Learning it the hard way:

Conflicts, economic sanctions and military aids*

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September 14, 2022

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*We thank the participants in the XI IBEO Conference in Alghero. We are grateful to Livio di Lonardo, Andrea Gallice, Ignacio Monzón, Massimo Morelli, Riccardo Saulle, and Scott Tyson for extremely useful discussions. The usual disclaimers apply.

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Abstract

We study the optimal intervention of a third country in a dispute between an attacker and a defender when there is uncertainty about the attacker’s military strength. The outcome of the dispute is informative about the attacker’s strength and affects its future aggressiveness. The third country has two tools: economic sanctions, that are public, and military aids, that the attacker does not observe. In equilibrium, the third country intervenes in one of three possible ways. It can (i) deter the attacker through a military pact with the defender; (ii) let the attacker conquer the defender’s territory, levying a moderate level of sanctions and providing no military aid; (iii) provide military aids and set a high level of sanctions. Whenever possible, the third country uses either (i) or (ii). By avoiding a militarized conflict, these types of intervention also avoid the increase in the attacker’s aggressiveness that a victory would cause. When the militarized conflict is unavoidable, the third country uses (iii): it sets a level of military aids that decreases the attacker’s probability of winning, while constraining its aggressiveness after a victory. We also show that fostering nationalism (defined as the commitment to fight for one’s independence) is a viable defensive strategy for the defender because it forces the third country to provide military aids.

Keywords: Conflict, economic sanctions, military aids.

JEL Codes: D74, D82.
1 Introduction

The Russian invasion of Ukraine fueled a heated debate on the role of the international community in this conflict. Should the US and the EU side with Ukraine or should they remain neutral? Were they to intervene, should they only levy economic sanctions or should they also provide military aids? Should NATO welcome new members and expand its boundaries to cope with Russian aggressiveness?

These questions are not new. Third parties’ intervention in interstate disputes are common and they have taken different forms throughout history. In the early 2000s, NATO enlarged to include countries that feared Russian aggressiveness. Before the onset of WWII, European democracies coped with German and Italian expansionary behaviors in a lenient way. This appeasement strategy, although ultimately ineffective, aimed at avoiding the escalation of local conflicts into a costly global war. In the summer of 1914, Great Britain was dragged into WWI by the German plan to attack Belgium and France. Economic sanctions complemented these various forms of military intervention. For instance, in the 1930s the League of Nations levied sanctions against the Italian fascist regime after it invaded Ethiopia. More recently, NATO countries reacted to Russian aggressiveness with increasingly severe sanctions.

In this paper, we study the scope of third country’s intervention in interstate disputes between an attacker and a defender. We characterize the optimal use of sanctions and military aids by the third country. We show that a defender under the threat of an aggression can boost its nationalism to obtain military aids from the third country.

In the model, an attacker and a defender engage in a dispute. The attacker must decide whether to attack or to remain peaceful. When there is an attack, the defender must decide whether to fight back or to surrender. If the defender fights back, a costly militarized conflict ensues. The outcome of the conflict depends on the military strength of the attacker, which is either high or low. Such military strength is uncertain to all countries, including the attacker itself. The relative strength of an army may be difficult to evaluate ex-ante, especially if the army seldom engages in
conflicts. In authoritarian regimes, leaders may also receive overly optimistic reports concerning the quality of their army, while other countries may lack reliable information due to secrecy and censorship.

The outcome of the conflict provides a noisy public signal about the attacker’s military strength. The attacker’s military confidence is the probability the attacker assigns to its military strength being high. A victory in the conflict inflates the attacker’s military confidence above the prior; a defeat dampens it below the prior.

A third country can intervene in the dispute to help the defender. The payoff of the third country depends negatively on the attacker’s military confidence. This negative impact becomes increasingly larger as the military confidence of the attacker grows. Indeed, when its military confidence increases, the attacker becomes more demanding in its interactions with the third country. The third country’s payoff is then decreasing and concave in the attacker’s military confidence. The attacker can also undergo a regime change with a moderate leader replacing an aggressive one. The third party’s payoff is increasing in the probability of such an event.

The third country has two different tools at its disposal: economic sanctions and military aids. Economic sanctions are often the result of a public discussion (e.g., a parliamentary debate) and they constitute a burden for the economies of the countries that levy them (e.g., they reduce trade flows with the target country). Furthermore, although sanctions weaken the long-run industrial production of the target country, they do not significantly disrupt its short-run military capability. Military aids, instead, are less costly for the international community, especially if they take a mild form (e.g., military training, sharing of intelligence briefings, provision of military equipment). In addition, the amount and specific details of such aids are often classified. Finally, military aids directly help the defender prevail in the military conflict. We model these features assuming that economic sanctions are costly for the third country, are perfectly observable, do not affect the outcome of the conflict, but they increase the likelihood of a regime change. Military aids, instead, are unobservable, have a negligible cost and directly reduce the probability that the attacker defeats
the defender.¹

An increase in military aids affects the payoff of the third country through two channels. First, as highlighted above, it increases the probability the attacker loses the conflict. When the attacker loses, its military confidence decreases and the payoff of the third country thus increases. Second, after an increase in military aids, the updated military confidence of the attacker increases more after a victory and decreases less after a defeat. This change in the updating lowers the expected payoff of the third country.

The attacker does not observe the level of military aids. The updating thus reacts to the attacker’s beliefs about the level of military aids rather than to their actual level. This poses a commitment problem. Holding the beliefs of the attacker constant, the third country wants to increase the level of military aids: higher military aids increase the third country’s payoff through the first channel, and they do not affect the second channel. The attacker then believes that military aids are high, which hurts the third country. To overcome this problem, the third country publicly commits at the beginning of the dispute to a maximum level of military assistance. These commitments are common. For instance, while pledging the support of the US to Ukraine, President Biden publicly declared: "We will not fight a war against Russia in Ukraine. Direct confrontation between NATO and Russia is World War Three, something we must strive to prevent".² At the same time, if the third country wants to credibly deter attacks, it must commit to a lower bound on the level of military aids. Defensive alliances such as the NATO are an example of this type of commitment.

In equilibrium, the third country intervenes in the conflict in one of three ways. When the attacker’s cost of fighting is high enough, the third country commits to a level of military aids that deters the attack and thus preserves peace. This is the logic behind strong and credible international defense alliances.

When the attacker’s cost from the militarized conflict is low, instead, deterrence is not a viable

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¹Section 2.1 discusses how to relax the assumptions concerning the cost and observability of military aids.
strategy: the attacker attacks even if the third country commits to help the defender. Two cases are then possible.

In the first case, the defender surrenders unless it receives sufficiently high military aids. We then deem the defender yielding. When the defender is yielding, the third country must choose whether to provide enough military aids to push the defender to fight back, or to let it surrender. The former option is a risky gamble for the third country: with some probability, the conflict ends with a victory (defeat) of the attacker and the attacker’s military confidence goes up (down). The latter option is instead safe: the military confidence of the attacker remains constant and equal to the prior. Because the payoff of the third country is decreasing and concave in the attacker’s military confidence, the third country prefers the latter option. The defender then surrenders, the attacker does not learn anything about its military strength and economic sanctions are optimally set based on the initial military confidence. This is an appeasement strategy: the international community makes concessions to the attacker and it does not help the defender. Such concessions avoid the gamble over the attacker’s military confidence.

In the second case, the defender fights back even if the third country provides no military aid. We deem the defender not yielding. The conflict between the attacker and the defender (hence, the gamble over the attacker’s military confidence) is unavoidable. The third country thus intervenes in the conflict with a level of military aids that trade-offs the increase in the attacker’s probability of losing the conflict against the risk of inflating its military confidence. As explained above, the military conflict is a risky gamble that hurts the third country in expectation. To compensate for this, the level of economic sanctions is higher than in the case in which the defender is yielding. The third country is dragged to war: the aggressiveness of the attacker and the readiness to fight back of the defender force the third party to intervene.

Our analysis delivers several predictions concerning the behavior of the countries. First, defense alliances deter attacks and preserve peace only when the cost of the militarized conflict for the attacker is sufficiently high.

Second, a third country may commit both to a minimum and to a maximum level of military
aids. The commitment to a minimum level deters attacks. The commitment to a maximum level constrains the attacker’s updating and thus prevents its military confidence to jump up excessively after the conflict.

Third, the level of economic sanctions and of military aids deployed in the conflict are positively correlated. When the third country sets a high level of military aids, the military confidence of the attacker jumps up significantly after a victory. The third country hedges against this risk by increasing the level of sanctions so as to favor a regime change.

Fourth, the third country’s engagement in the conflict is higher when the defender is willing to fight back; that is, when the defender is not yielding. This has an important implication. To obtain military aids and thus to improve the likelihood of preserving its independence, the defender can boost its willingness to fight back. For instance, it can adopt a rhetoric that highlights the value of fighting for the homeland and that reinforces national identity. This boost in nationalism turns the defender from yielding to not yielding and forces the third country to provide military aids.

Fifth, military aids are non-monotonic in the expected military strength of the attacker. When the defender is not yielding, the level of military aids increases with the attacker’s expected military strength. The third country wants to dampen the attacker’s military confidence through a defeat and it thus raises military aids. When the expected military strength of the attacker exceeds a certain threshold, though, the defender becomes yielding. The level of military aids then drops. The third country prefers the attacker to maintain its prior military confidence and it adopts an appeasement strategy. This non-monotonic comparative static can explain why the international community does not always play an active role in disputes and it sometimes disregards (or pays little attention to) conflicts in which the defender is weak.\(^3\)

Finally, we consider an extension of our model in which the efficacy of sanctions in overthrowing a regime also depends on the outcome of the conflict: it is lower when the attacker wins the conflict than when it loses it. In this case, the optimal intervention of the third country involves

\(^3\)For instance, the international community did not intervene to help Georgia against the Russian invasion in 2008.
less economic sanctions and more military aids. The level of economic sanctions is lower because sanctions are less effective after a victory of the attacker. In response to this lower efficacy, the third country increases the level of military aids: higher military aids improve the winning odds of the defender and thus improve the expected efficacy of sanctions.

1.1 Literature Review

Third party interventions aimed at ending conflicts and achieving lasting peaceful settlements have become a major priority for the international community. The literature identified three main instruments to achieve these goals: mediation, economic sanctions, and military intervention (including military aids or peace-keeping operations). Rohner (2022) offers a recent and comprehensive review of the state of art of third-party policies to foster peaceful settlements, in particular in the presence of civil conflicts.

Our paper fits in this broad literature and focuses on two of the three tools mentioned above: economic sanctions and military intervention. In this respect, it complements Meirowitz et al. (2019) that focuses on the role of mediation. In Meirowitz et al. (2019), a third party collects and optimally releases information to the parties involved in the dispute. This form of mediation can bring peaceful settlements, but only if it is designed carefully. Mediation protocols that reduce the likelihood of open wars may backfire: they can promote militarization by removing the penalty associated with this process, namely war. This can result in more conflicts.

Our paper studies how the international community can combine military aids and sanctions to dampen a country’s aggressiveness and it highlights an alternative channel through which military aids can foster aggressive behavior rather than dampening it.

When it comes to evaluate the effectiveness of the international community’s interventions in

\footnote{Other papers that study the role of mediators to avoid conflicts are Kydd (2003), Fey and Ramsay (2007) and Hörner et al. (2015). Instead, Baliga and Sjöström (2004) and Ramsay (2011) study the role of direct communication as a tool to avoid costly conflict.}

\footnote{On the interplay between militarization and the probability of conflict, see also Meirowitz et al. (2008) and Jackson and Morelli (2009).}

\footnote{Narang (2015) provides empirical evidence on the role of military aids in the presence of uncertainty concerning the strength of the contenders.}
conflicts, there are two opposing views. Some scholars believe that interventions (or their mere threat) can avoid wars and atrocities by discouraging aggressive behaviors. Others counterargue that the aids from a third-party encourage weak minorities moral hazard behavior, thus effectively prolonging the conflict and its costs. Kydd and Straus (2013) builds a model that encompasses these two opposite views and provides conditions under which the intervention of a third party reduces the likelihood of a conflict (see also Leeds, 2003, Spaniel, 2018, and Abu-Bader and Ianchovichina, 2019). In our paper, the commitment to military aids can deter aggressive behavior and preserve peace.\textsuperscript{7} When deterrence is not possible, however, the provision of military aids still helps the defender, but it could backfire against the international community inflating the attacker’s future aggressiveness.

In our analysis, the international community is a unitarian agent that benefits from a regime change in the attacker’s country. Eguia (2022) provides a theoretical model to study how multilateral interventions (or lack thereof) affect the stability of a regime.\textsuperscript{8} Insofar we focus on the role of third countries to favor regime changes, we complement Bueno de Mesquita (2010) that looks at the behavior of internal revolutionary vanguards.

In our model, the defender has an incentive to boost its nationalism. Indeed, as the defender becomes not yielding, the third country is dragged to war and provides military assistance. This links our paper to Smith (2019) that highlights how the aggressiveness of a country can drag its allies to war. For the same reason, our paper is also related to the literature that highlights how external threats favor the raise of nationalistic sentiments (see, for instance, Baum, 2002, Hjerm and Schnabel, 2010, Helms et al., 2020 and Gehring, 2022). Whereas most of this literature highlights psychological channels behind the raise of nationalism, our paper puts forward a complementary, strategic mechanism: a boost in nationalism is a viable defensive strategy to force the international community to intervene in the conflict.

Our model shows that the uncertainty about the attacker’s military strength can reduce third

\textsuperscript{7}In Spaniel (2020), the provision of military aids to a potentially weak defender reduces the temptation of an uninformed attacker to be aggressive. As a result, military aids help preserve peace.

\textsuperscript{8}On the role of the international community to weaken an hostile regime, see also De Bassa et al. (2021).
party’s incentives to intervene in the conflict. This relates our work to the extensive literature on the role of uncertainty in conflicts and crisis bargaining (see Fearon, 1995 for a fundamental contributions and Bas and Schub, 2017 for a recent review).\(^9\) Within this literature, we are particularly related to Powell (2004) that studies a bargaining model in which learning about military strength occurs as conflict unfolds. We differ from Powell (2004) in several respects.\(^10\) First, we assume that all players are uncertain about the military strength of the attacker. Second, we study how this uncertainty affects the intervention of a third country in the conflict. Third, we let the third country’s military aids affect the learning process and we show how this can inflate the attacker’s military confidence. Finally, we show that uncertainty concerning the attacker’s military strength can push a third country to avoid conflicts. In this respect, we complement the existing literature showing that the uncertainty about military capabilities may reduce the length of conflicts at the expenses of the defender rather than prolonging it.\(^11\)

Meirowitz et al. (2022) studies the link between third party intervention and militarization. In their bargaining model, militarization is a hidden action. They show that third party intervention can have negative, unintended consequences: interventions that lower the destructiveness of war may increase the probability of conflict. Furthermore, increasing the cost of arming may make destructive wars more likely. We share with Meirowitz et al. (2022) the idea that the deployment of military equipment is not fully observable and may be counterproductive (on a related note, Baliga et al., 2020 study a model in which the identity of the attacker is unobservable as well). Yet, we differ from them as we focus on the interaction between military aids and economic sanctions to prevent costly disputes.

In our model, the third country finds optimal to commit to a certain range of military aids

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\(^10\)For models in which learning about the contestants’ military strength occurs during the conflict, see also Spaniel and Bils (2018).  
\(^11\)War-of-attrition models also study the role of learning during conflicts. See Abreu and Gul (2000) for theoretical contribution and Ghosh et al. (2019), Krainin et al. (2020), and Menuet and Sekeris (2021) for application to political economy.
because military aids are not observable and they can inflate the attacker’s military confidence. This links our paper to Fearon (1997) and to the literature on signaling in international disputes that follows from this seminal contribution. We differ from these papers, because, in our model, all countries are equally informed about the military strength of the attacker. This enables us to abstract from signaling motives.

Our paper also discusses the role that military aids and defense pacts can play to deter aggressive behaviors. In this respect, we are close to the literature on strategic deterrence (see Snyder, 1984 for an early contribution and Yuen, 2009, Benson, 2012, Benson et al., 2014 for more recent work on this topic). In this literature, excessive military aids are counterproductive because they induce moral hazard behavior on the defender’s side. On a related vein, Di Lonardo and Tyson (2022) shows that the political instability of the attacker’s regime may dampen the efficacy of deterrence. In our model, military aids can turn the defender from yielding to not yielding and thus induce a costly conflict. This may foster the aggressiveness of the attacker in the future. The third party thus supplements military aids with economic sanctions that favor the regime change.12

The explicit account of economic sanctions relates our work to the extensive literature that regards them as a tool to weaken the stability of hostile regimes (see, among others, Marinov, 2005, and, specifically for authoritarian regimes, Escribà-Folch and Wright, 2010).13 A recent paper that investigates the role of sanctions in interstate disputes is Baliga and Sjöström (2022). In Baliga and Sjöström’s model, sanctions can be of two types. Targeted sanctions (e.g. blocking financial assets) hurt the leadership of a country, while comprehensive sanctions hurt citizens and create social turmoil. Targeted sanctions align the interests of the leadership with those of the citizens. This generates a “rally ’round the flag” effect and reduces social turmoil. Comprehensive sanctions, instead, increase social turmoil, but entail political and moral costs.14 Thus, targeted

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12 Another related paper is Chassang and Padró I Miquel (2010) that studies deterrence with and without strategic risk using a global game.
13 Foreign economic interventions can also occur via rewards rather than sanctions or military intervention. See, for instance, Aïdt and Albornoz (2011).
14 Several scholars highlighted how sanctions dampen living standards in the target country (Cortright et al., 1997) and increase violence (see Peksen, 2019 and Wood, 2008).
sanctions are optimal when social unrest is unlikely. This is the relevant case in totalitarian regimes. Our model differs insofar it focuses on the optimal combination of (comprehensive) sanctions and military aids.\textsuperscript{15} Finally, McCormack and Pascoe (2017) focuses on the effect of economic sanctions in weakening the future military power of a country. We share with McCormack and Pascoe (2017) the idea that sanctions have long-run effects, but we complement their analysis by adding another policy tool (military aids) that has a short-term impact on conflicts.

2 The model

Three risk-neutral countries engage in a dispute. An attacker (country $A$) and a defender (country $D$) are on the verge of a military conflict. A third country (country $I$) can intervene to support the defender.

At time $t = 0$, country $I$ publicly commits to a range of possible military aids in favor of country $D$, $[m, m] \subseteq [0, M]$, where $M$ is the highest feasible level of military aid. Military aids can take different levels. Country $I$ can train country $D$’s army, share intelligence information, supply military equipment, or include country $D$ in a military alliance.\textsuperscript{16} The commitment of country $I$ becomes common knowledge as soon as it occurs.

At time $t = 1$, country $A$ decides whether to remain peaceful ($a = 0$) or to attack country $D$ ($a = 1$). If country $A$ remains peaceful, the game ends. Countries $A$ and $I$ get a payoff of 0, while country $D$ enjoys a payoff of $\Psi > 0$. We refer to $\Psi$ as to the value of country $D$’s independence.

If country $A$ attacks, the game moves to time $t = 2$. Country $I$ chooses the level of sanctions $s$ to levy against country $A$ and the level of military aids $m$ to deploy in the conflict. The level of sanctions lies in the interval $[0, 1]$; the level of military aids, instead, is constrained by the previous

\textsuperscript{15}We differ from Baliga and Sjöström (2022) also in the source and the role of incomplete information. In Baliga and Sjöström (2022), citizens are imperfectly informed about the aggressiveness of a foreign country and this creates a conflict of interest with their own leader. When the foreign country is peaceful, citizens prefer an agreement, while their leader may still prefer to fight. In our model, instead, there is incomplete information about the attacker’s military strength and military aids affect the updating about this characteristic.

\textsuperscript{16}In the recent Ukraine conflict NATO’s public declarations against the enforcement of a no-fly zone or the deployment of troops set an upper bound $\overline{m}$ on the levels of military aids. On the contrary, the commitment to supply military equipment (e.g., drones) sets a lower bound $\underline{m}$ on the level of military aids.
commitment: $m \in [m, \bar{m}]$. Country I thus chooses a pair $(s, m) \in [0, 1] \times [m, \bar{m}]$. Sanctions and military aids differ in two dimensions. First, sanctions are observable and common knowledge, while military aids are observed by country $D$, but not by country $A$. We let $\tilde{m}$ denote the belief of country $A$ about the level of military aids $m$.\footnote{The belief $\tilde{m}$ is thus a cdf over the set $[m, \bar{m}]$.} Second, sanctions and military aids have different effects. Sanctions do not directly affect the outcome of the conflict and they are costly for country $I$. We represent the cost of sanctions through function $s \mapsto \frac{\kappa}{2}s^2$ with $\kappa \geq 1$. Military aids, instead, directly affect the outcome of the conflict (see below) and they do not entail any cost for country $I$ (on this latter assumption, see the discussion in Section 2.1).

At time $t = 3$, country $D$ observes the pair $(s, m)$ and decides whether to surrender ($d = 0$) or to defend ($d = 1$). If country $D$ defends, a conflict ensues. Country $A$ wins the conflict with probability $\max\{0, \theta - m\}$, where $\theta \in [0, 1]$ denotes the military strength of country $A$. Otherwise, country $A$ loses the conflict.

The conflict is costly for both countries. We let $c_A > 0$ and $c_D > 0$ denote the cost of the conflict for countries $A$ and $D$. If country $A$ wins the conflict, it gets a payoff of $1 - c_A$ and country $D$ gets a payoff of $-c_D$. If country $A$ loses the conflict, country $A$ gets a payoff of $-c_A$ and country $D$ gets a payoff equal to the value of independence net of the cost of fighting, $\Psi - c_D$. Finally, if country $D$ surrenders, country $A$ gets a payoff of 1 and country $D$ gets a payoff of 0. In this case we say there is an annexation. Country $I$ gets no payoff from the conflict. We focus on the interesting case in which country $D$’s value of freedom is higher than its cost of fighting: $\Psi > c_D$.\footnote{If the opposite inequality holds, country $D$ would surrender no matter how much military aids $m$ country $I$ deploys. The analysis would then be straightforward.}

The military strength of country $A$ is uncertain: it is equal to $\bar{\theta}$ with probability $p$ and to $\theta < \bar{\theta}$ with probability $1 - p$. The expected strength of country $A$ is thus equal to $\mathbb{E}[\theta] \equiv p\bar{\theta} + (1 - p)\theta$. The military confidence of country $A$ is the probability that country $A$’s military strength is equal to $\bar{\theta}$. The initial military confidence of country $A$ is equal to $p$. We assume that the outcome of the conflict is always uncertain: the probability that country $A$ wins the conflict, $\theta - m$, is always greater than 0 and lower than 1 no matter what $\theta$ and $m$ are.
A does not attack | A attacks
---|---
| D surrenders | D defends itself | A wins | D wins

| Country A | 0 | 1 + [1 − ϕs]v(p) | 1 − c_A + [1 − ϕs]v(ˆp) | −c_A + [1 − ϕs]v(ˆp) |
| Country D | Ψ | 0 | −c_D | Ψ − c_D |
| Country I | 0 | −[1 − ϕs]p − ˘s^2 | −[1 − ϕs]ˆp^2 − ˘s^2 | −[1 − ϕs]ˆp^2 − ˘s^2 |

**Table 1**: Payoffs of countries

**Assumption 1.** *The outcome of the conflict is uncertain: 0 < M < θ < 1.*

Once country D decided whether to defend itself and (possibly) the conflict is over, the game moves to time \( t = 4 \). At time \( t = 4 \) country D gets no additional payoff, while country A and country I do. In particular, country A may experience a regime change. This happens with a probability that depends on the level of sanctions. When the level of sanctions is equal to \( s \), the probability of a regime change is equal to \( ϕs \), with \( ϕ \in \mathbb{R}_+ \) representing the efficacy of sanctions.

**Assumption 2.** *Sanctions cannot guarantee a regime change: \( ϕ \in [0, 1) \).*

Throughout the paper, we assume that Assumptions 1-2 hold.

When country A experiences a regime change, its additional payoff (on top of the payoff derived from the conflict) is equal to 0 and the same is true for country I. If there is no regime change, the expected payoff of country A and country I depends on the updated military confidence of country A, denoted with \( ˆp \). Such updated military confidence plays a key role in our analysis. The payoff of country A is equal to \( v(ˆp) \) for some positive and increasing function \( v \). The payoff of country I is equal to \( −p^2 \) (see Section 2.1 for a discussion of this payoff structure).

The payoffs of the countries are then the sum of the payoffs obtained during the conflict between country A and country D and the payoff obtained once this conflict is over. Table 1 summarizes these payoffs.

The outcome of the conflict is informative about the military strength of country A. Let \( ˆp_0(m) \) and \( ˆp_1(m) \) be the updated military confidence of country A when it loses and wins the conflict.
Bayes rule implies that

\[ \hat{p}_0(m) = \frac{1 - \overline{\theta} + m}{1 - \mathbb{E}[\theta] + mp} \quad \text{and} \quad \hat{p}_1(m) = \frac{\overline{\theta} - m}{\mathbb{E}[\theta] - mp}. \]  

(1)

It is straightforward to show that \( \hat{p}_1(m) > p > \hat{p}_0(m) \): a victory in the conflict boosts country A’s military confidence, while a defeat dampens it. Moreover, both \( \hat{p}_0(m) \) and \( \hat{p}_0(m) \) are increasing in \( m \). If country D surrenders, countries do not learn anything about country A’s military strength. Thus, its military confidence is equal to the prior \( p \).

Figure 1 summarizes the timeline of the model. We use Perfect Bayesian Nash Equilibrium as solution concept and we refer to it simply as the equilibrium of the game.\textsuperscript{19}

2.1 Discussion

Military strength of country A. Country A’s military strength represents the probability that country A wins the conflict against country D absent any military aid. This linear formulation simplifies the algebra and enables us to derive a closed form solution for the optimal commitment of country I at time \( t = 0 \). Yet, the equilibrium characterization discussed in Section 3 extends to any setting in which (i) the outcome of the conflict depends on the military strength of country A, (ii) country A infers its military strength based on the outcome of the conflict, and (iii) country I cannot guarantee the victory of country D through military aids.\textsuperscript{20} For instance, we could use a

\textsuperscript{19}Because country A is uncertain about its own military strength, out-of-equilibrium beliefs are trivial.

\textsuperscript{20}In our model, this last point is guaranteed by Assumption 1. The logic of our results extends to the case in which military aids can guarantee the victory of country D against a weak country A (\( \theta = \overline{\theta} \)), but not against a
Tullock contest function. In this case, country A’s probability of winning in the conflict would be equal to: $\frac{\theta_A}{(\theta_A + \theta_D + m)}$, where $\theta_A \in \{\theta, \bar{\theta}\}$ is the (uncertain) military strength of country A and $\theta_D$ is the military strength of country D. Country D’s probability of winning in the conflict would instead be equal to $(\theta_D + m)/(\theta_A + \theta_D + m)$.

**Payoffs of country A and country I.** Country A and country I get an additional payoff at time $t = 4$ which depends on the military confidence of country A. In particular, the payoff of country A is increasing in its military confidence, $v(p)$, while the payoff of country I is decreasing and concave in military confidence, $-p^2$. The specific functional form for the expected payoff of country I provides analytical tractability, but our analysis extends to any other strictly decreasing and strictly concave function. The concavity of the payoff of country I in the military confidence of country A is sensible. Country A could demand increasingly higher concessions from country I as its military confidence increases. Or it can become increasingly more aggressive. The risk of this escalation in aggressiveness resonates into NATO Secretary General Jens Stoltenberg’s words: “So, even if you don’t care about the moral aspect of this, supporting the people of Ukraine, you should care about your own security interest. So therefore, you have to pay; pay for the support, pay for humanitarian aid, pay the consequences of the economic sanctions, because the alternative is to pay a much higher price later on”. In Appendix C, we provide a simple model of conflict that justifies the concavity of country I’s payoff in country A’s military confidence.

**Military aids.** In our model, military aids have no monetary cost. Nonetheless, they may inflate the military confidence of country A and thus lower country I’s payoff. In this respect, we provide a non-monetary rationale for the limited intervention of the international community in conflicts. Instead, if military aids can guarantee the victory of country D against both types of country A, country I would optimally set $m$ high enough to deter country A’s attack.


22 If the payoff of country I at time $t = 4$ was linear (respectively, strictly convex) in the military confidence of country A, our proofs techniques would extend. Differently from the results we derive below, country I would now be indifferent between deploying military aids and not doing so (respectively, would strictly prefer deploying military aids) when country D is yielding (see Definition 1 below). Furthermore, the level of sanctions would be the same (higher) after an annexation than after the conflict.
If military aids were costly, their cost would provide an additional reason to bound the level of military aids. Our insights would remain valid as long as country \( I \) cares about country \( A \)’s military confidence. In the model we also assume that country \( A \) does not observe the level of military aids actually deployed by country \( I \). Section 5.2 extends our analysis to the case in which country \( A \) imperfectly observes the level of military aids.

**Commitment of country \( I \)** Country \( I \) can commit both to a lower bound and to an upper bound on the level of military aids. These types of commitment occur in practice. The commitment to a lower bound happens, for instance, through contracts for the supply of military equipment, active sharing of intelligence reports, or international agreements and alliances. The commitment to an upper bound, instead, happens by setting clear bounds on the engagement in the conflict either formally, through votes in legislative assemblies, or informally, through public declarations that entail a popularity cost if reneged. We assume that commitment is full: country \( I \) cannot choose a level of military aids outside \([m, \bar{m}]\). Our analysis extends immediately to the case in which country \( I \) can renge on its previous commitment, but this behavior is sufficiently costly.

3 **Equilibrium Analysis**

At time \( t = 4 \), country \( D \)’s payoff depends only on its dispute with country \( A \) (see Table 1). Instead, the expected payoffs of country \( A \) and country \( I \) also depend on the level of sanctions and on the updated military confidence of country \( A \). The functions \( s \mapsto V_A(s \mid \hat{p}) \) and \( s \mapsto V_I(s \mid \hat{p}) \) represent these expected payoffs.

The expected payoff of country \( A \) is increasing in its updated military confidence and decreasing in the level of sanctions. The expected payoff of country \( I \) is instead decreasing and concave in the updated military confidence of country \( A \) and it is first increasing and then decreasing in the level of sanctions.\(^{23}\) These properties guarantee that the squared brackets in Table 1 are always

\(^{23}\)The U-shaped behavior of \( V_I(s \mid \hat{p}) \) with respect to the level of sanctions \( s \) follows from \( s \in [0, 1] \) and \( \phi < 1 \).
greater than 0 and lower than 1. They represent the probabilities with which country A’s regime survives. We solve the model backward starting from country D’s decision at time $t = 3$.

### 3.1 The choice of country D

At time $t = 3$, country D observes the choice $(s, m)$ of country I and decides whether to defend itself. Country D’s payoff depends only on the dispute with country A. It then compares the expected benefit from defending with the cost of fighting, $c_D$. Importantly, the expected benefit depends on the level of military aids, $m$, and on the value of independence, $\Psi$.

**Proposition 1.** Suppose country I chooses a pair $(s, m)$. Then, in equilibrium, country D defends itself if and only if $(1 - E[\theta] + m)\Psi \geq c_D$, or alternatively if and only if

$$m \geq \max \left\{ \frac{c_D}{\Psi} - 1 + E[\theta], 0 \right\} := m^\dagger.$$

All proofs are in the Appendix. By Proposition 1 country D can be one of two types. When $m^\dagger = 0$, country D defends itself independently of the level of military aids it receives. In this case, we say that country D is not yielding. On the contrary, when $m^\dagger > 0$, country D defends itself if and only if the level of military aids are sufficiently high. In this case, we say that country D is yielding. The type of country D is common knowledge.

**Definition 1.** Country D is yielding if $m^\dagger > 0$. Country D is not yielding if $m^\dagger = 0$.

### 3.2 The choice of country I on the level of sanctions and military aids.

At time $t = 2$, country I chooses the level of sanctions $s \in [0, 1]$ and of military aids $m \in [m, m]$. We start characterizing the optimal behavior of country I when country D is not yielding.

---

24 We assume that, whenever indifferent, country D defends itself. This tie-breaking rule plays no role in our analysis.
Proposition 2. Suppose that country I committed to a range of military aids equal to \([m, \bar{m}]\), and that country D is not yielding. Then, in equilibrium, country I chooses the pair \((\bar{m}, s^*_w(\bar{m}))\), where

\[
s^*_w(\bar{m}) = \frac{\phi}{k} \left[ (\hat{p}_1(\bar{m}))^2 (E[\theta] - \bar{m}) + (\hat{p}_0(\bar{m}))^2 (1 - E[\theta] + \bar{m}) \right].
\]

When country D is not yielding, country I sets military aids as high as possible; that is, \(m = \bar{m}\). To understand why, note that military aids affect country I’s payoff through two channels. First, military aids increase the probability that country A loses the conflict against country D. After a defeat, the updated military confidence of country A goes down. This improves country I’s expected payoff. Second, military aids modify the magnitude of the updating after the conflict. The updated military confidence of country A jumps up after a victory in the conflict and jumps down after a defeat. The size of the jump up is increasing in the level of military aids, while the size of the jump down is decreasing in it. This second channel thus lowers the expected payoff of country I. Because military aids are not observable, the first channel depends on the actual level of military aids deployed by country A, \(m\), while the second channel depends on the beliefs of country A about such level, \(\tilde{m}\). In equilibrium, country I then deploys a level of military aids equal to \(\bar{m}\). It this was not the case, country I could improve its payoff by deviating to \(\bar{m}\): by secretly deploying more military aids, country I would decrease the winning odds of country A without affecting the magnitude of the updating.

The optimal level of sanctions equates the marginal benefit of sanctions to their marginal cost. The marginal benefit of sanctions comes from the increase in the probability of a regime change. This benefit is large when the effectiveness of sanctions \(\phi\) is large and when the expected cost of country A’s future behavior for country I—the squared bracket in the expression for \(s^*_w(\bar{m})\)—is high. The marginal cost of sanctions is instead linear in the parameter \(k\).

Now suppose the country D is yielding; that is, \(m^\dagger > 0\). In this case, the optimal behavior of country I depends on its previous commitment. When \(\bar{m} < m^\dagger\), the military aids of country I cannot induce country D to defend itself. Thus country D surrenders. Because military aids are
costless, country $I$ is indifferent among every $m$ in the range $[\underline{m}, \overline{m}]$. We break this indifference assuming that country $I$ chooses the lowest possible level of military aids, $m = \underline{m}$. 25 Because country $D$ surrenders, the optimal level of sanctions equates the marginal benefit of levying sanctions to its marginal cost when the update military confidence of country $A$ is equal to the prior. To sum up, when $\overline{m} < m^\dagger$, country $I$ chooses a level of military aids equal to $\overline{m}$ and a level of sanctions equal to $s^*_\emptyset = \frac{\phi}{k} p^2$.

When $\underline{m} > m^\dagger$, the military aids of country $I$ always induce country $D$ to defend itself. The same logic behind Proposition 2 then applies. Country $I$ chooses a level of military aids equal to $\overline{m}$, and a level of sanctions equal to $s^*_w(\overline{m})$.

Consider now the most interesting case in which $m^\dagger \in [\underline{m}, \overline{m}]$. If country $I$ deploys enough military aids ($m \in [m^\dagger, \overline{m}]$), country $D$ defends itself. Otherwise ($m \in [\underline{m}, m^\dagger]$), country $D$ surrenders. What is the optimal behavior of country $I$? The conflict between country $A$ and country $D$ is a risky gamble for country $I$. This gamble pays off with expected probability $E[\theta] - m$. In this case, country $A$ loses the conflict and its military confidence falls below the prior $p$. This increases the expected payoff of country $I$. The gamble is not successful with complementary probability, $1 - E[\theta] + m$. In this case, country $A$ wins the conflict and its military confidence jumps above the prior $p$. This decreases the expected payoff of country $I$. When country $D$ surrenders, instead, the military confidence of country $A$ remains constant and equal to the prior $p$. Proposition 3 states that country $I$ prefers the safe option to the risky gamble.

**Proposition 3.** Suppose that country $I$ committed to a range of military aids equal to $[\underline{m}, \overline{m}]$ and that country $D$ is yielding. Then, in equilibrium, country $I$ chooses the level of sanctions and military aids as follows:

1. If $m^\dagger > \underline{m}$, then $(m, s) = (\overline{m}, s^*_\emptyset)$ where $s^*_\emptyset = \frac{\phi}{k} p^2$.

2. if $m^\dagger \leq \underline{m}$, then $(m, s) = (\overline{m}, s^*_w(\overline{m}))$.

25 We can justify this tie-breaking rule in several ways. For instance, we can impose some (small) cost on the provision of military aids. Or we can assume that when country $D$ surrenders, country $A$ seizes the military aids and uses them to inflate its military strength. Other tie-breaking rules would only make the statements of our results more cumbersome.
The message of Proposition 3 is simple. Country I prefers country D to surrender than to defend itself. Thus, country I never deploys a level of military aids \( m \geq m^\dagger \) unless its previous commitment forces it to do so. This happens even though military aids are costless and they decrease the probability that country A wins. As discussed above, country I regards the conflict as a gamble that can either inflate or dampen the military confidence of country A. Bayesian updating implies that the expected military confidence after this gamble is equal to the prior. The payoff of country I is decreasing and strictly concave in the military confidence of country A. Jensen inequality then implies that country I is worse off under the gamble than under the safe alternative option in which country A maintains its initial military confidence. To sum up, whenever the previous commitment allows it, country I prefers to deploy a level of military aids below \( m^\dagger \) and a level of sanctions equal to \( s^*_0 \). Our tie-breaking rule then implies \( m = m^\circ \). Instead, when the previous commitment forces country I to provide a level of military aids greater or equal to \( m^\dagger \), the logic of Proposition 2 applies: country I chooses the highest possible level of military aids and the level of sanctions is set equal to \( s^*_w(m) \).

Finally, country I levies a lower level of sanctions when country D surrenders than when it defends itself; that is, \( s^*_0 < s^*_w(m) \). To understand why, recall that the optimal level of sanctions equates country I’s marginal cost of sanctions with country I’s marginal benefit from a regime change. This benefit is decreasing in the payoff of country I at time \( t = 4 \), which, as discussed above, is lower after the conflict than after an annexation.\(^{26}\) Hence, the next corollary holds.

**Corollary 1.** Country I levies a higher level of sanctions when there is a conflict than when there is an annexation.

\(^{26}\)Formally, the squared bracket in Proposition 3 is the additive inverse of the expected payoff of country I when there is a conflict, while \( p^2 \) is the additive inverse of the payoff of country I when country D surrenders. Jensen inequality then implies:

\[
\left[(\hat{p}_1(m))^2(E[\theta] - m) + (\hat{p}_0(m))^2(1 - E[\theta] + m)\right] > \left[\hat{p}_1(m)(E[\theta] - m) + \hat{p}_0(m)(1 - E[\theta] + m)\right]^2 = p^2.
\]
3.3 The choice of country A.

At time $t = 1$, country $A$ takes the optimal behavior of country $I$ into account (see Proposition 2 and Proposition 3) and decides whether to attack or to remain peaceful.

**Proposition 4.** Suppose country $I$ committed to range of military aids equal to $[m, M]$. Country $A$ attacks with certainty if country $D$ is yielding and $\bar{m} < m^\dagger$. In the other cases, country $A$ attacks if and only if:

$$
(\mathbb{E}[\theta] - \bar{m}) + [1 - \phi s_w^*(\bar{m})] [(\mathbb{E}[\theta] - \bar{m})v(\hat{p}_1(\bar{m})) + (1 - \mathbb{E}[\theta] + \bar{m})v(\hat{p}_0(\bar{m}))] > c_A
$$

(2)

Country $A$ thus attacks with certainty when it expects country $D$ to surrender. This happens when country $D$ is yielding and the previous commitment allows country $I$ to deploy a level of military aids below $m^\dagger$. Otherwise, country $A$ decides whether to attack or not balancing out the expected benefit of an attack against its cost. The expected benefit of an attack is the sum of two components: the expected benefit associated to the conflict $(\mathbb{E}[\theta] - \bar{m})$ and the expected benefit associated to the continuation game $[(1 - \phi s_w^*(\bar{m})] [(\mathbb{E}[\theta] - \bar{m})v(\hat{p}_1(\bar{m})) + (1 - \mathbb{E}[\theta] + \bar{m})v(\hat{p}_0(\bar{m}))]$. The cost is instead equal to $c_A$.

3.4 The optimal commitment of country I.

We now consider the optimal commitment of country $I$ at time $t = 0$. The first best for country $I$ is to deter country $A$’s attack. In this case, country $I$ does not have to deal with country $A$’s aggressiveness and it does not incur any cost from levying sanction. Deterrence is feasible if there exists a level of military aids $\bar{m} \in [m^\dagger, M]$ such that inequality (2) fails. When this happens, country $I$ can optimally commit to a singleton $\bar{m} = \bar{m} = m_d$. This is the deterrence logic behind defense alliances like the NATO. The allies commit to provide military support to a country in case it is attacked. Such commitment deters the attack and guarantees a peaceful outcome.

When deterrence is not feasible, we can have one of two possible cases. When country $D$
is yielding, country $I$ commits to a set of military aids that allows him not to intervene in the conflict. In this case, we can assume without loss of generality that $\underline{m} = \overline{m} = 0$. This is the logic behind the *appeasement strategies* like the one enacted by European democracies with Germany before the onset of WWII. The international community does not intervene militarily in conflicts. An aggressive attacker easily achieves its expansionary goals and economic sanctions of moderate amount are the only punishment ($s = s^*_w$).

Instead, when country $D$ is not yielding, country $I$ commits to an upper bound on the level of military aids equal to what it will actually deploy in the conflict. The optimal commitment balances two countervailing forces. On the one hand, high military aids help country $D$ win the conflict and thus dampen country $A$’s military confidence. This increases the expected payoff of country $I$. On the other hand, the military confidence of country $A$ after the conflict is increasing in the level of military aids (see equations 1). The expected payoff of country $I$ then goes down. In this case, country $I$ is *dragged to war* like Britain in World War I. The international community intervenes in the conflict with military aids because if it does not, the attacker may obtain an easy win and this would boost its confidence in future interstate conflicts. In this setting, military aids are supplemented by a high level of economic sanctions ($s = s^*_w(\overline{m})$).

The next proposition summarizes the optimal commitment of country $I$ and concludes the equilibrium analysis.

**Proposition 5.** Consider the following inequality:

$$
(\mathbb{E}[\theta] - m) + [1 - \phi s^*_w(m)] [(\mathbb{E}[\theta] - m)v(\hat{p}_1(m)) + (1 - \mathbb{E}[\theta] + m)v(\hat{p}_0(m))] \leq c_A.
$$

(3)

The equilibrium commitment of country $I$ is described as follows.

1. **Deterrence.** If inequality (3) holds for some $m = m_d \in [m^\dagger, M]$, then country $I$ commits to $\underline{m} = \overline{m} = m_d$.

2. **Appeasement.** If inequality (3) fails for all $m \in [m^\dagger, M]$ and country $D$ is yielding, then
country I commits not to provide military aids: $m = \overline{m} = 0$.

3. **Dragged to war.** If inequality (3) fails for all $m \in [m^\dagger, M]$ and country $D$ is not yielding, then country I commits to a range of military aids equal to $[m, \overline{m}_w]$, where $\overline{m}_w = \min\{\max\{0, \mathbb{E}[\theta] - \frac{1}{2}\}, M\}$.

### 4 Comparative Statics and Testable Implications

The previous section characterizes the equilibrium behavior of countries in an international dispute. Our analysis highlights different ways in which the international community (country I) can intervene in the dispute. We now discuss some of the implications of our model.

Public opinion (see also Huth, 1998) often regards military aids as a rather extreme type of intervention in a dispute. As such, countries should provide military assistance only when the attacker (country $A$) is particularly aggressive and other, softer tools (e.g., economic sanctions) cannot restrain it. Our model suggests that this view may be too simplistic.

Economic sanctions and military aids are not just two types of intervention with different severity. Instead, they serve different purposes. Military aids help defeat the aggressor in the conflict. Economic sanctions foster discontent against the aggressor’s regime and thus contribute to its overthrown.

In our model, military aids are the main instrument to dampen the aggressor’s military confidence. The level of aids depends on the opportunity cost of fighting of the aggressor and on the defender’s (country $D$’s) resolve to defend itself. If the aggressor’s opportunity cost of fighting is high, then a defense pact between the international community and the defender can deter the aggressor’s attack. Instead, when its opportunity costs of fighting is low, the aggressor always attacks. In this case, the intervention of the international community depends on the defender’s resolve. If the defender is yielding, then the international community prefers to avoid a conflict and sets the level of military aids equal to zero. If the defender is not yielding and it defends itself in case of an attack, then the international community intervenes in the conflict to minimize the
probability that the aggressor’s prevails, while taking into account the effect that the intervention has on the aggressor’s updated military confidence.

**Remark 1.** The level of military aids deployed by the international community (country I) are increasing in the defender’s (country D’s) resolve to defend itself against an aggression. Furthermore, the international community never pushes a yielding country to defend itself against the aggressor’s (country A’s) attack.

Economic sanctions complement military aids. The international community sets higher economic sanctions against the aggressor when a conflict onsets than when there is an annexation (see Corollary 1). Indeed, when the conflict is unavoidable, the international community is dragged to war. In this case, the provision of military aids modifies the military confidence of the aggressor, which (in expectation) hurts the international community. The international community thus increases the level of sanctions to favor a regime change and partially offset this loss. On the contrary, when the international community does not provide military aids, the military confidence of the aggressor remains constant. The international community can thus set a lower level of sanctions.

**Remark 2.** When the aggressor attacks, the level of military aids and the level of economic sanctions are positively correlated.

How do the levels of sanctions and of military aids vary with the aggressor’s military strength? Previous research shows that the international community is less (more) likely to intervene when the aggressor (defender) is militarily strong (see Altfeld and Bueno De Mesquita, 1979, Werner and Lemke, 1997 and Corbetta, 2010). Although our model fundamentally confirms these findings, it also portrays a more complex picture. To see why, observe first that the international community deploys a positive level of military aids in the conflict if and only if the defender is not yielding; that is if and only if \( m^\dagger = 0 \). In this case, the level of economic sanctions is equal to \( s^\w (m) \). As the military strength of the aggressor increases, \( m^\dagger \) increases and the defender becomes yielding. In this case, the international community does not deploy military aids and it levies an intermediate level of economic sanctions (\( s^\w < s^\w (m) \)). Hence, in line with previous literature,
the international community is less likely to intervene in the conflict when the military strength of the aggressor increases. However, as long as the country $D$ is not yielding and the conflict ensues (i.e., as long as $m^\dagger = 0$), sanctions and military aids are both increasing in the expected military strength of the aggressor. When the international community intervenes in the conflict, its goal is to defeat the aggressor. The level of military aids thus increases with the aggressor’s expected military strength and the level of economic sanctions increases too (see Remark 2).

Overall, our model thus portrays a non-monotonic relationship between the aggressor’s strength and its level of intervention in the conflict: as the expected military strength increases, such level first increases and then decreases. This non-monotonicity can explain why the international community engages in lengthy and costly conflicts in some cases and it avoids to intervene in others. As such, it can guide empirical research on the type and scope of third party intervention in interstate disputes.

**Remark 3.** *If the aggressor’s attacks, the level of military aids and the level of economic sanctions deployed by the international community are non-monotonic in the aggressor’s military strength, $E[\theta]$: they first increase and then decrease in $E[\theta]$.***

Remark 1 above also has an important implication for the behavior of the defender. A country threatened by a hostile neighbor can react boosting its level of nationalism through propaganda (increase in $\Psi$) or increasing its military capacity (decrease in $c_D$). These behaviors fuel the defender’s resolve and increase the probability that a costly conflict ensues. However, their ultimate goal is actually defensive: nationalism and military build-up become a credible commitment device to fight back an aggression and to force the international community to provide military aids in case of an aggression. The conflict between Russian and Ukraine provides a recent example of this strategy. Following the Russian invasion of Crimea, Ukraine government significantly improved its military capacity and, at the same time, boosted nationalistic sentiments in the population and emphasized the cultural and historical differences with Russia (see Tamilina, 2021 and Yakymenko et al., 2019). In this respect, we highlight a strategic and non-psychological explanation for the
raise in nationalistic sentiment in the shadow of external threats (on the role of external threats on identity and nationalism, see Baum, 2002, Helms et al., 2020, and Gehring, 2022).

Remark 4. Increasing $\Psi$ or decreasing $c_D$ are viable defensive strategies for the defender. The level of intervention of the international community is increasing in $\Psi$ and decreasing in $c_D$.

Finally, our model highlights two separate reasons why the international community commits to the provision of military aids. On the one hand, the commitment to a lower bound on the level of military aids enables the international community to deter attacks. On the other hand, the commitment to an upper bound on the level of military aids disciplines the aggressor’s updating when military aids are unobservable. This twofold role for the commitment of the international community rationalizes not only the existence of credible defense pacts, but also public statements by international leaders that, while promising military assistance to a country under attack, explicitly bounds the extent of such assistance.

Remark 5. Commitments to minimum levels of military aids occur when the international community wants to deter attacks. Commitments to maximum levels of military aids occurs when military aids are unobservable and the international community provides military assistance.

5 Extensions

5.1 The effectiveness of sanctions

In our model, economic sanctions induce a regime change with probability $\phi_s$. The parameter $\phi \in (0,1)$ thus measures the marginal effectiveness of sanctions. A low value of $\phi$ corresponds to a situation in which the regime of country A is stable: the marginal impact of economic sanctions on the probability of a regime change is low.

The popularity of a regime may also depend on the regime’s foreign policy successes or failures. Foreign policy successes make a regime more popular and stable. Foreign policy failures have the opposite effect. We can capture the link between foreign policy and the regime’s stability as
follows. Let $\phi_s$ be the probability of a regime change when the level of sanctions is $s$ and country $A$ loses the conflict. Instead, let $\frac{\phi_s}{\lambda}$ with $\lambda > 1$ be the probability of a regime change when the level of sanctions is $s$ and either country $D$ surrenders or country $D$ defends itself and country $A$ wins. The parameter $\lambda$ captures the popularity boost of a foreign policy success. When the popularity boost is large, sanctions becomes less effective. The model in Section 2 corresponds to the case in which $\lambda = 1$.

Appendix B contains the description and analysis of the model with $\lambda \geq 1$. The equilibrium behavior of the countries follows the same logic highlighted in Section 3 with one important difference. It is now possible to build examples in which country $I$ pushes country $D$ to defend itself even when country $D$ is yielding. This happens only if the popularity boost of a foreign policy success is sufficiently large. The proof of Proposition 6 below follows from the analysis carried out in Appendix B.

**Proposition 6.** If $\lambda$ is sufficiently large, country $I$ may supply military aids to country $D$ even though country $D$ is yielding. When this happens, the upper bound on the amount of military aids $\overline{m}^*$ is increasing in $\lambda$.

When $\lambda > 1$, sanctions lose some of their effectiveness after an annexation. Instead, if the conflict ensues and country $A$ loses, sanctions preserve their effectiveness. Thus, when $\lambda$ is sufficiently large, country $I$ may decide to push country $D$ to defend itself so as to preserve the effectiveness of sanctions. It does so by deploying a level of military aids $m \geq m^\dagger$ that pushes a yielding country $D$ to defend itself.

Furthermore, when the conflict ensues, sanctions lose their effectiveness exactly when they are most valuable; that is, the ability of sanctions to induce a regime change decreases when country $A$ wins the conflict and its military confidence is highest. The optimal response of country $I$ is thus to increase the level of military aids so as to decrease the probability that country $A$ wins the conflict. Hence, $\overline{m}^*$ increases.
5.2 Partially Observable Military Aids

In the model discussed in Section 2, the level of sanctions is observable, while the level of military aids is not. In democratic regimes, economic sanctions are openly discussed in front of the public opinion and possibly ratified by parliaments. Military aids, instead, undergo a less transparent process and some specific details (e.g., the amount of intelligence assistance, or the specific characteristics of some weapons) are often classified.

Because military aids are not observable, country $I$ must commit to the upper bound $\overline{m}$. Without such commitment, if the conflict ensues, country $A$ would believe that military aids are high ($\hat{m} = M$). This would inflate country $A$’s military confidence and negatively impact country $I$’s payoff.

Does our analysis extend to the case in which military aids are partially observable? To answer this question, suppose that at time $t = 3$, country $A$ observes the actual level of military aids $m$ deployed by country $I$ with probability $\chi$. With complementary probability $1 - \chi$, country $A$ does not observe $m$. Everything else is as in Section 2.

When $\chi \leq \frac{1}{2}$, at time $t = 2$ country $I$ still sets the level of military aids as high as possible, $m = \overline{m}$. To understand why, suppose that there exists an equilibrium in which country $I$ committed to a range of military aids equal to $[m, \overline{m}]$ and deploys a level of military aids $m' < \overline{m}$ with positive probability. At time $t = 3$, country $A$ observes the level of military aids with probability $\chi$ and it does not observe it with probability $1 - \chi$. In this latter case, country $A$ holds beliefs about the level of military aids that, in equilibrium, must be correct. Thus, if the conflict between country $A$ and country $D$ ensues (which happens if $m' \geq \overline{m}^t$), $(m', s_w^*)$ must solve:

$$\max_{(m, s) \in [m, \overline{m}] \times [0, 1]} - \chi \left[ (1 - \phi s) \left( (\hat{p}_1(m))^2 (\mathbb{E}[\theta] - m) + (\hat{p}_0(m))^2 (1 - \mathbb{E}[\theta] + m) \right) \right]$$

$$- (1 - \chi) \left[ (1 - \phi s) \left( (\hat{p}_1(m'))^2 (\mathbb{E}[\theta] - m) + (\hat{p}_0(m'))^2 (1 - \mathbb{E}[\theta] + m) \right) \right] - \kappa \frac{s^2}{2}$$

The derivative with respect to $m$ of the $\chi$-term in this objective function is everywhere lower than
the one of the \((1 - \chi)\)-term. In equilibrium, country \(A\)'s beliefs must be correct. When country \(I\) deploys a level of military aids equal to \(m'\), its equilibrium payoff is thus equal to

\[
- \left[ (1 - \phi s^*_w(m')) \left( (\hat{p}_1(m'))^2 (\mathbb{E}[\theta] - m') + (\hat{p}_0(m'))^2 (1 - \mathbb{E}[\theta] + m') \right) \right] - \kappa \frac{(s^*_w(m'))^2}{2}, \tag{4}
\]

which is maximized whenever the \(\chi\)-term of the objective function is maximized. If \(m' < \overline{m}\) and \(\chi \leq \frac{1}{2}\), the previous discussion implies that country \(I\) could increase its payoff by marginally increasing \(m\) above \(m'\). In equilibrium, country \(I\) cannot then choose with positive probability a level of military aids lower than \(\overline{m}\).

The other steps in Section 3 immediately extend. All our insights thus hold true even when military aids are observable with a sufficiently low probability.

If the observability of military aids is very likely \((\chi > \frac{1}{2})\), country \(I\) does not necessarily choose \(\overline{m}\) at time \(t = 2\). The commitment to the upper bound on the level of military aids may lose its value. In equilibrium, country \(I\) may choose a level of military aids \(m < \overline{m}\) fearing that country \(A\) will observe it and thus update its military confidence based on it. For instance, in the extreme case in which military aids are perfectly observable, the commitment to the upper bound on the level of military aids is useless.\(^{27}\) However, all the other results and insights in the paper continue to hold. In particular, country \(I\) still deploys military aids only when country \(D\) is yielding. In response, country \(D\) has still an incentive to boost its military capability and nationalism so as to increase \(\Psi\) and decrease \(c_D\). Furthermore, when country \(I\) intervenes in the conflict, it still chooses the level of military aids as in the baseline model. It trades off the benefit of an increase in the likelihood of country \(A\)'s defeat in the conflict against the decrease in country \(I\)'s expected payoff due to the update in military confidence. The optimal level of military intervention is still equal to \(\overline{m}'\).

\(^{27}\)The commitment to the lower bound on the level of military aids is still valuable: it can still credibly deter country \(A\)'s attack.
6 Conclusion

Third countries often intervene in conflicts between an attacker and a defender. These interventions entail the use of several tools, including economic sanctions and military aids.

This paper studies the optimal intervention of a third country in a conflict when (i) the military strength of the attacker is unknown, (ii) economic sanctions are observable, weaken the stability of the attacker’s regime, but they do not immediately reduce its likelihood of prevailing in the conflict, and (iii) military aids are unobservable and reduce the attacker’s probability of winning in the conflict.

The paper highlights a key tradeoff in the provision of military aids. Military aids help the defender prevail in the conflict and thus dampen the military confidence of the attacker. This is beneficial for the third country as it limits the attacker’s aggressiveness once the conflict is over. Yet, military aids also make the attacker more optimistic about its strength after a victory, and less pessimistic after a defeat. This is detrimental for the third country as it boosts the attacker’s aggressiveness.

We find that the third country can intervene in the conflict according to three possible strategies. Whenever possible, the third country deters the attacker from initiating the conflict by committing to high military aids. This is the logic behind defense pacts. When deterrence is not feasible, the third country would like to avoid the conflict even though this means to let the defender surrender and lose its independence. The conflict is a risky gamble that can boost the attacker’s military confidence and thus lower the third country’s payoff in the future. Finally, if deterrence is not feasible and the conflict is unavoidable, the third country is dragged to war. In this case, it provides military aids to the defender so as to minimize the attacker’s future aggressiveness.

Economic sanctions complement military aids and protect the third country from the attacker’s aggressiveness once the dispute is over. In particular, economic sanctions are higher when the third country is dragged into war than when it is not.

Our model delivers testable implications on the factors that determine the level and the type
of military engagement of a third country in conflicts. Our analysis also suggests that the defender can adopt a nationalistic rhetoric or boost its military capability to force the third country to provide military assistance. Finally, when successes contribute to the stability of an aggressive regime, we show that the third country increases the deployment of military aids when dragged to war.
Appendix

A Omitted Proofs

A.1 Proof of Proposition 1

Country $D$ is uncertain about the strength of country $A$. In particular, it assigns probability $p$ to country $A$ having military strength $\theta$. Then, country $D$ expects to win the conflict with a probability equal to $1 - p\bar{\theta} - (1 - p)\theta + m = 1 - \mathbb{E}[\theta] + m$. In this case, it enjoys the value of freedom $\Psi$. Country $D$ thus defends itself if:

$$[1 - \mathbb{E}[\theta] + m] \Psi > c_D,$$

it surrenders if the opposite inequality holds, and it is indifferent if the two sides of the inequality are equal to each other. In this latter case, we assume that it breaks indifference defending itself. \qed

A.2 Proof of Proposition 2

Suppose country $D$ is not yielding. Let $\tilde{m}$ be a cumulative density function over the set $[\underline{m}, \bar{m}]$ representing the beliefs of country $A$ about the level of military aids deployed in the conflict by country $I$. In equilibrium, $\tilde{m}$ must be equal to the actual behavior of country $I$. When the beliefs of country $A$ are represented by $\tilde{m}$ and country $I$ deploys a level of military aids equal to $m$, the expected payoff of country $I$ is equal to:

$$\int_{\underline{m}}^{\bar{m}} - \left((1 - \phi s) \left[ (\hat{p}_1(x))^2(\mathbb{E}[\theta] - m) + (\hat{p}_0(x))^2(1 - \mathbb{E}[\theta] + m) \right] \right) \tilde{m}[x] dx - \kappa s^2$$

The previous expression is increasing in $m$ because $\hat{p}_0(x) < \hat{p}_1(x)$ for every $x$. Thus, country $I$ would optimally set $m$ as high as possible; that is, $m = \bar{m}$.

The convexity of the cost of sanctions implies that the payoff of country $I$ is convex in $s$ for
every level of military aids $m$. Hence, the optimal level of sanctions from the point of view of country $I$ equates the derivative of the expected payoff of country $I$ with respect to $s$ (computed at $m = \bar{m}$) to zero:

$$\phi \left[ \hat{p}_1(\bar{m}) \right]^2 \mathbb{E}[\theta] - \bar{m} + (\hat{p}_0(\bar{m}))^2 \mathbb{E}[\theta] - \bar{m} = \kappa s$$

The left-hand side of this expression is positive and lower than 1.\textsuperscript{28} Since $\kappa > 1$, the optimal level of sanctions lies in the interval $(0, 1)$.

\textbf{A.3 Proof of Proposition 3}

Suppose country $D$ is yielding. If $m < m^\dagger$, country $D$ always surrenders. The tie braking-rule then implies that country $I$ sets military aids as low as possible, $m = \bar{m}$. In this case, sanctions are set so as to maximize $V_I(s \mid p) = -[1 - \phi s] p^2 - \kappa \frac{s^2}{2}$. This function is concave in $s$. We can thus take the first order condition and conclude that the optimal level of sanctions is $s^*_\theta = \frac{\phi}{\pi} p^2 \in (0, 1)$.

Suppose $m \geq m^\dagger$. Then country $D$ always defends itself. The same logic of Proposition 2 implies that country $I$ chooses the pair $(\bar{m}, s^*_w(\bar{m}))$.

Consider now the most interesting case in which $m^\dagger \in (m, \bar{m}]$. In this case, country $D$ defends itself if $m \geq m^\dagger$ and surrenders if $m < m^\dagger$. Country $I$ must then decide whether to induce country $D$ to defend itself or to let it surrender. If country $I$ decides to let country $D$ surrender, our tie-breaking rule implies $m = \bar{m}$ and we can replicate the steps of the case in which $\bar{m} < m^\dagger$ to show that $s = s^*_\theta$. In this case, the payoff of country $I$ is equal to

$$V_I(s^*_\theta \mid p) = -[1 - \phi s^*_\theta] p^2 - \kappa \frac{(s^*_\theta)^2}{2}.$$ 

If country $I$ decides to push country $D$ to defend itself, then the same logic we used in the proof of Proposition 2 implies that country $I$ optimally chooses $m = \bar{m}$ and $s = s^*_w(\bar{m})$. In this case, the

\textsuperscript{28}To see this, note that $\phi \left[ \hat{p}_1(\bar{m}) \right]^2 \mathbb{E}[\theta] - \bar{m} + (\hat{p}_0(\bar{m}))^2 \mathbb{E}[\theta] - \bar{m} < \phi \left[ \hat{p}_1(\bar{m}) \mathbb{E}[\theta] - \bar{m} + \hat{p}_0(\bar{m}) \mathbb{E}[\theta] - \bar{m} \right] = \phi p < 1$, where the last inequality follows from $\phi < 1$. 

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expected payoff of country $I$ is equal to:

$$\mathbb{E}_p[V_I(s^*_w(\bar{m}) \mid \hat{p})] = -(1 - \phi s^*_w(\bar{m})) \left[ (\hat{p}_1(\bar{m}))^2 (\mathbb{E}[\theta] - \bar{m}) + (\hat{p}_0(\bar{m}))^2 (1 - \mathbb{E}[\theta] + \bar{m}) \right] - \frac{\kappa (s^*_w(\bar{m}))^2}{2}$$

Observe that

$$s^*_w(\bar{m}) = \frac{\phi}{\kappa} \left[ (\hat{p}_1(\bar{m}))^2 (\mathbb{E}[\theta] - \bar{m}_d) + (\hat{p}_0(\bar{m}))^2 (1 - \mathbb{E}[\theta] + \bar{m}_d) \right] > \frac{\phi}{\kappa} \left[ \mathbb{E}[\theta] - \bar{m}_d + \hat{p}_0(\bar{m})(1 - \mathbb{E}[\theta] + \bar{m}_d) \right]^2 = \frac{\phi}{\kappa} p^2 = s^*_0,$$

where the inequality follows from Jensen’s inequality, the first and third equalities are true by definition, and the second equality is true because, under Bayes rule, the expectation of the posterior is equal to the prior. Country $I$ thus levies a higher level of sanctions when it push country $D$ to defend itself rather than when it does not. Now, we can write

$$\mathbb{E}_p[V_I(s^*_w(\bar{m}) \mid \hat{p})] =$$

$$= -(1 - \phi s^*_w(\bar{m})) \left[ (\hat{p}_1(\bar{m}))^2 (\mathbb{E}[\theta] - \bar{m}) + (1 - \phi s''(\hat{p}_0(\bar{m})))^2 (1 - \mathbb{E}[\theta] + \bar{m}) \right] - \frac{\kappa (s^*_w(\bar{m}))^2}{2} <$$

$$< -(1 - \phi s^*_w(\bar{m})) p^2 - \frac{\kappa (s^*_w(\bar{m}))^2}{2} \leq -(1 - \phi s^*_0) p^2 - \frac{\kappa (s^*_0)^2}{2} = V_I(s^*_0 \mid p),$$

where the first inequality follows Jensens’ inequality and the second inequality follows from the optimality of $s^*_0$. Thus, $m^* \in (\underline{m}, \overline{m}]$ country $I$ prefers not to induce country $D$ to defend itself and it sets $m = \underline{m}$ and $s = s^*_0$. \hfill \Box

### A.4 Proof of Proposition 4

Suppose that country $D$ is yielding and that $\underline{m} < m^*$. Then Propositions 1 and 3 imply that if country $A$ attacks, country $I$ sets $m = \underline{m}$ and country $D$ surrenders. In this case country $A$ gets a payoff equal to $1 + (1 - \phi s^*_0) v(p)$. This payoff is greater than zero, the payoff from remaining peaceful. Country $A$ thus always attacks.
Suppose instead that either country $D$ is not yielding or $m \geq m^\dagger$. Then, Propositions 1, 2 and 3 imply that country $I$ sets $m = \overline{m}$ and country $D$ defends itself. If country $A$ attacks, it wins the conflict with probability $(\mathbb{E}[\theta] - \overline{m})$ and it loses it with complementary probability. The expected payoff from attacking is thus equal to

$$(\mathbb{E}[\theta] - \overline{m}) [1 + (1 - \phi s^*_w(\overline{m})) \hat{p}_1(\overline{m})] + (1 - \mathbb{E}[\theta] + \overline{m}) (1 - \phi s^*_w(\overline{m})) \nu(\hat{p}_0(\overline{m})) - c_A$$

Country $A$ attacks if the previous expression is strictly positive and it remains peaceful if the previous expression is non-positive.

**A.5 Proof of Proposition 5**

The payoff of country $I$ after an attack is bounded below 0 (see Table 1). Instead, the payoff of country $I$ when country $A$ remains peaceful is equal to 0. Then, the first best for country $I$ is to deter the attack of country $A$. This is feasible, if there exists a level $m_d \in [0, M]$ such that inequality (3) in the main text holds. In this case, we can assume without loss of generality that country $I$ sets $\underline{m} = \overline{m} = m_d$.

When the cost of conflict for country $A$ is sufficiently low, inequality (3) fails for all $m \in [0, M]$. In this case country $I$ knows that country $A$ attacks independently of its commitment on military aids. When country $D$ is yielding, Proposition 3 implies that country $I$ can optimally commit to $\underline{m} = \overline{m} = 0$. When country $D$ is not yielding, the conflict between country $A$ and country $D$ is unavoidable. Proposition 2 implies that if country $A$ commits to a set of military aids $[\underline{m}, \overline{m}]$ the level of military aids and the level of sanctions are set optimally equal to $(\overline{m}, s^*_w(\overline{m}))$. Then we can assume without loss of generality that the optimal commitment of country $I$ takes the form of $[0, \overline{m}]$ with $\overline{m} \in [0, M]$. In particular, country $I$ chooses $\overline{m}$ so as to maximize the following objective function:

$$- (1 - \phi s^*_w(\overline{m})) \frac{\nu^2(\overline{\theta} - \overline{m})^2}{\mathbb{E}[\theta] - \overline{m}} - (1 - \phi s^*_w(\overline{m})) \frac{\nu^2(1 - \overline{\theta} + \overline{m})^2}{1 - \mathbb{E}[\theta] + \overline{m}} - \frac{\kappa(s^*_w(\overline{m})^2)}{2},$$
In what follows, we proceed minimizing the opposite of this expression. Clearly, the two problems are identical.

Recall that \( s^*_w(m) = \frac{\phi}{k} \left[ \frac{p^2(\theta - m)^2}{\mathbb{E}[\theta] - m} + \frac{p^2(1 - \theta + m)^2}{1 - \mathbb{E}[\theta] + m} \right] \). To study the optimal commitment of country \( I \), it is convenient to define the auxiliary function \( m \in [0, \theta] \mapsto \varsigma(m) \), where

\[
\varsigma(m) = \frac{p^2(\theta - m)^2}{\mathbb{E}[\theta] - m} + \frac{p^2(1 - \theta + m)^2}{1 - \mathbb{E}[\theta] + m} \in (0, 1). \tag{A-1}
\]

The fact that \( \varsigma(m) < 1 \) for every \( m \) follows from the fact that

\[
\varsigma(m) = \left[ (\hat{p}_1(m))^2 \cdot \Pr(w = 1 \mid m) + (\hat{p}_0(m))^2 \cdot \Pr(w = 0 \mid m) \right] < \left[ \hat{p}_1(m) \cdot \Pr(w = 1 \mid m) + \hat{p}_0(m) \cdot \Pr(w = 0 \mid m) \right] = p < 1
\]

Exploiting this auxiliary function and the expression for \( s^*_w(m) \), we can write the objective function of country \( I \) as

\[
\left( 1 - \frac{\phi^2}{k} \varsigma(m) \right) \frac{p^2(\theta - m)^2}{\mathbb{E}[\theta] - m} + \left( 1 - \frac{\phi^2}{k} \varsigma(m) \right) \frac{p^2(1 - \theta + m)^2}{1 - \mathbb{E}[\theta] + m} + \frac{\phi^2}{2k} (\varsigma(m))^2 = \left( 1 - \frac{\phi^2}{2k} \varsigma(m) \right) \varsigma(m).
\]

The first order condition of this objective function is:

\[
\frac{\partial \varsigma(m)}{\partial m} \left( 1 - \frac{\phi^2}{k} \varsigma(m) \right) = 0.
\]

Such first order condition is satisfied in two cases. Either \( \frac{\partial \varsigma(m)}{\partial m} = 0 \) or \( \frac{\phi^2}{k} \varsigma(m) = 1 \). The second derivative of the objective function is:

\[
\frac{\partial^2 \varsigma(m)}{\partial m^2} \left( 1 - \frac{\phi^2}{k} \varsigma(m) \right) - \frac{\phi^2}{k} \left( \frac{\partial \varsigma(m)}{\partial m} \right)^2.
\]

Recall that \( \phi < 1 \), \( k > 1 \), and \( \varsigma(m) \in (0, 1) \).

Also note that \( \frac{\partial \varsigma(m)}{\partial m} = 0 \) if \( m = \mathbb{E}[\theta] - \frac{1}{2} := \overline{m}_1 \) and \( \frac{\partial \varsigma(m)}{\partial m} \) is negative (positive).
for values of \( m \) below (above) \( m^*_1 \). When \( m = m^*_1 \), the second derivative of \( \zeta(\cdot) \) is equal to 
\[ 32p^2(\theta - \mathbb{E}[\theta])^2 > 0 . \] Hence \( m = \mathbb{E}[\theta] - 1/2 \), is a local interior minimizer for our objective function.

The first order condition is also equal to 0 if \( \frac{\partial^2 \zeta}{m^*_1} = 1 \). In this case, the second order condition is equal to 
\[ -\frac{\phi^2}{k} \left( \frac{\partial \zeta}{\partial m} \right)^2 , \] which is negative. These values are not local minimizers of our objective function and we can ignore them. To prove that \( m^*_1 \) is a global minimizer, we need to rule out corner solution; that is, we need to show that the objective function is lower at \( m^*_1 \) than at \( m = 0 \) or \( m = \theta \). Pick \( m_c \in \{0, \theta\} \). The previous discussion implies that \( \zeta(m^*_1) < \zeta(m_c) \). We want to show that 
\[ (1 - \frac{\phi^2}{2k} \zeta(m^*_1)) \zeta(m^*_1) < (1 - \frac{\phi^2}{2k} \zeta(m_c)) \zeta(m_c) . \] Because \( \zeta(m_c) > \zeta(m^*_1) \), \( \phi < 1 \), and \( k > 1 \), the previous inequality holds if 
\[ \zeta(m_c) - \zeta(m^*_1) > \frac{\phi^2}{2k} \left( \zeta(m_c)^2 - (\zeta(m^*_1))^2 \right) . \] This always holds because the auxiliary function is everywhere lower than 1. Hence, the objective function of country \( I \) is maximized setting \( m \) equal to \( m^*_1 = \min \{ \max \{ \mathbb{E}[\theta] - \frac{1}{2}, 0 \}, M \} \).

\[ \square \]

**B  The efficacy of sanctions and the conflict**

In this section, we solve the model in which foreign policy successes affect the stability of country \( A \)'s regime. In particular, the probability of a regime overthrown is equal to \( \phi_s \) if country \( A \) loses the conflict, and it is equal to \( \frac{\phi}{X} s \) with \( \lambda > 1 \) if either country \( D \) surrenders or if country \( A \) wins the conflict. The parameter \( \lambda > 1 \) measures the popularity boosts of foreign policy successes.

|       | \( V_A(s | \hat{p}) \) | \( V_I(s | \hat{p}) \) |
|-------|----------------|----------------|
| if \( d = 0 \) | \( 1 - \frac{\phi}{X} s \) \( v(p) \) | \( -[1 - \frac{\phi}{X} s] (p)^2 - \kappa \frac{s^2}{2} \) |
| if \( d = 1 \) and country \( D \) wins | \( [1 - \phi s] \( \hat{v}(\hat{p}_0(m)) \) | \( -[1 - \phi s] (\hat{p}_0(m))^2 - \kappa \frac{s^2}{2} \) |
| if \( d = 1 \) and country \( A \) wins | \( [1 - \frac{\phi}{X} s] \( v(\hat{p}_1(m)) \) | \( -[1 - \frac{\phi}{X} s] (\hat{p}_1(m))^2 - \kappa \frac{s^2}{2} \) |

**Table B-1:** Expected payoffs after the conflict when the popularity boost is \( \lambda \).

The payoffs of country \( A \) and \( I \) at time \( t = 4 \) are represented in Table B-1. The behavior of country \( D \) at time \( t = 3 \) does not depend on the popularity boost \( \lambda \). At time \( t = 2 \), we can still
distinguish between two cases: the one in which country \( D \) is not yielding and the one in which country \( D \) is. When country \( D \) is not yielding, the same logic of Proposition 2 implies that country \( I \) optimally sets \( m = \bar{m} \) and \( s = s_{w,\lambda}^*(\bar{m}) \), where

\[
s_{w,\lambda}^*(\bar{m}) = \frac{\phi}{k} \left[ \frac{1}{\lambda} (\hat{p}_1(\bar{m}))^2(\mathbb{E}[\theta] - \bar{m}) + (\hat{p}_0(\bar{m}))^2(1 - \mathbb{E}[\theta] + \bar{m}) \right]
\]  

(B-1)

Coeteris paribus, the level of economic sanctions is now lower. If country \( D \) is not yielding, the conflict between country \( A \) and country \( D \) is unavoidable and country \( A \) may win it with positive probability. When country \( A \) wins the conflict, sanctions are less effective. Because the marginal cost of sanctions is unchanged, country \( I \) sets sanctions at a lower level.

When country \( D \) is yielding, the previous commitment still constraint the equilibrium behavior of country \( I \). If \( m > m^\dagger \), country \( I \) behaves as if country \( D \) was not yielding: \( m = \bar{m} \) and \( s = s_{w,\lambda}^*(\bar{m}) \). When \( m < m^\dagger \), country \( D \) surrenders with certainty. Country \( I \) then sets \( m = m \) and \( s = s_{w,\lambda}^* = \frac{\phi}{\kappa}\hat{p}^2 \). Also in this case the popularity boost lowers the efficacy of economic sanctions and their equilibrium level is then lower.

Consider now the case in which \( m^\dagger \in [m, \bar{m}] \). When the popularity boost \( \lambda \) is not too high, country \( I \) behaves as in the baseline model: it chooses not to provide military aids, \( m = m \) and \( s = s_{w,\lambda}^* \). When \( \lambda \) is large, however, country \( I \) may prefer to to push country \( D \) to defend itself by deploying military aids above \( m^\dagger \). The reason behind this intervention is simple. When \( \lambda \) grows, the choice of country \( D \) to surrender makes sanctions less effective. Sanctions instead preserve their effectiveness when country \( A \) loses the conflict. The desire to preserve the efficacy of sanction provides country \( I \) an incentive to deploy a level of military aids above \( m^\dagger \). This happens when the aggressiveness of country \( A \) after the conflict is not too high; that is when

\[
\left[ (\hat{p}_1(\bar{m}))^2(\mathbb{E}[\theta] - \bar{m}) + (\hat{p}_0(\bar{m}))^2(1 - \mathbb{E}[\theta] + \bar{m}) \right] < \frac{\mathbb{E}[\theta] - \bar{m}}{(\bar{\theta} - m)^2} \hat{p}^2.
\]  

(B-2)

To summarize, when \( m^\dagger \in [m, \bar{m}] \) and country \( D \) is not yielding, country \( I \) chooses \( m = m \) and
\( s = s^*_w,\lambda \) either if \( \lambda \in [1, \Lambda] \cup [\overline{\Lambda}, \infty) \), where \( \Lambda \) is possible equal to \( +\infty \), or if inequality (B-2) fails (or both). In the other case, country I chooses \( m = \overline{m} \) and \( s = s^*_{w,\lambda}(\overline{m}) \) (see the proof of Proposition B-1). In the former case, we say that country I does not want to trigger the conflict; in the latter case we say that country I wants to trigger the conflict.

We can then adapt the proof of Proposition 4 and show that country A attacks with certainty in one of two cases. First, country A attacks with certainty if \( \overline{m} < m^\dagger \), so that country D surrenders with certainty. Second, country A attacks with certainty if \( m^\dagger \in (\overline{m},\overline{m}] \), country D is yielding, and country I does not want to trigger the conflict. In the other cases, country A attacks if and only if

\[
(\mathbb{E}[\theta] - \overline{m}) + \phi \left( 1 - \frac{1}{\lambda} \right) s^*_w(\overline{m})(\overline{\theta} - \overline{m}) \left[ (\mathbb{E}[\theta] - \overline{m})v(\hat{p}_1(\overline{m})) + (1 - \mathbb{E}[\theta] + \overline{m})v(\hat{p}_0(\overline{m})) \right] + \\
+ (1 - \phi s^*_w(\overline{m})) \left[ (\mathbb{E}[\theta] - \overline{m})v(\hat{p}_1(\overline{m})) + (1 - \mathbb{E}[\theta] + \overline{m})v(\hat{p}_0(\overline{m})) \right] > c_A. \tag{B-3}
\]

The optimal commitment of country I is summarized in the following Proposition. Proposition 6 in the main text follows immediately from it.

**Proposition B-1.** Consider the following inequality:

\[
(\mathbb{E}[\theta] - m) + \phi \left( 1 - \frac{1}{\lambda} \right) s^*_w(m)(\overline{\theta} - m) \left[ (\mathbb{E}[\theta] - m)v(\hat{p}_1(m)) + (1 - \mathbb{E}[\theta] + m)v(\hat{p}_0(m)) \right] + \\
+ (1 - \phi s^*_w(m)) \left[ (\mathbb{E}[\theta] - m)v(\hat{p}_1(m)) + (1 - \mathbb{E}[\theta] + m)v(\hat{p}_0(m)) \right] \leq c_A. \tag{B-4}
\]

The equilibrium commitment of country I is described as follows.

1. **Deterrence.** If inequality (B-4) holds for some \( m = m_d \in [m^\dagger, M] \), then country I commits to \( m = \overline{m} = m_d \).

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\(^{29}\)For high values of \( \lambda (\lambda > \overline{\Lambda}) \) providing no military aids is again optimal. The choice to deploy a level of military aids \( m \geq m^\dagger \) must be paired with high and costly sanctions. As \( \lambda \) increases, these sanctions become more and more ineffective when country A wins the conflict. Country I can thus be better off choosing \( m < m^\dagger \), setting a lower level of economic sanctions and not gambling with the military confidence of country A.
2. **Appeasement.** If inequality (B-4) fails for all \( m \in [m^\dagger, M] \), country \( D \) is yielding and country \( I \) does not want to trigger the conflict, then country \( I \) commits not to provide military aids: \( m = \overline{m} = 0 \).

3. **Dragged to war.** If inequality (B-4) fails for all \( m \in [m^\dagger, M] \) and either country \( D \) is not yielding or country \( I \) wants to trigger the conflict, then country \( I \) commits to a range of military aids equal to \([m, \overline{m}^s_{w, \lambda}]\), where \( \overline{m}^s_{w, \lambda} > \overline{m}^s_w \) and \( \overline{m}^s_{w, \lambda} \) is increasing in \( \lambda \).

*Proof.* We start providing a proof of the equilibrium choice of \((m, s)\) when country \( A \) committed to \([\underline{m}, \overline{m}]\) and country \( D \) is not yielding. The same steps of Proposition 3 show that country \( I \) chooses \( m = \overline{m} \) and \( s = s^*_{w, \lambda}(\overline{m}) \) when \( m^\dagger > \overline{m} \), and \( m = \underline{m}, s = s^*_{0, \lambda} = \frac{\phi}{\lambda} p^2 \) when \( m^\dagger \leq \underline{m} \). When \( m^\dagger \in (\underline{m}, \overline{m}] \), country \( I \) can decide whether to deploy a level of military aids \( m \geq m^\dagger \) and push country \( D \) to defend itself, or a level \( m < m^\dagger \) and let country \( D \) surrender. In the former case, country \( I \)'s expected utility is equal to:

\[
V_I(s^*_{0, \lambda} \mid p) = -\left[1 - \frac{\phi}{\lambda} s^*_{0, \lambda}\right] p^2 - \kappa \frac{(s^*_{0, \lambda})^2}{2},
\]

while in the latter case it is equal to:

\[
\mathbb{E}_\hat{p}[V_I(s^*_{w, \lambda}(\overline{m}) \mid \hat{p})] = -\left(1 - \frac{\phi}{\lambda} (s^*_{w, \lambda}(\overline{m}))\right) (\hat{p}_1(\overline{m}))^2 (\mathbb{E}[^\theta] - \overline{m}) - (1 - \phi(s^*_{w, \lambda}(\overline{m}))) (\hat{p}_0(\overline{m}))^2 (1 - \mathbb{E}[^\theta] + \overline{m}) - \kappa \frac{(s^*_{w, \lambda}(\overline{m}))^2}{2}
\]

Consider the difference between these two quantities: \( V_I(s^*_{0, \lambda} \mid p) - \mathbb{E}_\hat{p}[V_I(s^*_{w, \lambda}(\overline{m}) \mid \hat{p})] \). The proof of Proposition 3 implies this difference is positive. The derivative of the difference with respect to \( \lambda \) is equal to:

\[
\frac{\partial}{\partial \lambda} \left[ V_I(s^*_{0, \lambda} \mid p) - \mathbb{E}_\hat{p}[V_I(s^*_{w, \lambda}(\overline{m}) \mid \hat{p})] \right] =
\frac{\phi^2}{k \lambda^3} p^2 \left[ \frac{(\overline{\theta} - m)^2}{\mathbb{E}[^\theta] - m} (\hat{p}_1(\overline{m}))^2 (\mathbb{E}[^\theta] - \overline{m}) + \lambda (\hat{p}_0(\overline{m}))^2 (1 - \mathbb{E}[^\theta] + \overline{m}) \right] - p^2
\]

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The term in squared bracket is increasing in $\lambda$. Thus, if such term is positive at $\lambda = 1$, country $I$ is always be better off setting military aids equal to zero. This is condition (B-2) in the main text.

If condition (B-2) fails, the difference in expected payoff starts positive and it first decreases and then increases in $\lambda$. As $\lambda \to +\infty$, the difference is equal to:

$$
(\hat{p}_1(m))^2(\mathbb{E}[\theta] - m) + (\hat{p}_0(m))^2(1 - \mathbb{E}[\theta] + m) - p^2 - \frac{\phi^2}{2k}(\hat{p}_0(m))^4(1 - \mathbb{E}[\theta] + m)^2.
$$

If this expression is negative, there exists a threshold $\lambda$ such that country $I$ prefers to deploy a level of military aids $m < m^\dagger$ if $\lambda \leq \lambda$ and $m = \bar{m}$ if $\lambda > \lambda$. If the previous expression is not negative, there exists a range $[\underline{\lambda}, \overline{\lambda}]$ with $\underline{\lambda}$ and $\overline{\lambda}$ possibly equal to $+\infty$ such that country $I$ is better off choosing $m = \bar{m}$ rather than $m < m^\dagger$ if $\lambda \in [\underline{\lambda}, \overline{\lambda}]$.

We now turn to the proof of the statement of the proposition. The proof of the cases 1 and 2 follows the same steps used in the proof Proposition 3. The only difference is that now, we need to explicitly take into account that the appeasement strategy (case 2) occurs when country $I$ does not want to trigger the conflict. We thus focus our attention on case 3.

Define the auxiliary function $m \in [0, \theta] \mapsto \varsigma_\lambda(m) =$, where

$$
\varsigma_\lambda(m) = \frac{1}{\lambda} \frac{p^2(\overline{\theta} - m)^2}{\mathbb{E}[\theta] - m} + \frac{p^2(1 - \theta + m)^2}{1 - \mathbb{E}[\theta] + m}. \quad (B-5)
$$

Obviously $\varsigma_1(m)$ is equal to the auxiliary function $\varsigma(m)$ we defined in the proof of Proposition 5. Note that $\varsigma_\lambda(m) \in (0, 1)$ for every $m$. Indeed we have

$$
\varsigma_\lambda(m) = \left[\frac{1}{\lambda} (\hat{p}_1(m))^2 \cdot \Pr(w = 1 \mid m) + (\hat{p}_0(m))^2 \cdot \Pr(w = 0 \mid m)\right] < \\
< \left[\frac{1}{\lambda} \hat{p}_1(m) \cdot \Pr(w = 1 \mid m) + \hat{p}_0(m) \cdot \Pr(w = 0 \mid m)\right] < p < 1.
$$
The objective function of country I when \( m \geq m^\dagger \) is equal to

\[
- \left( 1 - \frac{\phi}{\lambda} s^*_{w,\lambda}(m) \right) \frac{p^2(\tilde{\theta} - m)^2}{\mathbb{E}[\theta] - m} - (1 - \phi s^*_w(m)) \frac{p^2(1 - \tilde{\theta} + m)^2}{1 - \mathbb{E}[\theta] + m} - \frac{\kappa(s^*_w(m))^2}{2},
\]

Exploiting the auxiliary function and the expression for \( s^*_{w,\lambda}(m) \), we can rewrite the objective function as

\[
\left( 1 - \frac{\phi^2}{2k} \zeta_{\lambda}(m) \right) \zeta_{\lambda}(m) + \left( 1 - \frac{1}{\lambda} \right) \frac{p^2(\tilde{\theta} - m)^2}{\mathbb{E}[\theta] - m},
\]

The derivative with respect to \( m \) of the objective function is

\[
\left( 1 - \frac{\phi^2}{k} \zeta_{\lambda}(m) \right) \frac{\partial \zeta_{\lambda}(m)}{\partial m} + \left( 1 - \frac{1}{\lambda} \right) \frac{\partial}{\partial m} \left( \frac{p^2(\tilde{\theta} - m)^2}{\mathbb{E}[\theta] - m} \right) =
\]

\[
\left( 1 - \frac{\phi^2}{k} (A_{\lambda}(m) + B(m)) \right) \left[ \frac{\partial}{\partial m} A_{\lambda}(m) + \frac{\partial}{\partial m} B \right] + \left( 1 - \frac{1}{\lambda} \right) \frac{\partial}{\partial m} A_1(m),
\]

where \( A_{\lambda}(m) = \frac{1}{\lambda} \frac{p^2(\tilde{\theta} - m)^2}{\mathbb{E}[\theta] - m} \) and \( B(m) = \frac{p^2(1 - \tilde{\theta} + m)^2}{1 - \mathbb{E}[\theta] + m} \). Suppose \( m^* > 0 \). Then \( A_1 \) is first decreasing and then increasing in \( m \), while \( B \) is everywhere increasing. Moreover, \( \frac{\partial A_{\lambda}}{\partial m} = \frac{1}{\lambda} \frac{\partial A_{1}(m)}{\partial m} \); thus the derivative of the objective function with respect to \( m \) can be written as

\[
\left( 1 - \frac{\phi^2}{\lambda} \frac{1}{k} (A_{\lambda}(m) + B(m)) \right) \frac{\partial}{\partial m} A_{1}(m) + \left( 1 - \frac{\phi^2}{k} (A_{\lambda}(m) + B(m)) \right) \frac{\partial}{\partial m} B(m) \quad (B-6)
\]

Derivative (B-6) also applies to the case in which \( \lambda = 1 \). In particular, when \( \lambda = 1 \) and \( m^* > 0 \) (B-6) is negative at \( m = 0 \). When \( \lambda > 1 \), the derivative is still negative at \( m = 0 \) because the first term in (B-6) (the \( \frac{\partial A_{1}(m)}{\partial m} \)-term) receives a weight higher than the second term (the \( \frac{\partial B(m)}{\partial m} \)-term). For the very same reason, when \( \lambda > 1 \), (B-6) is still negative at \( m^* \). We conclude that the optimal commitment when \( \lambda > 1 \) and country I is dragged to war, \( m^*_w \), is greater than the optimal commitment when \( \lambda = 1 \). In other words, \( m^*_w > m^* \). The same logic immediately yields that \( m^*_{w,\lambda} \) is increasing in \( \lambda \). \( \square \)
C  Expected Payoffs at time t=4

In Section 2 we posit that the expected payoff of country I is decreasing and concave in country A’s military confidence. In what follows, we microfound this specific functional form through a simple model of conflict between country A and country I.

Suppose that if the regime in country A survives, country A can decide whether to escalate the conflict against the international community at cost $C_A > 0$. If country A does not escalate, both countries get a payoff equal to zero. We assume that $C_A$ is ex-ante unknown and that $C_A \sim U(0,1)$. Escalation is successful if the military strength of country A is $\theta$; in this case country A enjoys a payoff equal to 1. Escalation is not successful if the military strength of country A is $\bar{\theta}$; in this case country A enjoys a payoff equal to 0. When its military confidence is $\hat{p}$, the expected payoff of country A if it escalates is thus equal to $V_A(s|\hat{p}) = \hat{p} - C_A$. Hence, the probability that country A escalates when its updated perceived strength is $\hat{p}$ is equal to $\hat{p}$.

Next, we describe the expected payoff of country I. If country A escalates, country I incurs a cost equal to $C_I$. Moreover, if there is an escalation, country I gets a payoff of $-1$ if the escalation is successful and a payoff of 0 if the escalation is not successful. Then, when the attacker’s military confidence is $\hat{p}$, the expected payoff of country I is given by:

$$\hat{p} [\hat{p}(-1 - \ell) + (1 - \hat{p}(-\ell))] = -(\hat{p})^2 - \hat{p}\ell.$$ 

This expression is decreasing and concave in the military confidence, $\hat{p}$.

References


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