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Electoral Campaigns as Dynamic Contests*

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Abstract

We develop a model of electoral campaigns as dynamic contests in which two office-motivated candidates allocate their budgets over time to affect their odds of winning. We measure the candidates' evolving odds of winning using a state variable that tends to decay over time, and we refer to it as the candidates' "relative popularity." In our baseline model, the equilibrium ratio of spending by each candidate equals the ratio of their initial budgets; spending is independent of past realizations of relative popularity; and there is a positive relationship between the strength of decay in the popularity process and the rate at which candidates increase their spending over time as election day approaches. We use this relationship to recover estimates of the perceived decay rate in popularity leads in actual U.S. subnational elections.

Key words: campaigns, dynamic allocation problems, contests

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1 Introduction

When looking at patterns of electoral campaign spending, a key feature that stands out across races is that candidates tend to increase their spending over time in the run up to the election. This feature, reflected in Figure 1 below, may seem natural given the claims of the empirical literature on the positive but fleeting effects of campaign spending.[1] For example, the results in Gerber et al. (2011) and Hill et al. (2013) suggest that the marginal effects of spending dissipate in a matter of days, and no more than a couple of weeks.

But it is not obvious how a strategic candidate would optimize her spending plan knowing that the opposing candidate is also spending strategically. If the effects of campaign spending dissipate quickly over time, and two candidates save their entire budgets for the final stretch of the campaign, then either one of them may be better off pre-empting the other by spending some of her budget a little earlier. This raises the question: What is the optimal spending path for each candidate in a strategic setting, where both are optimizing?

We address this question by modeling electoral campaigns as dynamic contests in which two candidates allocate their campaign budgets across time ahead of an election that is held at a fixed future date. We develop a tractable framework to study this dynamic allocation problem.

In our model, the candidates (called 1 and 2) spend their budgets to influence the evolution of a random variable that we call *relative popularity*. Time runs discretely and in each period each candidate decides how much of her budget to spend to try to raise her relative popularity. The realization of relative popularity in any given period measures candidate 1's popularity lead over candidate 2, and thus her odds of eventually winning the election.

Candidates start with one being possibly more popular than the other. In each period, candidate 1's relative popularity may increase or decrease, evolving over time according to an AR(1) process that allows for decay in popularity leads. The candidates' spending decisions in any period affect the drift of this process between the current period and the next. The drift is strictly increasing and strictly concave in candidate 1's spending and strictly decreasing and strictly convex in candidate 2's spending. The candidate that is more popular at time T wins the election.

Our baseline model is a zero-sum game in which the candidates are purely office-motivated and have a fixed budget. This model has a unique equilibrium and the

[1] A key premise here is that money spent on advertising influences elections. For recent evidence on this, see Spenkuch and Toniatti (2018) and Martin (2014). On the effect of political advertising and political persuasion more generally, see DellaVigna and Gentzkow (2010), Kalla and Broockman (2018), Jacobson (2015), the references therein, as well as the related literature section below.

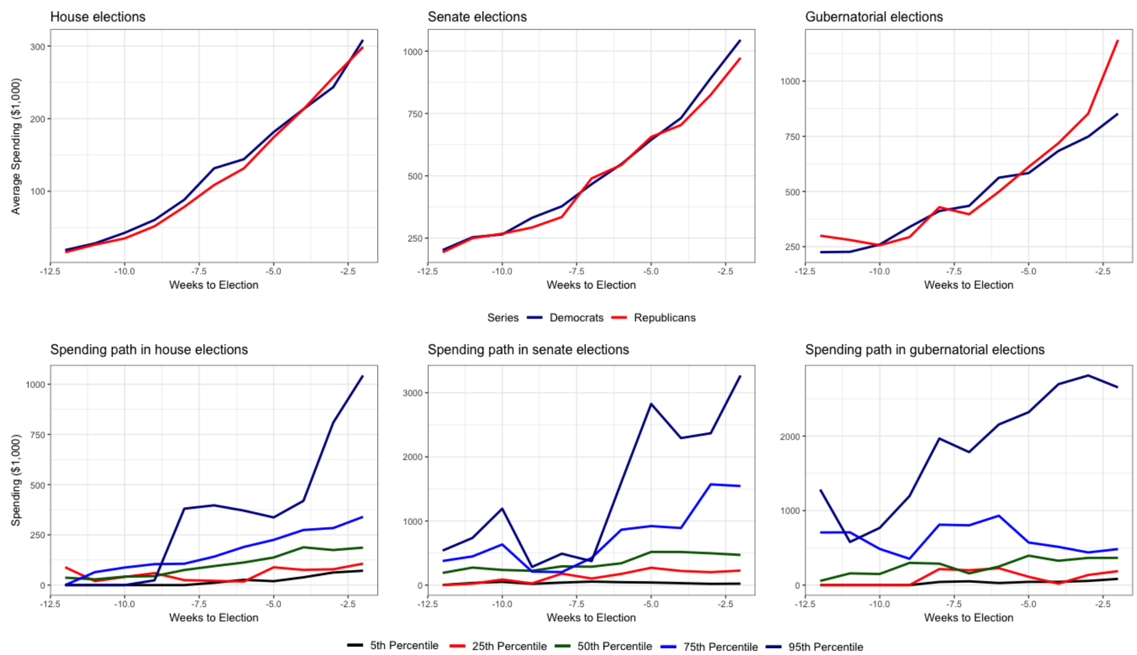

Figure 1: Upper figures are average spending paths by Democrats and Republicans on TV ads in "competitive" House, Senate and gubernatorial races in the period 2000-2014. These are elections in which both candidates spent a positive amount; see Section 4.1 for the source of these data, and more details. Bottom figures are spending paths for 5th, 25th, 50th, 75th, and 95th percentile candidates in terms of total money spent in the corresponding elections of the upper panel.

equilibrium path of spending is independent of the realizations of the stochastic process governing the evolution of relative popularity.

If the function that maps the candidates' spending levels to the drift of the popularity process is homogeneous, then an "equal spending ratio" property holds on the equilibrium path of play: the two candidates spend an equal share of their remaining budgets in every period. In addition, a "constant spending growth" property also holds: the rate of spending growth is (the same) constant over time for both candidates.

In fact, under the homogeneity assumption mentioned above, we can fully characterize the equilibrium rate of growth in spending over time as a function of the degree of homogeneity of the drift function and of the decay rate. We find that on the path of play, candidates increase their spending levels over time when popularity leads tend to decay; the rate of spending growth is increasing in the decay rate; and when there is no decay, they spread their budgets evenly across periods.

The tractability of our framework enables us to study several variants of the baseline model. First, we allow for the possibility that some voters turn out early, prior to the election date. Early voting has been an increasingly important phenomenon in American elections over the past decade. We characterize the candidates' spending paths under the assumption that voting commences prior to the election date. Early voting gives candidates an incentive to spend more resources in earlier stages. Once early voting starts, the growth rate of spending is no longer constant over time, and spending grows at a rate that is decreasing in the extent of early voting.

Second, we relax the zero-sum assumption of the model by having the candidates value money left over at the end of the race. Although election law restricts candidates from personally consuming campaign funds, they may still value money left over. For example, candidates may want to save money to spend on future elections. To characterize the equilibrium of this variant, we assume that the drift of the popularity processes is homogeneous of degree zero in the candidates' spending levels and that the marginal value of money left over is constant. We show that in every period, the ratio of the candidates' spending levels is constant and equal to the inverse ratio of their marginal values for money. However, in this variant, spending levels do vary with relative popularity: if the election is lopsided (in that one candidate develops a large popularity lead over the other), then both candidates spend less.

Third, we look at a variant of our model in which competition is over multiple districts, or possibly multiple media markets in a single district. We assume again that the drift is homogeneous of degree zero in the candidates' spending levels. This variant also covers the case in which candidates must decide how to target their spending across different groups of voters within a single district. We show that our equal spending ratio result holds district-by-district, and we characterize how resources are allocated not just over time but also across districts.

We end the paper by examining patterns of campaign spending in actual subnational American elections, focusing on TV ad spending. We first examine the extent to which the predictions of our model are violated in the data. We then fit the model to the data to obtain estimates of the candidates' perceived decay rates. Perceived decay rates are an important quantity of interest in practice because they tell us how candidates view a key factor that drives their spending decisions. They may also be useful as a benchmark for future candidates seeking to optimize their spending. We uncover substantial variation in perceived decay rates across races that is not explained by race characteristics such as incumbency vs. open seat, state-wide race vs. Congressional race, and the availability of early voting.

Related Literature— Our paper relates to the prior literature on campaigning. Kawai and Sunada (2015), for example, build on the work of Erikson and Palfrey (1993, 2000) to estimate a model of fund-raising and campaigning. While they assume that candidates allocate resources across different elections, we study the allocation problem across periods in the run-up to a particular election. In de Roos and Sarafidis (2018) candidates that won past races enjoy momentum, which results from a complementarity between prior successes and the current returns to spending.[2] In our setting, on the other hand, prior spending affects the popularity process but popularity leads decay over time.

Meirowitz (2008) studies a static model to show how asymmetries in the cost of effort can explain the incumbency advantage. Polborn and David (2004) and Skaperdas and Grofman (1995) also examine static campaigning models in which candidates choose between positive or negative advertising.[3] Iaryczower et al. (2017) estimate a model in which campaign spending weakens electoral accountability, assuming that the cost of spending is exogenous rather than subject to an inter-temporal budget constraint as in our model. Garcia-Jimeno and Yildirim (2017) estimate a dynamic model of campaigning in which candidates decide how to target voters in the presence of strategic media. Gul and Pesendorfer (2012) study a model of campaigning in which candidates provide information to voters over time, and face the strategic timing decision of when to stop. In our setting, by contrast, the date of the election is fixed, and spending affects the drift of the popularity process.

Our work is also related to the literature in marketing and operations research that models advertising as a stochastic control problem. In the seminal work of Nerlove and Arrow (1962), an agent controls the "stock of goodwill" over time by continuously deciding how much to spend on advertising while goodwill depreciates. More recently, Marinelli (2007) studies a problem similar to ours with a single advertiser facing an exogenous launch date for a product. The stock of goodwill is modeled as a Brownian motion whose evolution is controlled through spending. In the optimal control strategy the advertiser spends nothing until an intermediate time, and then she spends the maximum amount possible until the launch date.[4]

[2] Other models of electoral campaigns in which candidates enjoy momentum—such as Callander (2007), Knight and Schiff (2010), Ali and Kartik (2012)—entail sequential voting.

[3] Other static models of campaigning include Prat (2002) and Coate (2004), that investigate how one-shot campaign advertising financed by interest groups affects elections and voter welfare, and Krasa and Polborn (2010), that study a model in which candidates compete on the level of effort that they exert in different policy areas. Prato and Wolton (2018) study the effects of reputation and partisan imbalances on the electoral outcome.

[4] Feichtinger et al. (1994) provide a survey of the literature on stochastic control models in advertising. Several papers in this literature look at advertising for regular consumer goods (in the absence of a product launch), where advertisers use a "pulsing" strategy: short, high-intensity periods of ad spending followed by no spending at all. This pattern of spending is justified through a threshold-based (Dubé et al., 2005) or an S-shaped sales response curve to advertising (Feinberg, 2001, Aravindakshan and Naik, 2015). Using a model in which a stock of goodwill depreciates over time, Bronnenberg et al. (2012) study the long-term effects of marketing and brand images.

The effect of advertising in elections is also studied in the marketing literature (see Gordon et al., 2012, for an early contribution). For example, Gordon and Hartmann (2013) estimate that political advertising impacts the outcome of U.S. presidential elections, although advertising elasticities are smaller than for other branded goods.[5] Lovett and Peress (2015) estimate a model of targeted political advertising and find that TV ads mostly target swing voters. Chung and Zhang (2015) estimate the effectiveness of different campaign activities for the two major parties in U.S. presidential elections. Our model contributes to this literature by providing a tractable theoretical framework to study the allocation of advertising resources over time in a two-candidate generic electoral competition setting.

In a game similar to ours, Kwon and Zhang (2015) study a two-player model of stochastic control and strategic exit motivated by a duopolistic market where market shares are modeled as a diffusion process and the firms can choose to exit at any time.[6] Our paper also relates to Kamada and Kandori (2020) who study electoral campaigns through revision games, and to Kamada and Sugaya (2020) who study electoral campaigns as finite-horizon dynamic games.

Insofar as it studies campaigning as a dynamic strategic allocation problem, our paper relates to the vast literature on dynamic contests (see Konrad et al., 2009, and Vojnović, 2016 for reviews). Within this literature, Glazer and Hassin (2000) and Hinnosaar (2018) study contests in which multiple players move sequentially and only once, while we consider a setting in which the same two candidates move repeatedly over multiple periods.

Finally, our model relates to models of strategic races (see Harris and Vickers, 1985, 1987 for seminal contributions).[7] The papers that are most closely related to ours in this literature include Klumpp and Polborn (2006), Konrad and Kovenock (2009) and Klumpp et al. (2019). In fact, Klumpp et al. (2019) study a dynamic contest that is strategically similar to the special case of our baseline model in which there is no decay, finding that resource allocation is constant in time.[8]

[5]Gordon and Hartmann (2016) also focus on U.S. presidential elections and find that the electoral colleges skew the allocations of advertising resources toward battleground states and increase overall spending when the election is not tight.

[6]From a modeling standpoint, our approach in which two players simultaneously take actions in pre-determined periods makes our setup more tractable and it allows us to fully characterize the unique equilibrium path of spending.

[7]Lee and Wilde (1980) and Reinganum (1981, 1982) study races in the presence of uncertainty, but do not cover situations in which one competitor leads or trails against the others.

[8]Our use of the first order approach to characterize the equilibrium behavior also connects our paper to Cornes and Hartley (2005), Kolmar and Rommeswinkel (2013), Choi et al. (2016), Konishi et al. (2019) and Crutzen et al. (2020), who use CES functions in static contests to aggregate individual efforts.

2 Baseline Model

2.1 Setup

Consider the following complete information *dynamic campaigning game* between two candidates, $i = 1, 2$, ahead of an election. Time is discrete with a finite horizon and indexed by $t = 0, ..., T$. At the start of the game, each candidate is endowed with a budget: $X_0 > 0$ for candidate 1 and $Y_0 > 0$ for candidate 2.[9]

Candidates allocate their budgets across time to influence a state variable that we call *relative popularity*. We identify a *period* with the time $t = 0, 1, ..., T-1$ that candidates make spending decisions, and we use *time* to refer to the dates $t = 0, 1, ..., T$ at which relative popularity is measured. This includes the final date T at which the election takes place.

Let x_t be the amount of her remaining budget that candidate 1 spends in period t and y_t be the amount that candidate 2 spends. Candidate 1's remaining budget in period t is $X_t = X_0 - \sum_{t'<t} x_{t'}$ while candidate 2's is $Y_t = Y_0 - \sum_{t'<t} y_{t'}$. In every period t, budget constraints must hold: $x_t \leq X_t$ and $y_t \leq Y_t$.

Relative popularity at any time t is a random variable $Z_t \in \mathbb{R}$, whose realization we denote z_t. We interpret this random variable as a measure of candidate 1's lead in the polls. If $z_t > 0$, then candidate 1 is ahead of candidate 2; if $z_t < 0$, then candidate 2 is ahead; and if $z_t = 0$, it is a dead-heat. The campaign starts with relative popularity set to some arbitrary level $z_0 \in \mathbb{R}$.

The winner of the election at time T is the candidate that is more popular. So, if $z_T > 0$, then candidate 1 wins the election; if $z_T < 0$, then candidate 2 wins the election; and if $z_T = 0$, then the election is a tie and we assume that each candidate wins the election with probability 1/2. The winner accrues a payoff of 1 while the loser gets a payoff of 0.

Relative popularity evolves according to the following AR(1) process:

$$Z_{t+1} = p(x_t, y_t) + \delta Z_t + \varepsilon_t \tag{1}$$

Spending levels x_t and y_t in period t thus affect the evolution of relative popularity through the function $p : \mathbb{R}^2_+ \to \mathbb{R}$. $\delta \in (0, 1]$ is an inverse measure of the decay rate of the popularity process, and $\varepsilon_t \sim \mathcal{N}(0, \sigma^2)$ is a mean-zero normally distributed random shock with variance σ^2.

[9]This fixed budget assumption is tantamount to assuming that the candidates can forecast how much money will be available to them, they have access to credit, and they cannot end the race in debt. In actual elections, some large donors make pledges early on and disburse funds over time. In section OA3 of the Online Appendix, we further allow candidates' budget to change stochastically over time in response to changes in the popularity process.

We assume throughout that the shocks $\{\varepsilon_t\}$ are iid and that each shock ε_t is realized after the candidates make their period t spending choices. We note that by allowing for $\delta = 1$, we cover the case in which popularity leads do not decay.

Our solution concept is pure strategy subgame perfect equilibrium (SPE), which we refer to succinctly as "equilibrium." In the following section, we introduce an assumption on the popularity process—specifically, the function p—to establish equilibrium existence and uniqueness, and we show that on-path equilibrium spending levels are independent of the past realizations of relative popularity. In the sections that follow, we progressively strengthen assumptions on the function p to derive additional properties of the equilibrium path under these assumptions.

2.2 Equilibrium Analysis

Recursive substitution of equation (1) yields the following expression for relative popularity at the time of the election:

$$Z_T = \sum_{t=0}^{T-1} \delta^{T-1-t} p(x_t, y_t) + \delta^T z_0 + \sum_{t=0}^{T-1} \delta^{T-1-t} \varepsilon_t \tag{2}$$

Note that Z_T is the sum of three additively separable terms: the (weighted sum of the) impact of candidates' spending levels, the (discounted) level of the initial popularity, and the (weighted sum of the) normal mean-zero popularity shocks.

In any period t, candidate 1 maximizes $\Pr[Z_T > 0 \mid (z_{t'}, X_{t'}, Y_{t'})_{t' \leq t}]$, while candidate 2 minimizes this. Since the coefficients of the normal shocks in (2) do not depend on the candidates' choices, the variance of Z_T is independent of the candidates' strategies. Therefore, we can assume that candidate 1 maximizes the expected value of Z_T, while candidate 2 minimizes it. Because the game is zero-sum, given candidate 2's on-path spending levels $(y_0, ..., y_{T-1})$, the on-path equilibrium spending levels $(x_0, ..., x_{T-1})$ of candidate 1 solve the following maximization problem:

$$\max_{x_0,\dots,x_{T-1}} \sum_{t=0}^{T-1} \delta^{T-1-t} p(x_t, y_t)$$

$$\text{s.t. } x_t \geq 0,\ \forall t = 0, ..., T-1, \text{ and } \sum_{t=0}^{T-1} x_t = X_0 \tag{3}$$

Given candidate 1's on-path spending levels $(x_0, ..., x_{T-1})$, candidate 2's on-path equilibrium spending levels minimize $\sum_{t=0}^{T-1} \delta^{T-1-t} p(x_t, y_t)$ subject to the corresponding constraints. The following assumption ensures equilibrium existence and uniqueness.

Assumption 1. *The function p is twice continously differentiable, and*

(a) $p(\cdot, y)$ is strictly increasing for all y, and $p(x, \cdot)$ is strictly decreasing for all x;

(b) $p(\cdot, y)$ is strictly concave for all y, and $p(x, \cdot)$ is strictly convex for all x;

(c) p satisfies the Inada-0 conditions:

$$\lim_{x \to 0} \frac{\partial p(x, y)}{\partial x} = \infty \text{ for all } y \quad \text{and} \quad \lim_{y \to 0} \frac{\partial p(x, y)}{\partial y} = -\infty \text{ for all } x.$$

Assumption 1(a) states that each candidate's spending has a positive effect on her popularity. Assumption 1(b) implies that each candidate has a unique spending level that maximizes her relative popularity given the spending level of the other candidate. Finally, Assumption 1(c) says that the marginal benefit of spending is very large when a candidate is spending close to zero.

Assumption 1 guarantees that problem (3) for candidate 1 and the corresponding problem for candidate 2 are both concave. The candidates' equilibirum on-path spending levels can thus be found by solving the system of first order conditions to these problems. Our first proposition records this observation and the fact that the equilibrium spending path is independent of past realizations of relative popularity.

Proposition 1. *Suppose Assumption 1 holds. Then,*

(i) the dynamic campaigning game has a unique equilibrium, and the on-path spending levels satisfy the first order conditions of the optimization problem (3), and

(ii) for all periods t, the equilibrium on-path spending levels (x_t, y_t) are independent of the past history of relative popularity $(z_{t'})_{t' \leq t}$.

The intuition for part (ii) of the proposition is as follows. In equilibrium, candidates allocate resources over time based on the marginal rate of substitution between spending in different periods. When a popularity shock occurs at time t, the probability a candidate wins changes, but the marginal benefit of spending in all periods after time t also changes by the same amount. The marginal rate of substitution between spending in different periods is then independent of the popularity shocks. This result holds because the popularity process in (1) is additively separable. If we relax this additive separability, spending decisions are not necessarily popularity independent.[10]

[10] However, under some additional conditions, the ratio of candidates' spending decisions is independent of the popularity shocks and this can be enough to study the evolution of the electoral competition. See Section OA1 in the Online Appendix for details.

The results of Proposition 1—particularly, the history-independence property reported in part (ii)—have further notable consequences for the robustness of the equilibrium path to changing the structure of the game. Although the dynamic campaigning game has complete information, the equilibrium path of the game is robust to candidates having incomplete information about the popularity process or to the candidates moving sequentially in each period, as the following remark clarifies.

Remark 1. *The equilibrium of the game has the same path of play as*

(i) any equilibrium of the alternative version of the game where candidates imperfectly (and possibly asymmetrically) observe the realization of the path of relative popularity, and

(ii) every Nash equilibrium of a game where candidates move sequentially within a period with arbitrary (and possibly stochastic) order of moves.

These observations follow from equation (2) and known results in the literature on zero-sum games. In particular, because on-path spending levels do not depend on past realizations of popularity, the candidates' equilibrium spending paths would be the same even if popularity was not fully observable. Furthermore, because the game is zero-sum, the equilibrium path of play is unique and robust to allowing candidates to move sequentially within a period, with arbitrary order of moves (see, for example, Mertens et al., 2015).

2.3 Equilibrium Spending Ratios

To say more about the equilibrium spending paths and the candidates' equilibrium probabilities of winning, we need to impose additional assumptions on how spending levels affect the popularity process. Under the following assumption, we can fully specify the equilibrium evolution of the popularity process.

Assumption 2. *The function p is homogeneous of degree $\beta \geq 0$.*

The function $p(x, y) = \alpha_1 x^\beta - \alpha_2 y^\beta$ satisfies this assumption and further satisfies Assumption 1 when $\beta \in (0, 1)$ and $\alpha_1, \alpha_2 > 0$. Another example that satisfies the assumption is the function $p(x, y) = \alpha_1 \log x - \alpha_2 \log y$, which is homogeneous of degree 0 and satisfies Assumption 1 when $\alpha_1, \alpha_2 > 0$.[11]

[11] For this example, the model is not closed since p is not defined when either $x = 0$ or $y = 0$. However, we can close the model by assuming that: (i) if any candidate i spends 0 at any time t, then the game ends immediately with candidate $j \neq i$ winning so long as j spends a positive amount, and (ii) if both candidates simultaneously spend 0, then the game ends with each candidate winning with probability 1/2. The results of Proposition 1 and 2 continue to hold under this amendment.

We define the *spending ratio* of a candidate in period t to be the ratio between her spending level in period t and the remaining budget available to her that period: in period t, candidate 1's spending ratio is x_t/X_t and candidate 2's is y_t/Y_t. We refer to the ratio of spending in period $t+1$ to spending in period t for a candidate as the *consecutive period spending ratio*, and we use $r_{1,t} := x_{t+1}/x_t$ and $r_{2,t} := y_{t+1}/y_t$ from here on to denote them.

We now show that Assumption 2 implies two key results that inform our analysis of on-path spending patterns in actual elections. The first is an equal spending ratio result that says that the candidates' spending ratios equal each other on the path of play. The second is a constant spending growth result that says that the candidates' consecutive period spending ratios equal the same constant in all periods.

Proposition 2. *Suppose Assumptions 1 and 2 hold. Then, in the unique equilibrium path of the dynamic campaigning game,*

(i) the candidates' spending ratios equal each other every period: $x_t/X_t = y_t/Y_t$ for all periods t.

(ii) the candidates' consecutive period spending ratios equal each other and are constant through time; in particular, $r_{1,t} = r_{2,t} = \delta^{1/(\beta-1)}$ for all periods $t < T-1$.

The intuition behind the equal spending ratio result is as follows. To maximize the probability of winning the election, candidates equalize the (decay-weighted) marginal benefit of spending at any period $t < T-1$ with the (decay-weighted) marginal benefit of spending in period $T-1$, just ahead of the election. Assumption 2 implies that the first order conditions of each candidate depend on her opponent's spending only through the ratio of their spending levels. The equal spending ratio result then follows from taking the ratio of these first order conditions for each period.

The equal spending ratio result is notable because it implies a number of additional equilibrium properties. For example, it implies that the candidates' on-path consecutive period spending ratios are also equal to each other, i.e. when $x_t/X_t = y_t/Y_t$ we have $r_{1,t} = r_{2,t}$ for all periods t. Because budgets are fixed, it also implies that the ratio x_t/y_t of the candidates' spending levels in any period t (which we refer to as the *cross-candidate spending ratio*) is a constant that is equal to the ratio of the candidates' starting budgets; that is, $x_t/y_t = X_0/Y_0$ for all periods t. Showing that the cross candidate spending ratio is constant over time is a key step to deriving the equilibrium consecutive period spending ratios.

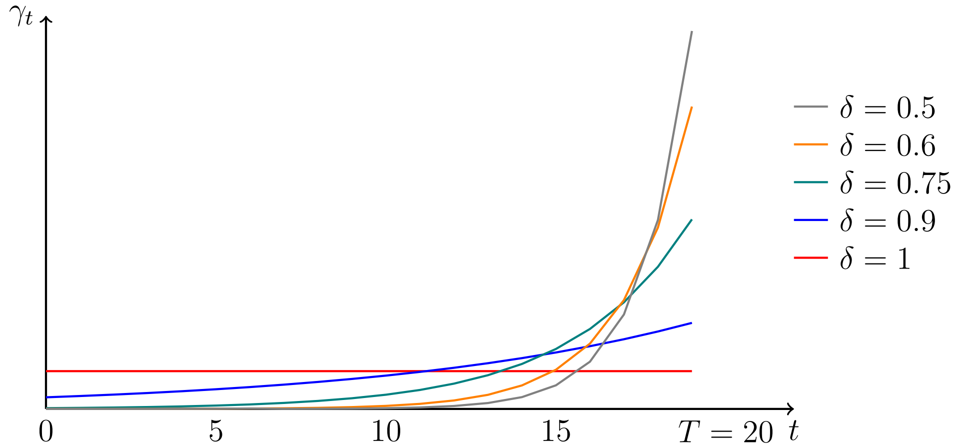

Figure 2: Equilibrium share of the initial budget $\gamma_t = x_t/X_0$ that the candidates spend over time when $T = 20$, $\beta = 0$ and δ takes different values.

It is clear from Proposition 2 that for spending to actually grow over time it must be that both $\beta, \delta < 1$ given our specification that δ does not exceed 1.[12] The expression in the proposition verifies that if $\delta = 1$ (popularity leads do not decay) then the candidates spread their budgets evenly across periods. Since p is concave, the candidates want to smooth their spending over time. The lack of decay further implies that this smoothing is full: candidates allocate the same share of their initial budgets to each period. On the other hand, if $\delta < 1$ then spending increases over time, and the fraction of the initial budget each candidate spends at time t is

$$\gamma_t = \frac{x_t}{X_0} = \frac{y_t}{Y_0} = \frac{r-1}{r^T - 1} r^t, \tag{4}$$

where $r = \delta^{1/(\beta-1)}$ is the common consecutive period spending ratio. The assumption that popularity leads decay ($\delta < 1$) generates a force that pushes candidates towards spending more in later periods.

The comparative statics of γ_t reflect these countervailing forces. If β increases, the marginal return to spending decreases at a slower rate within each period. Candidates thus spend even more towards the end of the campaign and less in the early stages. As $\beta \to 1^-$ candidates spend all of their resources in the final period. As δ decreases, popularity leads decay more and candidates have an incentive to invest less in the

[12] Although we have assumed $\delta \leq 1$, none of the above results rely on this assumption. They hold even if $\delta > 1$, in which case popularity leads tend to amplify over time; and on the equilibrium path the candidates will decrease their spending over time if $\beta < 1$.

early stages of the race and more in the later stages. Figure 2 depicts these features, plotting γ_t for $\beta = 0$ and different values of δ.

Strategic Spending Considerations. A candidate's best response varies with the spending behavior of the other candidate only if the effects of the candidates' spending levels on the drift of the popularity process (i.e., function p) are not additively separable. So consider the function $p(x, y) = (x - y)/2(x + y)$ which is not additively separable.[13] Given any behavior by candidate 2, the first order condition for candidate 1 implies that the marginal benefit to spending in period $t < T - 1$ equals the marginal benefit to spending in the final period $T - 1$, or

$$\delta^{T-1-t} \frac{y_t}{(x_t + y_t)^2} = \frac{y_{T-1}}{(x_{T-1} + y_{T-1})^2}.$$

Both the left and the right hand sides of this equation feature expressions of the form $y/(x+y)^2$, whose partial derivative in y is $(x-y)/(x+y)^3$ and in x is $-2y/(x+y)^3$. With this in mind, suppose that candidate 2 marginally lowers his spending in period t and to keep his budget balanced increases his spending in a later period, say the final period. The previous observation implies that candidate 1's best response would be to either increase her spending (this happens if $x_t \leq y_t$), or to lower it as well but by a factor smaller than candidate 2's (this happens if $x_t > y_t$).[14] In both cases, the cross-candidate spending ratio x_t/y_t increases. Analogously, if candidate 2 raises her spending in any period t relative to the equilibrium level, and candidate 1 best responds, then the cross-candidate spending ratio x_t/y_t decreases.

Suppose that candidate 2 naively spends all of his budget in the final period. The observations above imply that a strategic candidate 1 would best respond by spending a positive amount in all periods and increasing her spending over time at a rate that is faster than the equilibrium rate, i.e. the rate stated in Proposition 2 for $\beta = 0$. If candidate 2 naively allocates his budget evenly across all periods, a strategic candidate 1 would best respond by increasing her spending over time at a slower rate than the equilibrium rate.

[13] To close the model when both candidates spend 0, see footnote 11. In addition, although this function does not satisfy Assumption 1(c), the results of Propositions 1 and 2 hold with $\beta = 0$; in particular, the first order conditions are satisfied at an interior equilibrium, and since the function is homogeneous of degree 0 the common consecutive period spending ratio is $r = 1/\delta$.

[14] To see why, note that if candidate 2 lowers his spending in period t from y_t to αy_t with $\alpha < 1$ and candidate 1 also lowers her spending from x_t to αx_t (or to an even lower amount) then $y_t/(x_t + y_t)^2$ drops to $y_t/[\alpha(x_t + y_t)^2]$ (or even lower) and the FOC is violated. For the FOC to hold, candidate 1's best response spending level must necessarily be larger than αx_t.

2.4 Discussion

Our baseline model is general enough to account for several factors that influence campaigning. For instance, advantages (or disadvantages) due to incumbency, to prior legislative records, or to a candidate's name recognition can affect the initial lead in relative popularity, z_0, or starting budgets, X_0 and Y_0.

Candidates can also differ in the effectiveness of their campaign spending. These differences may depend on differences in how their campaigns are organized, or on the fact that one candidate is simply better than the other at campaigning. A candidate's policy platform may also be more popular than that of the other candidate. We can capture these features through asymmetries in the partial first derivatives of p.

Although the payoffs we have assumed imply a winner-take-all electoral rule, our equilibrium analysis also immediately extends to the case where the candidates' payoffs are linear (or piecewise linear) in relative popularity on election day, z_T. Therefore, it covers the case in which margin of victory also matters to the candidates.

The results in the previous sections hold if relative popularity evolves according to the AR(1) process in equation (1). In the Online Appendix, we examine nonseparable popularity processes, imposing additional assumptions to guarantee that the first-order approach is still sufficient to characterize the equilibrium evolution of relative popularity.

In our baseline model candidates have fixed budgets, or equivalently can forecast exactly how much money they will have by the end, and they are not allowed to finish in debt. In the Online Appendix, we consider a variant of the model in which budgets are uncertain and evolve over time in response to fluctuations in relative popularity. We show for a specification of that model that the equal spending ratio result continues to hold but the constant spending growth prediction does not. Because spending decisions depend on the candidates' expectations of how their budgets evolve and because these expectations vary with fluctuations in the relative popularity, equilibrium spending also evolves stochastically.

Finally, in the Online Appendix we present a model in which we allow voters to react to campaign spending differently, following the approach of the marketing literature. Our model of the electorate gives rise to a popularity process for the candidates that is equivalent to equation (1). We then demonstrate how this approach can be used to derive policy implications; specifically, we study the welfare effects of campaign silence laws and spending caps.

In the next section, we look at three additional variants of our model.

3 Variants

3.1 Early Voting

In the baseline model, the candidates' payoffs depend only on their relative popularity on election day, i.e at time T. But in many elections voters can and do cast their votes prior to election day, which suggests that the candidates' payoffs should depend on realizations of relative popularity even prior to time T. We now analyze how early voting affects the candidates' spending decisions.

Consider the baseline model, but now suppose that voters can vote from period $\hat{T} < T$ onwards. Suppose that the difference in votes cast for the two candidates in each period $t \geq \hat{T}$ is proportional to their relative popularity in that period, Z_t, and let the number of votes cast in period $t \geq \hat{T}$ be a proportion $\xi \in (0, 1)$ of the number of votes cast in period $t + 1$.[15] As ξ converges to zero, almost all votes are cast at time T and the model converges to the baseline model. Finally, assume that despite the possibility of early voting, either candidate is still able to eventually win the election if she is sufficiently more popular than her opponent at T no matter how low her popularity was in previous periods.[16]

Candidate 1 thus maximizes $\Pr[\sum_{t=\hat{T}}^{T} \xi^{T-t} Z_t \geq 0 \mid (z_{t'}, X_{t'}, Y_{t'})_{t' \leq t}]$, while candidate 2 minimizes this expression. An analogue to problem (3) in the baseline model holds in this variant as well. In particular, candidate 1's equilibrium spending path $\{x_0, ..., x_{T-1}\}$ now maximizes

$$\sum_{t=0}^{\hat{T}-1} \sum_{t'=0}^{T-\hat{T}} \xi^{t'} \delta^{T-1-t-t'} p(x_t, y_t) + \sum_{t=\hat{T}}^{T-1} \sum_{t'=0}^{T-1-t} \xi^{t'} \delta^{T-1-t-t'} p(x_t, y_t), \tag{5}$$

subject to the same nonnegativity and budget constraints as in problem (3). Candidate 2's spending path correspondingly minimizes this expression subject to her own nonnegativity and budget constraints.

Proposition 3. *Suppose Assumptions 1 and 2 hold. Then in the unique equilibrium path of the game with early voting,*

(i) $x_t/X_t = y_t/Y_t$ *for all periods* t.

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[15] We thus postulate constant growth in turnout. This assumption simplifies the notation and the statement of our result, but our proof techniques extend to other assumptions about turnout so long as candidates anticipate the turnout rates and cannot manipulate them.

[16] This condition holds if $\xi(2 - \xi^{T-\hat{T}}) < 1$, which is implied by $\xi < 1/2$. Alternatively, we could also assume that candidate 1 maximizes (and candidate 2 minimizes) the difference in candidate 1 and 2's vote shares, $\sum_{t=\hat{T}}^{T} \xi^{T-t} Z_t$. The results of Proposition 3 extend to this case.

(ii) the consecutive period spending ratios equal each other: $r_{1,t} = r_{2,t} = r_t$ *for all periods* t*, and in particular,* $r_t = \delta^{1/(\beta-1)}$ *if* $t < \hat{T} - 1$*, and*

$$r_t = \left[\delta\left(1 + \frac{1}{\sum_{t'=0}^{T-2-t} \xi^{-(T-1-t-t')}\delta^{T-2-t-t'}}\right)\right]^{1/(\beta-1)} \qquad \textit{if } t \geq \hat{T} - 1$$

Proposition 3 asserts that under early voting candidates continue to allocate the same share of their budgets on the path of play. But early voting modifies the spending path. Because the term in large parentheses in the $t \geq \hat{T} - 1$ case is larger than 1 and because $\beta < 1$, $r_t < r$ when $t \geq \hat{T} - 1$: the spending path is now flatter. As some voters vote early, candidates now have a new incentive to allocate a larger share of their budget to earlier periods, relative to the baseline model. Moreover, the consecutive period spending ratio is decreasing in ξ after early voting begins. If a larger share of voters vote early (higher ξ), the candidates' spending levels will be more evenly distributed in these periods (lower r_t for $t > \hat{T} - 1$).

3.2 Valuing Money Left Over

In the variants studied so far, the two candidates are purely office-motivated and fully deplete their budgets by the end of the race because they do not value money left over. However, in reality money left over may be valuable: a candidate may want to save money for future campaigns, or for investment opportunities outside politics—to the extent that this is legally allowed.

To capture this possibility, let X_T and Y_T be money left over at the end of the campaign for candidates 1 and 2 respectively. Assume that at each time t candidate 1 maximizes $\Pr[Z_T \geq 0 \mid (z_{t'}, X_{t'}, Y_{t'})_{t' \leq t}] + \kappa_1 X_T$, while candidate 2 maximizes $(1 - \Pr[Z_T \geq 0 \mid (z_{t'}, X_{t'}, Y_{t'})_{t' \leq t}]) + \kappa_2 Y_T$. The parameter $\kappa_i > 0$ captures candidate i's marginal value for money. On top of saving money and benefiting from this at rate κ_i, we also assume that each candidate i can overspend his budget by borrowing money at a cost equal to κ_i.[17] Thus, X_T and Y_T can be negative.

In addition, for tractability we assume that Assumption 2 holds with $\beta = 0$ and we define the function q so that $p(x, y) = p(x/y, 1) =: q(x/y)$ for $y > 0$. To close the model in the case of $y = 0$, see footnote 11.

In this setting, the candidates' spending ratios (and spending levels) are not constant in time, and in fact depend on the realized path of relative popularity. Spending by both candidates decreases as the race becomes more lopsided. To state this pop-

[17] To simplify the analysis, we abstract from the time dimension when we model borrowing: a unit of money borrowed at any point during the race has the same cost κ_i.

ularity dependence formally, define the following quantity for every time t:

$$\zeta((\varepsilon_{t'})_{t'=0}^{t-1}) = \frac{\sum_{t'=0}^{T-1} \delta^{T-1-t'} q\left(\kappa_1/\kappa_2\right) + \delta^T z_0 + \sum_{t'=0}^{t-1} \delta^{T-1-t'} \varepsilon_{t'}}{\sigma\sqrt{\sum_{t'=t}^{T-1} \delta^{T-1-t'}}} \tag{6}$$

where ε_t is the popularity shock at time t. This quantity measures the expected electoral advantage that one candidate has over the other at time t: when one candidate has a large popularity advantage over the other, $|\zeta((\varepsilon_{t'})_{t'=0}^{t-1})|$ is large.

Proposition 4. *Suppose Assumptions 1 and 2 hold with $\beta = 0$. Then in the unique equilibrium path of the game in which candidates' marginal valuations for money left over are $\kappa_1, \kappa_2 > 0$,*

- *(i) $x_t/y_t = \kappa_2/\kappa_1$ for all periods t, and*
- *(ii) x_t and y_t are both decreasing in $|\zeta((\varepsilon_{t'})_{t'=0}^{t-1})|$.*

Part (i) of Proposition 4 says that the equilibrium cross-candidate spending ratios x_t/y_t all equal the ratio of marginal valuations of money left over, κ_2/κ_1. (Recall that in the baseline model the cross-candidate spending ratios are all equal to the ratio of starting budgets X_0/Y_0.) The intuition is as follows. In equilibrium candidates equalize the marginal benefit of spending with the opportunity cost, which now is equal to the marginal value of money left over. When the drift is homogeneous of degree 0, the ratio of the candidates' marginal benefits to spending depends only on the ratio of their spending levels.

Part (ii) of the proposition says that spending by both candidates decreases as the election becomes more lopsided, implying that the candidates' spending levels are no longer independent of relative popularity. A popularity shock at time t still affects the probability of winning and, hence, the marginal value of spending in all subsequent periods. But the shock does not change the marginal value of saving money, which remains equal to κ_i. The marginal rate of substitution between spending at a given time and saving the budget thus depends on the popularity shock and spending decisions are no longer independent of the shock.

3.3 Multi-district Competition

In any campaign, candidates choose not just when to spend their resources, but also how to target these resources across voters—for example by targeting specific geographic areas. Suppose that the two candidates compete over multiple districts, or any other targetable subpopulation. The set of districts is $\{1, 2, ..., S\}$ and the payoffs of the candidates depend on how these different districts aggregate. With a specific aggregation rule, this setting covers the electoral college in U.S. presidential elections, competition between two parties seeking to control a majoritarian legislature with representatives elected in winner-take-all single-member districts, and the case where candidates compete in a single winner-take-all race but must choose how to allocate spending across different media markets within a single district.

Popularity in each district s is represented by the random variable Z_t^s with realizations z_t^s. We assume that $(Z_t^s)_s$ are distributed according to a multivariate normal distribution with arbitrary variance-covariance matrix. In each district s, the popularity process is

$$Z_{t+1}^s = p(x_t^s, y_t^s) + \delta^s Z_t^s + \varepsilon_t^s, \tag{7}$$

where $\varepsilon_t^s \sim \mathcal{N}(0, (\sigma^s)^2)$ and these shocks are iid over time. Each district s thus has its own decay parameter δ^s, and its own variance $(\sigma^s)^2$. In addition, as in the previous section we assume that the function p satisfies Assumptions 1 and 2 with $\beta = 0$ so that $p(x, y) = p(x/y, 1) = q(x/y)$ for some function q.[18]

The aggregation rule for the outcomes in the various districts is arbitrary, but we impose the following assumptions: the candidates' payoffs depend only on the vector $(Z_T^s)_{s=1}^S$, the game is still zero sum, and candidate 1's payoff is strictly increasing in each Z_T^s, while candidate 2's is strictly decreasing in each Z_T^s. More formally, denote candidate 1's payoff $u((Z_T^s)_{s=1}^S)$ so that candidate 2's payoff is $-u((Z_T^s)_{s=1}^S)$, and assume that

$$\frac{\partial u((Z_T^s)_{s=1}^S)}{\partial Z_T^s} > 0, \qquad \text{for every } s. \tag{8}$$

For this model, we can show that the equal spending ratio result hold district by district, which is stated in part (i) of Proposition 5 below. However, unlike in the baseline model, spending decisions may depend on the history of the popularity processes. If the competition in some districts becomes lopsided (in terms of the candidates' relative popularity), the marginal benefit of spending money in those districts decreases for both candidates. Candidates react by concentrating their spending in other, more competitive districts. Relative popularity within districts thus plays a role in the spending decisions.

This popularity-dependence does not arise in the special case in which payoffs are a weighted sum of relative popularity in each district at time T. In this case, candidate 1's marginal benefit of increasing her popularity in a specific district is constant and equal to the marginal benefit of candidate 2. Moreover, under this assumption, we can characterize the expected inverse consecutive period spending ratios for this model as well as the optimal allocation of resources across districts in each period—results

[18]We extend the assumption in footnote 11 as follows: if a candidate spends an amount equal to 0 in any district, then the game ends and the candidate wins with probability $1/2$ if the other candidate is also spending an amount equal to 0 in some district, and loses with probability 1 otherwise.

that are stated in parts (ii) and (iii) of Proposition 5 respectively. The following assumption, which strengthens the monotonicity assumption in equation (8), states the condition formally.

Assumption 3. *For weights* $\{w^s\}_{s=1}^S$ *such that* $w^s > 0$ *and* $\sum_{s=1}^S w^s = 1$,

$$u\left((Z_T^s)_{s=1}^S\right) = \sum_{s=1}^S w^s Z_T^s.$$

Assumption 3 fits either a setting where candidates allocate resources across multiple media markets, or one in which the candidates are two parties that compete to maximize the number of seats in a legislature, seats are allocated proportionally in each district, and the number of seats assigned to each district depends on the district population reflected in w^s.

To state Proposition 5, let h_t denote histories up to period t prior to the candidates choosing their period t spending levels. Let the consecutive period spending ratios for the two candidates in any district s be $r_{1,t}^s = x_{t+1}^s / x_t^s$ and $r_{2,t}^s = y_{t+1}^s / y_t^s$.

Proposition 5. *Suppose Assumptions 1 and 2 hold with* $\beta = 0$. *In any equilibrium of this multi-district extension,*

(i) for all districts s, $x_t^s / X_t = y_t^s / Y_t$.

(ii) if Assumption 3 also holds, then for all districts s, *the candidates' expected consecutive period spending ratios conditional on any on-path history* h_t *equal each other:* $\mathbb{E}[1/r_{1,t}^s | h_t] = \mathbb{E}[1/r_{2,t}^s | h_t] = \delta^s$ *for all* s *and all such histories* h_t.

(iii) if Assumption 3 also holds, then for all periods t *and any pair of districts* s, s',

$$\frac{x_t^s}{x_t^{s'}} = \frac{y_t^s}{y_t^{s'}} = \frac{w^s}{w^{s'}} \left(\frac{\delta^s}{\delta^{s'}}\right)^{T-t-1}.$$

When Assumption 3 holds, Proposition 5 states that the allocation of resources across districts given the total spending in the current period is independent of the popularity process and that candidates spend more in districts that get greater electoral weight and where popularity decays at a slower rate. Furthermore, the differences in spending due to differences in the decay rates are maximal at the beginning of the campaign and decrease as election day approaches. These results hold even if the candidates' investments in any one district also affect popularity in others.

4 TV Ad Spending in Actual Elections

We now look at actual campaign spending data through the lens of our baseline model. Under Assumptions 1 and 2, the pattern of spending is given by $r_{1,t} = r_{2,t} = r = \delta^{1/(\beta-1)}$; see Proposition 2, equation (4), and Figure 2. Our main goal is to use this relationship to recover election-specific estimates of δ (for fixed β) from the observed values of $r_{1,t} = x_{t+1}/x_t$ and $r_{2,t} = y_{t+1}/y_t$ in spending data. Since β and δ cannot be separately identified from these data alone, we focus mainly on the $\beta = 0$ case and comment on how the estimates of δ vary as we fix β at progressively higher values. For fixed values of β, this gives us estimates of how the candidates perceive the decay rate $1 - \delta$ when making their spending decisions.

Before embarking on this estimation, we first introduce the data we use and then investigate the extent to which two important implications of our baseline model are violated in the data: the equal spending ratio result ($x_t/X_t = y_t/Y_t$ for all t) and the constant spending growth result ($r_{1,t}$ and $r_{2,t}$ are both constant in t).

4.1 Data

We focus on subnational American elections, namely U.S. House, Senate, and gubernatorial elections in the period 2000 to 2014.

Spending in our model refers to all spending—TV ads, calls, mailers, door-to-door canvasing visits—that directly affects the candidates' relative popularity. But for some of these categories, it is not straightforward to separate out the part of spending that has a direct impact on relative popularity from the part that does not (e.g. fixed administrative costs). For television ads, it is straightforward to do this, so we focus exclusively on TV ad spending. Television advertising constitutes around 35% of the total expenditures by congressional candidates, and around 90% of all advertisement expenditure during the period we study (see, e.g., Albert, 2017). Furthermore, for TV ads, we have access to the exact timing of the candidates' expenditures, which is not the case for other types of spending. We proceed under the assumption that any spending on other types of campaign activities that directly affect relative popularity is proportional to spending on TV ads.

Our TV ad spending data are from the Wesleyan Media Project and the Wisconsin Advertising Database.[19] For each election in which TV ads were bought, the database

[19] The dataset we use from these sources draws information directly from TV channels, and it includes all ads aired on TV which support a candidate. Unfortunately, it does not include any information on the source of spending (whether by PACs or the candidates themselves) but the vast majority of TV advertising expenditures are made through PACs and other entities. Martin and Peskowitz (2018) provide evidence on this, showing the share of TV ad spending coming directly from candidates is 0.8 percent between 2010-2014.

contains information about the candidate that each ad supports, the date it was aired, and the estimated cost. For the year 2000, the data covers only the 75 largest Designated Market Areas (DMAs), and for years 2002-2004 it covers the 100 largest DMAs. The data from 2006 onward covers all 210 DMAs. We obtain the amount spent on ads from total ads bought and price per ad. Ad price data are missing for 2006, so for that year we estimate prices using ad prices in 2008.[20]

We focus on races where the leading two candidates in terms of vote share are from the Democratic and the Republican party. We label the Democratic candidate as candidate 1 and the Republican candidate as candidate 2, so that x_t, X_0, etc. refer to the Democrat's spending, budget, etc. and y_t, Y_0, etc. refer to the Republican's.

We aggregate ad spending made on behalf of the two major parties' candidates by week and focus on the 12 weeks leading to election day, though we drop the final week which is typically incomplete since elections are held on Tuesdays. For our main analysis, we exclude elections that are clearly not genuine contests to which our model does not apply, defining these to be elections in which one of the candidates did not spend anything for more than half of the period studied. This leaves us with 346 House, 122 Senate, and 133 gubernatorial elections.[21] We focus on the last 12 weeks mainly to restrict attention to the general election campaign. We define the total budgets of the candidates to be the total amount that they spend over these 12 weeks.[22] However, in the Online Appendix, we also include the replication of our analyses with a larger dataset excluding fewer elections (leaving us with 1163 elections over 14 years), and a longer time period (20 weeks instead of 12).

In our model spending decisions are made at discrete moments in time defined in such a way as that the inter-period decay rate $1 - \delta$ is constant. This raises the question of how to define a period of spending in the data, given that spending data are reported somewhat irregularly. To address this issue we examine in the Online Appendix an equivalent continuous time formulation of our model in which candidates make spending decisions at fixed intervals of time and the decay rate is constant. There, we prove an identification result that implies that the level of aggregation of spending is irrelevant: e.g., if candidates make their spending decisions daily but the data are aggregated weekly, then the sum of what they spend over seven days is the

[20]One concern with this approach could be that if prices increase as the election approaches, then the increase in total spending over time confounds the price increase with increased advertising. However, federal regulations limit the ability of TV stations to increase ad prices as the election approaches, requiring them to charge political candidates "the lowest unit charge of the station for the same class and amount of time for the same period" (Chapter 5 of Title 47 of the United States Code 315, Subchapter III, Part 1, Section 315, 1934). This fact allays some of this concern.

[21]A tabulation of these elections is given in the Appendix.

[22]In some cases, primaries are held less than 12 weeks before the general election, but ad spending for the general election before the primaries is typically zero. In the rare cases where ad spending for primary elections happens, we exclude it and focus only on spending for the general election.

same as in a setting in which they make spending decisions weekly. Given this result, we choose to aggregate and examine spending data at the weekly level.

Summary statistics for our baseline sample are given in Table 1. There is considerable difference in the amount of spending between state-wide and House elections, with another key difference being the time at which candidates start spending positive amounts. For statewide races, candidates spend on average about \$6 million on TV ads, with most candidates already spending positive amounts 12 weeks prior to the election. For House races, they spend \$1.5 million on average and the majority of candidates start spending 9 weeks out.

In addition, there is variation in the amount spent by candidates competing in the same race. The average difference in the amount spent by the candidates competing in the same congressional election is one third of the average total spending for those races, while for gubernatorial elections the analogous difference exceeds 50%. Finally, candidates tend to spend more in more competitive elections: the overall amount spent by candidates is higher in elections where there is no incumbent, and in elections where the final margin of victory is thin.

Because of these differences, we will include these disaggregations in the models that we use to estimate perceived decay rates.

4.2 Diagnostics

How well do the predictions of the baseline model under Assumptions 1 and 2 agree with actual spending patterns in the data?

The first main prediction given in Proposition 1(ii) that spending is independent of popularity cannot be tested because publicly available polling data are too sparse.[23] So we proceed to investigate the predictions of Proposition 2. The main predictions are that of part (i), the equal spending ratio result ($x_t/X_t = y_t/Y_t$ for all weeks t, also implying $r_{1,t} = r_{2,t}$ in our dataset where total budget is defined as the sum of total spending), and of part (ii), the constant spending growth result ($r_{1,t} = r_{2,t} = r_t$ is constant in t). Below, we investigate the extent to which these predictions are violated in actual spending data.

Equal Spending Ratios. In Table 2 we look at the extent to which the equal spending ratio result is violated in the data. We find that the candidates' weekly

[23] To the best of our knowledge, FiveThirtyEight and Pollster provide the largest publicly available database on polls. We collected data from these sources and identified only 24 elections (all state-wide races) with more than 3 weeks of polling data, which constitute a sample that is too sparse and potentially not representative of the full set of races in our dataset to conduct a systematic analysis of how spending decisions are affected by changes in relative popularity.

Table 1: Descriptive Statistics

	N	Open Seat Election	Incumbent Competing	No Excuse Early Voting	Average total spending	Average spending difference
Senate	122	68	54	82	6019 (5627)	1962 (2921)
Governor	133	59	74	92	5980 (9254)	3173 (6,337)
House	346	97	249	223	1533 (1304)	521 (615)
Overall	601	224	377	397	3428 (5581)	1401 (3,461)

Average Spending and Standard Deviations in Parentheses by Week and Election Type

Week	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1
Senate	196	250	266	314	357	477	545	652	716	860	1,002
	(291)	(328)	(403)	(487)	(401)	(505)	(577)	(724)	(803)	(947)	(1,047)
Share spending 0	0.270	0.180	0.123	0.082	0.008	0	0	0	0	0	0
Governor	262	253	258	316	420	416	530	597	701	800	1,019
	(632)	(468)	(424)	(581)	(865)	(579)	(1,249)	(1,015)	(1,305)	(1,523)	(1,956)
Share spending 0	0.297	0.207	0.139	0.068	0.030	0	0	0	0	0	0
House	17	27	38	56	83	120	137	177	212	250	303
	(41)	(55)	(57)	(85)	(93)	(134)	(134)	(182)	(219)	(270)	(340)
Share spending 0	0.653	0.545	0.386	0.246	0.095	0	0	0	0	0	0
Early Voting	113	123	128	168	223	262	320	390	449	526	624
	(324)	(256)	(256)	(348)	(488)	(404)	(694)	(663)	(775)	(895)	(1033)
No Early Voting	99	122	144	162	194	250	283	321	373	436	569
	(213)	(246)	(286)	(314)	(259)	(317)	(348)	(394)	(473)	(524)	(866)
Open Seat	164	183	191	217	279	324	362	445	521	602	729
	(404)	(311)	(325)	(352)	(476)	(445)	(485)	(635)	(800)	(892)	(1046)
Incumbent	75	87	99	135	174	219	275	320	366	432	532
	(189)	(202)	(218)	(324)	(386)	(324)	(657)	(550)	(606)	(716)	(931)
Close Election	122	131	154	200	250	292	383	479	544	661	858
	(318)	(236)	(320)	(421)	(539)	(407)	(915)	(791)	(746)	(884)	(1266)
Not Close Election	103	120	125	152	199	245	278	322	376	430	506
	(280)	(259)	(242)	(297)	(369)	(364)	(412)	(476)	(659)	(741)	(820)
Close Budgets	97	118	129	150	196	264	301	362	411	477	587
	(190)	(209)	(255)	(225)	(246)	(339)	(385)	(442)	(488)	(550)	(710)
Not Close Budgets	117	127	137	178	227	254	313	370	434	510	620
	(351)	(282)	(275)	(404)	(525)	(405)	(727)	(680)	(813)	(938)	(1149)

Note: The upper panel reports the breakdown of elections that are open seat versus those that have an incumbent running, the numbers in which voters can vote early without an excuse to do so, average spending levels by the candidates, and the average difference in spending between the two candidates, all by election type. The lower panel presents average spending for each week in our dataset, by election type. Standard deviations are in parentheses. All monetary amounts are in units of \$1,000. Close elections are races where the final difference in vote shares between two candidates is less than 5 percentage points. Close budget races are those in which the ratio of budgets of the two candidates lies in the interval $(0.75, 1.25)$.

Table 2: $x_t/X_t - y_t/Y_t$

Week	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1
% ∈ (-0.1,0.1)	0.963	0.953	0.938	0.902	0.879	0.847	0.829	0.754	0.676	0.622	0.797
Senate	0.943	0.934	0.975	0.926	0.934	0.885	0.844	0.787	0.746	0.648	0.803
Governor	0.932	0.910	0.887	0.820	0.812	0.812	0.767	0.774	0.639	0.624	0.782
House	0.983	0.977	0.945	0.925	0.884	0.847	0.847	0.734	0.665	0.613	0.801
Early Voting	0.970	0.955	0.942	0.912	0.884	0.844	0.816	0.753	0.673	0.612	0.798
No Early Voting	0.951	0.951	0.931	0.882	0.868	0.853	0.853	0.755	0.681	0.642	0.794
Open Seat	0.942	0.933	0.920	0.897	0.857	0.862	0.866	0.795	0.705	0.656	0.804
Incumbent Competing	0.976	0.966	0.950	0.905	0.891	0.838	0.806	0.729	0.658	0.602	0.793
Close Election	0.976	0.965	0.935	0.941	0.947	0.924	0.906	0.882	0.776	0.706	0.788
Not Close Election	0.958	0.949	0.940	0.886	0.852	0.817	0.798	0.703	0.636	0.589	0.800
Close Budgets	0.974	0.974	0.959	0.925	0.914	0.895	0.883	0.812	0.763	0.695	0.838
Not Close Budgets	0.955	0.937	0.922	0.884	0.851	0.809	0.785	0.707	0.606	0.564	0.764
% ∈ (-0.05,0.05)	0.865	0.815	0.757	0.727	0.661	0.599	0.554	0.468	0.418	0.369	0.562
Average x_t/X_t	0.021	0.028	0.039	0.054	0.075	0.109	0.134	0.184	0.251	0.377	0.728
	(0.032)	(0.036)	(0.044)	(0.051)	(0.054)	(0.067)	(0.073)	(0.085)	(0.095)	(0.108)	(0.076)
Average y_t/Y_t	0.021	0.029	0.038	0.049	0.074	0.105	0.133	0.184	0.249	0.380	0.733
	(0.035)	(0.041)	(0.046)	(0.053)	(0.063)	(0.073)	(0.080)	(0.094)	(0.097)	(0.111)	(0.073)

Note: The table reports the share of elections in which the candidates' spending ratios are within 10 percentage points of each other for every week, across election types. See the note below Table 1 for definitions of close elections and close budget elections.

spending ratios are within 10 percentage points (pp) of each others' in 80% of election-weeks, and within 5 pp of each others' in 56% (see the Online Appendix for disaggregations of the 5pp analysis). Even in the final six weeks of the campaign when candidates spend larger amounts, they are within 10 pp of each others' in 75% of election-weeks, and within 5 pp of each others' in about half.[24]

Violations of the equal spending ratio result do not seem to be more pronounced in open seat elections versus ones with an incumbent running, or in those where voters are able to cast their ballots early without an excuse to do so. This is consistent with our early voting extension in which the equal spending ratio result continues to hold

[24]Since spending ratios are defined as the share of remaining (rather than total) budget that is spent, they can take any value between 0 and 1 in every week in the data prior to the final (partial) week, where by construction they will be 100% for both candidates. Recall that we do not include this final partial week anyway, so we are not biasing the results in the direction of fewer and smaller violations of the equal spending ratio result.

analytically. On the other hand, we do see more pronounced violations in elections that are lopsided in terms of money spent and final vote shares. If these correspond to elections in which one candidate (e.g. the better resourced one) frequently has large leads against the other, then these more pronounced violations could be explained by the variant of our model in which candidates value money left over.

Finally, the extent to which our equal spending ratio result appears violated in the data is increasing as the election approaches. Part of this is due to the fact that the result holds trivially when both candidates spend zero, and the percent of zero spending is decreasing over time as Table 1 indicates. Another reason could be that spending decisions close to the election are affected more by disturbances resulting from factors outside our model.[25]

Constant Spending Growth. Recall that the consecutive period spending ratio (CPSR) is x_{t+1}/x_t for the Democrat and y_{t+1}/y_t for the Republican candidate, which is defined for ten consecutive week pairs in our dataset. If the constant spending growth prediction holds, then these ratios should be relatively stable over time. However, since there are candidates who spend zero in some of the earlier weeks, this ratio cannot be calculated for certain periods.

Given this, we calculate CPSRs using two approaches: (i) dropping all elections with zero spending in any week, and (ii) dropping all pairs of consecutive weeks that would include a week with zero spending.[26] These constitute two rules for focusing on different subsets of data. Approach (i) leaves us with only 221 (out of the total 601) elections where no zero spending occurs, and in approach (ii) we drop 1,692 consecutive week pairs out of a total of 13,222, which is only 12.8% of consecutive week pairs. We note that in our sample there is no instance of zero spending following positive spending: once a candidate starts spending a positive amount, she continues to do so until the election.

The distribution of average CPSRs for every candidate, along with their 95% confidence intervals from each of the two approaches are depicted in Figure 3. The distributions obtained from approaches (i) and (ii) are very similar, as are the confidence intervals. The reported CPSRs for the second approach can be interpreted as growth rates conditional on having started positive spending during an electoral campaign. This approach uses all of the available sample in getting estimates of CPSRs and discards less data so we proceed with analyzing the growth rates obtained using

[25] One such disturbance that could occur close to an election is an "October surprise"—the surfacing of new information, like a scandal, that affects the candidates' spending decisions. Examples from the 2016 presidential race are the Access Hollywood tape and Comey letter, but even lower level races can feature such events. Another factor outside our model is the idea that close to election day, trailing candidates may simply give up because of threshold effects.

[26] If zero spending occurs in week t, both x_{t+1}/x_t and x_t/x_{t-1} are dropped.

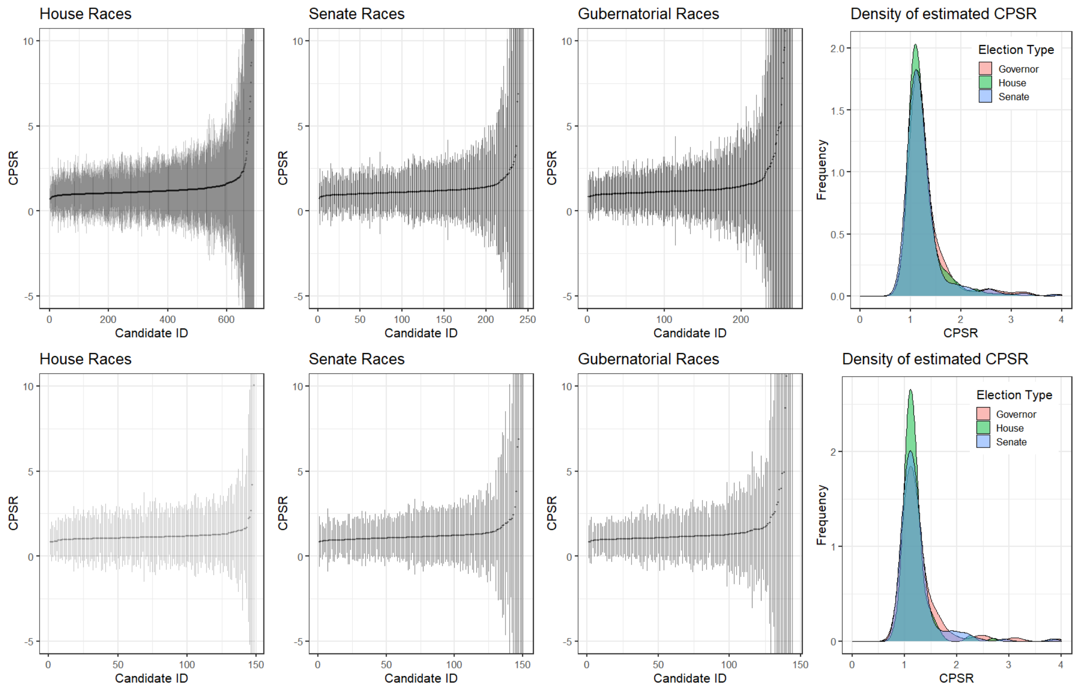

Figure 3: Estimated CPSR values for candidates in our dataset, along with 95% confidence intervals. The upper display row depicts the estimates that we get from dropping all elections with zero spending. The bottom depicts the estimates that we get from dropping all pairs of consecutive weeks that include zero spending. We also depict the densities of the CPSRs across election types from both approaches.

the second method. Hereafter, when we say "growth rates," we will be referring to growth rates conditional on having started spending positive amounts.

Since our baseline model predicts a constant spending growth rate over time, we focus on empirically reporting how CPSRs change over the course of an election. In the data, we find that spending increases from one week to the next for 85% of election-weeks. To examine the extent to which growth rates are close to each other, Table 3 displays a specific measure of central tendency for CPSRs: the share of candidates that remained within half a standard deviation of their election's CPSR average, for every week in our dataset.

Looking at Figure 3, the majority of the candidates in our dataset have relatively stable spending growth rates. The middle 90% of the distribution of CPSR values (i.e. the 5th to 95th percentile) spans $[0.98, 1.9]$. For the candidate with the median value, we get an average CPSR of 1.16, meaning that their spending increased by 16% on average every week after they started spending positive amounts. The majority

Table 3: Consecutive Period Spending Ratios

	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	Overall
$\%\in(\mu_r - 0.5\sigma_r, \mu_r + 0.5\sigma_r)$	0.469	0.480	0.450	0.509	0.547	0.628	0.632	0.699	0.688	0.683	0.542
Senate	0.461	0.465	0.458	0.496	0.558	0.664	0.664	0.725	0.680	0.730	0.554
Governor	0.529	0.483	0.463	0.516	0.558	0.673	0.635	0.741	0.711	0.650	0.559
House	0.429	0.489	0.440	0.511	0.538	0.600	0.620	0.675	0.684	0.679	0.532
Early Voting	0.485	0.511	0.467	0.518	0.557	0.630	0.617	0.698	0.685	0.675	0.544
No Early Voting	0.441	0.421	0.418	0.491	0.529	0.627	0.662	0.703	0.696	0.699	0.539
Open Seat	0.462	0.447	0.448	0.474	0.545	0.629	0.625	0.685	0.719	0.690	0.536
Incumbent Competing	0.475	0.507	0.452	0.532	0.548	0.629	0.637	0.708	0.671	0.679	0.546
Close Election	0.455	0.476	0.462	0.549	0.571	0.676	0.656	0.709	0.688	0.691	0.552
Not Close Election	0.476	0.483	0.446	0.492	0.537	0.610	0.623	0.696	0.689	0.680	0.538
Close Budgets	0.490	0.487	0.501	0.528	0.546	0.656	0.662	0.722	0.759	0.726	0.566
Not Close Budgets	0.451	0.474	0.405	0.492	0.548	0.607	0.609	0.682	0.633	0.649	0.523

Note: The table reports the share of candidates for which the CPSRs are less than 0.5 standard deviations away from that candidate's average CPSR over 11 weeks. Week -2 is missing because the final week is not included in the analysis. See the note under Table 2 for the definition of close elections and close budgets.

of candidates have relatively low standard deviations, with 62% having a standard deviation below 1 and 87% having a standard deviation below 2. Any variation in CPSR values is usually driven by only a few weeks of volatile growth, rather than volatility in the entire spending path. On average, candidates remain within half a standard deviation of their mean CPSR value for 5.25 out of 10 weeks. 62% of the candidates remain within this range for more than 5 weeks, and 43% for more than 6 weeks. On an average week, about 54% of the candidates are in this range.

Table 3 shows that the constant spending growth prediction is violated to a smaller extent as the election approaches and candidates begin to spend more substantial amounts. It also shows that statewide races, which typically see greater spending, generally have smaller/fewer violations than House races, though the differences are small. For example, in the last eight weeks of the elections, the CPSRs remain within half a standard deviation of their means for each candidate in 62.1%, 61.8%, and 59.3% of Senate, gubernatorial and House races, respectively.

One possible explanation for these deviations is given by our early voting model in which spending growth is constant until the time early voting starts, which is typically anywhere from a few days prior to the election to up to eight weeks from election day. While Table 3 does reveal somewhat greater deviations from constant

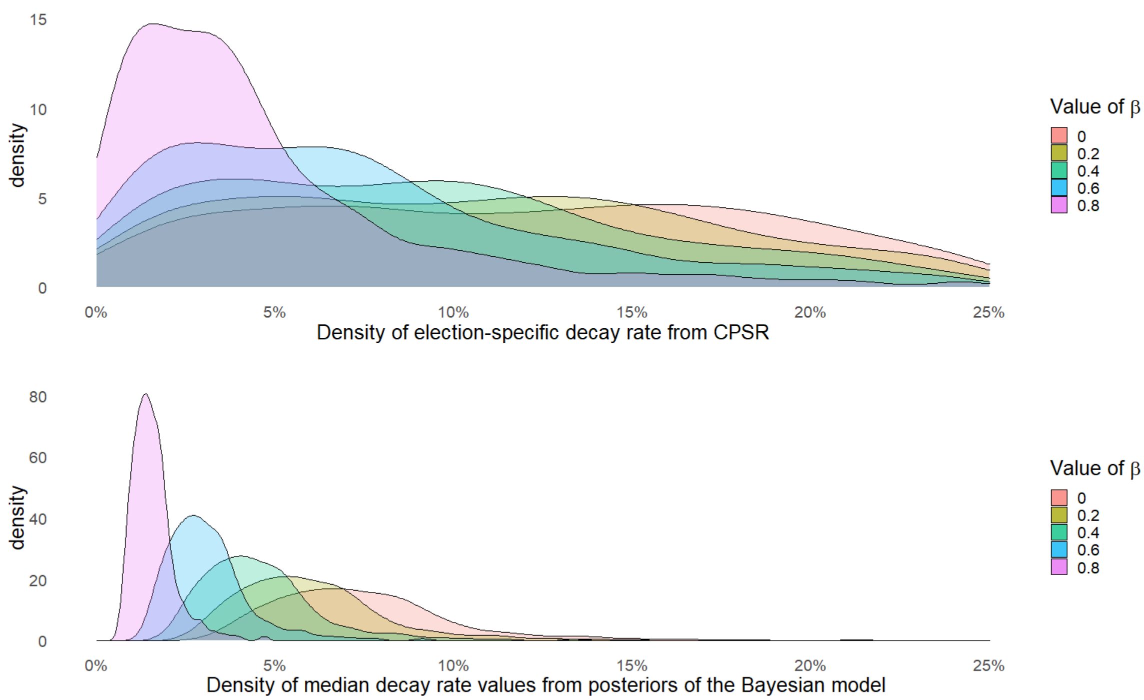

Figure 4: The top display shows distributions of our direct estimates of the candidates' perceived decay rates from the candidates CPSRs. The bottom display shows distributions of the estimates of candidates' perceived decay rates from estimates of the hierarchical Bayes model. In both approaches, we estimate different distributions for values of β ranging from 0 to 1.

spending growth at six weeks from election day, the differences are not substantial across weeks, and early voting does not appear to a major driver of violations to the constant spending growth prediction overall.

Another possible explanation for these deviations from constant spending growth is that candidates value money left over as in our extension. Though we cannot directly test this, we can reason that if House candidates are more likely to value money left over than Senate or gubernatorial candidates (because the value of office is lower, or their future political ambitions—perhaps to become Senators or governors—are greater, or because they compete more frequently in future elections) this appears to be reflected only to a limited extent in the disaggregation by election type.[27]

[27]A third possibility is that the candidates have uncertain budgets that react to their polling performance, as in our evolving budgets model in the Online Appendix. Unfortunately, we cannot investigate whether the spending path predicted by this model could account for these violations since data on when candidates receive money or pledges from donors are not available.

4.3 Perceived Decay Rates

The decay rate is $1 - \delta$, and the perceived decay rate is the value of $1 - \delta$ that is "most consistent" with the candidates' spending behavior in an election given that the CPSR in the baseline specification of our model is $r = \delta^{1/(\beta-1)}$. Since the perceived decay rate cannot be separately identified from the parameter β using spending data alone, we fix a grid of values of β ranging from 0 to 1 and report how the distribution of estimated perceived decay rates varies as β varies.[28]

Direct Estimates from CPSRs. The most straightforward way to estimate a perceived decay rate is to fix r_j to be the mean of the candidates' CPSRs estimated from their actual spending levels in election j (these are given in Figure 3) and then use the relationship $1 - \delta_j = 1 - (r_j)^{\beta-1}$ where $1 - \delta_j$ is the estimated perceived decay rate for election j using approach (ii) in which we drop all candidate-weeks with zero spending.[29] To increase the precision of our estimates, for each election we pool the two candidates' CPSRs which gives us potentially up to 20 total values (if there are no weeks with zero spending) and we estimate a common perceived decay rate for the two candidates.

The upper display in Figure 4 shows the distributions of point estimates of these decay rates for five different values of β, indicating that most of the mass in decay rates is below 25% no matter what value of β we fix. The decay rate estimates along with their 95% confidence intervals are plotted in the Online Appendix.

Hierarchical Bayes Model. The direct estimates of election-specific decay rates from CPSRs discards weeks with zero spending and produces estimates that are noisy and unreliable for elections with sparse positive spending data. We now estimate election-specific perceived decay rates from spending data using a hierarchical Bayes model that allows us to estimate election-specific decay rates while specifying certain parameters of the model to be common across elections.

Let δ_j denote the perceived value of parameter δ in election j. (We continue to add the election identifier j as a subscript to all election-specific parameters and observations.) Given that we observe zero spending by some candidates in the early

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[28]Assumption 2 requires $\beta \geq 0$, but Assumption 1 requires p to be strictly concave in candidate 1's spending and strictly convex in candidate 2's. Thus, we constrain β to be lower than 1.

[29]More specifically, δ_j can be estimated directly from the first moment of the distribution of observed CPSRs. Since $r_{i,t} = (i\text{'s spending in week } t+1)/(i\text{'s spending in week } t)$ is observed for $t = 0, 1, ..., T-2$, for any candidate i, we can compute the first moment of these consecutive period spending ratios as $\sum_{i=1,2} \hat{r}_i = \frac{1}{|\mathcal{T}|} \sum_{t \in \mathcal{T}} r_{i,t}$ where $\mathcal{T}$ is the set of weeks for which $r_{i,t}$ can be computed. Then, as our model predicts $r_{i,t} = r = (\delta_i)^{1/(\beta-1)}$ for all i and t, we can estimate the perceived δ_i from the first moment alone (if we fix β to a pre-specified value) as $\hat{\delta}_i = (\hat{r}_i)^{\beta-1}$.

weeks, and $x_{t,j}/X_{t,j}, y_{t,j}/Y_{t,j} \in [0,1]$, we model this data generating process using a zero-inflated truncated normal distribution with mean

$$\gamma_t(\delta_j) = \frac{(\delta_j)^{\frac{t}{\beta-1}} - (\delta_j)^{\frac{t+1}{\beta-1}}}{1-(\delta_j)^{\frac{T}{\beta-1}}} \tag{9}$$

which is equation (4) for the share of initial budget spent by each candidate in the baseline model when $p(x,y)$ satisfies Assumptions 1 and 2. We specify the following statistical model for the likelihood of observed spending and for the priors of the underlying parameters:

$$\begin{aligned}
\frac{x_{t,j}}{X_{t,j}}, \frac{y_{t,j}}{Y_{t,j}} &\sim (1-\varsigma_t) \times \textsf{Truncated [0,1] Normal}\left(\gamma_t(\delta_j), \sigma^2_{\textsf{spend}}\right) \\
\varsigma_t &\sim \textsf{Bernoulli}((T-t)^2 \varrho_{\textsf{type}}), \quad \textsf{type} = \textsf{S, H, G} \\
\varrho_{\textsf{S}}, \varrho_{\textsf{H}}, \varrho_{\textsf{G}} &\sim \textsf{Uniform}(0, 0.00826) \\
\delta_j &\sim \textsf{Normal}(\beta_{\textsf{O}}\textsf{Open}_j + \beta_{\textsf{E}}\textsf{Early}_j + \beta_{\textsf{S}}\textsf{Senate}_j + \beta_{\textsf{H}}\textsf{House}_j + \beta_{\textsf{G}}\textsf{Gov}_j, \sigma^2_\delta) \\
\beta_{\textsf{O}}, \beta_{\textsf{E}}, \beta_{\textsf{S}}, \beta_{\textsf{H}}, \beta_{\textsf{G}} &\sim \textsf{Normal}(0,1) \\
\sigma^2_\delta &\sim \textsf{Scale-inv-}\chi^2(5,1) \\
\sigma^2_{\textsf{spend}} &\sim \textsf{Scale-inv-}\chi^2(1,10)
\end{aligned}$$

Notation is as follows. ς_t is a Bernoulli random variable for observing zero spending at time t. Its mean decreases over time as election day approaches.[30] $\varrho_{\textsf{type}}$ is the baseline probability of observing zero spending during election week $t = 11$, which we allow to be different across election types (S for Senate, H for House, and G for governor races), motivated by our observations in Table 1. Since $T - t$ can take a maximum value of 11, $\varrho_{\textsf{type}}$ can take a maximum value of 0.00826. We posit a uniform prior over [0,0.00826] for these baseline probabilities. The parameter δ_j has a normal distribution whose parameters depend on election characteristics: Open indicates an open-seat election, Early indicates early voting was available, and the Senate, House and Gov variables here are also indicators for election type. The β variables are the corresponding coefficients. The interpretation of our theoretical model (see Section 2.4) allows δ to vary depending on election characteristics, and the inclusion of these variables as potential determinants of the decay rate are driven by what we see in Table 2. Finally, σ^2_δ and $\sigma^2_{\textsf{spend}}$ are common variance parameters for, respectively, the election specific δ_j, and the observed spending ratios $x_{t,j}/X_{t,j}$ and $y_{t,j}/Y_{t,j}$, with scaled

[30]Thus, we model the truncation process with a time-dependent parameter. Decoupling the truncation from the spending process allows us to estimate the underlying decay rate parameters, without our choice of time horizon affecting them mechanically.

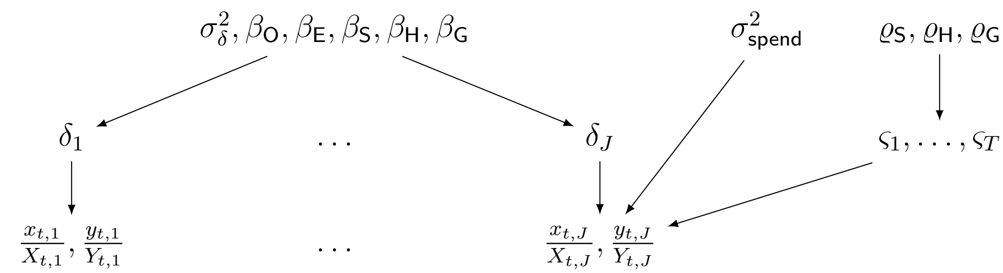

Figure 5: The picture above depicts the hierarchical structure of the Bayes model. At the highest level are the coefficients for election characteristics (open seat, early voting, and election type), the common variance parameters for the speed of reversion and for weekly spending, and the baseline zero-spending probabilities. These determine the distributions of election specific mean reversion parameters, and weekly zero-spending probabilities in the middle layer. Finally, all of these parameters determine the distribution of weekly spending at the lowest level. Elections are $j = 1, ..., J$.

inverse chi-square priors (a common choice for variance parameters). The scales are chosen appropriately for these variables. See also Figure 5.[31]

Therefore, we assume that the posteriors of the average decay rate parameters for each type of election are determined by the average values of δ_j belonging to each election type. The posteriors for the baseline zero spending probabilities $\varrho_{\mathsf{S}}, \varrho_{\mathsf{H}}, \varrho_{\mathsf{G}}$ along with ς_t are determined through the fraction of candidates in each time period spending zero for each type of election. Finally, the posterior for $\sigma^2_{\mathsf{spend}}$ is determined by the unexplained variation in $x_{t,j}/X_{t,j}, y_{t,j}/Y_{t,j}$ relative to what is captured by the mean $\gamma_t(\delta_j)$, and the posterior for σ_δ^2 is determined by the unexplained variation in estimated δ_j values beyond what can be explained using the election-type mean parameters $\beta_{\mathsf{O}}\mathsf{Open}_j + \beta_{\mathsf{E}}\mathsf{Early}_j + \beta_{\mathsf{S}}\mathsf{Senate}_j + \beta_{\mathsf{H}}\mathsf{House}_j + \beta_{\mathsf{G}}\mathsf{Gov}_j$.

We use No-U-Turn sampling, a Hamiltonian Monte Carlo based method, to get our posteriors for the parameters (see Hoffman and Gelman, 2014). We report the Bayesian credible intervals from the posterior distributions for the key model parameters in Table 4. As the table shows, we do not find meaningful differences in the estimates between open seat elections and elections in which an incumbent competes, or between elections with early voting and elections without. The differences across Senate, House and gubernatorial elections are also minimal. Although House candidates start spending on TV ads later, the estimates for these races are similar to those for state-wide races. The lack of a statistically meaningful difference in the

[31] Under mild regularity conditions, the posterior Bayesian credible intervals from this hierarchical Bayes model asymptotically approach the confidence intervals of the same parameters obtained by standard frequentist approaches using any efficient estimation method, e.g. maximum likelihood. This is the Bernstein-von Mises Theorem; see, e.g., Section 10.2 of Van der Vaart (2000).

Parameter	$\hat{R}$	n_{eff}	mean	s.d.	2.5%	50%	97.5%
β_{O}	1.001	5491	-0.017	0.095	-0.203	-0.017	0.172
β_{E}	1.002	5972	-0.175	0.092	-0.357	-0.175	0.007
β_{S}	1.003	4418	0.915	0.126	0.894	0.915	0.933
β_{H}	1.003	3537	0.925	0.090	0.911	0.924	0.937
β_{G}	1.001	4290	0.918	0.122	0.898	0.918	0.935
ϱ_{S}	1.000	26717	0.003	0.002	0.000	0.003	0.007
ϱ_{H}	1.000	25982	0.003	0.000	0.003	0.003	0.003
ϱ_{G}	1.000	29000	0.003	0.002	0.000	0.003	0.007
$\sigma^2_{\mathsf{spend}}$	1.000	15799	0.012	0.000	0.012	0.012	0.012
σ^2_{δ}	1.004	1698	0.378	0.060	0.272	0.374	0.504

Table 4: Model parameters with convergence diagnostics and 95% Bayesian credible intervals for the 601 elections in our sample, and 12 weeks of data.

estimates between statewide elections and congressional district elections is a notable qualitative result.[32]

We transform the posteriors of election-specific δ_j to posteriors on perceived weekly decay rates using the fact that the decay rate is equal to $1 - \delta_j$. The distribution of point-estimates of the perceived decay rate is given in the lower display of Figure 4. These estimates complement our direct estimates in the upper display. They are more tightly distributed and generally lower, typically falling below 10%. They are presented along with their 95% credible intervals in the Online Appendix.

The direct CPSR-based estimates are noisy and difficult to distinguish from the Bayesian estimates given their typically large confidence intervals. The direct approach only uses information from a specific election when estimating the decay rates. The Bayesian model has a partial pooling property (Gelman, 2006): it uses information from other elections when estimating the decay rates for a particular election. In particular, as depicted in Figure 5, the hierarchical model assumes that elections with similar properties have decay rates coming from the same distribution. This additional structure results in more precise estimates.

Table 4 reveals how chains have mixed in our model, with $\hat{R} < 1.01$ for all parameters. The posteriors for $\beta_{\mathsf{S}}, \beta_{\mathsf{H}}, \beta_{\mathsf{G}}$ show no substantial difference between the average decay rate parameters for House, Senate or gubernatorial elections, with all of these having 95% credible intervals that span the range 0.89 to 0.93. In addition, the credi-

[32]These results also hold in the replication using 20 weeks of spending data and a larger dataset of 1163 elections in which we keep all contests where two candidates spend non-zero amounts for at least two weeks. Overall, our estimates are robust to the choice of time horizon and to throwing away fewer elections. These results are reported in the Online Appendix.

ble interval for σ_δ^2, the variance of decay rates, spans 0.27 to 0.5. Taken together, the lack of a meaningful difference between the β parameters and the magnitude of σ_δ^2 suggests a considerable degree of heterogeneity that is not well explained by election type.[33] The distributions of the decay rates for open seat and early voting elections do not seem to differ meaningfully from those with incumbents and no early voting, which is a finding that deserves more attention in future work.[34]

Comparison with the Experimental Literature. Previous literature estimates actual (as opposed to perceived) decay rates using survey and experimental data. For example, using survey data and an exponential decay model similar to ours, Hill et al. (2013) recover an average daily decay rate in the persuasive effects of political advertising of 52.4% in 2006 U.S. elections. This corresponds to a 99% weekly decay, though their 95% confidence interval for this estimate covers the $[0, 100\%]$ interval. Similarly, using a field-experimental approach, Gerber et al. (2011) recover a weekly decay rate of 88%, though in their case, the estimates vary substantially according to the specification of their model. For example, their 3rd order polynomial distributed lag model estimates show that the standing of the advertising candidate increases by 4.07 percentage points in the week that the ad is aired, and the effect goes down to 3.05 percentage points the following week (25% decay). In another specification, the first week effect is 6.48%, and goes down to 0.44% in the second (94% decay).[35]

If we take the point estimates from these prior studies at face value, Figure 4 shows that the perceived weekly decay rates—which are typically below 15%—are considerably lower than previous estimates of actual decay rates. Our parameterized baseline model therefore suggests that candidates spend more in earlier weeks compared to what the decay rates estimated from the past literature would imply. On the other hand, since our estimates of the perceived decay rates are within the large margins of error of prior estimates of actual decay rates, we can make no conclusive inferences on this.

There are several possible reasons for why our estimates are lower than the point estimates found by the experimental literature. One is that candidates are irrationally spending too much money in the early stages of the campaign. Another

[33]This can also be seen in the plots for estimated perceived decay rates split by election type that we present in the Online Appendix.

[34]Since early voting gives candidates incentives to spend comparatively more money in earlier periods, one might expect that the perceived decay rates estimated from the baseline model would be higher in races that do allow for early voting. The fact that they are not suggests that we have not missed an important driver of early spending by estimating the baseline model (rather than the early voting model) for elections that allow for early voting.

[35]The volatility of these estimates may be due to data limitation, as well as sensitivity to the parametric specifications; see Lewis and Rao (2015).

is that candidates are spending rationally but prior point estimates are off because they measure decay rates only for marginal spending, which could differ substantially from the global average.[36] This is certainly a possibility as prior work in the (non-political) marketing literature finds decay rates that are more in line with our estimates; e.g., Dubé et al. (2005) estimate the weekly decay of goodwill from ads in the frozen food industry to be only around 12%. Yet another possibility is that candidates are spending rationally, actual decay rates are quite high, but our baseline model is failing to capture the full benefits to early spending. One of these benefits is experimentation—campaigns spend early to try to learn what kind of ad targeting works best given their characteristics and political platforms. Another is the increase in the support from donors due to improvements in early polling leads.[37]

5 Conclusion

We have developed a model of electoral campaigns as dynamic contests and used it to study the optimal allocation of campaign resources over time when popularity leads tend to decay. The model provides a tractable framework to analyze the dynamics of campaign spending and we identify conditions under which spending decisions are independent of popularity and satisfy an equal spending ratio condition.

Our framework is flexible enough to allow for arbitrary initial advantages, early voting, candidates valuing money left over at the end the campaign, and multi-district competition. We have analyzed the main predictions of our baseline model by looking at spending data from U.S. elections, and we recovered estimates of the perceived rate of decay of popularity leads.

To focus on the budget allocation problem, we have abstracted away from some important considerations in campaigning like the incentives of donors, and the candidates' trade-off between campaigning and fundraising. These considerations are natural complements to our analysis.[38] Embedding the strategic behavior of donors

[36]The political consultant David Shor told one of the present authors that he advises campaigns to perceive a weekly decay rate in ad spending in the ballpark of 15%. Moreover, the timing of the field experiments conducted by the experimental literature vary considerably and do not always coincide with the 12 week period we are focusing on in this paper. Decay rates may be different for ad spending that happens even before the general election period starts, as voters pay less sustained attention to political ads on the whole.

[37]We take this possibility seriously by estimating perceived decay rates using the model with evolving budgets introduced in section OA3 of the Online Appendix. See section OA5 in the Online Appendix for these estimates.

[38]Mattozzi and Michelucci (2017) analyze a two-period dynamic model in which donors decide how much to contribute to each of two possible candidates without knowing ex-ante who is the more likely winner. Bouton et al. (2018) study the strategic choice of donors who try to affect the electoral

in a model of dynamic campaign spending is a particularly interesting and promising avenue for future research.

We have also abstracted from the fact that candidates may not know the return to spending or the decay rate of popularity leads at various stages of the campaign. These quantities may be specific to the characteristics of the candidates or to the political environment, including the "mood" of voters. Real-life candidates thus face an optimal experimentation problem whereby they try to learn about the campaign environment through early spending. There is no doubt that well-run campaigns spend resources to acquire valuable information about how voters are engaging with and responding to the candidates over time. These are interesting and important questions that ought to be addressed in subsequent work.

outcome and highlight that donor behavior depends on the competitiveness of the election. Bouton et al. (2022) provide an empirical analysis of small donors' contribution decisions.

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Appendix

A Proofs

A.1 Proof of Proposition 1

Equilibrium existence follows from Debreu-Fan-Glicksberg Theorem, given the compactness and convexity of the set of candidates' strategies and the continuity and concavity (convexity) of p with respect to x_t (y_t). Uniqueness follows from Assumption 1(b) and the minmax theorem (see Theorem 10 in Rockafellar, 1971).

In equilibrium, spending profiles must be interior: candidates must spend a positive amount at every history. Suppose to the contrary that there exists an equilibrium spending profile in which one of the candidates spends 0 at some history h_t. Assumption 1(a) implies that this candidate spends a positive amount at some history $h_{t'}$ that includes history h_t. By Assumption 1(b)-(c), this candidate will then be better off moving some spending from history $h_{t'}$ to history h_t.

Thus, the equilibrium spending profile from time t onwards must satisfy the set of first-order conditions with respect to x_t and y_t obtained from problem (3). These first order conditions are:

$$\delta^{T-1-t} p_x (x_t, y_t) = p_x (x_{T-1}, y_{T-1})$$
$$\delta^{T-1-t} p_y (x_t, y_t) = p_y (x_{T-1}, y_{T-1})$$

where p_x denotes the partial derivative with respect to the first component and p_y with respect to the second. These conditions do not depend on the past realizations of relative popularity $(z_{t'})_{t'<t}$. These observations establish the claims made in parts (i) and (ii) of the proposition. □

A.2 Proof of Proposition 2

To show part (i), let $h_t = ((x_{t'}, y_{t'}, z_{t'})_{t'<t}, z_t)$ denote the history of candidates' spending decisions up to period $t-1$ and of the relative popularity process up to time t. The budgets available to candidates at history h_t are $X[h_t] = X_0 - \sum_{t'=0}^{t-1} x_{t'}$ and $Y[h_t] = Y_0 - \sum_{t'=0}^{t-1} y_{t'}$. Optimality implies that for any period t and any h_t, candidate 1 maximizes $\Pr[Z_T \geq 0 \mid h_t]$ under the constraint $\sum_{t'=t}^{T} x_{t'} \leq X_t[h_t]$, while candidate 2 minimizes this probability under the constraint $\sum_{t'=t}^{T-1} y_{t'} \leq Y_t[h_t]$.

Using equation (1), we can recast the objective of maximizing $\Pr[Z_T \geq 0 \mid h_t]$ as problem (3). Under Assumption 2, for every $t < T-1$ and every h_t, the candidates' first order conditions with respect to x_t and y_t are thus respectively:

$$\delta^{T-1-t} p_x(x_t, y_t) = p_x(x_{T-1}, y_{T-1})$$
$$\delta^{T-1-t} p_y(x_t, y_t) = p_y(x_{T-1}, y_{T-1})$$

Taking the ratio of these two first order conditions and noting that the partial derivatives of p are homogeneous of degree $\beta - 1$, we get:

$$\frac{p_x\left(\frac{x_t}{y_t}, 1\right)}{p_y\left(\frac{x_t}{y_t}, 1\right)} = \frac{p_x\left(\frac{x_{T-1}}{y_{T-1}}, 1\right)}{p_y\left(\frac{x_{T-1}}{y_{T-1}}, 1\right)}$$

Assumption 1 implies that equilibrium spending levels are interior and unique. Thus, we must have $x_t/y_t = x_{T-1}/y_{T-1}$ for every period t. Using the candidates' budget constraints, we get that for all periods t, $x_t/X_t = y_t/Y_t$. This immediately implies $x_0/y_0 = X_0/Y_0$. Suppose for the sake of the induction argument that $x_{t'}/y_{t'} = X_0/Y_0$ for every period $t' \leq t$; then

$$\frac{x_{t+1}}{y_{t+1}} = \frac{X_{t+1}}{Y_{t+1}} = \frac{X_t - x_t}{Y_t - y_t} = \frac{X_t - x_t}{\frac{X_t}{x_t} y_t - y_t} = \frac{X_t - x_t}{\frac{X_t - x_t}{x_t} y_t} = \frac{x_t}{y_t} = \frac{X_0}{Y_0}$$

where the first and third equalities hold because $x_t/X_t = y_t/Y_t$ for every t and the last equality holds by the inductive hypothesis. Hence, by induction $x_t/y_t = X_0/Y_0$ for every t. (This also implies that $r_{1,t} = r_{2,t} = r_t$ for every $t < T - 1$.)

Now consider part (ii). For any two consecutive periods t and $t+1$ take candidate 1's first order condition among the following pair

$$\delta p_x(x_t, y_t) = p_x(x_{t+1}, y_{t+1})$$
$$\delta p_y(x_t, y_t) = p_y(x_{t+1}, y_{t+1})$$

and note that because the partial derivatives of p are homogeneous of degree $\beta - 1$ we have

$$\delta(x_t)^{\beta-1} p_x(1, y_t/x_t) = (x_{t+1})^{\beta-1} p_x(1, y_{t+1}/x_{t+1})$$

The equal spending ratio result proven above says that that $y_t/x_t = y_{t+1}/x_{t+1} = X_0/Y_0$. Substituting this into the centered equation above and simplifying we get

$$r_{1,t} = x_{t+1}/x_t = \delta^{1/(\beta-1)}.$$

The result for candidate 2 follows from the fact that $r_{1,t} = r_{2,t}$. $\square$

A.3 Proof of Proposition 3

Consider the periods in which voters cast their votes: $\hat{T}$,..., T. We can write the popularity processes at the beginning of these periods as:

$$
\begin{aligned}
Z_T &= \sum_{t=0}^{T-1} \delta^{T-1-t} p(x_t, y_t) + \delta^T z_0 + \sum_{t=0}^{T-1} \delta^{T-1-t} \varepsilon_t, \\
Z_{T-1} &= \sum_{t=0}^{T-2} \delta^{T-2-t} p(x_t, y_t) + \delta^{T-1} z_0 + \sum_{t=0}^{T-2} \delta^{T-2-t} \varepsilon_t, \\
&\vdots \qquad\qquad\qquad \vdots \qquad\qquad\qquad \vdots \\
Z_{\hat{T}} &= \sum_{t=0}^{\hat{T}-1} \delta^{\hat{T}-1-t} p(x_t, y_t) + \delta^{\hat{T}} z_0 + \sum_{t=0}^{\hat{T}-1} \delta^{\hat{T}-1-t} \varepsilon_t.
\end{aligned}
$$

Substituting these expressions into the candidates' objective function, we get:

$$
\Pr\left[\sum_{t=\hat{T}}^{T} \xi^{T-t} Z_t \geq 0\right] = \Pr\left[\sum_{t=\hat{T}}^{T} \xi^{T-t} E_t \geq -\sum_{t=\hat{T}}^{T} \xi^{T-t} B_t\right],
$$

where $E_t := \sum_{t'=0}^{t-1} \delta^{T-1-t'} \varepsilon_{t'}$ and $B_t := \sum_{t'=0}^{t-1} \delta^{t-1-t'} p(x_{t'}, y_{t'}) + \delta^t z_0$.

Each E_t is the sum of normally distributed shocks with zero mean and with a variance that does not depend on candidates' spending. We can thus assume that candidate 1 maximizes (and candidate 2 minimizes) $\sum_{t=\hat{T}}^{T} \xi^{T-t} B_t$, or equivalently

$$
\sum_{t=0}^{\hat{T}-1} \sum_{t'=0}^{T-\hat{T}} \xi^{t'} \delta^{T-1-t-t'} p(x_t, y_t) + \sum_{t=\hat{T}}^{T-1} \sum_{t'=0}^{T-1-t} \xi^{t'} \delta^{T-1-t-t'} p(x_t, y_t)
$$

The same steps of the proof of Proposition 2 allow us to show that $x_t/X_t = y_t/Y_t$ for every t, and that the cross-candidate spending ratio, x_t/y_t, is constant over time. In particular, $x_t/y_t = X_0/Y_0$. This establishes part (i).

For part (ii) consider a period $t < \hat{T} - 1$. The same steps used in the proof of Proposition 2 yield that the consecutive period spending ratio for every t is constant across players and it is equal to the one derived for the baseline model; that is, $r_t = \delta^{1/(\beta-1)}$ for all $t < \hat{T} - 1$. Next, consider a period $t \geq \hat{T} - 1$. The result of part (i) implies that even in this case $r_{1,t} = r_{2,t}$, so we can focus on candidate 1's first order conditions. If we equate her first order conditions for two consecutive periods and use the homogeneity of function p we get

$$\left(\frac{x_{t+1}}{x_t}\right)^{\beta-1} = \frac{\sum_{t'=0}^{T-1-t} \xi^{t'} \delta^{T-1-t-t'}}{\sum_{t'=0}^{T-2-t} \xi^{t'} \delta^{T-2-t-t'}}.$$

From this we obtain:

$$r_t = \left[\delta\left(1 + \frac{1}{\sum_{t'=0}^{T-2-t} \xi^{-(T-1-t-t')} \delta^{T-2-t-t'}}\right)\right]^{1/(\beta-1)}.$$

A.4 Proof of Proposition 4

Under Assumptions 1 and 2 the function $q(x/y)$ defined as $p(x/y, 1)$ is strictly increasing and strictly quasiconcave. Pick an arbitrary history h_{T-1} up to period $T-1$ and let $(\hat{x}_t)_{t=0}^{T-2}$ and $(\hat{y}_t)_{t=0}^{T-2}$ be the amounts spent by the candidates along this history. Denote the choice variable for candidate 1's spending at history h_{T-1} by x_{T-1} and for candidate 2 by y_{T-1}. Candidate 1 maximizes $\mathbb{E}[\mathbf{1}_{\{Z_T \geq 0\}} + \kappa_1 X_T \mid h_{T-1}]$ and candidate 2 maximizes $\mathbb{E}[(1 - \mathbf{1}_{\{Z_T \geq 0\}}) + \kappa_2 Y_T \mid h_{T-1}]$. Let

$$L[h_{T-1}] = \sum_{t=0}^{T-2} \delta^{T-1-t} q\left(\frac{\hat{x}_t}{\hat{y}_t}\right) + q\left(\frac{x_{T-1}}{y_{T-1}}\right) + \delta^T z_0 + \sum_{t=0}^{T-2} \delta^{T-1-t} \varepsilon_t.$$

Given that $\varepsilon_{T-1} \sim \mathcal{N}(0, \sigma^2)$, we have that $Z_T \mid h_{T-1} \sim \mathcal{N}(L[h_{T-1}], \sigma^2)$. Hence, the first order conditions of the two candidates candidates are respectively:

$$\phi_{(0,1)}\left(-\frac{\hat{L}[h_{T-1}]}{\sigma}\right)\frac{1}{\sigma} q'\left(\frac{\hat{x}_{T-1}}{\hat{y}_{T-1}}\right)\frac{1}{\hat{y}_{T-1}} = \kappa_1$$
$$\phi_{(0,1)}\left(-\frac{\hat{L}[h_{T-1}]}{\sigma}\right)\frac{1}{\sigma} q'\left(\frac{\hat{x}_{T-1}}{\hat{y}_{T-1}}\right)\frac{\hat{x}_{T-1}}{(\hat{y}_{T-1})^2} = \kappa_2$$

where $\phi_{(0,1)}$ is the pdf of the standard normal, $\hat{x}_{T-1}$ and $\hat{y}_{T-1}$ are equilibrium values of x_{T-1} and y_{T-1} following history h_{T-1}, and $\hat{L}[h_{T-1}]$ is the value that $L[h_{T-1}]$ takes when $x_{T-1}/y_{T-1} = \hat{x}_{T-1}/\hat{y}_{T-1}$. Taking the ratio of these first order conditions gives

$\hat{x}_{T-1}/\hat{y}_{T-1} = \kappa_2/\kappa_1$, which is independent of the history h_{T-1}. Thus

$$\hat{x}_{T-1} = \frac{\kappa_2}{(\kappa_1)^2}\phi_{(0,1)}\left(-\frac{\hat{L}[h_{T-1}]}{\sigma}\right)\frac{1}{\sigma}q'\left(\frac{\kappa_2}{\kappa_1}\right)$$
$$\hat{y}_{T-1} = \frac{1}{\kappa_1}\phi_{(0,1)}\left(-\frac{\hat{L}[h_{T-1}]}{\sigma}\right)\frac{1}{\sigma}q'\left(\frac{\kappa_2}{\kappa_1}\right)$$

Both spending decisions are decreasing in $|\hat{L}[h_{T-1}]|$, which depends on history.

Now assume for the sake of an inductive argument that for all histories h_t with $t \in \{\tilde{t}+1, \tilde{t}+2, ..., T-1\}$, we have that in an interior equilibrium, (i) $\hat{x}_t/\hat{y}_t = \kappa_2/\kappa_1$ where $\hat{x}_t$ and $\hat{y}_t$ are the equilibrium amounts spent following history h_t, and (ii) spending decisions are given by:

$$\hat{x}_t = \frac{\kappa_2}{(\kappa_1)^2}\phi_{(0,1)}\left(-\frac{\hat{L}[h_t]}{\sigma\sqrt{\sum_{t'=t}^{T-1}\delta^{T-1-t'}}}\right)\frac{\delta^{T-1-t}}{\sigma\sqrt{\sum_{t'=t}^{T-1}\delta^{T-1-t'}}}q'\left(\frac{\kappa_2}{\kappa_1}\right)$$
$$\hat{y}_t = \frac{1}{\kappa_1}\phi_{(0,1)}\left(-\frac{\hat{L}[h_t]}{\sigma\sqrt{\sum_{t'=t}^{T-1}\delta^{T-1-t'}}}\right)\frac{\delta^{T-1-t}}{\sigma\sqrt{\sum_{t'=t}^{T-1}\delta^{T-1-t'}}}q'\left(\frac{\kappa_2}{\kappa_1}\right),$$

where

$$\hat{L}[h_t] = \sum_{t'=0}^{t-1}\delta^{T-1-t'}\left[q\left(\frac{\hat{x}_{t'}}{\hat{y}_{t'}}\right) + \varepsilon_{t'}\right] + \delta^T z_0 + \sum_{t'=t}^{T-1}\delta^{T-1-t'}q\left(\frac{\kappa_2}{\kappa_1}\right),$$

and $(\hat{x}_{t'})_{t'=0}^{t-1}$ and $(\hat{y}_{t'})_{t'=0}^{t-1}$ are the spending choices of candidates along history h_t. Obviously, spending decisions $\hat{x}_t$ and $\hat{y}_t$ are decreasing in $|\hat{L}[h_t]|$.

Consider period t and pick an arbitrary history h_t. Since $(\varepsilon_{t'})_{t'=0}^{T-1}$ are iid shocks distributed according to $\mathcal{N}(0,\sigma^2)$ and (by the inductive hypothesis) the ratios of spending decision in subsequent periods are history independent and equal to κ_2/κ_1, we have that $Z_T \mid h_t \sim \mathcal{N}(L[h_t], \sigma^2\sum_{t'=t}^{T-1}\delta^{2(T-1-t')})$, where

$$L[h_t] = \sum_{t'=0}^{t-1}\delta^{T-1-t'}\left[q\left(\frac{\hat{x}_{t'}}{\hat{y}_{t'}}\right) + \varepsilon_t\right] + \delta^T z_0 + \delta^{T-1-t}q\left(\frac{x_t}{y_t}\right) + \sum_{t'=t+1}^{T-1}\delta^{T-1-t'}q\left(\frac{\kappa_2}{\kappa_1}\right),$$

and $(\hat{x}_{t'})_{t'=0}^{t-1}$ and $(\hat{y}_{t'})_{t'=0}^{t-1}$ are the amounts spent by candidates along history h_t, and x_t and y_t are the choice variables for the candidates' spending levels at history h_t.

The first order conditions for an interior optimum are

$$\phi_{(0,1)}\left(-\frac{\hat{L}[h_t]}{\sigma\sqrt{\sum_{t'=t}^{T-1}\delta^{2(T-1-t')}}}\right)\frac{\delta^{T-1-t}}{\sigma\sqrt{\sum_{j=t}^{T-1}\delta^{2(T-1-j)}}}q'\left(\frac{\hat{x}_t}{\hat{y}_t}\right)\frac{1}{\hat{y}_t}=\kappa_1$$
$$\phi_{(0,1)}\left(-\frac{\hat{L}[h_t]}{\sigma\sqrt{\sum_{t'=t}^{T-1}\delta^{2(T-1-t')}}}\right)\frac{\delta^{T-1-t}}{\sigma\sqrt{\sum_{j=t}^{T-1}\delta^{2(T-1-j)}}}q'\left(\frac{\hat{x}_t}{\hat{y}_t}\right)\frac{\hat{x}_t}{(\hat{y}_t)^2}=\kappa_2$$

where $\hat{L}[h_t]$ is equal to $L[h_t]$ after replacing the ratio x_t/y_t with $\hat{x}_t/\hat{y}_t$. Taking the ratio of these expressions, we get $\hat{x}_t/\hat{y}_t=\kappa_2/\kappa_1$, which is independent of the past, and the candidates' equilibrium spending decisions are

$$\hat{x}_t=\frac{\kappa_2}{(\kappa_1)^2}\phi_{(0,1)}\left(-\frac{\hat{L}[h_t]}{\sigma\sqrt{\sum_{t'=t}^{T-1}\delta^{2(T-1-t')}}}\right)\frac{\delta^{T-1-t}}{\sigma\sqrt{\sum_{j=t}^{T-1}\delta^{2(T-1-j)}}}q'\left(\frac{\hat{x}_t}{\hat{y}_t}\right)$$
$$\hat{y}_t=\frac{1}{\kappa_1}\phi_{(0,1)}\left(-\frac{\hat{L}[h_t]}{\sigma\sqrt{\sum_{t'=t}^{T-1}\delta^{2(T-1-t')}}}\right)\frac{\delta^{T-1-t}}{\sigma\sqrt{\sum_{j=t}^{T-1}\delta^{2(T-1-j)}}}q'\left(\frac{\hat{x}_t}{\hat{y}_t}\right)$$

Given the condition $\hat{x}_t/\hat{y}_t=\kappa_2/\kappa_1$, we have $\hat{L}[h_t]=\zeta((\varepsilon_{t'})_{t'=0}^{t-1})$, where $(\varepsilon_{t'})_{t'=0}^{t-1}$ are the shocks along history h_t. Thus, the candidates' equilibrium spending levels are decreasing $|\zeta((\varepsilon_{t'})_{t'=0}^{t-1})|$. The argument that we have given establishes both parts (i) and (ii) of the proposition by induction. □

A.5 Proof of Proposition 5

We start observing that there cannot be an equilibrium in which both candidates spend an amount equal to 0 in some district in the same period. In this case, footnote 18 implies that either candidate would have an incentive to deviate and spend a positive amount, securing victory with probability 1.

Furthermore, there cannot be an equilibrium in which one of the two candidates spend an amount equal to 0 in a district, say district s, in a given period. In this case, the candidate would lose with certainty and she would be better off saving a small amount from each of the other districts and investing the saved amount in district s. In equilibrium spending must then be interior, satisfying the first order conditions for any district and in any period.

We now prove part (i) of the proposition by induction. Consider the final period. Fix $(z_{T-1}^s)_{s=1}^S$ arbitrarily. Suppose candidates 1 and 2 have resource stocks equal to X_{T-1} and Y_{T-1} at the beginning of the last period. Fix an equilibrium strategy profile

$(\hat{x}^s_{T-1}, \hat{y}^s_{T-1})_{s=1}^S$. We will show that if the candidates have budgets ϑX_{T-1} and ϑY_{T-1}, then $(\vartheta \hat{x}^s_{T-1}, \vartheta \hat{y}^s_{T-1})_{s=1}^S$ is an equilibrium, which in turn implies that the equilibrium payoff in the last period is determined by $(z^s_{T-1})_{s=1}^S$ and X_{T-1}/Y_{T-1} only. Suppose otherwise. Without loss of generality, assume that there exists $(\tilde{x}^s_{T-1})_{s=1}^S$ satisfying $\sum_{s=1}^S \tilde{x}^s_{T-1} \le \vartheta X_{T-1}$ that gives a higher probability of winning to candidate 1 given $(z^s_{T-1})_{s=1}^S$ and $(\vartheta \hat{y}^s_{T-1})_{s=1}^S$. The distribution of $(Z^s_T)_{s=1}^S$ is determined by $(z^s_{T-1})_{s=1}^S$ and $(x^s_{T-1}/y^s_{T-1})_{s=1}^S$ only. This means that the distribution of $(Z^s_T)_{s=1}^S$ given $(z^s_{T-1})_{s=1}^S$ and $(\tilde{x}^s_{T-1}/\vartheta \hat{y}^s_{T-1})_{s=1}^S$ is more favorable to candidate 1 than that given $(z^s_{T-1})_{s=1}^S$ and $(\hat{x}^s_{T-1}/\vartheta \hat{y}^s_{T-1})_{s=1}^S$. Obviously, $(\vartheta \hat{x}^s_{T-1}/\vartheta \hat{y}^s_{T-1})_{s=1}^S = (\hat{x}^s_{T-1}/\hat{y}^s_{T-1})_{s=1}^S$ and candidate 1 could spend $(\frac{1}{\vartheta}\tilde{x}^s_{T-1})_{s=1}^S$ when the budgets are (X_{T-1}, Y_{T-1}). Because $(\hat{x}^s_{T-1}, \hat{y}^s_{T-1})_{s=1}^S$ is an equilibrium, the distribution of $(Z^s_T)_{s=1}^S$ given $(z^s_{T-1})_{s=1}^S$ is more favorable to candidate 1 under $(\hat{x}^s_{T-1}/\hat{y}^s_{T-1})_{s=1}^S$ than under $(\frac{1}{\vartheta}\tilde{x}^s_{T-1}/\hat{y}^s_{T-1})_{s=1}^S = (\tilde{x}^s_{T-1}/\vartheta \hat{y}^s_{T-1})_{s=1}^S$. This establishes a contradiction.

Now, we prove the inductive step. The inductive hypotheses are (i) that the continuation payoff for either candidate in period $t' \ge t+1$ can be written as a function of only the budget ratio $X_{t'}/Y_{t'}$ and the vector $(z^s_{t'})_{s=1}^S$, and (ii) second that $x^s_{t'}/X_{t'} = y^s_{t'}/Y_{t'}$ for every district s and every period $t' \ge t+1$. We want to show that $x^s_t/X_t = y^s_t/Y_t$ in each district s and that the continuation value at time t can be written as a function of only the budget ratio X_t/Y_t and the vector $(z^s_t)_{s=1}^S$. For each period t, let $x_t = \sum_s x^s_t$, $y_t = \sum_s y^s_t$ and let $z_t = (z^s_t)_{s=1}^S$. Let $V_{t+1}\big(\frac{X_{t+1}}{Y_{t+1}}, z_{t+1}\big)$ denote the continuation payoff of candidate 1 starting in period $t+1$. Candidate 1's objective is

$$\max_{(x^s_t)_{s=1}^S} \int V_{t+1}\left(\frac{X_t - x_t}{Y_t - y_t}, z_{t+1}\right) \phi_t \left(z_{t+1} \mid \left(\frac{x^s_t}{y^s_t}\right)_{s=1}^S, z_t \right) dz_{t+1}$$

where $\phi_t(\cdot|\cdot)$ is the conditional distribution of the vector z_{t+1}. For each district s, the first order conditions for an interior optimum for candidate 1 is then

$$\frac{1}{Y_t - y_t} \int \frac{\partial V_{t+1}\left((X_t - x_t)/(Y_t - y_t), z_{t+1}\right)}{\partial (x^s_t/y^s_t)} \phi_t \left(z_{t+1} \mid (x^s_t/y^s_t)_{s=1}^S, z_t \right) dz_{t+1}$$
$$= \frac{1}{y^s_t} \int V_{t+1}\left(\frac{X_t - x_t}{Y_t - y_t}, z_{t+1}\right) \frac{\partial \phi_t \left(z_{t+1} \mid (x^s_t/y^s_t)_{s=1}^S, z_t \right)}{\partial (x^s_t/y^s_t)} dz_{t+1}.$$

Similarly, the objective function for candidate 2 is

$$\min_{(y^s_t)_{s=1}^S} \int V_{t+1}\left(\frac{X_t - x_t}{Y_t - y_t}, z_{t+1}\right) \phi_t \left(z_{t+1} \mid \left(\frac{x^s_t}{y^s_t}\right)_{s=1}^S, z_t \right) dz_{t+1}$$

and the corresponding first order condition for each s is

$$\frac{X_t - x_t}{(Y_t - y_t)^2} \int \frac{\partial V_{t+1}\left((X_t - x_t)/(Y_t - y_t), z_{t+1}\right)}{\partial (x_t^s/y_t^s)} \phi_t\left(z_{t+1} \mid (x_t^s/y_t^s)_{s=1}^S, z_t\right) dz_{t+1}$$

$$= \frac{x_t^s}{(y_t^s)^2} \int V_{t+1}\left(\frac{X_t - x_t}{Y_t - y_t}, z_{t+1}\right) \frac{\partial \phi_t\left(z_{t+1} \mid (x_t^s/y_t^s)_{s=1}^S, z_t\right)}{\partial (x_t^s/y_t^s)} dz_{t+1}.$$

Dividing the candidate 1's first order condition by candidate 2's, we have

$$\frac{X_t - x_t}{Y_t - y_t} = \frac{x_t^s}{y_t^s},$$

which implies $x_t^s/y_t^s = X_t/Y_t$ for all s. As a result, the continuation value of candidates in period t is a function of only the budget ratio X_t/Y_t and the vector $(z_t^s)_{s=1}^S$. Part (i) of the proposition follows by induction.

Now for part (ii), consider candidate 2's problem from the history h_t perspective. Let $(\hat{x}_t^s)_{s=1}^S$ be the equilibrium spending strategy of candidate 1. Candidate 2 solves

$$\min_{(y_t^s)} \mathbb{E}[u((Z_T^s)_{s=1}^S) \mid (\hat{x}_t^s)_{s=1}^S]$$

Given the law of motion of each Z_T^s, we can equalize the first order necessary conditions for an interior optimum associated with spending at time t and at time $t+1$ in a given district s and get

$$\mathbb{E}\left[(\delta^s)^{T-t} \frac{\partial q\left(x_t^s/y_t^s\right)}{\partial\left(x_t^s/y_t^s\right)} \frac{x_t^s}{(y_t^s)^2} \frac{\partial u\left((Z_T^s)_{s=1}^S\right)}{\partial Z_T^s} \mid h_t\right]$$

$$= \mathbb{E}\left[(\delta^s)^{T-t-1} \frac{\partial q\left(x_{t+1}^s/y_{t+1}^s\right)}{\partial\left(x_{t+1}^s/y_{t+1}^s\right)} \frac{x_{t+1}^s}{(y_{t+1}^s)^2} \frac{\partial u\left((Z_T^s)_{s=1}^S\right)}{\partial Z_T^s} \mid h_t\right].$$

From the equal spending ratio result in part (i) we have $x_t^s/y_t^s = X_0/Y_0$ for all s and t. Using this fact to cancel terms on both sides of the equation above, we have

$$\frac{\mathbb{E}[\partial u\left((Z_T^s)_{s=1}^S\right)/\partial Z_T^s \mid h_t]}{\mathbb{E}[(1/r_{2,t}^s)\partial u\left((Z_T^s)_{s=1}^S\right)/\partial Z_T^s \mid h_t]} = \frac{1}{\delta^s}$$

By Assumption 3, $\partial u\left((Z_T^s)_{s=1}^S\right)/\partial Z^s = w^s$, which is constant. Therefore, we have $1/\mathbb{E}[(1/r_{2,t}^s)] = 1/\delta^s$. The analogous result for candidate 1 follows from a symmetric argument. This establishes part (ii) of the proposition.

For part (iii), under Assumption 3 we can write:

$$\begin{aligned}\mathbb{E}[u_1((Z_T^s)_{s=1}^S)|h_t] &= \sum_{s=1}^S w^s \mathbb{E}[Z_T^s|h_t] \\ &= \sum_{s=1}^S w^s \mathbb{E}\left[\sum_{t'=t}^{T-t-1} (\delta^s)^{T-t-1} q\left(\frac{x_{t'}^s}{y_{t'}^s}\right) + (\delta^s)^{T-t} z_t^s \mid h_t\right].\end{aligned}$$

For every realization of the random shock, this expression is increasing and strictly quasiconcave in each $x_{t'}^s$ and decreasing and strictly quasiconvex in each $y_{t'}^s$, $t' \geq t$. Hence, we can assume that candidate 1 maximizes

$$\sum_{s=1}^S w^s \mathbb{E}[Z_T^s|h_t] = \sum_{s=1}^S w^s \mathbb{E}\left[\sum_{t'=t}^{T-t-1} (\delta^s)^{T-t-1} q\left(\frac{x_{t'}^s}{y_{t'}^s}\right) \mid h_t\right],$$

while candidate 2 minimizes it. The left over budgets are $X_t = X_0 - \sum_{t'<t}\sum_{s=1}^S x_{t'}^s$ and $Y_t = Y_0 - \sum_{t'<t}\sum_{s=1}^S y_{t'}^s$. If we equate the first order necessary conditions for an interior optimum associated with spending at time t in districts s and s', we get

$$w^s(\delta^s)^{T-t-1}\frac{\partial q(x_t^s/y_t^s)}{\partial(x_t^s/y_t^s)}\frac{x_t^s}{(y_t^s)^2} = w^{s'}(\delta^{s'})^{T-t-1}\frac{\partial q(x_t^{s'}/y_t^{s'})}{\partial(x_t^{s'}/y_t^{s'})}\frac{x_t^{s'}}{(y_t^{s'})^2}.$$

In equilibrium, $x_t^s/y_t^s = X_t/Y_t$ for all t and s. The above expression thus simplifies to

$$\frac{y_t^s}{y_t^{s'}} = \frac{w^s}{w^{s'}}\left(\frac{\delta^s}{\delta^{s'}}\right)^{T-t-1} \qquad \text{for every } s, s' \text{ and every } t.$$

□

Senate Elections in our Baseline Sample

Year	State
2000	DE, FL, IN, ME, MI, MN, MO, NE, NV, NY, PA, RI, VA, WA
2002	AL, AR, CO, GA, IA, LA, ME, NC, NH, NJ, OK, OR, SC, TN, TX
2004	CO, FL, GA, KY, LA, NC, OK, PA, SC, WA
2006	AZ, MD, MI, MO, NE, OH, PA, RI, TN, VA, WA, WV
2008	AK, CO, GA, ID, KS, KY, LA, ME, MS, NC, NE, NH, NM, OK, OR, SD
2010	AL, AR, CA, CO, CT, IA, IL, IN, KY, LA, MD, MO, NH, NV, NY, OR, PA, VT, WA
2012	AZ, CT, FL, HI, IN, MA, MO, MT, ND, NE, NM, NV, OH, PA, RI, VA, WI, WV
2014	AK, AR, CO, GA, IA, IL, KY, LA, ME, MI, MT, NC, NH, NM, OR, SD, VA, WV

Gubernatorial Elections in our Baseline Sample

Year	State
2000	IN, MO, NC, NH, WA, WV
2002	AL, AR, AZ, CA, CT, FL, GA, HI, IA, IL, KS, MA, MD, ME, MI, NM, NY, OK, OR, PA, RI, SC, TN, TX, WI
2004	IN, MO, NC, NH, UT, VT, WA
2006	AL, AR, AZ, CO, CT, FL, GA, IA, IL, KS, MD, ME, MI, MN, NH, NV, NY, OH, OR, PA, RI, TN, VT, WI
2008	IN, MO, NC, WA
2010	AK, AL, AR, AZ, CA, CT, FL, GA, HI, IA, ID, IL, MA, MD, MI, MN, NH, NM, NV, NY, OH, OK, OR, PA, SC, SD, TN, TX, UT, VT, WI
2012	IN, MO, MT, NC, ND, NH, WA, WV
2014	AL, AR, AZ, CO, CT, FL, GA, HI, IA, ID, IL, KS, MA, MD, ME, MI, MN, NE, NH, NM, NY, OH, OK, OR, PA, SC, TX, WI

House Elections in our Baseline Sample

Year	State-District
2000	AL-4, AR-4, CA-20, CA-49, CO-6, CT-5, FL-12, FL-22, FL-8, GA-7, KS-3, KY-3, KY-6, MI-8, MN-6, MO-2, MO-3, MO-6, NC-11, NC-8, NH-1, NH-2, NM-1, NV-1, OH-1, OH-12, OK-2, PA-10, PA-13, PA-4, TX-25, UT-2, VA-2, WA-1, WA-5, WV-2
2002	AL-1, AL-3, AR-4, CT-5, FL-22, IA-1, IA-2, IA-3, IA-4, IL-19, IN-2, KS-3, KS-4, KY-3, ME-2, MI-9, MS-3, NH-1, NH-2, NM-1, NM-2, OK-4, PA-11, PA-17, SC-3, TX-11, UT-2, WV-2
2004	CA-20, CO-3, CT-2, CT-4, FL-13, GA-12, IA-3, IN-8, KS-3, KY-3, MO-5, MO-6, NC-11, NE-2, NM-1, NM-2, NV-3, NY-27, OK-2, OR-1, TX-17, WA-5, WV-2
2006	AZ-5, AZ-8, CO-4, CO-7, CT-2, CT-4, CT-5, FL-13, FL-22, GA-12, HI-2, IA-1, IA-3, ID-1, IL-6, IN-2, IN-8, IN-9, KY-2, KY-3, KY-4, MN-6, NC-11, NH-2, NM-1, NV-3, NY-20, NY-24, NY-25, NY-29, OH-1, OH-12, OH-15, OH-18, OR-5, PA-10, SC-5, TX-17, VA-2, VA-5, VT-1, WA-5, WI-8
2008	AK-1, AL-2, AL-3, AL-5, AZ-3, AZ-5, AZ-8, CA-11, CA-4, CO-4, CT-4, CT-5, FL-16, FL-24, FL-8, GA-8, ID-1, IL-10, IN-3, KY-2, KY-3, LA-4, LA-6, MD-1, MI-7, MO-6, NC-8, NH-1, NH-2, NM-1, NM-2, NV-2, NV-3, NY-20, NY-24, NY-25, NY-26, NY-29, OH-1, OH-15, PA-10, PA-11, SC-1, VA-2, VA-5, WI-8, WV-2
2010	AL-2, AL-5, AR-2, AZ-1, AZ-5, AZ-8, CA-20, CA-45, CO-3, CO-4, CT-4, CT-5, FL-2, FL-22, FL-24, FL-8, GA-12, GA-8, HI-1, IA-1, IA-2, IA-3, IN-2, IN-8, KS-4, KY-6, MA-1, MD-1, MD-2, MI-1, MI-3, MI-7, MI-9, MN-6, MO-3, MO-4, MO-8, MS-1, NC-2, NC-5, NC-8, NE-2, NH-1, NH-2, NM-1, NM-2, NV-3, NY-20, NY-23, NY-24, NY-25, OH-1, OH-12, OH-13, OH-15, OH-16, OH-9, OK-5, OR-3, OR-5, PA-10, PA-11, PA-4, SC-2, SC-5, SD-1, TN-1, TN-4, TN-8, TN-9, TX-17, VA-2, VA-5, VA-9, WA-2, WI-8, WV-3
2012	AZ-2, CA-10, CA-24, CA-3, CA-36, CA-52, CA-7, CA-9, CO-3, CO-6, CO-7, CT-5, FL-18, GA-12, HI-1, IA-1, IA-2, IA-3, IA-4, IL-12, IL-13, IL-17, IL-8, IN-2, IN-8, KY-6, MA-6, ME-2, MI-6, MN-6, MN-8, MT-1, NC-7, ND-1, NH-1, NH-2, NM-1, NV-3, NY-19, NY-21, NY-24, NY-25, NY-27, OH-16, OH-6, PA-12, RI-1, SD-1, TX-23, UT-4, VA-2, VA-5, WI-8, WV-3
2014	AR-2, AZ-1, AZ-2, CA-21, CA-36, CA-52, CA-7, CO-6, CT-5, FL-18, FL-2, FL-26, GA-12, HI-1, IA-1, IA-2, IA-3, IL-10, IL-12, IL-13, IL-17, IN-2, ME-2, MI-7, MN-7, MN-8, MT-1, ND-1, NE-2, NH-2, NM-2, NV-3, NY-19, NY-21, NY-23, NY-24, VA-10, VA-2