $c^h - c^l - \psi'(e^h_o) > 0$. However, by the first-order condition with respect to $e^h_o$, we have that:

$$0 < e_b(c^h - c^l - \psi'(e^h_o)) + \lambda_{hh} e^h_o \psi''(e^h_o) = \lambda_{hl}(1 - e^h_o) \psi''(e^h_o) = 0,$$
which brings us to a contradiction. Thus, $\lambda_{hl} > 0$ and $\psi'(e^h_o) = c_i - c^l$ (hence, $e^{hpf}_o = e^*_o$).

Assume now that $\lambda_{hl} > 0$ at the optimum. Then, by the same argument, it necessarily follows that $\lambda_{hh} > 0$ and $e^{hpf}_o = e^*_o$.

Lemma I.5 The solution of the problem (44) is such that $\mu > 0$ (i.e., $e^{lpf}_o = e^s_o$).

Proof. By the the first-order condition of the problem (44) with respect to $e^l_o$,

$$c^h - c^l - \psi(e^l_o) - e^l_o\psi'(e^l_o) - e_b(c^h - c^l - \psi(e^l_o)) - (\lambda_{hh} + \lambda_{hl})e^l_o\psi''(e^l_o) + \mu = 0.$$ 

It is straightforward to see that, if $\mu = 0$ (i.e., $e^{lpf}_o > e^s_o$), $e^{lpf}_o < e^l_p$. However, this brings us to a contradiction given that, by Proposition 4, $e^l_p < e^s_o$. Hence, at the optimum, $\mu > 0$ and $e^{lpf}_o = e^s_o$.

Proof of Proposition 6. By Lemmas I.4 and I.5, we have that the solution of the problem (44) is such that $e^{hpf}_o = e^*_o$ and $e^{lpf}_o = e^*_o$, respectively. Moreover, by Lemma I.4, the BCs are binding under both the partnership and sequential contracts in the states of the world \{q^h, c^h\} and \{q^h, c^l\}. Thus, substituting the binding BCs in the optimization conditions (40) and (45), we can write:

$$\phi'(e^{lpf}_o) - \phi'(e^{sf}_o) = l_c - l_b - l_o + (e^*_o - e^s_o)(c^h - c^l) - \psi(e^*_o) + \psi(e^s_o).$$

By the proof of Proposition 3, we know that $\delta > 0$. Thus, $e^{lpf}_o > e^{sf}_o$ if $l_c - l_b - l_o > -\delta$.

Proof of Corollary 3. By the assumption of Proposition 5, $l_c - l_b - l_o \geq 0$. Thus, by Proposition 6, $e^{lpf}_o > e^{sf}_o$ and $(1 - e^{sf}_o)(l_c - l_b - l_o) \geq 0$. Let us remark that, by the expression
(48) and by the optimization condition (45),

\[(S - k)(q^h - q^l) - F - l_c + kq^h + c^h - e_o^s(c^h - c^l - \psi'(e_o^s)) =
\]
\[= (S - k)(q^h - q^l) - \phi'(e_b^{pf}) + (e_o^s - e_o^{ps})(c^h - c^l) + \psi(e_o^s) - \psi(e_o^s) =
\]
\[= (e_b^{pf} + \lambda_{bh} + \lambda_{hh})\phi''(e_b^{pf}) > 0.
\]

Therefore, the social welfare differential (49) is strictly positive. \[■\]