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Superstar Exclusivity in Two-Sided Markets*

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In most platform environments, the exclusive provision of premium content from leading creators (Superstars) is employed as a strategy to boost user participation and secure a competitive edge *vis-à-vis* rivals. In this article, we study the impact of Superstar exclusive content provision on platform competition and complementors' homing decisions. Two competing platforms facilitate interactions between consumers and suppliers, of which the latter are identified by the Superstar and a fringe of complementors (*e.g.,* independent developers, amateurs). When platform competition is intense, more consumers become affiliated with the platform *favored* by Superstar exclusivity. This mechanism is self-reinforcing as it generates an entry cascade of complementors and some complementors singlehome on the *favored* platform. We find that cross-group externalities are key in shaping market outcomes. First, exclusivity benefits complementors and might make consumers better off when cross-group externalities are large enough. Second, contrary to conventional wisdom, vertical integration (platform-Superstar) may make exclusivity less likely than vertical separation under reasonable conditions. Finally, we discuss implications for the strategies of platform owners, managers of Superstars and complementors, and antitrust enforcers.

JEL Classification: L13, L22, L86, K21.

Keywords: exclusivity, platforms, two-sided markets, vertical integration, network externalities.

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1. Introduction

In most digital markets, platforms orchestrate interactions between different groups of agents (Caillaud & Jullien 2003, Rochet & Tirole 2003, Parker & Van Alstyne 2005, Rochet & Tirole 2006, Armstrong 2006, Jullien 2011). The extant literature has mostly focused on how platforms compete to reach a critical size, attracting a large number of consumers and suppliers. Because of cross-group externalities, a critical mass on one side of the market is fundamental for winning the other side as well.

However, little attention has been paid to the heterogeneity that exists on the supplier side of the market. Most platforms feature a mix of amateurs and professionals, of which the latter are typically more attractive for consumers (Boudreau 2019). Heterogeneity even exists among professionals, for example, between popular and non-popular artists. In the music-on-demand streaming market, Spotify and Apple Music feature Superstar artists (*e.g.,* Beyoncé, Taylor Swift) alongside emerging talents. Similarly, in the mobile app market, large and famous developers (*e.g.,* Whatsapp and Instagram) co-exist with a fringe of small, mostly independent developers. Other examples can be found in open-source software (*e.g.,* Pivotal), games (*e.g.,* Call of Duty, Fortnite), news (*e.g.,* Sean Penn interviewing El Chapo), and sports broadcasting (*e.g.,* PSG, Real Madrid, Juventus).

A "Superstar" can be a valuable asset when she can confer a significant competitive edge on a platform over its rivals, and exclusivity is a critical way of reaching this goal (Cennamo & Santalo 2013, Förderer & Gutt 2021). The power of exclusivity is boosted by the presence of cross-group externalities, as user participation attracts agents on the other side (*i.e.,* complementors), which in turn consolidates the platform's competitive position. While the importance of such indirect effects is not new to the literature (Ambrus & Argenziano 2009), a Superstar can play a key role in enabling them, underscoring the significance of homing decisions in the light of a Superstar's market power or marquee status (Rochet & Tirole 2003, Biglaiser et al. 2019).

This paper studies exclusivity decisions by Superstars and their implications for platform competition, complementors' participation in the market, and consumers. Notable instances of Superstar exclusivity include "The Joe Rogan Experience" podcast on Spotify and the professional gamer Tyler "Ninja" Blevins on the streaming platform Mixer, both of whom received multi-year and multi-million dollar contracts. Examples also include the release of the remake of *Demon's Souls* exclusively for PlayStation 5, and the first-party exclusive provision of successful video games (Cennamo & Santalo 2013, Lee 2013) and content (*e.g.,* Disney/Marvel on Disney+ or Netflix Originals on Netflix).[1]

We develop a tractable model with two horizontally differentiated platforms acting as intermediaries between consumers and firms. The firm side is composed of a fringe of complementors, while the Superstar acts as a monopolist supplier of her product and has full bargaining power *vis-à-vis* the platforms. We analyze two market structures. First, when the platform and Superstar are independent, the Superstar's decision is between offering her product to one platform (exclusivity) or both (non-exclusivity). Second, when the Superstar is integrated within a platform, the decision of the owner of the merged entity becomes a matter of whether to provide the rival platform with the Superstar's content.

To understand how exclusivity reshapes platform competition, we focus on an ex-ante symmetric market configuration and isolate the Superstar's contribution net of any coordination domino effect linked to externalities.[2] Under non-exclusivity, consumer demand is equally split between the two platforms, while complementors are either active on both platforms (multihoming) or inactive (zerohoming). In contrast, exclusivity renders a platform *favored* in the competition with the rival *unfavored* platform. In particular, some consumers follow the Superstar and more complementors become active in the market and agglomerate on the *favored* platform, with some zerohomers and some multihomers becoming singlehomers. As a result, an exclusive contract between the Superstar and the *favored* platform enables direct and indirect asymmetries and externalities that are capitalized upon by the Superstar due to the extra value she creates. The agglomeration of consumers and complementors on the *favored* platform under exclusivity might lead to welfare gains when cross-group externalities are large enough.

Although exclusivity entails the above-discussed gains, the move does require the Superstar to limit her market reach. Indeed, the surplus extracted from the *favored* platform must be sufficiently large to compensate for the foregone revenues otherwise obtained under non-exclusivity. We find that exclusivity is more likely when platform competition is sufficiently intense that the Superstar has the potential to affect the homing decisions of a large mass of consumers and complementors on the *favored* platform. Adapting the contractual setting of Ordover et al. (1990), the Superstar can extract this surplus through an exclusive contract resulting from an auction with a reserve price (see Bounie et al. 2021). On the

[1] In the Online Appendix (Section 1), we present examples of the exclusive provision in several different digital markets, including e-sports, audiobooks, mobile apps and publishers. We also present examples of exclusive contracts in the shopping mall industry with "anchor stores."

[2] In markets with network externalities, a coordination problem leads to the emergence of multiple equilibria (*e.g.,* Caillaud & Jullien 2003, Hagiu 2006, Jullien 2011), when agents have different beliefs regarding participation on the other side. We discuss this issue in Section 6.

other hand, non-exclusivity emerges when the market is less competitive, as consumers are less mobile. In this case, because the surplus to be capitalized by the Superstar under exclusivity is not high enough, reaching the entire market with a non-exclusive contract becomes more profitable.

The main trade-off critically changes under vertical integration. Under exclusivity, the merged entity internalizes the network benefits the Superstar obtains from her interactions with consumers. This exerts downward pressure on prices and on the rival's profits. Because the platform's owner is ex-post *tougher* in the market under exclusivity, at the initial contractual stage the merged entity can leverage its market power and induce the other platform to accept a higher tariff for non-exclusive access to the Superstar. This generates a novel effect that contrasts with the findings of the traditional literature on vertical integration and foreclosure, according to which exclusivity is more likely in the presence of vertical integration (see *e.g.,* Rasmusen et al. 1991, Bernheim & Whinston 1998, Fumagalli & Motta 2006, *inter alia*). Specifically, as long as exclusivity under vertical integration generates a sufficiently large (respectively small) demand asymmetry compared to vertical separation, the merged entity is more likely to grant the rival platform access to the Superstar content (resp. be exclusive). In this case, the higher non-exclusive tariff more than compensates for any direct demand expansion effect. This case always prevails in the presence of a uniform distribution of preferences.

Our analysis highlights how cross-group externalities might lead to conclusions that are different from those in traditional one-sided markets, with important implications for the strategies employed by platforms, complementors, and Superstars, as well as for antitrust enforcement.[3] For example, platforms may want to engage in exclusive dealing with Superstars or first-party exclusive provision of the Superstar content to facilitate coordination among users. From a policy perspective, meanwhile, these results suggest that overlooking the role of network externalities might lead to an overestimation of the potential harm and, thus, excessive limitations placed on exclusivity arrangements.[4]

[3] Our analysis may be relevant to the antitrust proceedings against Microsoft's planned acquisition of Activision Blizzard, the developer and publisher of "AAA" games such as Call of Duty, Diablo and Overwatch. In 2022, the US Federal Trade Commission (FTC) and several other antitrust authorities raised concerns that the acquisition of Activision could increase Microsoft's incentive to disadvantage rivals by withholding or degrading content. For the lawsuit filed by the US FTC, see Docket No. 9412.

[4] In 2019, the Chinese regulator started an investigation against Tencent Music for its exclusive deals with some labels and considered policies such as bans on exclusive deals in this market. See mLex, September 13, 2019. "Tencent Music probe opens up whole new avenue for China antitrust enforcement in digital sector".. Indeed, when Tidal and Apple Music signed exclusive deals with some Superstar artists in 2016, Spotify complained about the negative impact on consumers, artists, and the entire industry. See *e.g.,* Rolling Stone, October 5, 2016. "How Apple Music, Tidal Exclusives Are Reshaping Music Industry".

The remainder of the paper is organized as follows. In the next section, we discuss the related literature. In Section 3, we introduce the preliminaries of the model, before turning, in Section 4, to an examination of the impact of exclusivity on platform competition and welfare. In Section 5, we endogenize the exclusivity decision, and then in Section 6 we present and discuss several extensions. In Section 7, we present the managerial implications of our results. Finally, we provide concluding remarks in Section 8.

2. Related Literature

Our paper relates to a long-standing body of literature on two-sided markets (Caillaud & Jullien 2003, Rochet & Tirole 2003, Parker & Van Alstyne 2005, Rochet & Tirole 2006, Armstrong 2006, Jullien 2011). Studies in this area have largely considered atomistic players and the platform's need to reach a critical size on one side to activate cross-group externalities. We add to these works by explicitly modeling heterogeneity in market power on the supply side of the market, with Superstars who have full bargaining power and complementors who are price takers.[5] The Superstar is therefore pivotal for the homing strategies of consumers and complementors, which amplify the scope for exclusivity. In doing so, we complement work by Bedre-Defolie & Biglaiser (2020), who study the impact of exclusive dealing by a *marquee agent* on the quality and variety available on the platform. Our paper also relates to Markovich & Yehezkel (2022), who present a model of platform competition with direct rather than indirect externalities. The authors study how grouping users may facilitate the migration from a less efficient focal platform to a more efficient one. Our paper differs, however, in that coordination of users and complementors is facilitated by the exclusivity decision of the Superstar.

We furthermore build on the argument that indirect network effects are critical for the success of a platform (see, among others, Ambrus & Argenziano 2009, Karle et al. 2020). Previous work has focused on how exclusive contracts are strategic tools used by platforms to influence the homing decisions of complementors. For example, platforms might discourage seller multihoming by making an exclusive contract more attractive than a non-exclusive one. Armstrong & Wright (2007) show that when this strategy is adopted, there is a partial (respectively complete) foreclosure as all users on this side (resp. both sides) would prefer to singlehome. However, exclusivity clauses might be detrimental to at least one side of the market, while multihoming could make all market participants better

[5]To a different extent, heterogeneity in market power is present in Lee (2014) and Adachi & Tremblay (2020), who consider oligopolistic firms contracting with the platform(s).

off (Belleflamme & Peitz 2019). In a similar vein to Hagiu & Lee (2011), who study the emergence of exclusivity under outright sale and content affiliation, we are interested in the impact of exclusivity arrangements involving a premium player in markets in which there are spillovers for complementors.[6] In contrast to these studies, however, our analysis has the Superstar facing a trade-off between exclusivity and non-exclusivity, a choice that depends on the intensity of platform competition. Our results partly resemble those obtained in a model without network externalities by Weeds (2016).

We also add to the literature on the anti- and pro-competitive effects of exclusivity. Many contributions have highlighted how exclusivity might entail anti-competitive effects by deterring entry or causing the foreclosure of more efficient rivals (Aghion & Bolton 1987, Rasmusen et al. 1991, Fumagalli & Motta 2006, Abito & Wright 2008, Fumagalli et al. 2009, 2012). Similar practices can also arise in the presence of network externalities when an incumbent can make exclusive introductory deals and prevent more efficient platforms from entering the market (Doganoglu & Wright 2010), as well as in the presence of interlocking bilateral relationships between upstream and downstream firms (Nocke & Rey 2018), or when a marquee agent signs exclusive contracts with a dominant platform that reduces the variety and quality in the market (Bedre-Defolie & Biglaiser 2020). Nevertheless, exclusive dealing might also entail pro-competitive effects such as effort provision (Segal & Whinston 2000, De Meza & Selvaggi 2007) or deterring the entry of inefficient firms (Innes & Sexton 1994). A major difference between our framework and those of existing studies is that cross-group externalities amplify the impact of exclusivity. In equilibrium, this generates entry cascades of complementors and, in some circumstances, higher consumer surplus.[7]

Our analysis also relates to the literature on vertical integration and input foreclosure.[8] D'Annunzio (2017) offers one of the first studies to address the issue of competing platforms and the decision to provide premium content. She shows that while premium content is always offered exclusively, vertical integration between the provider and platform may change the incentives to invest in quality. In our study, non-exclusivity may arise to a greater extent in the presence of vertical integration, which might prevent the aggressive pricing strategies that cross-group externalities trigger.

This paper rationalizes the behavior of digital platforms in several markets. Recent empir-

[6] In a recent article, Ishihara & Oki (2021) focus on the amount of content being offered exclusively by a monopolistic multi-product content provider.

[7] This approach is reminiscent of Kourandi et al. (2015), who study the contractual decision made by internet service providers to content providers. In their case, however, exclusivity can be welfare-enhancing when the competition of content providers over informative ads is sufficiently intense.

[8] For a more recent contribution on this topic see Padilla et al. (2022) who study foreclosure of competing third-party device makers within the platform and their main focus is intra-platform foreclosure.

ical contributions have demonstrated that exclusive deals are key to competitiveness. For example, in the gaming industry, empirical evidence shows that exclusive deals between platforms and producers might help small platforms challenge the incumbents (Lee 2013). Cennamo & Santalo (2013) argue that exclusivity is a *winner-take-all* strategy that can help a platform improve its performance when this strategy is taken in isolation. Our model provides direct empirically testable implications related to the entry of complementors caused by exclusivity and to how the agglomeration of complementors and consumers on the *favored* platform depends on the intensity of inter-platform competition.

Relatedly, recent work has focused on the heterogeneity of complementors. Boudreau (2019) studies the presence of amateurs and professionals in the Apple App Store, demonstrating that their presence relates to changes in app development costs. In our setting, entry by complementors with high development costs is facilitated by the Superstar's exclusivity decisions. Ershov (2020) also examines the mobile app market, focusing more on the externality generated by Superstar applications and their role in enabling entry cascades by low-quality entrants. While the author suggests that the Superstar's presence on the app store is the source of the positive demand-discovery effect for complementors, we identify another source of positive spillovers stemming from the type of contract which the Superstar signs. Specifically, when the Superstar is exclusively available on the *favored* platform, she improves the competitive position of that platform and, in turn, facilitates agglomeration and entry by complementors. These results corroborate recent findings from Förderer & Gutt (2021), who look at the effects of Superstar complementors on platform competition and content production. They study content provision by professional gamers and find that when Ninja — a Superstar gamer — unexpectedly moved from Twitch to Microsoft Mixer, viewers followed him and complementors sought content differentiation.

3. The Model

We adapt the generalized Hotelling model of Fudenberg & Tirole (2000) to a two-sided market setting. We assume two competing and horizontally differentiated platforms $i = \{1, 2\}$, located at the endpoints of the Hotelling line. We assume that consumers singlehome and active firms can either multihome or singlehome. There are two types of firms: the Superstar (she) and a fringe of complementors.

Consumers. There is a unit mass of consumers, whose preferences are quasi-linear in money and are indexed by $m \in [\underline{m}, \overline{m}]$, which is symmetric around 0 with $\underline{m} = -\overline{m} < 0$.

The parameter m denotes the measure of the relative preference for 2 relative to 1, and it is distributed according to a cumulative distribution function $F(\cdot)$ with density $f(\cdot)$. We assume full market coverage on this side of the market and $\overline{m}$ to be large enough such that an equilibrium with two competing platforms always exists.

When consumers join a given platform, they obtain a standalone utility, $v > 0$, and also enjoy some positive network externalities. If the Superstar is present on a given platform, she generates a value ϕ for the consumers, whereas complementors generate a network benefit $\theta > 0$.[9] The indicator function $g_i \in \{0, 1\}$ expresses the presence of the Superstar and N_i^e is the expected mass of complementors on platform i.

The utility of a type-m consumer joining platform 1 at price p_1 is $u_1(g_1) \triangleq v + \phi g_1 + \theta N_1^e - p_1 - m/2$, whereas the utility from joining platform 2 at price p_2 is $u_2(g_2) \triangleq v + \phi g_2 + \theta N_2^e - p_2 + m/2$. Consumers join platform 1 over platform 2 whenever $u_1(g_1) \geq u_2(g_2)$ or

$$m \leq \tilde{m}(p_1, p_2, N_1^e, N_2^e, g_1, g_2) \triangleq \phi(g_1 - g_2) - (p_1 - p_2) + \theta(N_1^e - N_2^e). \tag{1}$$

The demand for platform 1 is represented by all consumers with $m \leq \tilde{m}(p_1, p_2, N_1^e, N_2^e, g_1, g_2)$, i.e., $D_i(p_1, p_2, N_1^e, N_2^e, g_1, g_2) = F(\tilde{m}(p_1, p_2, N_1^e, N_2^e, g_1, g_2))$, whereas the demand for platform 2 is $D_2(p_2, p_1, N_2^e, N_1^e, g_2, g_1) = 1 - F(\tilde{m}(p_1, p_2, N_1^e, N_2^e, g_1, g_2))$.

Complementors. There is a fringe of small complementors receiving value $\gamma > 0$ when interacting with consumers through the platform. These complementors have heterogeneous opportunity costs of joining each platform, $k \in [0, \infty)$, with k distributed according to a cumulative distribution function $\Lambda(\cdot)$ and density $\lambda(\cdot)$. The opportunity cost can be interpreted as an entry, development, and porting cost that each complementor incurs when joining a platform. The utility of a type-k complementor joining platform i is $u_i^k = \gamma D_i^e - k$, with D_i^e representing the expected mass of consumers at platform i. A complementor joins a platform if $k \leq \gamma D_i^e$. Throughout the paper, we assume that complementors do not incur any subscription fee when joining the platform.[10] Thus, the mass of complementors on platform i is $N_i(D_i^e) = \Lambda(\gamma D_i^e)$.

Platforms. Platforms collect revenues from the consumers who pay a subscription price. For ease of exposition, we assume that marginal costs to serve consumers are normalized

[9] To ensure full market coverage, we assume v to be sufficiently high. Note that some of our insights carry over when relaxing the full market coverage assumption. More details are provided in Section 6 and in the Online Appendix.

[10] In the Online Appendix, we show that our results also apply when there is a subscription price for complementors.

to zero. Denote T_i as the tariff paid to the Superstar to distribute her premium content and assume that when one platform is vertically integrated with the Superstar, only the non-integrated rival pays the tariff when hosting the Superstar. The net profit of platform i is given by:

$$\Pi_i(p_i, p_j, N_i^e, N_j^e, g_i, g_j) - g_i T_i = p_i D_i(p_i, p_j, N_i^e, N_j^e, g_i, g_j) - g_i T_i. \tag{2}$$

The Superstar. She has all the bargaining power over her product and she receives a network benefit $\gamma^S > 0$ when interacting with consumers via a platform. This is a measure of the cross-group externality and can comprise merchandising, royalties from participation in live events, in-app purchases, or other forms of ancillary revenues. The Superstar, therefore, cares about her total market reach.

We consider two market structures. Under vertical separation, the Superstar offers her premium content to platform i and/or j in exchange for a tariff T_i and/or T_j. Since the Superstar also makes ancillary revenues, her total profit is:

$$\Pi^S(p_1, p_2, N_1^e, N_2^e, T_1, T_2, g_1, g_2) = \gamma^S \sum_{i=1,2} g_i D_i(p_i, p_j, N_i^e, N_j^e, g_i, g_j) + g_1 T_1 + g_2 T_2.$$

When the Superstar is exclusively on one platform, say platform 1, $g_1 = 1$ and $g_2 = 0$, meaning that she neither receives a tariff from platform 2 nor interacts with consumers on platform 2. Non-exclusivity instead implies $g_1 = g_2 = 1$.

Under vertical integration with platform 1, $g_1 = 1$ always. In this case, the owner of the merged entity (he) can decide to be either the sole distributor of the Superstar's content (*i.e.,* $g_2 = 0$) or to license her content to the rival platform (*i.e.,* $g_2 = 1$).[11] In the latter case, the owner of the merged entity sets T_2 and the associated payoff is denoted by Π^S, which includes the revenues made in the downstream market and those made directly by the Superstar. Formally,

$$\begin{aligned}\Pi^S(p_1, p_2, N_1^e, N_2^e, T_2, 1, g_2) = & D_1(p_1, p_2, N_1^e, N_2^e, 1, g_2)\left(p_1 + \gamma^S\right) \\ & + g_2 \gamma^S D_2(p_2, p_1, N_2^e, N_1^e, g_2, 1) + g_2 T_2.\end{aligned} \tag{3}$$

Throughout the analysis, we make the following assumptions.

[11]In our setup, this is captured by $g_1 = 1$. Note that the merged entity would never have incentives to offer the premium content exclusively to its rival ($g_1 = 0, g_2 = 1$). The reason is that such a strategy would put the merged entity in an unfavorable competitive position *vis-à-vis* the rival.

Assumption 1. $\Lambda(\cdot)$ *is smooth, twice continuously differentiable, with a strictly positive density function* $\lambda(\cdot)$ *and weakly positive second derivative* $\lambda'(\cdot) \geq 0$.

Assumption 2. $F(\cdot)$ *is smooth, twice continuously differentiable, with strictly positive density function* $f(\cdot)$, *symmetric around zero, and its second derivative* $f'(\cdot)$ *is bounded from above and from below, i.e.,* $\underline{f}' < f'(\cdot) < \overline{f}'(\cdot)$.

Assumption 3. $1 > f(\cdot)\gamma\theta[\lambda(\gamma F(\cdot)) + \lambda(\gamma(1 - F(\cdot)))]$.

Assumption 1 and 2 give regularity conditions on the distributions. Moreover, Assumption 2 also gives a sufficient condition for an equilibrium to exist and to ensure concavity in profits for the two platforms and for prices to be strategic complements.[12] Assumption 3 generalizes the corresponding assumption in Armstrong (2006) to rule out market tipping and ensure that the demands with fulfilled expectations decrease in own prices. Together with Assumption 2, this means that cross-group externalities $\{\gamma, \theta\}$ are sufficiently small relative to the differentiation parameter, which in our model is normalized to 1.

Timing. The timing of the game is as follows. In the first stage, exclusivity decisions are taken by the Superstar or the owner of the merged entity. In the second stage, conditional on hosting the Superstar, each platform simultaneously and independently sets a price for consumers. Finally, consumers (respectively complementors) form expectations regarding the mass of complementors (resp. consumers) on each platform and decide to join platforms. The equilibrium concept is subgame perfect rational expectations equilibrium.[13]

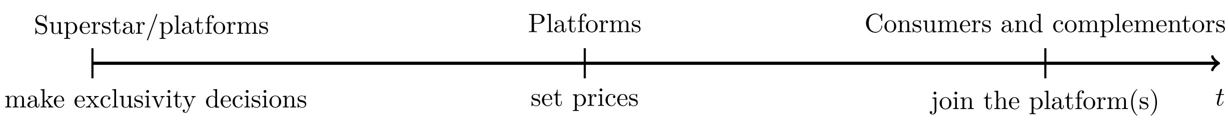

Figure 1: Timing of the model

The model is analyzed by backward induction. In Section 4, we study platform competition in the presence of the Superstar and provide welfare implications on the desirability of exclusivity in the two market structures considered. In Section 5, we endogenize the Superstar's exclusivity decision and discuss how it changes in these two market structures.

[12] A detailed analysis is available in the Proof of Lemma 1.

[13] Note that including a stage zero, in which the platform and the Superstar can decide to merge, provides the intuitive result that a merger will always occur. By internalizing the network benefits of the Superstar, the merged entity will be able to make higher (joint) profits than under separation.

4. Platform competition: the effect of exclusivity

In this section, we first consider the vertical separation market structure, studying price competition under exclusivity and non-exclusivity. We then consider the vertical integration market structure, highlighting the main differences in platform competition.

4.1. Vertical separation

Suppose platforms are vertically separated. In the last stage of the game, consumers and complementors make their homing decisions, deciding whether to join and on which platform. Since their decisions are made simultaneously, this requires coordination among agents and, hence, expectations, which inherently lead to a multiplicity of equilibria. Under non-exclusivity, we focus on the symmetric scenario in which consumers believe that the market will be equally split between the ex-ante symmetric platforms at equal prices. Under exclusivity, we isolate the contribution of the Superstar, net of any coordination domino effect linked to network externalities.[14]

In the third stage of the game, imposing fulfilled expectations about the participation of complementors and consumers, *i.e.*, $D_i^e = D_i$ and $N_i^e = N_i$ for $i \in \{1, 2\}$, we solve the system of equations of the demands at the two platforms for the two sides of the markets. This yields the indifferent consumer, and the consumers' and complementors' demands at two platforms, as functions of prices and exclusivity decisions (i.e., g_1 and g_2). Formally,

$$\begin{aligned} \tilde{\tilde{m}}(p_1, p_2, g_1, g_2) &\triangleq \tilde{m}(p_1, p_2, \tilde{N}_1(p_1, p_2, g_1, g_2), \tilde{N}_2(p_2, p_1, g_2, g_1), g_1, g_2), \\ \tilde{D}_1(p_1, p_2, g_1, g_2) &\triangleq F(\tilde{\tilde{m}}(p_1, p_2, g_1, g_2)), \\ \tilde{N}_i(p_i, p_j, g_i, g_j) &\triangleq N_i(\tilde{D}_i(p_i, p_j, g_i, g_j)). \end{aligned} \tag{4}$$

represent the solution of the system of equations, for $j \neq i \in \{1, 2\}$, with $\tilde{D}_2(p_2, p_1, g_1, g_2) \triangleq 1 - \tilde{D}_1(p_1, p_2, g_1, g_2)$. The associated gross profit of each platform (before paying any tariff to the Superstar) is $\tilde{\Pi}_i(p_i, p_j, g_i, g_j) \triangleq p_i \tilde{D}_i(p_i, p_j, g_i, g_j)$ for $i, j = 1, 2$ and $j \neq i$.

In the second stage of the game, each platform i sets its price p_i to maximize profits for given exclusivity decisions of the Superstar. The following lemma provides the equilibrium price conditions for a given g_1 and g_2.

[14] Recall that $\overline{m}$ is sufficiently large to avoid tipping even under exclusivity. This implies that it is too costly to coordinate on one platform, no matter the belief structure.

Lemma 1. *For any (g_1, g_2), the equilibrium prices denoted by $p_i^\star(g_i, g_j)$, for $i, j \in \{1, 2\}$ and $j \neq i$, are implicitly given as follows:*

$$p_1^\star(g_1, g_2) = F(m^\star(g_1, g_2)) \left\{ \frac{1}{f(m^\star(g_1, g_2))} - \gamma\theta \Big[\lambda(\gamma D_1^\star(g_1, g_2)) + \lambda(\gamma D_2^\star(g_2, g_1)) \Big] \right\},$$
$$p_2^\star(g_2, g_1) = \Big(1 - F(m^\star(g_1, g_2))\Big) \left\{ \frac{1}{f(m^\star(g_1, g_2))} - \gamma\theta \Big[\lambda(\gamma D_1^\star(g_1, g_2)) + \lambda(\gamma D_2^\star(g_2, g_1)) \Big] \right\}.$$

where $m^\star(g_1, g_2) \triangleq \tilde{\tilde{m}}(p_1^\star(g_1, g_2), p_2^\star(g_2, g_1), g_1, g_2)$ is the indifferent consumer at equilibrium, $D_1^\star(g_1, g_2) \triangleq F(m^\star(g_1, g_2))$, $D_2^\star(g_2, g_1) \triangleq 1 - F(m^\star(g_1, g_2))$ and $N_i^\star(g_i, g_j) \triangleq \Lambda(\gamma D_i^\star(g_i, g_j))$.

Proof. See Appendix A.1. □

The optimal prices account for heterogeneous consumer preferences and cross-group externalities. Assumption 3 ensures that, in the market-sharing equilibrium, the equilibrium prices of the two platforms are positive. The critical value $m^\star$, which is a function of (g_1, g_2), captures the impact of the Superstar on prices. If $g_1 = g_2$, platforms are symmetric and the price is equal to the one specified in the standard competitive-bottleneck model (Armstrong 2006).[15] This case is summarized by the following lemma.

Lemma 2. *Under non-exclusivity ($g_1 = g_2 = 1$), the two platforms charge symmetric prices equal to*

$$p_1^\star(1, 1) = p_2^\star(1, 1) = \frac{1}{2f(0)} - \gamma\theta\lambda(\gamma/2).$$

The platforms split the market equally i.e., $m^\star(1, 1) = 0$. All active complementors multihome ($N_1^\star(1, 1) = N_2^\star(1, 1) = \Lambda(\gamma/2)$), whereas all complementors with $k > \gamma/2$ zerohome.

Proof. See Appendix A.2. □

Lemma 2 describes a symmetric scenario in which neither platform enjoys the competitive advantage of the premium content. Indeed, both platforms host the Superstar and the final consumer demand is symmetric $F(m^\star(1, 1)) = F(0) = 1/2$. Figure 2 provides a graphical representation of the consumer and complementor participation. The following lemma presents the market outcome when the Superstar is exclusively on platform 1.

[15] Due to the Hotelling structure, there is an equivalence result between the case in which the Superstar is not present at all (i.e., $g_1 = g_2 = 0$) and the case in which the Superstar is non-exclusive (i.e., $g_1 = g_2 = 1$). This is no longer the case when considering elastic demand participation in Section 6.

Lemma 3. *Under exclusivity ($g_1 = 1$, $g_2 = 0$), the equilibrium prices are such that*

$$p_1^\star(1,0) > p_1^\star(1,1) = p_2^\star(1,1) > p_2^\star(0,1).$$

The platform hosting the Superstar attracts a larger mass of consumers and complementors than the rival, i.e., $D_1^\star(1,0) > D_2^\star(0,1)$ and $N_1^\star(1,0) > N_2^\star(0,1)$.

Proof. See Appendix A.3. □

Lemma 3 highlights important differences relative to the symmetric case of non-exclusivity. First, exclusivity renders the final prices asymmetric: the platform *favored* by the Superstar sets a price higher than that of the rival and than that set under non-exclusivity, *i.e.,* $p_1^\star(1,0) > p_1^\star(1,1)$, whereas the price of the *unfavored* platform decreases, *i.e.,* $p_2^\star(0,1) < p_2^\star(1,1)$. Second, and most importantly, because the magnitude of the price change is lower than the value generated by the Superstar, $\frac{\partial p_1^\star(1,0)}{\partial \phi} - \frac{\partial p_2^\star(0,1)}{\partial \phi} \in [0,1]$, there is some surplus left over for consumers, which triggers a demand expansion for the *favored* platform. Such a (first-order) *business-stealing effect* gives rise to positive cross-group externalities on the other side, which then feeds back into the consumer utility. In turn, the Superstar agglomerates consumers and complementors on the *favored* platform. The following proposition discusses the impact of exclusivity on the homing decisions of the complementors.

Proposition 1. *Superstar exclusivity fosters entry in the market and induces singlehoming of some complementors. Specifically,*

- *complementors with $k \in (0, \gamma D_2^\star(0,1)]$ multihome;*
- *complementors with $k \in (\gamma D_2^\star(0,1), \gamma D_1^\star(1,0)]$ singlehome on platform 1;*
- *complementors with $k \in (\gamma D_1^\star(1,0), \infty)$ zerohome.*

The intuition is as follows. Under exclusivity, the impact on complementors is twofold relative to non-exclusivity. First, there is an entry cascade of complementors as more become active in the market because of the higher value generated by the *favored* platform for those previously zerohoming. Second, some complementors become exclusively active on the *favored* platform endogenously, creating more exclusivity due to fewer complementors multihoming and more complementors singlehoming.

For a graphical representation of this mechanism, consider Figure 2, which depicts the case of non-exclusivity. The consumer side is equally split between the two platforms and all complementors with low $k \leq \gamma D_1^\star(1,1) = \gamma/2$ multihome, while complementors with a high outside option remain inactive. Under exclusivity, a larger mass of consumers becomes active on the *favored* platform relative to the rival ($D_1^\star(1,0) > D_1^\star(1,1) = 1/2$). Since the number of complementors on a platform depends on the number of consumers on that platform, some complementors that were zerohomers in the non-exclusive case are now singlehomers. Moreover, some of the multihomers (in the non-exclusive case) now singlehome on the *favored* platform. Figure 3 provides a graphical representation of the effect of exclusivity on both sides of the market.

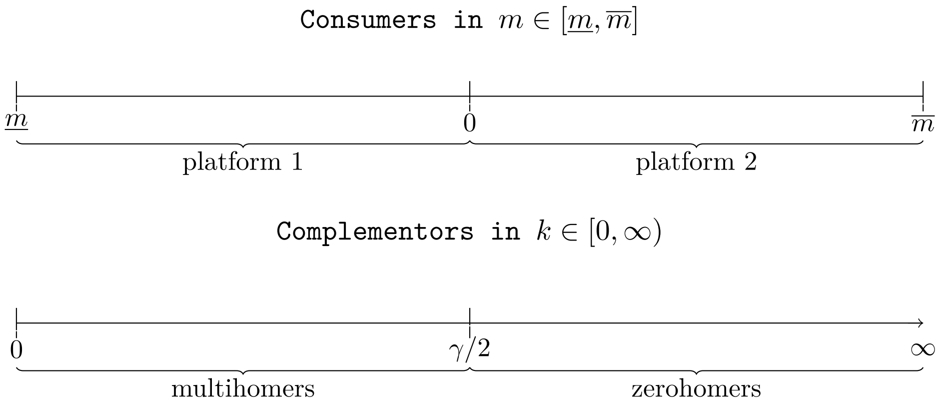

Figure 2: Non-exclusivity

Under non-exclusivity, the consumer side is equally split and symmetric around 0. All complementors with $k \leq \gamma/2$ are multihomers, whereas the others are zerohomers.

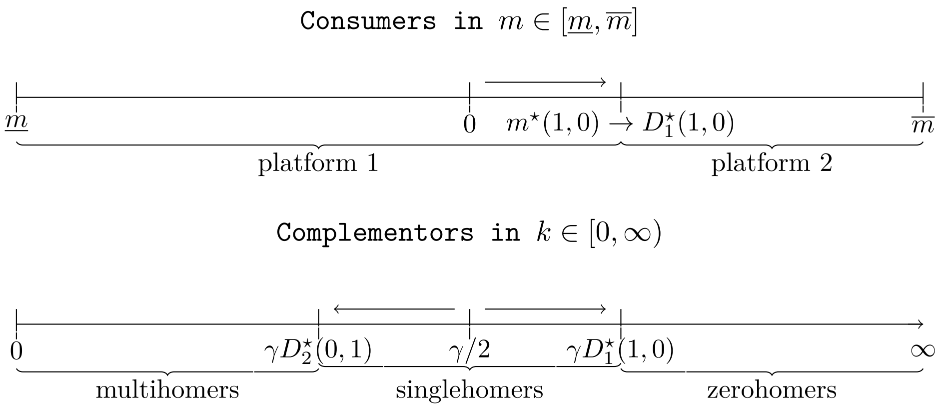

Figure 3: Exclusivity with platform 1.

Under exclusivity on platform 1, more consumers join platform 1 ($D_1^\star(1,0) > 1/2$). Complementors with $k \leq \gamma D_2^\star(0,1)$ multihome, complementors with $k \in (\gamma D_2^\star(0,1), \gamma D_1^\star(1,0))$ singlehome on platform 1 and complementors with $k \geq \gamma D_1^\star(1,0)$ zerohome.

4.2. Vertical Integration

In this section, we modify the previous model by assuming that the Superstar is integrated with platform 1 and decisions are made by the owner of the merged entity. In the third stage of the game, the demands of the two platforms are obtained following the same steps as under vertical separation to obtain the expressions in (4), at $g_1 = 1$ as a function of prices and exclusivity decisions. The associated profit of the merged entity, for a given g_2, is $\tilde{\Pi}^S(p_1, p_2, 1, g_2) \triangleq (p_1 + \gamma^S)\tilde{D}_1(p_1, p_2, 1, g_2) + g_2 T_2$. The net profit of platform 2 remains unchanged and equal to $\tilde{\Pi}_2(p_2, p_1, g_2, 1) - g_2 T_2 = p_2 \tilde{D}_2(p_2, p_1, g_2, 1) - g_2 T_2$.

Intuitively, under non-exclusivity, prices are unaffected by the vertically integrated nature of the Superstar. As such, the results from Lemma 2 apply.[16] Under exclusivity, the problem of the merged entity is now different, since the effect of prices on the revenues of both the platform and the Superstar is now considered. From the first-order condition with respect to p_1 we obtain the following:

$$\frac{\partial \tilde{\Pi}^S(p_1, p_2, 1, 0)}{\partial p_1} = \tilde{D}_1(p_1, p_2, 1, 0) + p_1 \frac{\partial \tilde{D}_1(p_1, p_2, 1, 0)}{\partial p_1} + \underbrace{\gamma^S \frac{\partial \tilde{D}_1(p_1, p_2, 1, 0)}{\partial p_1}}_{\substack{\text{Network internalization} \\ \text{effect } (-)}}, \tag{5}$$

where the *network internalization effect* represents the primary difference between the market structure in the presence of vertical separation and its variation in the presence of a merged entity. The above expression leads to the following conclusion.

Proposition 2. *Conditional on exclusivity, the favored platform sets a lower price under vertical integration than under vertical separation.*

The above proposition provides a novel result. The merged entity internalizes the benefit that the Superstar obtains when reaching consumers, which exerts downward pressure on prices.[17] Specifically, substituting the equilibrium price under vertical separation into equation (5), the following relationship on prices is immediate

$$\left.\frac{\partial \tilde{\Pi}^S(p_1, p_2, 1, 0)}{\partial p_1}\right|_{p_1^\star(1,0)} = \gamma^S \left.\frac{\partial \tilde{D}_1(p_1, p_2, 1, 0)}{\partial p_1}\right|_{p_1^\star(1,0)} < 0.$$

[16] In Section 6, we show that when allowing for partial market coverage (*i.e.,* demand expansion), Lemma 2 no longer applies. Yet, some of our main insights continue to hold qualitatively.

[17] Note that in the case of uncovered demand, these results may be nuanced since the owner of the merged entity would be able to expand its downstream demand keeping the price constant. We discuss this case in Section 6.

The reason is that, under vertical integration, consumer participation is more salient. Thus, the merged entity is more *aggressive* in the market in order to reach more consumers. Since prices are strategic complements, the price of the rival platform also falls. Such a downstream pricing externality in the presence of vertical integration is reminiscent of the downstream externality (caused by high investments) in the seminal paper by Bolton & Whinston (1993). Note that the price reduction we observe is independent of any efficiency gains resulting from the avoidance of double marginalization or other merger-specific efficiencies. Thus, the cause of this price reduction is solely due to the presence of the *network internalization effect*.[18]

4.3. Welfare implications of exclusivity

In this section, we compare the surplus of complementors and consumers under non-exclusivity and exclusivity to highlight welfare implications. Note that this welfare analysis applies to both vertical separation and vertical integration, though with certain differences between the two cases, which will be discussed.

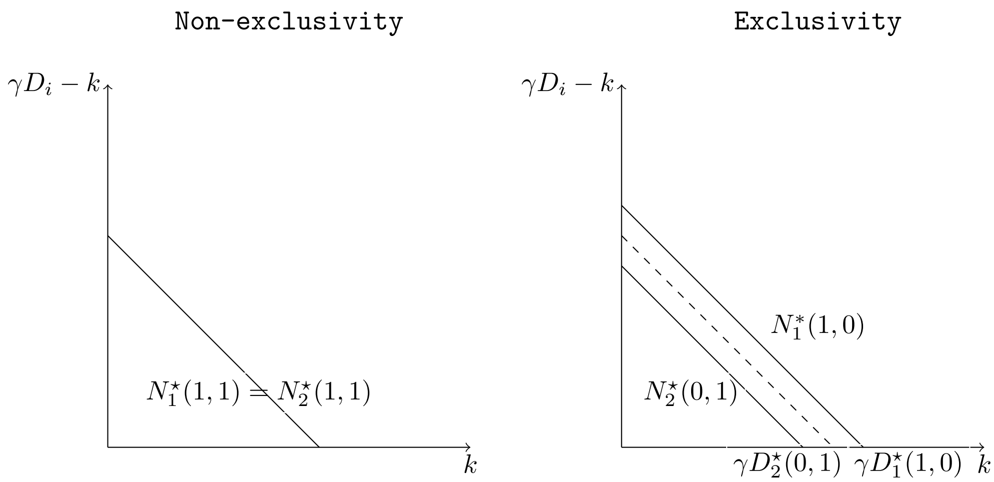

Figure 4: Surplus of the complementors

The figure depicts the surplus of complementors under both regimes. The exclusive case always achieves a higher total surplus.

Impact on Complementors. As highlighted by Proposition 1, under exclusivity, the Superstar grants an advantage to the *favored* platform in terms of market reach, while

[18] In the absence of network externalities, a merger would be neutral in our setting due to the absence of linear wholesale fees charged by the Superstar. In the case of non-exclusive contracts and two-part tariffs, the rival is expected to obtain higher wholesale fees post-merger.

some complementors find it optimal to join that platform relative to the case of non-exclusivity. We denote by $\Delta FS \triangleq FS^{\star}(1,0) - FS^{\star}(1,1)$ the net gain from exclusivity. After some arithmetical manipulation, provided in the Appendix, we decompose ΔFS:

$$\Delta FS = \underbrace{0 \cdot \int_{0}^{\gamma D_2^{\star}(0,1)} \lambda(k)dk}_{\text{multihomers}} + \underbrace{\int_{\gamma D_2^{\star}(0,1)}^{\gamma/2} [(\gamma(D_1^{\star}(1,0)-1)+k)\lambda(k)]dk}_{\text{from multi- to single-homers}}$$
$$+ \underbrace{\int_{\gamma/2}^{\gamma D_1^{\star}(1,0)} [(\gamma D_1^{\star}(1,0)-k)\lambda(k)]dk}_{\text{entrants}}.$$

In the above expression, the first term compares the difference in the surplus of multi-homers under exclusivity and non-exclusivity, which is equal to zero since the complementors that multihome in both scenarios are not impacted by exclusivity. The second term describes the change in surplus for the complementors that multihome under non-exclusivity and singlehome on the *favored* platform under exclusivity. Under exclusivity, these complementors reach fewer consumers than under non-exclusivity but save on entry and development cost k. Finally, the third term represents the gains of complementors that zerohome under non-exclusivity and singlehome on the *favored* platform under exclusivity. The following proposition can be stated.

Proposition 3. *The surplus of complementors is higher under exclusivity than under non-exclusivity.*

Proof. See Appendix A.4. □

Proposition 3 suggests that the contractual decision of the Superstar causes significant externalities for other players. The Superstar increases the value perceived by the consumers joining the *favored* platform rather than its rival, and this value creation makes it possible for complementors to save entry costs on the *unfavored* platform and earn more on the *favored* platform. In turn, complementors benefit from exclusivity. This result comes from the complementarity and the positive spillover that exclusivity entails. This is consistent with empirical evidence by Ershov (2020), who found that 'Superstar apps' in the mobile industry exert positive, substantial, and persistent *demand-discovery* spillovers on small app developers. In our framework, the positive spillover comes from the ability of the Superstar to coordinate and agglomerate users on both sides onto the *favored* platform.

Figure 4 provides a graphical intuition of the above result and plots the surplus of complementors according to the opportunity cost, k. The triangles represent the mass of complementors on each platform. Moving from the case of non-exclusivity (left panel) to the case of exclusivity (right panel), the intercept increases (respectively decreases) for complementors on platform 1 (resp. platform 2). The net effect is positive.

Impact on Consumer Surplus. Here, we discuss whether or not consumers should welcome exclusivity. Let us denote $\Delta CS \triangleq CS^{\star}(1,0) - CS^{\star}(1,1)$ as the net gain (or loss) from exclusivity at equilibrium. After some arithmetical manipulation, one can decompose ΔCS as follows:

$$\Delta CS = \underbrace{\theta[\bar{N} - N^{\star}(1,1)]}_{\Delta \text{ externalities}} - \underbrace{\phi D_2^{\star}(0,1)}_{\text{prevented access}} - \underbrace{[\bar{p} - p^{\star}(1,1)]}_{\Delta \text{ prices}} - \underbrace{\int_0^{m^{\star}(1,0)} m f(m) dm}_{\text{preference mismatch}}, \tag{6}$$

where $\bar{N} \triangleq F(m^{\star}(1,0))N_1^{\star}(1,0) + (1 - F(m^{\star}(1,0)))N_2^{\star}(0,1)$ and $\bar{p} \triangleq F(m^{\star}(1,0))p_1^{\star}(1,0) + (1 - F(m^{\star}(1,0))p_2^{\star}(0,1)$ are the average mass of complementors and the average prices under exclusivity, respectively.

Equation (6) presents the four elements that impact consumer surplus under the two exclusivity regimes. The first relates to the positive externalities that the presence of entrant complementors generates on consumers under exclusivity. The second element captures the negative effect of the lack of access to the Superstar for consumers on the *unfavored* platform (i.e., *prevented access).* The third identifies the negative effect that exclusivity has on the average price. The final element refers to the augmented preference mismatch, as there are consumers who inefficiently buy from their least-preferred platform.

It follows that ΔCS can be positive only if the first (positive) effect outweighs the other (negative) effects, highlighting the importance of the magnitude of cross-group externalities in driving up consumer surplus under exclusivity. In the extreme case in which consumers do not benefit from the presence of complementors (i.e., $\theta = 0$), the net effect of exclusivity is negative. Indeed, the presence of cross-group externalities creates value from exclusivity, which makes consumers better off when externalities are sufficiently large. What is critical in determining the sign of the net effect is how many consumers the Superstar moves toward the *favored* platform, which depends on the distribution of consumer preferences. If a large mass of consumers is concentrated around zero, the market is very competitive, suggesting that many consumers would follow the Superstar to the *favored* platform, decreasing the importance of the prevented access due to exclusivity.[19]

[19] In the Online Appendix, we provide an example of how consumer surplus changes with exclusivity using uniform distributions. Consumers at the *favored* platform always benefit from exclusivity whereas consumers at the *unfavored* platform are always worse off with exclusivity relative to the non-exclusive

The mechanism is the same regardless of whether exclusivity arises in the form of exclusive dealing or vertical integration. In the latter case, ΔCS in (6) is larger than in the presence of vertical separation because, as laid out in Proposition 2, exclusivity entails downward pressure on prices. We, therefore, conclude the following.

Proposition 4. *For sufficiently large cross-group externalities, exclusivity increases consumer surplus relative to non-exclusivity*

Proof. See Appendix A.5. □

A direct implication of Proposition 4, together with Proposition 3, is that exclusivity is socially desirable when cross-group externalities are large enough. This result follows from the agglomeration effect that exclusivity entails, therefore amplifying value creation.

5. Contractual stage: exclusivity vs. non-exclusivity

So far, we have highlighted the effects of exclusivity on market outcomes. These effects are independent of contractual arrangements, the allocation of bargaining power among agents, and even the vertical structure. However, it is important to understand the conditions under which exclusivity emerges. In this section, we endogenize the exclusivity decision of the Superstar and the merged entity under vertical separation and integration, respectively. Given that a comprehensive analysis of contractual arrangements is beyond the scope of this study, we follow Ordover et al. (1990), adapting their auction mechanism to our framework. Specifically, we assume that the Superstar, or the merged entity, has all the bargaining power *vis-à-vis* the platform(s) and can potentially allocate her exclusivity via a second-price sealed-bid auction. This is also consistent with recent contributions on important input provision (*e.g.,* Montes et al. 2019, Bounie et al. 2021). We focus on a fixed tariff since doing so ensures that our results are not biased by any other distortion.

Vertical separation. In light of her bargaining power, the Superstar can allocate her product under vertical separation by designing an auction with a reserve price as in Bounie

setting. The positive effect of exclusivity on consumer surplus dominates the negative effect only if cross-group externalities are large enough.

et al. (2021) and extract the highest value in each exclusivity scenario.[20] Under non-exclusivity, the maximum the Superstar can obtain is the difference between the profit a platform obtains under non-exclusivity and the profit obtained by the *unfavored* platform. In order to reach this outcome and extract this surplus entirely, the outside option of creating an unfavorable condition for the platform rejecting the contract needs to be credible. One way to bring this about is to threaten the platform(s) with an exclusivity auction if a non-exclusive contract is rejected.[21] In turn, a contract with a tariff equal to $T^\star(1,1) \triangleq \Pi_i^\star(1,1) - \Pi_i^\star(0,1)$, for $i = \{1,2\}$, is incentive-compatible for both platforms.

Under exclusivity, the Superstar can, at most, extract the difference between the profit a platform expects as the *favored* platform and that of the *unfavored* platform. To this end, the Superstar can design an auction for her exclusivity with a reserve price equal to $T^\star(1,1)$.[22] This way, the Superstar introduces competition for exclusivity and each platform bids to be the *favored* platform.[23] Accordingly, as desired by the Superstar, each platform will bid the difference between the gross profit obtained by the *favored* platform and that of the *unfavored* one, i.e. $T_i^\star(1,0) \triangleq \Pi_i^\star(1,0) - \Pi_j^\star(0,1)$. Due to symmetry, the two platforms bid the same amount and, hence, $T_1^\star(1,0) = T_2^\star(1,0) = \Pi^\star(1,0) - \Pi^\star(0,1)$.

In summary, the profit of the Superstar under exclusivity is

$$\Pi^S(1,0) \triangleq \gamma^S D_1^\star(1,0) + \Pi^\star(1,0) - \Pi^\star(0,1),$$

whereas her profit under non-exclusivity is

$$\Pi^S(1,1) \triangleq \gamma^S + 2[\Pi^\star(1,1) - \Pi^\star(0,1)].$$

By comparing profits in the two scenarios, if $\Pi^S(1,1) \geq \Pi^S(1,0)$, the Superstar will offer both platforms a non-exclusive contract, threatening the launch of an auction if the

[20]In the same vein, Bernheim & Whinston (1998) endogenize exclusive dealing via an auction mechanism. A similar bidding stage is also used in Hagiu & Lee (2011), with platforms making simultaneous take-it-or-leave-it offers to content providers. However, the way the auction is run in their model is starkly different and incompatible with our setup.

[21]If an auction is run, each platform will bid the difference between its profit with exclusivity and its profit if their rival gains exclusivity (i.e., $\Pi_i^\star(0,1)$), with the latter being equal to the outside option in the non-exclusive contractual stage.

[22]Note that the reserve price is key to inducing the desired outcome of the Superstar as it ensures that both platforms would accept the non-exclusive contract at tariff $T^\star(1,1)$, knowing that, if they do not accept the offer, an auction with a reserve price equal to the same tariff will be run. Moreover, it is also reasonable to believe that if a Superstar launches an auction for exclusivity, she would ask for no less than what she could achieve under non-exclusivity.

[23]For a similar mechanism, see Bounie et al. (2021) and the profits of the data broker under exclusivity and non-exclusivity in Lemma 3. Their mechanism is akin to ours, with the difference that the Superstar also earns ancillary revenues from their interactions with consumers.

contract is rejected. Otherwise, a second-price sealed-bid auction with a reserve price equal to $T^{\star}(1,1)$ will be run and platform 1 will be awarded exclusivity. The following proposition summarizes this result.

Proposition 5. *Under vertical separation, there exists a threshold*

$$\tilde{\gamma}^S \triangleq \frac{\Pi^{\star}(1,0)+\Pi^{\star}(0,1)-2\Pi^{\star}(1,1)}{1-D_1^{\star}(1,0)}$$

such that non-exclusivity emerges in equilibrium if, and only if, $\gamma^S \geq \tilde{\gamma}^S$*. Else, exclusivity emerges.*

Under exclusivity, two forces are at stake. First, a *rent extraction effect*, which is captured by the numerator of $\tilde{\gamma}^S$, and represents the difference between the tariffs in the two regimes, which is always positive.[24] Second, a *competition effect* due to the increased demand of the customer base of the *favored* platform, which is captured by the denominator of $\tilde{\gamma}^S$. This effect gets larger as the degree of differentiation between platforms decreases. When competition is intense, consumers are more responsive to the presence of the Superstar, which increases $D_1^{\star}(1,0)$. In turn, the denominator of $\tilde{\gamma}^S$ shrinks, thereby expanding the parameter range in which exclusivity occurs. In contrast, when γ^S is large enough, the Superstar highly benefits from interactions with consumers and finds it optimal not to be exclusive. By remaining non-exclusive, she has access to the entire market, which provides gains that outweigh any rent-extraction effect.

Corollary 1. *Under vertical separation, exclusivity becomes more likely the larger the value generated by the Superstar, i.e.,* $\frac{d\tilde{\gamma}^S(\phi)}{d\phi} > 0$.

Proof. See Appendix A.6. □

Corollary 1 states that when the value of ϕ gets larger for the Superstar, consumers become more responsive to it, with many migrating from one platform to another, and this might make exclusivity welfare-enhancing if network externalities are large enough. For ease of exposition, we provide an example for an exclusive contract to be welfare-enhancing when $F(\cdot)$ and $\Lambda(\cdot)$ follow a uniform distribution. Details can be found in the Online Appendix.

[24] Note that a positive numerator $T_1^{\star}(1,0) - 2T_1^{\star}(1,1)$ fulfills the implementability constraint in Bounie et al. (2021), which guarantees, in their setup, that the buyer will always participate in the market.

Example. Suppose $F(\cdot)$ and $\Lambda(\cdot)$ are uniform and consider vertical separation. Exclusivity emerges if

$$\gamma^S \leq \frac{4\phi^2}{3(3-2\phi-6\gamma\theta)} \equiv \tilde{\gamma}^S,$$

which is increasing in ϕ (Corollary 1). Importantly, the critical threshold is also increasing in the size of the cross-group externalities, γ and θ. Hence, the rent extraction and competition effects are larger, driving the critical value $\tilde{\gamma}^S$ up and making exclusivity more likely to emerge. Consumer surplus increases with exclusivity if the following two conditions are jointly satisfied:

$$\theta > \frac{1}{2\gamma} - \frac{\sqrt{\frac{\phi}{\gamma^2}}}{6} > 0, \qquad \phi < 1/4.$$

Note that the above conditions are independent of whether exclusivity arises. It follows that consumers benefit from exclusivity if the value generated by the Superstar is not extremely large and the cross-group externalities are large enough. The two conditions should be jointly satisfied. When the cross-group externalities, θ, is large exclusivity is more likely to emerge in equilibrium, as the cutoff $\tilde{\gamma}^S$ increases, and also consumer surplus is more likely to improve with exclusivity. When the value of the Superstar, ϕ, increases, exclusivity is again more likely to emerge in equilibrium, but now consumer surplus is more likely to decrease with exclusivity as this would entail a major disutility for consumers who cannot access the Superstar.

Vertical integration. Unlike the scenario under vertical separation, the Superstar does not need to run an auction for her product as exclusivity is the default outcome. However, if non-exclusivity is more profitable, the owner of the merged entity can offer an incentive-compatible contract to platform 2 with a tariff equal to $T_2^\star(1) \triangleq \Pi^\star(1) - \Pi_2^\star(0)$. Indeed, under non-exclusivity, the net profit of platform 2 is the same as it obtains under vertical separation, *i.e.,* $\Pi_2^\star(1)$, with $\Pi_2^\star(1) = \Pi_1^\star(1) = \Pi^\star(1)$ by symmetry.

For the merged entity, the decision to prevent access to platform 2 implies giving up demand on the rival platform, $1 - D_1^\star(0)$, as well as the collection of $T_2^\star(1)$. As a consequence, non-exclusivity is preferred by the owner of the merged entity when his profits are larger than those obtained under exclusivity. It follows that non-exclusivity emerges if the following inequality holds at equilibrium:

$$\Pi^\star(1) + \gamma^S + T_2^\star(1) > \Pi_1^\star(0) + \gamma^S D_1^\star(0).$$

where $\Pi^\star(0) = p_1^\star D_1^\star(0)$ represents the profit obtained by the merged entity in the downstream market only. As in the case of vertical separation, the decision to offer the Superstar's product to the rival platform hinges upon rent extraction and competition effects. Notably, this happens if $\gamma^S \leq \tilde{\gamma}^{vi}$, with $\tilde{\gamma}^{vi}$ is implicitly determined as follows:

$$\tilde{\gamma}^{vi} \triangleq \frac{\Pi_1^\star(0) + \Pi_2^\star(0) - 2\Pi^\star(1)}{1 - D_1^\star(0)}.$$

Comparing the above threshold with $\tilde{\gamma}^S$ in Proposition 5, we can state the following.

Proposition 6. *Unless exclusivity generates a large demand asymmetry under vertical integration relative to vertical separation, vertical integration leads to less (more) exclusivity than vertical separation, i.e.,* $\tilde{\gamma}^{vi} < (>) \tilde{\gamma}^S$.

Proof. See Appendix A.7. □

These results are novel in the literature. Specifically, contrary to the theory of input foreclosure in traditional markets, Proposition 6 highlights that exclusivity can be less likely in the presence of cross-group externalities.[25] The intuition behind this result is the following. Under exclusivity, the merged entity internalizes the network benefits the Superstar obtains and, hence, she is *tougher* in the market. This price reduction also lowers the rival's price and profits. Under non-exclusivity, profits are identical to those under vertical separation. Since the merged entity can offer a tariff equal to the difference in the rival's profits under non-exclusivity and those it obtains under exclusivity, a non-exclusive contract becomes relatively more profitable. This is always the scenario that arises if $F(\cdot)$ and $\Lambda(\cdot)$ are both uniform distributions. On the other hand, the opposite effect is verified when there is a large demand asymmetry associated with exclusivity under vertical integration. In this case, the aggressive pricing would not offset the gains from increased demand and, therefore, exclusivity would arise more often than under vertical separation.

6. Discussion

In this section, we relax some of the assumptions that were made in the baseline model and provide a discussion of features present in real markets but not considered so far. We then discuss some implications for antitrust enforcers concerning input foreclosure.

[25] Note that in our framework, absent network externalities, a vertical merger would have no effect. This is because we abstract away from the presence of wholesale prices.

Two-sided pricing. Often, platforms set prices on both sides of the market. For example, in the music industry, artists are remunerated by platforms such as Spotify and Tidal. In the app market, developers pay an annual fee to have their account, a practice that is replicated in many other markets. We are able to show that, under vertical separation, the Superstar's decision is only affected by the degree of downstream competition. When the ancillary revenues of the complementors are small relative to consumer cross-group externalities, the platform prefers to attract complementors with a negative price. Under exclusivity, complementors are less responsive to additional consumer moving from the *unfavored* to the *favored* platform. As a result, the *favored* platform subsidizes complementors even more. In the opposite case, consumers are more valuable to the complementors and the platform extracts more surplus by charging the latter a higher price under exclusivity. In the Online Appendix (Section 3.1), using a uniform distribution of consumer preferences and complementors' cost, we provide an example under vertical separation showing that, as in the baseline model, there exists a threshold value of γ^S below which exclusivity arises and above which non-exclusivity arises.

Asymmetric platforms. In the real world, platforms can be ex-ante asymmetric, e.g., one platform provides a higher standalone utility compared to the rival platform. In this case, the Superstar faces three choices. First, she can be non-exclusive and patterns of platform dominance would not change. Second, she can join the high-quality platform. The rationale, in this case, would be to ensure the largest market reach but she can extract surplus from the competitive edge granted to the high-quality platform in terms of agglomeration of consumers and complementors. Third, she can join the low-quality platform, possibly overturning market dominance. In the Online Appendix (Section 3.2), we provide an example using uniform distributions. We focus on the case in which one of the two platforms provides a higher standalone utility than the rival. We show that the main insights stemming from our baseline model remain valid and there is a threshold value of γ^S below (resp. above) which exclusivity (resp. non-exclusivity) arises. Moreover, regardless of the market structure considered, it is possible to identify a parameter range in which exclusivity is chosen by the platform or by the merged entity and this choice leads to a higher consumer surplus relative to non-exclusivity provided that the magnitude of the cross-group externalities is large enough. However, this parameter range is likely to be very small, suggesting that caution is required when deriving policy implications.

Moreover, relative to platform's strategies, under vertical separation, the high-quality platform always provides a higher bid for the Superstar exclusivity than the low-quality platform and can secure exclusivity if the Superstar finds it optimal to launch the auction. Thus, the decision of the Superstar lies between exclusivity on the high-quality platform and non-exclusivity.[26] Compared to the baseline model with platform symmetry, under asymmetry gains from exclusivity and non-exclusivity are lower for the Superstar. Under non-exclusivity, the Superstar has to set a non-exclusive tariff (i.e., used as a reserve price) which is now lower in order to satisfy the participation constraint of the low-quality platform. Under exclusivity, the high-quality platform can win the auction for exclusivity by matching the bid of the low-quality rival, which implies that the Superstar does not extract the highest surplus. Nevertheless, exclusivity emerges when γ^S is low enough and non-exclusivity emerges otherwise. Results under vertical integration qualitatively hold when the merged entity is of higher quality. Although the gains from non-exclusivity are smaller when the merged entity is larger than the independent rival, we show that non-exclusivity might still arise if γ^S is large enough, whereas exclusivity remains the default choice otherwise.

Alternative mode of competition and elastic demand participation. The baseline model relies on a generalized Hotelling setup with a covered market. While ensuring tractability, this model has key limitations in identifying general welfare effects for the impossibility to generate a demand expansion. In the Online Appendix (Section 3.3), we adapt our setting to a different model of competition in which a (representative) consumer exhibits preferences *à la* Singh & Vives (1984). If the market were uncovered, both exclusivity and non-exclusivity could affect the extensive margin. However, exclusivity would now generate a market-shrinking effect relative to non-exclusivity, which would be taken into account in the first instance by the Superstar when making her contractual decision. We show that, under vertical separation, exclusivity continues to be chosen whenever γ^S is sufficiently low. On the contrary, if γ^S is large enough, serving a larger demand, further amplified by the market expansion effect, is more profitable. When considering vertical integration, the merged entity internalizes the network benefit related to the presence of the Superstar both under exclusivity and non-exclusivity.[27] Thus, there is downward pressure on prices under both regimes. A higher γ^S makes the demand obtained by the Superstar more salient, thereby creating a complementarity between the two platforms under non-exclusivity while also lowering market prices due to the network internalization effect. Under non-exclusivity, the downward pressure on prices lowers the tariff paid to the merged entity. Nevertheless, if γ^S is large, the positive effect from increased network benefits on the merged entity more than compensates for the negative effect on the tariff. This makes non-exclusivity more profitable. The opposite occurs, instead, for sufficiently

[26] These considerations also apply in the presence of multiple premium agents that make their exclusivity decisions sequentially.

[27] Note that Lemma 2, derived under vertical separation no longer applies under vertical integration.

low γ^S. Notably, the demand-shrinking effect associated with exclusivity might be detrimental to consumers. Our analysis shows that, in the two market structures, it is still possible to have a parameter range in which either the Superstar or the merged entity chooses exclusivity and this is beneficial to consumers when the cross-group externality θ is sufficiently large. However, the parameter range in which this case is verified can be very small. This suggests that, also in this case, caution is required when deriving policy implications.

Coordination Problem. As in any model with network externalities (*e.g.,* Caillaud & Jullien 2003, Hagiu 2006, Damiano & Li 2007, Jullien 2011, Biglaiser & Crémer 2020, Markovich & Yehezkel 2022), a coordination problem arises. In particular, if consumers believe that a sufficiently large number of other consumers and complementors will follow the Superstar, then the market may tip. In our model, a tipping scenario towards the *favored* platform may lead to efficiency gains due to cross-group effects. On the negative side, consumers may bear the cost of preference mismatches and pay a higher price. When cross-group externalities become more substantial, these efficiency gains outweigh the consumer welfare losses, making exclusivity welfare-enhancing. In the Online Appendix (Section 3.4), we study the case of market tipping in the presence of exclusivity.

Multihoming consumers. In most markets, consumer multihoming is quite common, and platforms have overlapping market shares. This can increase the likelihood of non-exclusivity, since Superstar exclusivity would attract fewer consumers and complementors. However, the central insights of the baseline model also hold in this case. The only difference is that exclusivity on the *favored* platform only affects the consumers' choice between multihoming and singlehoming on the rival platform. The *favored* platform would generate a smaller demand expansion, mitigating the threat of exclusivity for the rival platform. Meanwhile, non-exclusivity would only arise free of charge, as the threat of exclusivity with the rival would be absent. Because these two forces go in opposite directions, the critical value below which exclusivity arises moves accordingly. In the Online Appendix (Section 3.5), we formally provide conditions for the emergence of exclusivity under vertical separation in the presence of multihoming consumers.

Competition between the Superstar and complementors. In our model, the Superstar and complementors are not competing for consumer attention. This is consistent with most of the markets this paper considers. Suppose, however, that complementors are small firms that compete with the Superstar, with the latter creating negative externalities for the former such as competition or congestion (see *e.g.,* Karle et al. 2020, Bedre-Defolie & Biglaiser 2020). In such a setting, the network benefit for the small firms when joining the *favored* platform would be lowered. Thus, we can speculate that if the reduction is large enough, Superstar exclusivity would lead some complementors to join the *unfavored* platform in order to avoid being crowded out by the Superstar. This would reduce the rent extraction of the Superstar under exclusivity, making exclusive arrangements less likely. However, non-exclusivity may lead some complementors to exit the market. In turn, exclusivity might be welfare-enhancing but it would emerge less often.

Input foreclosure and vertical integration. In this paragraph, we present our results through the lens of their antitrust implications. Vertical mergers are generally presumed pro-competitive due to their inherent efficiency effects (*e.g.,* the elimination of double marginalization). However, vertical mergers can also lead to anti-competitive effects, and these effects may prevail for some of them (see *e.g.,* Salinger 1988, Ordover et al. 1990, Bourreau et al. 2011 and, for a survey, Rey & Tirole 2007). For instance, the European Commission, in its merger control, follows the Non-Horizontal Merger Guidelines (NHMG) for assessing a vertical merger. The Commission looks at the ability and incentive of a vertically integrated entity to foreclose rivals and the ensuing impact of such a strategy on the actual competition. Accordingly, foreclosure is a concern when the upstream firm *(i)* has a significant degree of market power, *(ii)* is an important supplier of inputs, *e.g.,* it represents a significant cost factor for downstream firms (NHMG 2008, para 35), and *(iii)* the merged entity would be able to negatively affect the availability of inputs to its rivals (NHMG 2008, para 36). In our case, the Superstar fulfills these three conditions.

Thus, according to the NHMG (2008), a vertical merger can (input) foreclose a rival platform, leading to higher prices for consumers. In our model, the presence of cross-group externalities and, more importantly, vertical integration, makes exclusivity (input foreclosure) less likely and entails an aggressive pricing strategy unless there is a large demand asymmetry under vertical integration. This result is an additional efficiency argument since we control for the elimination of double marginalization. Our results apply in the presence of two essential factors, which require due diligence by antitrust enforcers. First, *cross-group externalities* should be taken into account when defining a market. Second, *exclusivity should not lead to market tipping* and, indeed, input foreclosure should not prevent the rival platform from attracting consumers and complementors.

7. Managerial implications

Our analysis also has important managerial implications for platforms, Superstar players, and complementors.

When is it profitable for a Superstar to sign exclusive deals? Our analysis provides direct insights for a Superstar's manager about the profitability of exclusivity. The findings indicate that Superstar managers should pay attention to the competitiveness of the platform market in which products are provided, taking care to understand consumer preferences towards each platform. When consumers have strong preferences, exclusive deals may be less profitable, since they would be unlikely to attract many consumers and complementors. Instead, exclusivity should be pursued whenever the Superstar has the ability to induce agglomeration of consumers and entry of complementors, which is more likely when preferences are not very strong. Under these conditions, the presence of an exclusive Superstar can be pivotal in the market. Recent evidence shows that increased exclusivity can be observed in the on-demand streaming music market. For example, competition between Apple Music and Spotify has become more intense in recent years and exclusive deals with Superstar artists and podcasters (*e.g.,* "The Joe Rogan Experience" podcast on Spotify) have gained prominence. Similar trends can be observed in the game streaming market, which features intense competition for viewers between Twitch and Microsoft Mixer, the latter of which signed an exclusive contract with Ninja before being discontinued in 2020.

As shown in an extension, multihoming also matters for the Superstar's exclusivity decision. While multihoming consumers limit the bargaining position of the Superstar because of the reduced market expansion, also non-exclusivity reduces profitability as it does not allow the Superstar to extract surplus from the platforms. This suggests that Superstars' managers should take consumer behavior into account.

Should platform owners engage in exclusivity? In 2016, Spotify claimed that Superstar exclusives were bad for artists, consumers, and platforms. Nevertheless, in 2018, Spotify began working with exclusivity as well (*e.g.,* with Taylor Swift's *Delicate* and the acoustic version of Earth, Wind & Fire's *September*). Likewise, the company struck a multi-year deal with Higher Ground Audio, a podcast company, to produce podcasts with Barack and Michelle Obama, and Joe Rogan. Our paper suggests that exclusivity can benefit the industry and can help a platform sustain market expansion on both sides of the market (even though exclusivity can be expensive). Signing a contract with a Superstar or announcing the first-party provision of premium content can help users and complementors form favorable expectations about their level of participation on the platform (see *e.g.,* Chellappa & Mukherjee 2021). The results herein imply that exclusivity can represent a way to expand the user base, generate self-reinforcing effects due to the higher participation of complementors, and ultimately outperform rivals. Thus, focusing on exclusivity in the provision of Superstar content can help to reach the same goal, in terms of market penetration, usually achieved via a more traditional aggressive pricing strategy.

Should Superstar first-party provision be exclusive? If a platform develops or acquires a Superstar product, the most profitable strategy is not necessarily the most intuitive one, namely keeping her exclusive. Our findings identify conditions in which the platform's owner can profitably license the Superstar to the rival. The trade-off the platform's owner faces is as follows. On the one hand, keeping the Superstar exclusive expands the market reach and increases platform revenues. On the other hand, ancillary revenue from the Superstar content is lower due to exclusivity. Intuitively, non-exclusivity implies lower revenues on the platform market and higher revenues from licensing the Superstar content to the rival. Our findings suggest that platform managers engaging in the production/acquisition of first-party Superstar content should account for this trade-off. Exclusivity should be maintained whenever the per-user ancillary revenues obtained by the Superstar are not very large. This way, the platform's owner can position its platform as being more attractive so as to reach a large audience of users and complementors (*e.g.,* artists and podcasters) in the typical feedback loop that characterizes markets with externalities. Meanwhile, Superstar content should be licensed to rivals if it can generate high enough per-user ancillary value. In this case, the platform's owner should be willing to sacrifice platform revenues and (static) market positioning in favor of larger revenues from licensing to the rival. We observe that a dynamic use of both strategies is frequently followed by platform owners and Superstar managers, with a product that has the potential to be pivotal for a platform being released exclusively on a platform before being licensed to others. This allows the positive effects of exclusivity to be maximized in the early stages, exploiting the Superstar's access by a large audience in the subsequent stages.

How does the presence of the Superstar impact the market of complementors? There are also meaningful insights for managers of complementors, artists, game producers, or developers. When complementors are not in direct competition with the Superstar, exclusivity can help them to break into the market and be accessible to consumers. One typical example is the demand-discovery effect that can be generated by playlists with Superstar artists. In a study of exclusivity and complementors in the app market, Cennamo & Santalo (2013) note that market consolidation by platforms through exclusivity arrangements should be weighed against the costs of the hostile market environment which exclusivity brings about. Our analysis suggests that a hostile environment is less likely to emerge when Superstars and complementors are not in direct competition with one another. In such a case, the total effect for complementors is unambiguously positive. Moreover, by encouraging entry cascades of complementors, our results also suggest that the exclusive presence of Superstars can generate important supply-side effects such as an increase in variety and differentiation (as recently shown in Förderer & Gutt 2021).

8. Concluding Remarks

Exclusivity is commonly observed in markets with cross-group externalities. This article studies the rationale behind its emergence, in the form of exclusive dealing and first-party provision, and its impact on the different market participants. We find that exclusivity emerges when platform competition is more severe because consumers are very responsive to the presence of the Superstar. This effect is further magnified by the two-sidedness of the market as the *favored* platform becomes more appealing for a large mass of complementors, with some zerohomers and multihomers becoming singlehomers. Importantly, when vertical integration takes place, either because of first-party provision or acquisition via vertical mergers, exclusivity might emerge less often than under vertical separation.

In contrast to existing theories intended for one-sided markets, our results suggest that exclusivity does not necessarily translate into harm to consumers and complementors. Under certain conditions, exclusivity might represent a welfare-enhancing choice for the industry. In these cases, bans on exclusive dealing would be detrimental to complementors and possibly to consumers. Moreover, typical arguments, related to input foreclosure associated with vertical integration, may not apply in these markets under reasonable conditions.

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Appendix A.

A.1. Proof of Lemma 1

Consider the last stage of the game. Imposing fulfilled expectations $D_i^e = D_i$ and $N_i^e = N_i$ for $i = 1, 2$, we solve the system of equations for the demands at the two platforms, with equation (4) presenting the solutions of such a system. For brevity, we suppress the arguments of the functions and we use $(\cdot)$ instead of (p_i, p_j, g_i, g_j).

In the second stage of the game, platform i makes a decision on p_i for the given presence of the Superstar to maximize the following gross profit (before any payment to the Superstar)

$$\tilde{\Pi}_i(p_i, p_j, g_i, g_j) = p_i \tilde{D}_i(\cdot),$$

with $\tilde{D}_1(\cdot) = F(\tilde{\tilde{m}}(p_1, p_2, g_1, g_2))$ and $\tilde{D}_2(\cdot) = 1 - \tilde{D}_1(\cdot)$.

From the first-order condition, we obtain

$$\frac{\partial \tilde{\Pi}_i(\cdot)}{\partial p_i} = \tilde{D}_i(\cdot) + p_i \frac{\partial \tilde{D}_i(\cdot)}{\partial p_i} = 0, \tag{A-1}$$

where

$$\begin{aligned}
\frac{\partial \tilde{D}_1(\cdot)}{\partial p_1} =& f(\tilde{\tilde{m}}(\cdot)) \frac{\partial \tilde{\tilde{m}}(\cdot)}{\partial p_1} \\
=& f(\tilde{\tilde{m}}(\cdot)) \Big[\theta \Big(\frac{\partial N_1(\cdot)}{\partial D_1^e} \frac{\partial \tilde{D}_1(\cdot)}{\partial p_1} - \frac{\partial N_2(\cdot)}{\partial D_2^e} \frac{\partial \tilde{D}_2(\cdot)}{\partial p_1} \Big) - 1 \Big], \\
=& f(\tilde{\tilde{m}}(\cdot)) \Big[\gamma\theta \frac{\partial \tilde{D}_1(\cdot)}{\partial p_1} \Big(\lambda(\gamma D_1^e) + \lambda(\gamma D_2^e) \Big) - 1 \Big],
\end{aligned} \tag{A-2}$$

as (i) $\frac{\partial N_i(\cdot)}{\partial D_i^e} = \lambda(\gamma D_i^e)\gamma$ and (ii) $\frac{\partial \tilde{D}_j(\cdot)}{\partial p_i} = -\frac{\partial \tilde{D}_i(\cdot)}{\partial p_i}$ as $D_2(\cdot) = 1 - D_1(\cdot)$. Since $\tilde{D}_j(\cdot) = D_j^e$ and $\tilde{D}_i(\cdot) = D_i^e$, simplifying and solving for $\frac{\partial \tilde{D}_i(\cdot)}{\partial p_i}$, we obtain

$$\frac{\partial \tilde{D}_1(\cdot)}{\partial p_1} = f(\tilde{\tilde{m}}(\cdot)) \frac{\partial \tilde{\tilde{m}}(\cdot)}{\partial p_1} = -\frac{f(\tilde{\tilde{m}}(\cdot))}{1 - f(\tilde{\tilde{m}}(\cdot))\theta\gamma[\lambda(\gamma \tilde{D}_1(\cdot)) + \lambda(\gamma \tilde{D}_2(\cdot))]}, \tag{A-3}$$

which is negative by Assumption 3. We follow similar steps to get $\frac{\partial \tilde{D}_2(\cdot)}{\partial p_2} = -f(\cdot)\frac{\partial \tilde{\tilde{m}}(\cdot)}{\partial p_2} < 0$ as $\frac{\partial \tilde{\tilde{m}}(\cdot)}{\partial p_2} = -\frac{\partial \tilde{\tilde{m}}(\cdot)}{\partial p_1} > 0$, whose expression is identical to the one in equation (A-3).

Following the same steps, we also obtain the following result:

$$\begin{aligned}
\frac{\partial \tilde{D}_i(\cdot)}{\partial \phi} =& f(\tilde{\tilde{m}}(\cdot)) \Big[\theta \Big(\frac{\partial N_i(\cdot)}{\partial D_i^e} \frac{\partial \tilde{D}_i(\cdot)}{\partial \phi} - \frac{\partial N_j(\cdot)}{\partial D_j^e} \frac{\tilde{D}_j(\cdot)}{\partial \phi} \Big) + 1 \Big], \\
=& \frac{\partial \tilde{D}_i(\cdot)}{\partial p_j} = \frac{f(\tilde{\tilde{m}}(.))}{1 - f(\tilde{\tilde{m}}(.))\theta\gamma[\lambda(\gamma \tilde{D}_i(\cdot)) + \lambda(\gamma \tilde{D}_j(\cdot))]},
\end{aligned} \tag{A-4}$$

which is positive by Assumption 3.

Plugging (A-3) into (A-1), we implicitly determine the equilibrium prices, which are denoted by $p_i^\star(g_i, g_j)$, for given (g_1, g_2):

$$
\begin{aligned}
p_1^\star(g_1,g_2) &= F(m^\star(g_1,g_2))\left\{\frac{1}{f(m^\star(g_1,g_2))} - \gamma\theta\Big[\lambda(\gamma D_1^\star(g_1,g_2)) + \lambda(\gamma D_2^\star(g_2,g_1))\Big]\right\},\\
p_2^\star(g_2,g_1) &= \Big(1 - F(m^\star(g_1,g_2))\Big)\left\{\frac{1}{f(m^\star(g_1,g_2))} - \gamma\theta\Big[\lambda(\gamma D_1^\star(g_1,g_2)) + \lambda(\gamma D_2^\star(g_2,g_1))\Big]\right\},
\end{aligned}
$$

with $m^\star(g_1,g_2) \triangleq \tilde{\tilde{m}}(p_1^\star(g_1,g_2), p_2^\star(g_2,g_1), g_1, g_2)$, $D_1^\star(g_1,g_2) \triangleq F(m^\star(g_1,g_2))$, $D_2^\star(g_2,g_1) \triangleq 1 - F(m^\star(g_1,g_2))$ and $N_i^\star(g_i,g_j) \triangleq \Lambda(\gamma D_i^\star(g_i,g_j))$. Note that Assumption 3 ensures that these prices are positive.

Uniqueness of the equilibrium. To ensure the uniqueness of the equilibrium, we show that there is at most one intersection of the two reaction functions, *i.e.,* a sufficient condition for this is that the best responses have a positive slope of less than 1. To this end, we formally provide the conditions that ensure concavity in profits and those that ensure that prices are strategic complements.

Deriving the marginal profits ($\frac{\partial \tilde{\Pi}_i(\cdot)}{\partial p_i}$) with respect to own and rival's prices, we obtain

$$
\begin{aligned}
\frac{\partial^2 \tilde{\Pi}_1(\cdot)}{\partial p_1^2} &= \frac{\partial \tilde{\tilde{m}}(\cdot)}{\partial p_1}\Big[2f(\cdot) - \frac{F(\cdot)\Big(f'(\cdot) + f^3(\cdot)\theta\gamma^2[\lambda'(\gamma\tilde{D}_1(\cdot)) - \lambda'(\gamma\tilde{D}_2(\cdot))]\Big)}{f(\cdot)[1-\theta\gamma f(\cdot)(\lambda(\gamma\tilde{D}_1(\cdot)) + \lambda(\gamma\tilde{D}_2(\cdot))]}\Big],\\
\frac{\partial^2 \tilde{\Pi}_2(\cdot)}{\partial p_2^2} &= -\frac{\partial \tilde{\tilde{m}}(\cdot)}{\partial p_1}\Big[2f(\cdot) + \frac{[1-F(\cdot)]\Big(f'(\cdot) + f^3(\cdot)\theta\gamma^2[\lambda'(\gamma\tilde{D}_1(\cdot)) - \lambda'(\gamma\tilde{D}_2(\cdot))]\Big)}{f(\cdot)[1-\theta\gamma f(\cdot)(\lambda(\gamma\tilde{D}_1(\cdot)) + \lambda(\gamma\tilde{D}_2(\cdot))]}\Big],\\
\frac{\partial^2 \tilde{\Pi}_1(\cdot)}{\partial p_1 \partial p_2} &= -\frac{\partial \tilde{\tilde{m}}(\cdot)}{\partial p_1}\Big[f(\cdot) - \frac{F(\cdot)\Big(f'(\cdot) + f^3(\cdot)\theta\gamma^2[\lambda'(\gamma\tilde{D}_1(\cdot)) - \lambda'(\gamma\tilde{D}_2(\cdot))]\Big)}{f(\cdot)[1-\theta\gamma f(\cdot)(\lambda(\gamma\tilde{D}_1(\cdot)) + \lambda(\gamma\tilde{D}_2(\cdot))]}\Big],\\
\frac{\partial^2 \tilde{\Pi}_2(\cdot)}{\partial p_2 \partial p_1} &= -\frac{\partial \tilde{\tilde{m}}(\cdot)}{\partial p_1}\Big[f(\cdot) + \frac{[1-F(\cdot)]\Big(f'(\cdot) + f^3(\cdot)\theta\gamma^2[\lambda'(\gamma\tilde{D}_1(\cdot)) - \lambda'(\gamma\tilde{D}_2(\cdot))]\Big)}{f(\cdot)[1-\theta\gamma f(\cdot)(\lambda(\gamma\tilde{D}_1(\cdot)) + \lambda(\gamma\tilde{D}_2(\cdot))]}\Big],
\end{aligned}
\tag{A-5}
$$

A sufficient condition to ensure concavity in profits is to ensure $\frac{\partial^2 \tilde{\Pi}_1(\cdot)}{\partial p_1 \partial p_2} > 0$ and $\frac{\partial^2 \tilde{\Pi}_2(\cdot)}{\partial p_2 \partial p_1} > 0$.[28]

[28] Note that ensuring that the derivatives of the marginal profit with respect to the rivals' prices are positive also ensures that the second derivative of profits with respect to prices are negative.

Recall that $\frac{\partial \tilde{\tilde{m}}(\cdot)}{\partial p_1} < 0$, then the following two conditions should jointly hold

$$\begin{aligned} f(\cdot) - \frac{F(\cdot)\Big(f'(\cdot) + f^3(\cdot)\theta\gamma^2[\lambda'(\gamma\tilde{D}_1(\cdot)) - \lambda'(\gamma\tilde{D}_2(\cdot))]\Big)}{f(\cdot)[1-\theta\gamma f(\cdot)(\lambda(\gamma\tilde{D}_1(\cdot)) + \lambda(\gamma\tilde{D}_2(\cdot))} &> 0, \\ f(\cdot) + \frac{[1-F(\cdot)]\Big(f'(\cdot) + f^3(\cdot)\theta\gamma^2[\lambda'(\gamma\tilde{D}_1(\cdot)) - \lambda'(\gamma\tilde{D}_2(\cdot))]\Big)}{f(\cdot)[1-\theta\gamma f(\cdot)(\lambda(\gamma\tilde{D}_1(\cdot)) + \lambda(\gamma\tilde{D}_2(\cdot))]} &> 0. \end{aligned} \tag{A-6}$$

Solving for $f'(\cdot)$, we obtain

$$\begin{aligned} \underline{f}' \equiv & \frac{f^2(\cdot)\Big[f(\cdot)\gamma\theta\Big(\lambda(\gamma\tilde{D}_1(\cdot)) + \lambda(\gamma\tilde{D}_2(\cdot)) - \gamma(1-F(\cdot))[\lambda'(\gamma\tilde{D}_1(\cdot)) - \lambda'(\gamma\tilde{D}_2(\cdot))]\Big) - 1\Big]}{1-F(\cdot)} \\ & < f'(\cdot) < \\ & \frac{f^2(\cdot)\Big[(1-\gamma\theta f(\cdot)\Big(F(\cdot)\gamma[\lambda'(\gamma\tilde{D}_1(\cdot)) - \lambda'(\gamma\tilde{D}_2(\cdot))] + \lambda(\gamma\tilde{D}_1(\cdot)) + \lambda(\gamma\tilde{D}_2(\cdot))\Big)\Big]}{F(\cdot)} \equiv \overline{f}' \end{aligned}$$

In the paper, to save on notation, we assume that $f'(\cdot)$ is bounded from above and from below, such that $\underline{f}' < f'(\cdot) < \overline{f}'(\cdot)$. This is reported in Assumption 2.

Therefore, under Assumption 2, we have

$$\frac{\partial^2\tilde{\Pi}_1(\cdot)}{\partial p_1 \partial p_2} > 0 \qquad \frac{\partial^2\tilde{\Pi}_2(\cdot)}{\partial p_2 \partial p_1} > 0 \qquad \frac{\partial^2\tilde{\Pi}_1(\cdot)}{\partial p_1^2} < 0 \qquad \frac{\partial^2\tilde{\Pi}_2(\cdot)}{\partial p_2^2} < 0. \tag{A-7}$$

Denoting $p_i^{BR}(p_j)$, the slope of best response of platform i as follows

$$\frac{\partial p_i^{BR}(p_j)}{\partial p_j} = \frac{\frac{\partial^2\tilde{\Pi}_i(\cdot)}{\partial p_i \partial p_j}}{\frac{\partial^2\tilde{\Pi}_i(\cdot)}{\partial p_i^2}},$$

for $i = 1,2$ and $j \neq i$. Thus, we conclude that $\frac{\partial p_i^{BR}(p_j)}{\partial p_j} \in (0,1)$ and this ensures the uniqueness of the equilibrium. This concludes the proof.

A.2. Proof of Lemma 2

Consider the case of non-exclusivity, i.e., $g_1 = g_2 = 1$. We focus on the symmetric scenario in which each participant expects the market to be symmetric. Using results in Lemma 1 and imposing symmetry, we obtain $p_1^\star(1,1) = p_2^\star(1,1)$ as $D_1^\star(1,1) = D_2^\star(1,1) = F(m^\star =$

$0) = \frac{1}{2}$ and $N_1^\star(1,1) = N_2^\star(1,1)$. The equilibrium prices are denoted by

$$\begin{aligned} p_1^\star(1,1) = p_2^\star(1,1) =& \frac{F(0)}{f(0)}\Big[1 - f(0)\gamma\theta\Big(\lambda(\gamma D_1^\star(1,1)) + \lambda(\gamma D_2^\star(1,1))\Big)\Big] \\ =& \frac{1}{2f(0)} - \gamma\theta\lambda(\gamma/2). \end{aligned}$$

All complementors with $k \leq \gamma/2$ are active on both platforms, whereas all complementors with $k > \gamma/2$ zerohome. This concludes the proof.

A.3. Proof of Lemma 3

In this proof, we demonstrate that the equilibrium prices, consumers' demands and complementors' participation on two platforms are respectively ordered as follows: $p_1^\star(1,0) > p_i^\star(1,1) > p_2^\star(0,1)$, $D_1^\star(1,0) > D_2^\star(0,1)$ and $N_1^\star(1,0) > N_2^\star(0,1)$.

To this end, it should be noted that due to the Hotelling setup and symmetry between platforms, for any $\phi = 0$ there is a symmetric outcome, which implies $m^\star(g_1, g_2) = 0$, and this results in $D_1^\star(g_1, g_2) = D_2^\star(g_1, g_2) = 1/2$. Thus, a sufficient condition for $m^\star(1,0)$ to be strictly positive for $\phi > 0$, which means that the demand of the *favored* platform is strictly greater than the demand of the *unfavored* platform, is that $\frac{dm^\star(1,0)}{d\phi} > 0$.

In what follows, we assess the *total* impact of a change in ϕ on $m^\star(1,0)$, decomposing it into the direct effect (Step 1) and the indirect effect through changes in prices (Step 2.a and 2.b). Formally, we determine the sign of the following term and, to facilitate the analysis, we provide different steps:[29]

$$\begin{aligned} \frac{dm^\star(1,0)}{d\phi} = \quad & \underbrace{1 + \theta\Big(\frac{\partial \tilde{N}_1(\cdot)}{\partial \phi} - \frac{\partial \tilde{N}_2(\cdot)}{\partial \phi}\Big)}_{\textbf{Step 1.}\text{ Direct effect}} \\ & \underbrace{+\theta\Big[\Big(\frac{\partial \tilde{N}_1(\cdot)}{\partial p_1} - \frac{\partial \tilde{N}_2(\cdot)}{\partial p_1}\Big)\frac{\partial p_1^\star(1,0)}{\partial \phi} + \Big(\frac{\partial \tilde{N}_1(\cdot)}{\partial p_2} - \frac{\partial \tilde{N}_2(\cdot)}{\partial p_2}\Big)\frac{\partial p_2^\star(0,1)}{\partial \phi}\Big]}_{\substack{\textbf{Step 2a.}\text{ Indirect price effect} \\ \text{on complementors}}} \\ & \underbrace{-\frac{\partial(p_1^\star(1,0) - p_2^\star(0,1))}{\partial \phi}}_{\textbf{Step 2b.}\text{ Direct price effect}}. \end{aligned} \quad \text{(A-8)}$$

Step 1: The direct effect of ϕ. The direct effect of a change in ϕ, for given prices, can be obtained by differentiating $\tilde{\tilde{m}}(p_1, p_2, 1, 0)$ with respect to ϕ. This is equivalent in

[29]We thank an anonymous referee for helpful comments that have significantly improved the exposition of this part of the proof.

sign to what is shown in (A-4), as $\frac{\partial \tilde{\tilde{D}}_1(p_1,p_2,1,0)}{\partial \phi} = f(\tilde{\tilde{m}}(p_1,p_2,1,0))\frac{\partial \tilde{\tilde{m}}(p_1,p_2,1,0)}{\partial \phi}$, so that

$$\frac{\partial \tilde{\tilde{m}}(p_1,p_2,1,0)}{\partial \phi} \quad = \quad \frac{1}{1-\gamma\theta f(\tilde{\tilde{m}}(\cdot))[\lambda(\gamma\tilde{D}_1(\cdot))+\lambda(\gamma\tilde{D}_2(\cdot))]} \tag{A-9}$$

which is larger than 1 by Assumption 3.

Step 2a: The impact of ϕ on complementors due to a price change. The effect of a change in price on complementors can be written as

$$\begin{aligned}\frac{\partial \tilde{N}_1(\cdot)}{\partial p_1} - \frac{\partial \tilde{N}_2(\cdot)}{\partial p_1} =& \frac{\partial N_1(\cdot)}{\partial D_1^e}\frac{\partial \tilde{D}_1(\cdot)}{\partial p_1} - \frac{\partial N_2(\cdot)}{\partial D_2^e}\frac{\partial \tilde{D}_2(\cdot)}{\partial p_1}\\ =& \frac{\partial \tilde{D}_1(\cdot)}{\partial p_1}\left(\frac{\partial N_1(\cdot)}{\partial D_1^e} + \frac{\partial N_2(\cdot)}{\partial D_2^e}\right),\end{aligned}$$

where the second line results from (A-4) and (A-2) in the proof of Lemma 1, i.e., $\frac{\partial \tilde{D}_1(\cdot)}{\partial p_1} = -\frac{\partial \tilde{D}_2(\cdot)}{\partial p_1}$. Using the same rationale, we have $\frac{\partial \tilde{N}_1(\cdot)}{\partial p_2} - \frac{\partial \tilde{N}_2(\cdot)}{\partial p_2} = -\frac{\partial \tilde{D}_2(\cdot)}{\partial p_2}\left(\frac{\partial N_1(\cdot)}{\partial D_1^e} + \frac{\partial N_2(\cdot)}{\partial D_2^e}\right)$.

Recalling that $\frac{\partial N_i(\cdot)}{\partial D_i^e} = \gamma\lambda(\gamma D_i^e)$, $D_i^e = \tilde{D}_i$, and $\frac{\partial \tilde{D}_1(\cdot)}{\partial p_1} = \frac{\partial \tilde{D}_2(\cdot)}{\partial p_2}$, we write the following

$$\begin{aligned}&\left(\frac{\partial \tilde{N}_1(\cdot)}{\partial p_1} - \frac{\partial \tilde{N}_2(\cdot)}{\partial p_1}\right)\frac{\partial p_1^\star(1,0)}{\partial \phi} + \left(\frac{\partial \tilde{N}_1(\cdot)}{\partial p_2} - \frac{\partial \tilde{N}_2(\cdot)}{\partial p_2}\right)\frac{\partial p_2^\star(0,1)}{\partial \phi}\\ =&\theta\frac{\partial \tilde{D}_1(\cdot)}{\partial p_1}\left(\frac{\partial N_1(\cdot)}{\partial D_1^e} + \frac{\partial N_2(\cdot)}{\partial D_2^e}\right)\frac{\partial (p_1^\star(1,0)-p_2^\star(0,1))}{\partial \phi}\\ =&-\frac{\theta\gamma f(\tilde{\tilde{m}}(\cdot))[\lambda(\gamma\tilde{D}_1(\cdot))+\lambda(\gamma\tilde{D}_2(\cdot))]}{1-f(\tilde{\tilde{m}}(\cdot))\theta\gamma[\lambda(\gamma\tilde{D}_1(\cdot))+\lambda(\gamma\tilde{D}_2(\cdot))]}\frac{\partial (p_1^\star(1,0)-p_2^\star(0,1))}{\partial \phi},\end{aligned} \tag{A-10}$$

We note that the sign of the expression in (A-10) is the opposite of that of $\frac{\partial (p_1^\star(1,0)-p_2^\star(0,1))}{\partial \phi}$.

Step 2b: The impact of ϕ on equilibrium prices. To identify the effect of ϕ on $p_1^\star(1,0)$ and $p_2^\star(0,1)$, we totally differentiate the first-order conditions in (A-1) with respect to ϕ:

$$\begin{aligned}\frac{\partial^2 \tilde{\Pi}_1(\cdot)}{\partial p_1^2}\frac{\partial p_1^\star(1,0)}{\partial \phi} + \frac{\partial^2 \tilde{\Pi}_1(\cdot)}{\partial p_1 \partial \phi} + \frac{\partial^2 \tilde{\Pi}_1(\cdot)}{\partial p_1 \partial p_2}\frac{\partial p_2^\star(0,1)}{\partial \phi} = 0\\ \frac{\partial^2 \tilde{\Pi}_2(\cdot)}{\partial p_2 \partial p_1}\frac{\partial p_1^\star(1,0)}{\partial \phi} + \frac{\partial^2 \tilde{\Pi}_2(\cdot)}{\partial p_2 \partial \phi} + \frac{\partial^2 \tilde{\Pi}_2(\cdot)}{\partial p_2^2}\frac{\partial p_2^\star(0,1)}{\partial \phi} = 0.\end{aligned} \tag{A-11}$$

To simplify matters, we use results in (A-3), (A-4), and concavity conditions in (A-7), to establish the following relationships

$$\frac{\partial^2 \tilde{\Pi}_1(\cdot)}{\partial p_1 \partial \phi} = \frac{\partial^2 \tilde{\Pi}_1(\cdot)}{\partial p_1 \partial p_2} > 0, \qquad \frac{\partial^2 \tilde{\Pi}_2(\cdot)}{\partial p_2 \partial \phi} = -\frac{\partial^2 \tilde{\Pi}_2(\cdot)}{\partial p_2 \partial p_1} < 0,$$

Moreover, we also exploit the fact that $\frac{\partial^2 \tilde{D}_i(\cdot)}{\partial p_i^2} = -\frac{\partial^2 \tilde{D}_i(\cdot)}{\partial p_i \partial p_j}$ because $\frac{\partial \tilde{D}_i(\cdot)}{\partial p_j} = -\frac{\partial \tilde{D}_i(\cdot)}{\partial p_i}$, so we can establish that $\frac{\partial^2 \tilde{\Pi}_i(\cdot)}{\partial p_i^2} = -\frac{\partial^2 \tilde{\Pi}_i(\cdot)}{\partial p_i \partial p_j} + \frac{\partial \tilde{D}_i(\cdot)}{\partial p_i}$ and, in turn,

$$\frac{\partial^2 \tilde{\Pi}_1(\cdot)}{\partial p_1^2} = -\frac{\partial^2 \tilde{\Pi}_1(\cdot)}{\partial p_1 \partial \phi} + \frac{\partial \tilde{D}_1(\cdot)}{\partial p_1} \qquad \frac{\partial^2 \tilde{\Pi}_2(\cdot)}{\partial p_2^2} = \frac{\partial^2 \tilde{\Pi}_2(\cdot)}{\partial p_2 \partial \phi} + \frac{\partial \tilde{D}_2(\cdot)}{\partial p_2}. \tag{A-12}$$

Putting things together, we rewrite (A-11) as

$$\begin{aligned}
\Big(-\frac{\partial^2 \tilde{\Pi}_1(\cdot)}{\partial p_1 \partial \phi} + \frac{\partial \tilde{D}_1(\cdot)}{\partial p_1} \Big) \frac{\partial p_1^\star(1,0)}{\partial \phi} + \frac{\partial^2 \tilde{\Pi}_1(\cdot)}{\partial p_1 \partial \phi} + \frac{\partial^2 \tilde{\Pi}_1(\cdot)}{\partial p_1 \partial \phi} \frac{\partial p_2^\star(0,1)}{\partial \phi} &= 0 \\
-\frac{\partial^2 \tilde{\Pi}_2(\cdot)}{\partial p_2 \partial \phi} \frac{\partial p_1^\star(1,0)}{\partial \phi} + \frac{\partial^2 \tilde{\Pi}_2(\cdot)}{\partial p_2 \partial \phi} + \Big(\frac{\partial^2 \tilde{\Pi}_2(\cdot)}{\partial p_2 \partial \phi} + \frac{\partial \tilde{D}_2(\cdot)}{\partial p_2} \Big) \frac{\partial p_2^\star(0,1)}{\partial \phi} &= 0.
\end{aligned}$$

Further, exploiting that $\frac{\partial \tilde{D}_2(\cdot)}{\partial p_2} = \frac{\partial \tilde{D}_1(\cdot)}{\partial p_1}$ and solving the system of equations, we finally identify the effect of ϕ on prices as follows

$$\begin{aligned}
\frac{\partial p_1^\star(1,0)}{\partial \phi} &= -\frac{\frac{\partial^2 \tilde{\Pi}_1(\cdot)}{\partial p_1 \partial \phi}}{\frac{\partial \tilde{D}_1(\cdot)}{\partial p_1} - \frac{\partial^2 \tilde{\Pi}_1(\cdot)}{\partial p_1 \partial \phi} + \frac{\partial^2 \tilde{\Pi}_2(\cdot)}{\partial p_2 \partial \phi}} \\
\frac{\partial p_2^\star(0,1)}{\partial \phi} &= -\frac{\frac{\partial^2 \tilde{\Pi}_2(\cdot)}{\partial p_2 \partial \phi}}{\frac{\partial \tilde{D}_1(\cdot)}{\partial p_1} - \frac{\partial^2 \tilde{\Pi}_1(\cdot)}{\partial p_1 \partial \phi} + \frac{\partial^2 \tilde{\Pi}_2(\cdot)}{\partial p_2 \partial \phi}}
\end{aligned} \tag{A-13}$$

Note that the denominator of both terms is negative. Therefore, the sign of $\frac{\partial p_1^\star(1,0)}{\partial \phi}$ is the same as the sign of $\frac{\partial^2 \tilde{\Pi}_1(\cdot)}{\partial p_1 \partial \phi}$, which is positive. Likewise, the sign of $\frac{\partial p_2^\star(0,1)}{\partial \phi}$ is the same as the sign of $\frac{\partial^2 \tilde{\Pi}_2(\cdot)}{\partial p_2 \partial \phi}$, which is negative. This proves that

$$\frac{\partial (p_1^\star(1,0) - p_2^\star(0,1))}{\partial \phi} = \frac{-\Big(\frac{\partial^2 \tilde{\Pi}_1(\cdot)}{\partial p_1 \partial \phi} - \frac{\partial^2 \tilde{\Pi}_2(\cdot)}{\partial p_2 \partial \phi} \Big)}{\frac{\partial \tilde{D}_1(\cdot)}{\partial p_1} - \frac{\partial^2 \tilde{\Pi}_1(\cdot)}{\partial p_1 \partial \phi} + \frac{\partial^2 \tilde{\Pi}_2(\cdot)}{\partial p_2 \partial \phi}} > 0$$

Moreover, both the numerator and the denominator are negative. To prove that $0 < \frac{\partial p_1^\star(1,0)}{\partial \phi} - \frac{\partial p_2^\star(0,1)}{\partial \phi} < 1$, we verify that

$$-\frac{\partial^2 \tilde{\Pi}_1(\cdot)}{\partial p_1 \partial \phi} + \frac{\partial^2 \tilde{\Pi}_2(\cdot)}{\partial p_2 \partial \phi} > \frac{\partial \tilde{D}_1(\cdot)}{\partial p_1} - \frac{\partial^2 \tilde{\Pi}_1(\cdot)}{\partial p_1 \partial \phi} + \frac{\partial^2 \tilde{\Pi}_2(\cdot)}{\partial p_2 \partial \phi} \Leftrightarrow \frac{\partial \tilde{D}_1(\cdot)}{\partial p_1} < 0.$$

This implies that $0 < \frac{\partial p_1^\star(1,0)}{\partial \phi} - \frac{\partial p_2^\star(0,1)}{\partial \phi} < 1$.

The total effect. Summing up Step 1, Step 2a, and Step 2b and simplifying, we rewrite the total effect in (A-8) at equilibrium as

$$\frac{\partial m^\star(1,0)}{\partial \phi} = \frac{1-\left(\frac{\partial p_1^\star(1,0)}{\partial \phi}-\frac{\partial p_2^\star(0,1)}{\partial \phi}\right)}{1-\gamma\theta f(\tilde{\tilde{m}}(\cdot))\Big[\lambda(\gamma \tilde{D}_1(\cdot))+\lambda(\gamma \tilde{D}_2(\cdot))\Big]} > 0.$$

The above expression is always positive as $0 < \frac{\partial p_1^\star(1,0)}{\partial \phi} - \frac{\partial p_2^\star(0,1)}{\partial \phi} < 1$ by Step 2. This, combined with the fact that $m^\star(1,0) = 0$ if $\phi = 0$, implies that for $\phi > 0$, $m^\star(1,0) > 0$ and therefore $D_1^\star(1,0) > D_2^\star(0,1)$ and $N_1^\star(1,0) > N_2^\star(0,1)$. Moreover, $\frac{\partial p_1^\star(1,0)}{\partial \phi} - \frac{\partial p_2^\star(0,1)}{\partial \phi} > 0$ implies $p_1^\star(1,0) > p_2^\star(0,1)$ because $\frac{\partial p_1^\star(1,0)}{\partial \phi} - \frac{\partial p_2^\star(0,1)}{\partial \phi} = 0$ for $\phi = 0$.

Finally note that for the limit case in which $\phi = 0$, we have $p_1^\star(1,0) = p_1^\star(1,1) = p_2^\star(1,1) = p_2^\star(0,1)$. Moreover, we have already shown that $\frac{\partial p_1^\star(1,0)}{\partial \phi} > 0$ for any $\phi > 0$ and $\frac{\partial p_2^\star(0,1)}{\partial \phi} < 0$ for any $\phi > 0$. This, combined with the fact that $\frac{\partial p_1^\star(1,1)}{\partial \phi} = \frac{\partial p_2^\star(1,1)}{\partial \phi} = 0$ for any ϕ, implies that $p_1^\star(1,0) > p_1^\star(1,1) = p_2^\star(1,1) > p_2^\star(0,1)$. This concludes the proof.

A.4. Proof of Proposition 3

Denote the surplus of complementors on platform i at equilibrium by $FS_i^\star(g_i, g_j)$, for $i = 1, 2$. Under non-exclusivity, let $FS^\star(1,1)$ be the sum of $FS_1^\star(1,1)$ and $FS_2^\star(1,1)$, then

$$FS^\star(1,1) \triangleq \int_0^{\gamma/2} [(\gamma - 2k)\lambda(k)]dk.$$

Under exclusivity on platform 1, the surplus of complementors on platform 2 is:

$$FS_2^\star(0,1) \triangleq \int_0^{\gamma D_2^\star(0,1)} [(\gamma D_2^\star(0,1) - k)\lambda(k)]dk,$$

whereas the surplus of complementors on platform 1 is:

$$FS_1^\star(1,0) \triangleq \int_0^{\gamma D_1^\star(1,0)} [(\gamma D_1^\star(1,0) - k)\lambda(k)]dk.$$

The total surplus of complementors under exclusivity is:

$$FS^\star(1,0) = \int_0^{\gamma D_2^\star(0,1)} [(\gamma D_2^\star(0,1) - k)\lambda(k)]dk + \int_0^{\gamma D_1^\star(1,0)} [\lambda(k)(\gamma D_1^\star(1,0) - k)]dk.$$

Denote $\Delta FS \triangleq FS^\star(1,0) - FS^\star(1,1)$ the difference in total complementors' surplus between exclusivity and non-exclusivity:

$$\begin{aligned}
\Delta FS =& \int_0^{\gamma D_2^\star(0,1)} [0 \cdot \lambda(k)]dk + \int_{\gamma D_2^\star(0,1)}^{\gamma/2} [(\gamma D_1^\star(1,0) - k - (\gamma - 2k))\lambda(k)]dk + \\
& \int_{\gamma/2}^{\gamma D_1^\star(1,0)} [(\gamma D_1^\star(1,0) - k)\lambda(k)]dk, \\
=& \int_{\gamma D_2^\star(0,1)}^{\gamma/2} [(\gamma(D_1^\star(1,0) - 1) + k)\lambda(k)]dk + \int_{\gamma/2}^{\gamma D_1^\star(1,0)} [(\gamma D_1^\star(1,0) - k)\lambda(k)]dk.
\end{aligned}$$

Exploiting that $D_1^\star(1,0) + D_2^\star(0,1) = 1$, each element in the first integral can be rewritten as $\gamma D_1^\star(1,0) - \gamma D_1^\star(1,0) - \gamma D_2^\star(0,1) + k$, which is simplified as $-\gamma D_2^\star(0,1) + k$, which is always greater than zero in the interval considered — $k \in [\gamma D_2^\star(0,1), \gamma/2]$. The second term is unambiguously positive. Therefore $\Delta FS > 0$. This concludes the proof.

A.5. Proof of Proposition 4

We begin by identifying consumer surplus in the two regimes. Under non-exclusivity, consumers surplus at platform i is denoted by $CS_i(1,1)$, for $i = 1, 2$, such that

$$CS_1^\star(1,1) \triangleq \int_{\underline{m}}^0 \left[v + \phi + \theta N_1^\star(1,1) - p_1^\star(1,1) - \frac{m}{2}\right] f(m)dm.$$

Integration by parts implies that

$$CS_1^\star(1,1) = \frac{1}{2}\left[v + \phi + \theta N_1^\star(1,1) - p_1^\star(1,1)\right] + \int_{\underline{m}}^0 \frac{F(m)}{2}dm,$$

as $F(0) = 1/2$ and $F(\underline{m}) = 0$. Consumer surplus at platform 2 is

$$CS_2^\star(1,1) \triangleq \frac{1}{2}\left[v + \phi + \theta N_2^\star(1,1) - p_2^\star(1,1) + \overline{m}\right] - \int_0^{\overline{m}} \frac{F(m)}{2}dm.$$

Exploiting symmetry, $p_1^\star(1,1) = p_2^\star(1,1)$, $N_1^\star(1,1) = N_2^\star(1,1) = \Lambda(\gamma/2)$, $F(m^\star(1,1)) = F(0) = 1/2$, and $\int_0^{\overline{m}} \frac{F(m)}{2}dm = -\int_{\underline{m}}^0 \frac{F(m)}{2}dm$, then

$$CS^\star(1,1) \triangleq v + \phi + \theta\Lambda(\gamma/2) - p_1^\star(1,1) + \frac{\overline{m}}{2}. \tag{A-14}$$

Under exclusivity on platform 1, denote consumer surplus at platform 1 (resp. 2) $CS_1(1,0)$ (resp. $CS_2(0,1)$). Then, total consumer surplus under non-exclusivity is:

$$CS_1^\star(1,0) \triangleq \int_{\underline{m}}^{m^\star(1,0)} \frac{F(m)}{2}dm + \left[v + \phi + \theta N_1^\star(1,0) - p_1^\star(1,0) - \frac{m^\star(1,0)}{2}\right]F(m^\star(1,0)).$$

whereas consumer surplus on platform 2 is:

$$CS_2^\star(0,1) \triangleq (1 - F(m^\star(1,0))[v + \theta N_2^\star(0,1) - p_2^\star(0,1)] + \frac{1}{2}\Big[\overline{m} - m^\star(1,0)F(m^\star(1,0))\Big] - \int_{m^\star(1,0)}^{\overline{m}} \frac{F(m)}{2} dm.$$

Total consumer surplus under exclusivity, denoted by $CS^\star(1,0)$, is

$$\begin{aligned} CS^\star(1,0) \triangleq & \frac{1}{2}\Big[\int_{\underline{m}}^{m^\star(1,0)} F(m)dm - \int_{m^\star(1,0)}^{\overline{m}} F(m)dm\Big] \\ & + \Big[v + \phi + \theta N_1^\star(1,0) - p_1^\star(1,0) - \frac{m^\star(1,0)}{2}\Big] F(m^\star(1,0)) \\ & + (1 - F(m^\star(1,0))\Big[v + \theta N_2^\star(0,1) - p_2^\star(0,1)\Big] \\ & + \frac{1}{2}\Big[\overline{m} - m^\star(1,0)F(m^\star(1,0))\Big]. \end{aligned} \tag{A-15}$$

As $F(\cdot)$ is symmetric around 0, then

$$\int_{\underline{m}}^{m^\star(1,0)} F(m)dm - \int_{m^\star(1,0)}^{\overline{m}} F(m)dm \equiv \int_{\underline{m}}^{m^\star(1,0)-\overline{m}} F(m)dm + \int_{m^\star(1,0)-\overline{m}}^{m^\star(1,0)} F(m)dm - \int_{m^\star(1,0)}^{\overline{m}} F(m)dm.$$

The first and third terms of the RHS cancel out. The second term can be simplified by exploiting the symmetry of $F(\cdot)$ around 0. Hence, the above expression can be rewritten as follows:

$$\int_{m^\star(1,0)-\overline{m}}^{m^\star(1,0)} F(m)dm = \int_{m^\star(1,0)-\overline{m}}^{0} F(m)dm + \int_{0}^{m^\star(1,0)} F(m)dm = 2\int_{0}^{m^\star(1,0)} F(m)dm.$$

Indeed, consumer surplus under exclusivity in (A-15) is

$$\begin{aligned} CS^\star(1,0) = & \int_0^{m^\star(1,0)} F(m)dm + \Big[v + \phi + \theta N_1^\star(1,0) - p_1^\star(1,0) - \frac{m^\star(1,0)}{2}\Big] F(m^\star(1,0)) \\ & + (1 - F(m^\star(1,0))\Big[v + \theta N_2^\star(0,1) - p_2^\star(0,1)\Big] + \frac{1}{2}\Big[\overline{m} - m^\star(1,0)F(m^\star(1,0))\Big]. \end{aligned} \tag{A-16}$$

Impact of exclusivity on total CS. Denote $\Delta CS \triangleq CS^{\star}(1,0) - CS^{\star}(1,1)$, where $CS^{\star}(1,0)$ and $CS^{\star}(1,1)$ are determined by (A-15) and (A-14), respectively. Then,

$$\Delta CS = \int_0^{m^{\star}(1,0)} F(m)dm + \left[v + \phi + \theta N_1^{\star}(1,0) - p_1^{\star}(1,0) - \frac{m^{\star}(1,0)}{2}\right] F(m^{\star}(1,0) \\ + (1 - F(m^{\star}(1,0)))\left[v + \theta N_2^{\star}(0,1) - p_2^{\star}(0,1)\right] + \frac{1}{2}\left[\overline{m} - m^{\star}(1,0)F(m^{\star}(1,0))\right] - \\ \left[v + \phi + \theta \Lambda(\gamma/2) - p_1^{\star}(1,1)\right] - \frac{\overline{m}}{2}.$$

Using the above, we rewrite it to get the expression in equation (6) as follows:

$$\Delta CS = \underbrace{\theta[\bar{N} - N^{\star}(1,1)]}_{\Delta \text{ externalities}} - \underbrace{\phi D_2^{\star}(0,1)}_{\text{prevented access}} - \underbrace{[\bar{p} - p^{\star}(1,1)]}_{\Delta \text{ prices}} - \underbrace{\int_0^{m^{\star}(1,0)} mf(m)dm}_{\text{preference mismatch}},$$

where $\bar{N} \triangleq F(m^{\star}(1,0))N_1^{\star}(1,0) + (1 - F(m^{\star}(1,0)))N_2^{\star}(0,1)$ and $\bar{p} \triangleq F(m^{\star}(1,0))p_1^{\star}(1,0) + (1 - F(m^{\star}(1,0))p_2^{\star}(0,1)$ are the average mass of complementors and the average prices under exclusivity, respectively.[30] It follows that for sufficiently large θ, exclusivity is beneficial to consumers and intuitions. The continuation of the proof is in the main text and an example with a uniform distribution of $F(\cdot)$ and $\Lambda(\cdot)$ is provided in the Online Appendix. This concludes the proof.

A.6. Proof of Corollary 1

We prove how the threshold $\tilde{\gamma}^S$ in Proposition 5 changes with ϕ. First, we write the threshold as follows

$$\tilde{\gamma}^S \triangleq \frac{p_1^{\star}(1,0)D_1^{\star}(1,0) + p_2^{\star}(0,1)D_2^{\star}(0,1) - p_1^{\star}(1,1)}{D_2^{\star}(0,1)}$$

as $2\Pi^{\star}(1,1) = 2D_1^{\star}(1,1)p_1^{\star}(1,1) = p_1^{\star}(1,1)$ as, by symmetry, $D_1^{\star}(1,1) = 1/2$ and $D_2^{\star}(0,1) = 1 - D_1^{\star}(1,0)$, $p_2^{\star}(0,1) = p_1^{\star}(0,1)$.

[30] The preference mismatch is determined as follows

$$pref\_mism = \int_{m^{\star}(1,0)}^{\overline{m}} \frac{m}{2} f(m)dm - \int_{\underline{m}}^{m^{\star}(1,0)} \frac{m}{2} f(m)dm + \int_{\underline{m}}^{0} \frac{m}{2} f(m)dm - \int_0^{\overline{m}} \frac{m}{2} f(m)dm \\ = - \int_0^{m^{\star}(1,0)} \frac{m}{2} f(m)dm - \int_0^{m^{\star}(1,0)} \frac{m}{2} f(m)dm = - \int_0^{m^{\star}(1,0)} mf(m)dm.$$

Totally differentiating $\tilde{\gamma}^S$ at equilibrium values with respect to ϕ, we obtain the following

$$\begin{aligned}\frac{d\tilde{\gamma}^S(\phi)}{d\phi} =& \frac{p_1^\star(1,0)\frac{\partial D_1^\star(1,0)}{\partial\phi} + \frac{\partial p_1^\star(1,0)}{\partial\phi}D_1^\star(1,0)}{D_2^\star(0,1)} \\ &- \frac{p_1^\star(1,0)D_1^\star(1,0) + p_2^\star(0,1)D_2^\star(0,1) - p_1^\star(1,1)}{[D_2^\star(0,1)]^2}\frac{\partial D_2^\star(0,1)}{\partial\phi}\end{aligned}$$

Recall that $\frac{\partial D_2^\star(0,1)}{\partial\phi} = -\frac{\partial D_1^\star(1,0)}{\partial\phi}$. We rewrite the above expression as follows

$$\begin{aligned}\frac{d\tilde{\gamma}^S(\phi)}{d\phi} =& \frac{D_2^\star(0,1)\Big(D_1^\star(1,0)\frac{\partial p_1^\star(1,0)}{\partial\phi} + \frac{\partial p_2^\star(0,1)}{\partial\phi}D_2^\star(0,1)\Big)}{[D_2^\star(0,1)]^2} \\ &+ \frac{\frac{\partial D_2^\star(0,1)}{\partial\phi}\Big(p_1^\star(1,1) - D_1^\star(1,0)p_1^\star(1,0) - D_2^\star(0,1)p_1^\star(1,0)\Big)}{[D_2^\star(0,1)]^2},\end{aligned}$$

which has the same sign as the numerator. Focusing on the numerator, we note that

- The first line is positive $D_1^\star(1,0) > D_2^\star(0,1)$ and $|\frac{\partial(p_1^\star(1,0)}{\partial\phi}| > |\frac{\partial(p_2^\star(0,1)}{\partial\phi}|$ which follows by the proof of Lemma 3.
- The second line is positive as $\frac{\partial D_2^\star(0,1)}{\partial\phi} < 0$ and the term within the brackets is:

  $$\begin{aligned}&p_1^\star(1,1) - p_1^\star(1,0)\Big(D_1^\star(1,0) + D_2^\star(0,1)\Big) = \\ =&p_1^\star(1,1) - p_1^\star(1,0) < 0\end{aligned}$$

  because $p_1^\star(1,1) < p_1^\star(1,0)$ by Lemma 3 and $D_1^\star(1,0) + D_2^\star(0,1) = 1$.

As a result, $\frac{d\tilde{\gamma}^S(\phi)}{\partial\phi} > 0$. This concludes the proof.

A.7. Proof of Proposition 6

Denote $\Delta\Pi^S \triangleq \Pi^{S,\star}(1,1) - \Pi^{S,\star}(1,0)$ as the net gain from non-exclusivity under vertical separation and $\Delta\Pi^{S,vi} \triangleq \Pi^{S,\star}(1) - \Pi^{S,\star}(0)$ as the net gain under vertical integration, i.e.,

$$\begin{aligned}\Delta\Pi^S =&\gamma^S\Big(1 - D_1^\star(1,0)\Big) + 2\Pi^\star(1,1) - \Pi_1^\star(1,0) - \Pi_1^\star(0,1), \\ \Delta\Pi^{S,vi} =&\gamma^S\Big(1 - D_1^\star(0)\Big) + 2\Pi^\star(1) - \Pi_1^\star(0) - \Pi_2^\star(0).\end{aligned}$$

Note that, in the baseline model with full market coverage, $\Pi^\star(1,1) = \Pi^\star(1)$.

In what follows, we identify the conditions under which $\Delta\Pi^S < \Delta\Pi^{S,vi}$. Note that when $\gamma^S = 0$, $\Delta\Pi^{S,vi} = \Delta\Pi^S < 0$ as $\Pi^\star(1,1) = \Pi^\star(1)$. Therefore, it is sufficient to show that $0 < \frac{\partial\Delta\Pi^S}{\partial\gamma^S} < \frac{\partial\Delta\Pi^{S,vi}}{\partial\gamma^S}$ to identify conditions under which $\Pi^\star(1,1) = \Pi^\star(1)$. To this end, we first observe that

$$\frac{\partial \Delta\Pi^S}{\partial \gamma^S} = 1 - D_1^\star(1,0). \tag{A-17}$$

The above arises directly from the fact that, under vertical separation, the platform market does not internalize the network benefit of the Superstar, γ^S. As a consequence of this, platform profits are independent of γ^S and, therefore, a change in γ^S impacts $\Delta\Pi^S$ directly via γ^S.

Second, we observe that

$$\frac{\partial \Delta\Pi^{S,vi}}{\partial \gamma^S} = 1 - D_1^\star(0) - \gamma^S \frac{\partial D_1^\star(0)}{\partial \gamma^S} - \left(\frac{\partial \Pi_1^\star(0)}{\partial \gamma^S} + \frac{\partial \Pi_2^\star(0)}{\partial \gamma^S}\right), \tag{A-18}$$

with $\frac{\partial \Pi_1^\star(0)}{\partial \gamma^S} < 0$ and $\frac{\partial \Pi_2^\star(0)}{\partial \gamma^S} < 0$ because of the downward pressure on prices exerted by γ^S.[31] Using (A-17) and (A-18),

$$\frac{\partial \Delta\Pi^S}{\partial \gamma^S} - \frac{\partial \Delta\Pi^{S,vi}}{\partial \gamma^S} < 0 \Longleftrightarrow 1 - D_1^\star(1,0) - \left(1 - D_1^\star(0) - \gamma^S \frac{\partial D_1^\star(0)}{\partial \gamma^S} - \left(\frac{\partial \Pi_1^\star(0)}{\partial \gamma^S} + \frac{\partial \Pi_2^\star(0)}{\partial \gamma^S}\right)\right) < 0.$$

Denote $\Delta D \triangleq D_1^\star(0) - D_1^\star(1,0)$, then $\frac{\partial \Delta\Pi^S}{\partial \gamma^S} - \frac{\partial \Delta\Pi^{S,vi}}{\partial \gamma^S} > (<) 0$

$$\Delta D < (>) \underbrace{-\left(\frac{\partial \Pi_1^\star(0)}{\partial \gamma^S} + \frac{\partial \Pi_2^\star(0)}{\partial \gamma^S}\right)}_{(+)} - \gamma^S \frac{\partial D_1^\star(0)}{\partial \gamma^S},$$

which then implies that there exists a threshold of ΔD such that $\Delta\Pi^S < (>) \Delta\Pi^{S,vi}$ if this is sufficiently low (resp. high). This concludes the proof.

[31] Recall that the merged entity internalizes the network benefit of the Superstar and, in turn, lowers its platform's price. Due to strategic complementarity, also the rival's price and profits decrease in γ^S.

Superstar exclusivity in two-sided markets

Online Appendix

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In this Appendix, we complement the analysis in the paper. In Section 1, we provide supporting evidence of industry practices. In Section 2, we provide an example with uniform distributions. In Section 3, we provide robustness checks and extensions whose discussion is presented in the paper.

1 Industry Background

Although practices and contract types may differ on a case-by-case basis, the industries discussed below are all characterized by interactions between different sides of the market, cross-group externalities, Superstar content provision, exclusive dealing, and/or some degree of vertical integration. Table 1 presents a summary.

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Music on-demand industry. In the music streaming market, the global growth rate reached 34% in 2019. The streaming market accounted for almost half of the music revenues (IFPI 2019). Starting with Apple Music and Tidal, exclusive dealing in this market has often emerged in the form of windowed releases. Windowed releases represent a new practice in the music industry market under which songs or albums are released exclusively on a platform for a limited period. Notable examples refer to Drake (*Views, Hotline Bling, Summer Sixteen*), Frank Ocean (*Blonde*, followed by his album *Endless*), Chance the Rapper (*Coloring Book*), and more recently PNL (with the *Deux Fréres album*) on Apple Music, Kanye West (*The Life of Pablo*), Rihanna (*Anti*) or Beyoncé (*Lemonade* and *Die With You*) on Tidal. Revenues from exclusive deals can be highly lucrative, ranging from \$ 500,000 obtained by Chance The Rapper to \$ 20 million by Drake, and equity stakes obtained by Rihanna, Kanye, Beyoncé.

Whereas these artists opted for exclusives, others continued to offer their records to their largest possible audience. For instance, Lady Gaga expressed her strong opinion against exclusive contracts. Opposition to these contracts also mounted on the platform side, with Spotify claiming in 2016 that Superstar exclusives were bad for artists, consumers, and platforms. In 2018, Spotify turned into exclusives as well (*e.g.,* with Taylor Swift's *Delicate* and the acoustic version of *Earth, Wind & Fire's September)* and, more recently, stuck a multi-year deal with Higher Ground Audio, a podcast company, to produce a series of podcasts with Barack and Michelle Obama, and with Joe Rogan. The exclusive deal of the author of "The Joe Rogan Experience" was worth more than \$100 million.[1] Moreover, the industry features several cases of vertical integration (*e.g.,* Tidal was launched as an artist-owned streaming platform by Jay-Z) and acquisitions (*e.g.,* Spotify acquired podcast producers Gimlet and Parcast).

Gaming industry. The gaming industry, which is expected to hit \$300 billion by 2025,[2] has been historically characterized by a large proportion of exclusive agreements, negotiations, and a high degree of vertical integration (Lee 2013). In

[1] See The New York Times, "Joe Rogan Strikes an Exclusive, Multiyear Deal With Spotify "
[2] See Variety, "Video Games Could Be a \$300 Billion Industry by 2025 (Report)"

this context, exclusivity may be console- or/and PC-specific, permanent or limited in time, or only related to some features of the videogame. In 2019, Epic Store, the gaming house producing the popular *Fortnite*, announced that "*store exclusives are the only way to improve Steam and the PC market*". Thanks to that game, Epic Store attracted as many as 85 million users on the platform and additional exclusive developers due to generous revenue split (*e.g., Metro Exodus*, initially planned to be released on Steam).[3]. In the same year, several small indie games, including *Ooblets*, announced exclusivity on that platform and an agreement was signed with Ubisoft on selected exclusive titles.

Most titles are developed in-house as first-party content, e.g., Epic's Fortnite was a publisher turned into a distributor. In the home console market, *MLB The Show 19, Gran Turismo Sport, The Last of Us, God of War,* are developed by Sony and only available on Sony's own console PlayStation (PS) 4. Nintendo released exclusively *Super Mario Odyssey* and *Pokemon: Sword and Shield* for its Switch, while in 2020 Electronic Arts (a gaming producer vertically integrated with Origin) announced the release of *Battlefield* non-exclusively for the competing platform, Twitch. Third-party developers are heterogeneous in their homing decisions, with some available exclusively on some consoles (*e.g.,* Marvel's *Spider Man* on PS), and others available non-exclusively (*e.g., Grand Theft Auto V* on Xbox and PS or Electronic Arts' *FIFA 2019* on Xbox, Switch, and PS).

E-sport Market. This market is worth $10.1 billion by the end of 2019 and consists of streaming live or pre-recorded games. Two platforms (YouTube Gaming and Amazon's Twitch) dominate the market, followed by fast-growing platforms such as Facebook Live and Microsoft's Mixer (StreamLab 2018). The most played game is *Fortnite*. Platforms compete by attracting game streamers and users paying a monthly subscription fee to have access to the platform. In 2019, a significant change in the industry concerned the decision of the most followed player (with more than 14 million followers), Ninja, to leave Twitch for an exclusive contract with Mixer. The unexpected decision of Ninja led to a drop in the number

[3] See *e.g.,* The Verge, "Epic Games Store chief says they'll eventually stop paying for exclusive PC games"

of viewers on the platform of origin and induced complementors to differentiate their content production (Förderer & Gutt 2021).[4]

Publishing Industry. In the publishing industry, audiobooks are on the rise, with revenues growing by 24.5% and more than 44,685 titles published in the US in 2018 (APA 2019). Platforms such as Amazon's Audible and Storytel charge consumers a fixed monthly fee for access to their audio-book catalog. This market is characterized by several exclusive titles. For instance, Audible has an exclusive agreement with Italian publishers (*e.g., Garzanti, Loganesi*) and so Storytel (with Gruppo Giunti's *Disney/Bompiani*). The former has also launched "Originals", a series of exclusives produced in-house by the platform and narrated by celebrated storytellers. In the US, Audible struck a deal directly with some best-selling authors by-passing major publishers (*e.g., Robert Caro, Jeffery Deaver, Michael Lewis*)[5] and Amazon's own distribution channel, ACX, allows right-holders (*e.g.,* authors, publishers) to distribute their rights exclusively to its network or non-exclusively to other retailers.

Apps and Developers Industry. The app market is characterized by two platforms, Apple iOs, and Android. Whereas most apps are available on both platforms, there are several others that are either exclusive on Apple iOS (*e.g., Bear, Timepage, Overcast*) or on Android (*e.g., Steam Link, Tasker*). Both platforms charge a fee to developers to get an account and publish their apps (*e.g.,* Google charges a one-time fee, whereas Apple a yearly fee). Developers can offer their apps for free and earn from in-app ads, ask for an upfront payment, or have

[4]Following Ninja's decision, the number of downloads of the app increased by 650,000 in five days. According to Streamlabs & Newzoo Q3 2019 Statistics, "Ninja's move may have spurred a significant migration of users to Mixer" and stimulated an influx of new streamers on the platform. Amongst others, in October 2019, another famous streamer, Shroud, left Twitch for Mixer. In June 2020, Microsoft shut down Mixer for a partnership with Facebook Gaming and later for an exclusive contract with Twitch. See e.g 'Twitch Streamers React to Ninja's Exclusive Move to Mixer' and The Business Insider, "Ninja became the first Mixer streamer to reach 1 million subscribers, less than a week after announcing he was ditching Twitch for Microsoft", August 7, 2019.. F

[5]See *e.g.,* The New York Times, "Want to Read Michael Lewis's Next Work? You'll Be Able to Listen to It First", June 2, 2019.

in-app purchases.[6] This market features a long tail of apps and a few tops and best-sellers (*e.g., Angry Birds, WhatsApp*) whose appearance might generate more entry in the market by similar apps (Ershov 2020) and act as discovery facilitators. Several apps are also built in-house or acquired by platforms, so featuring a certain degree of vertical integration (*e.g., Apple's Arcade* and *Shazam, Google's Suite*).

Shopping Mall Industry. Shopping malls are an example of non-digital platforms characterized by externalities. Consumers decide which mall to shop at depending on the number of retailers and their preferences (*e.g.,* distance), and retailers may sign exclusive/non-exclusive contracts with the mall. This market features the presence of anchor stores that can benefit from favorable contractual terms because of their demand externalities to non-anchor stores (Pashigian & Gould 1998, Gould et al. 2005). Moreover, exclusive dealing in the industry is common, and lease agreements often feature radius clauses, i.e., contractual arrangements which prohibit the opening of the same shopping activity within a given distance (Lentzner 1977). These clauses have attracted the attention of several competition agencies because of their potentially negative effect on market competition (*e.g.,* the German Federal Cartel Office, the UK Competition and Markets Authority).[7]

2 Example with uniform distributions

In this section, we provide an example of the results contained in the paper using uniform distribution for m and k. Specifically, we assume that m is uniformly distributed $\in (-\frac{1}{2}, \frac{1}{2})$, with $f(m) = 1$ and k is uniformly distributed $\in (0, 1)$ with $\lambda(k) = 1$. For ease of exposition, we further assume that $\gamma < 1$ and $\phi < 1$.

[6] Whereas exclusivity is common in this market, we are not currently aware of exclusive *contracts* between developers and platforms.

[7] See *e.g.,* Kluwer Competition Law Blog, "Property leases and competition law: Some clarity on restrictions in leases".

2.1 Vertical separation

Exclusivity. Following the baseline model, platforms set prices equal to

$$p_1^\star(1,0) = \frac{1}{2} - \gamma\theta + \frac{1}{3}\phi > \frac{1}{2} - \gamma\theta - \frac{1}{3}\phi = p_2^\star(0,1)$$

and the associated demands are $D_1^\star(1,0) = \frac{1}{2} + \frac{\phi}{3(1-2\gamma\theta)}$ and $D_2^\star(0,1) = 1 - D_1^\star(1,0)$. The number of complementors on each platform is $N_1^\star(1,0) = \gamma D_1^\star(1,0)$ and $N_2^\star(0,1) = \gamma D_2^\star(0,1)$, with $N_1^\star(1,0) > N_2^\star(0,1)$. The associated net platform's profits are

$$\Pi_1^\star(1,0) - T_1 = \frac{(2\phi + 3(1-2\gamma\theta))^2}{36(1-2\gamma\theta)} - T_1 \text{ and } \Pi_2^\star(0,1) = \frac{(2\phi - 3(1-2\gamma\theta))^2}{36(1-2\gamma\theta)}.$$

Under exclusivity, $T_1^\star(1,0) = \Pi_1^\star(1,0) - \Pi_1(0,1) = \frac{2\phi}{3}$. The Superstar obtains $\Pi^{S,\star}(1,0) = \gamma^S D_1^\star(1,0) + T_1^\star(1,0)$. Consumer surplus at the favored platform is

$$CS_1^\star(1,0) = \frac{(6\gamma\theta - 2\phi - 3)(2\phi(12\gamma\theta - 7) + 3(2\gamma\theta - 1)(12\gamma\theta + 8v - 3))}{144(1-2\gamma\theta)^2}$$

Consumer surplus at the unfavored platform is

$$CS_2^\star(0,1) = \frac{(6\gamma\theta + 2\phi - 3)(2\phi(12\gamma\theta - 5) + 3(2\gamma\theta - 1)(12\gamma\theta + 8v - 3))}{144(1-2\gamma\theta)^2}$$

Total consumer surplus under exclusivity is:

$$CS^\star(1,0) = v + \frac{1}{8}\phi\left(9 + \frac{\phi}{(1-2\gamma\theta)^2}\right) + \frac{3\gamma\theta}{2} - \frac{3}{8}.$$

Non-exclusivity. The platforms set prices equal to

$$p_1^\star(1,1) = p_2^\star(1,1) = \frac{1}{2} - \gamma\theta.$$

The associated demand is $D_1^\star(1,1) = D_2^\star(1,1) = \frac{1}{2}$, with $N_1^\star(1,1) = N_2^\star(1,1) = \gamma/2$. Gross platform's profits (before paying the non-exclusive tariff) are

$$\Pi_i^\star(1,1) = \frac{1-2\gamma\theta}{4}.$$

Under non-exclusivity, the Superstar collects $T^\star(1,1) = T_1^\star(1,1) = T_2^\star(1,1) = \phi\frac{3-6\gamma\theta-\phi}{9(1-2\gamma\theta)}$ and overall she obtains $\Pi^{S,\star}(1,1) = \gamma^S + 2T^\star(1,1)$. Consumer surplus under non-exclusivity is

$$CS^\star(1,1) = v + \phi + \frac{3\gamma\theta}{2} - \frac{3}{8}.$$

The consumer surplus at each platform is $CS_1^\star(1,1) = CS_2^\star(1,1) = CS^\star(1,1)/2$.

Exclusivity decision. Comparing the Superstar's profits in the two exclusivity scenarios, then $\Pi^{S,\star}(1,0) > \Pi^{S,\star}(1,1)$ if, and only if, $\gamma^S \leq \tilde{\gamma}^S$ where $\tilde{\gamma}^S \equiv \frac{4\phi^2}{3(3-2\phi-6\gamma\theta)}$. Indeed, exclusivity is chosen by the Superstar if γ^S is sufficiently low and non-exclusivity is chosen if γ^S is sufficiently large.[8]

Welfare-enhancing exclusivity. Because complementors always benefit from exclusivity, a sufficient condition for exclusivity to be welfare-enhancing is that it raises consumer surplus relative to non-exclusivity. Comparing consumer surplus in the two exclusivity regimes, consumers at the favored platform benefit from exclusivity as

$$CS_1^\star(1,0) - CS_1^\star(1,1) = \frac{\phi(-12\gamma\theta(\phi+1) + v(12-24\gamma\theta) + 7\phi + 6)}{36(1-2\gamma\theta)^2} > 0,$$

in the relevant parameter space. Second, consumers at the unfavored platform always suffer from exclusivity as

$$CS_2^\star(0,1) - CS_2^\star(1,1) = \frac{\phi(\phi(12\gamma\theta-5) + 12(2\gamma\theta-1)(3\gamma\theta+v-1))}{36(1-2\gamma\theta)^2} < 0,$$

[8] To ensure concavity and rule out market tipping, we assume that $\phi < \frac{3}{2}$ and $\theta < \frac{3-2\phi}{\gamma^S}$.

in the relevant parameter space.[9] Considering both gains and losses from exclusivity, we compare total consumer surplus and observe that

$$\Delta CS = CS^{\star}(1,0) - CS^{\star}(1,1) = \frac{1}{18}\phi\Big(\frac{\phi}{(1-2\gamma\theta)^2} - 9\Big),$$

which is positive for $\theta > \frac{1}{2\gamma} - \frac{1}{6}\sqrt{\frac{\phi}{\gamma^2}} > 0$ and $\phi < 1/4$. The positive effect of exclusivity on the consumer surplus at the favored platform dominates the negative effects on consumer surplus at the unfavored platform provided that cross-group externalities are large enough. This result confirms Proposition 3.

Together with the conditions under which the Superstar finds it optimal to choose exclusivity, then exclusivity arises and it is welfare-enhancing when the following conditions are jointly satisfied:

$$\theta > \frac{1}{2\gamma} - \frac{\sqrt{\frac{\phi}{\gamma^2}}}{6} > 0, \quad \phi < 1/4, \quad \gamma^S \leq \frac{4\phi^2}{3(3-2\phi-6\gamma\theta)}.$$

These intervals of parameters form a non-empty set. In the paper, we provide intuitions for these results.

2.2 Vertical integration

Consider the case with vertical integration, such that $g_1 = 1$.

Exclusivity. The merged entity and the independent platform set prices equal to

$$p_1^{\star}(0) = \frac{3-4\gamma^S-6\gamma\theta+2\phi}{6} > p_2^{\star}(0) = \frac{3-2\gamma^S-6\gamma\theta-2\phi}{6}$$

and the associated demands are $D_1^{\star}(0) = \frac{1}{2} + \frac{\phi+\gamma^S}{3(1-2\gamma\theta)}$ and $D_2^{\star}(0) = 1 - D_1^{\star}(0)$. The number of complementors on each platform is $N_1^{\star}(1,0) = \gamma D_1^{\star}(0)$ and $N_2^{\star}(0) =$

[9] Recall that v is large enough to ensure full market coverage and positive consumer surplus at the two platforms.

$\gamma D_2^\star(0)$, with $N_1^\star(0) > N_2^\star(0)$. The associated profits are

$$\Pi^{S,\star}(0) = \frac{(2(\gamma^S + \phi) + 3(1 - 2\gamma\theta))^2}{36(1 - 2\gamma\theta)} \text{ and } \Pi_2^\star(0) = \frac{(2(\gamma^S + \phi) - 3(1 - 2\gamma\theta))^2}{36(1 - 2\gamma\theta)}.$$

Non-exclusivity. The merged entity and the independent platform set prices equal to

$$p_1^\star(1) = p_2^\star(1) = \frac{1}{2} - \gamma\theta$$

and obtain $D_1^\star(1) = D_2^\star(1) = \frac{1}{2}$, with $N_1^\star(1) = N_2^\star(1) = \gamma/2$. The associated profits are

$$\Pi^{S,\star}(1) = \frac{1 - 2\gamma\theta}{4} + \gamma^S + T_2 \text{ and } \Pi_2^*(1) = \frac{1 - 2\gamma\theta}{4} - T_2.$$

The merged entity's non-exclusive tariff is $T_2^\star = \frac{(\gamma^S+\phi)(3-6\gamma\theta-\phi-\gamma^S)}{9(1-2\gamma\theta)}$ and the total profit is

$$\Pi^{S,\star}(1) = \frac{1 - 2\gamma\theta}{4} + \gamma^S + \frac{(\gamma^S + \phi)(3 - 6\gamma\theta - \phi - \gamma^S)}{9(1 - 2\gamma\theta)}$$

Exclusivity decision. Comparing profits, $\Pi^{S,\star}(0) > \Pi^{S,\star}(1)$ if, and only if, $\gamma^S \leq \tilde{\gamma}^{VI}$ where $\tilde{\gamma}^{VI} = \min\left\{ \frac{\left(9-4\phi-18\gamma\theta-3\sqrt{(1-2\gamma\theta)(9-18\gamma\theta-8\phi)}\right)}{4}, \frac{(3-6\gamma\theta-2\phi)}{2} \right\}$.

Comparing $\tilde{\gamma}^{VI}$ and $\tilde{\gamma}^S$, it is immediate that $\tilde{\gamma}^{VI} < \tilde{\gamma}^S$ holds always. In turn, with a uniform distribution, exclusivity is always less likely to arise under vertical integration than under vertical separation. This is a special case of what is considered in Proposition 6. This concludes the analysis.

3 Discussion and Extensions

In this section, we relax some of the assumptions that were made in the baseline model and provide formal analyses of the discussion presented in Section 6.

3.1 Two-sided pricing

There are several platforms that set prices on both sides of the market. In this extension, we provide an example of how our results are robust to a setting in which a platform charges both complementors and consumers. For the variation with two-sided pricing, we follow the same approach as in this Online Appendix - Section 2 and provide an example using uniform distributions.

Consumers. Consumer demands are derived as in the baseline mode. The indifferent consumer is denoted by $\tilde{m}(\cdot) = \phi(g_1 - g_2) - (p_1 - p_2) + \theta(N_1^e - N_2^e)$, which is a function of the expected mass of complementors on the two platforms and related prices. The demand on platform 1 is $D_1(\cdot) \triangleq \tilde{m}(\cdot) + \frac{1}{2}$, whereas the demand on platform 2 is $D_2(\cdot) \triangleq 1 - D_1(\cdot)$.

Complementors. Complementors' payoff from affiliating with platform i is $u_i^s = \gamma \cdot D_i^e - l_i - k$, where l_i is the price charged to complementors and D_i^e is the expected mass of consumers at platform i. Complementors are active on a platform if, and only if, $u_i^s \geq 0$. The mass of complementors on platform i is, therefore, $N_i(\cdot) = \gamma D_i^e - l_i$. For $l_i < 0$, complementors are subsidized.

Platform profits. Platform i's profits, before any payment to the Superstar, are $\Pi_i(\cdot) = p_i D_i(\cdot) + l_i N_i(\cdot)$. To ensure concavity and rule out market tipping, we assume $0 < \phi < 1/2(3 - \gamma^2 - 4\gamma\theta - \theta^2)$, and $\theta < \min\left\{\frac{2-\gamma^2}{3\gamma}, 2\sqrt{1+2\gamma^2} - 3\gamma\right\}$.

Analysis. In the third stage, imposing fulfilled expectations — $\tilde{N}_i = N_i^e$ and $\tilde{D}_i = D_i^e$ — and solving for consumers' and complementors' demand, we obtain

$$\begin{aligned}
\tilde{D}_1(p_1, p_2, l_1, l_2, g_1, g_2) &= \frac{1}{2} + \frac{\theta(l_2 - l_1) + (p_2 - p_1) + \phi(g_1 - g_2)}{1 - 2\gamma\theta}, \ \tilde{N}_1(\cdot) = \gamma \tilde{D}_1(\cdot) - l_1 \\
\tilde{D}_2(p_2, p_1, l_2, l_1, g_2, g_1) &= 1 - D_1(\cdot), \ \tilde{N}_2(\cdot) = \gamma \tilde{D}_2(\cdot) - l_2.
\end{aligned}$$

Given (g_i, g_j), in stage 2, each platform sets prices to maximize the following profits

$$\max_{p_i, l_i} \tilde{\Pi}_i(\cdot) = p_i \tilde{D}_i(\cdot) + l_i \tilde{N}_i(\cdot). \tag{1}$$

Differentiating the profit of each platform with respect to p_i and l_i and solving the first-order conditions simultaneously, we obtain prices as functions of g_i and g_j only

$$p_i^\star(g_i, g_j) = \frac{(2 - \gamma^2 + 3\gamma\theta)(3 - \gamma^2 - 4\gamma\theta - \theta^2 - 2\phi(g_i - g_j))}{4\eta} \tag{2}$$

$$l_i^\star(g_i, g_j) = \frac{(\gamma - \theta)}{4}\left(1 + \frac{2\phi(g_j - q_i)}{\eta}\right), \tag{3}$$

where $\eta \triangleq 3 - \gamma^2 - 4\gamma\theta - \theta^2 > 0$.

We now identify the equilibrium outcomes under exclusivity and non-exclusivity. Under non-exclusivity ($g_i = g_j = 1$), platforms are symmetric. Therefore $p_1^\star(1,1) = p_2^\star(1,1) = 1/2 - \gamma(\gamma + 3\theta)/4$ for consumers and $l_1^\star(1,1) = l_2^\star(1,1) = (\gamma - \theta)/4$ for the complementors. Demands are given by $D_1^\star(1,1) = D_2^\star(1,1) = 1/2$ and $N_1^\star(1,1) = N_2^\star(1,1) = (\gamma + \theta)/4$.

Under exclusivity with platform 1 ($g_1 = 1$ and $g_2 = 0$), equilibrium prices on the consumer side are:

$$p_1^\star(1,0) = p^\star(1,1)\left(1 + \frac{2\phi}{\eta}\right), \qquad p_2^\star(0,1) = p^\star(1,1)\left(1 - \frac{2\phi}{\eta}\right).$$

Equilibrium prices for complementors are:

$$l_1^\star(1,0) = l^\star(1,1)\left(1 + \frac{2\phi}{\eta}\right), \qquad l_2^\star(0,1) = l^\star(1,1)\left(1 - \frac{2\phi}{\eta}\right),$$

Note that $p_1^\star(1,0) > p_1^\star(1,1) > p_2^\star(0,1) > 0$. If $\gamma > \theta$, the price for complementors is positive and increases with the value generated by the Superstar ϕ. If $\gamma < \theta$, complementors are subsidized and the subsidy increases with ϕ.

Regardless of the pricing strategy, as in the baseline model, there is an agglomeration of complementors on the *favored* platform, such that

$$N_1^\star(1,0) = N_1^\star(1,1)\Big(1 + \frac{2\phi}{\eta}\Big), \qquad N_2^\star(0,1) = N_1^\star(1,1)\Big(1 - \frac{2\phi}{\eta}\Big)$$

with $N_1^\star(1,0) > N_1^\star(1,1) > N_2^\star(0,1)$ as in Proposition 1. The exclusivity decision of the Superstar in the two regimes follows the same reasoning as in the main model. Under exclusivity, the Superstar's profit is

$$\Pi^S(1,0) = \frac{\gamma^S}{2} + \frac{\phi(2-\gamma^2+\gamma^S-3\gamma\theta)}{\eta}.$$

Under non-exclusivity, the Superstar's profit is

$$\Pi^S(1,1) = \gamma^S + \frac{\phi(2-\gamma^2-3\gamma\theta)(\eta-\phi)}{\eta^2}.$$

Therefore, $\Pi^S(1,0) > \Pi^S(1,1)$ if, and only if, $\gamma^S < \tilde{\gamma}^S$, where:

$$\tilde{\gamma}^S = \frac{2\phi^2(2-\gamma^2-3\gamma\theta)\phi^2}{\eta(\eta-2\phi)}.$$

Otherwise, non-exclusivity is chosen. The mechanism behind this result is identical to the one discussed in the baseline model.

3.2 Asymmetric platforms

In this section, we relax the assumption of symmetry between platforms and prove that our main results may hold qualitatively under some conditions. To this end, we first consider the case in which there is vertical separation. Then, we consider the case of a merged entity that includes the Superstar and the high-quality platform.

Formally, we capture asymmetry between platforms by assuming asymmetry in the standalone utility that each platform provides to consumers. We denote platform 1 (resp. 2) as the high-quality (resp. low-quality) platform. Consumers are uniformly distributed according to their preferences, with $m \sim \mathcal{U}(-1/2, 1/2)$ denoting their type. Consumers affiliating to platform 1 obtain the following utility

$$u_1 = v + \Delta + \theta N_1^e + g_1\phi - p_1 - \frac{m}{2},$$

with $\Delta > 0$ representing the additional value provided by the high-quality platform relative to platform 2, whose consumers obtain

$$u_2 = v + \theta N_2^e + g_2\phi - p_2 + \frac{m}{2}.$$

Consumers join platform 1 if $u_1(m) \geq u_2(m)$, and the consumer indifferent between the two platforms is denoted by

$$\tilde{m}(p_1, p_2, N_1^e, N_2^e, g_1, g_2) = \Delta + \phi(g_1 - g_2) + \theta(N_1^e - N_2^e) - (p_1 - p_2).$$

Consumers' demand is denoted by $D_1(p_1, p_2, N_1^e, N_2^e, g_1, g_2) = 1/2 + \tilde{m}(p_1, p_2, N_1^e, N_2^e, g_1, g_2)$ and $D_2(p_2, p_1, N_2^e, N_1^e, g_2, g_1) = 1 - D_1(p_1, p_2, N_1^e, N_2^e, g_1, g_2)$.

Complementors are uniformly distributed according to their outside option as $k \sim \mathcal{U}(0, 1)$ with the associated density of 1. Thus, the number of complementors at each platform is $N_i(D_i^e, g_i, g_j) = \gamma D_i^e$. As in the baseline model, we assume fulfilled expectations $N_i(D_i^e, g_i, g_j) = N_i^e$ and $D_i(p_1, p_2, N_1^e, N_2^e, g_1, g_2) = D_i^e$. Solving the system of equations, we obtain the demands at each platform

$$\begin{aligned}
\tilde{D}_1(p_1, p_2, g_1, g_2) =& \frac{1 - 2\rho + 2\Delta + 2\phi(g_1 - g_2) - 2(p_1 - p_2)}{2(1 - 2\rho)}, \\
\tilde{D}_2(p_2, p_1, g_2, g_1) =& 1 - \tilde{D}_1(p_1, p_2, g_1, g_2), \\
\tilde{N}_1(p_1, p_2, g_1, g_2) =& \gamma \tilde{D}_1(p_1, p_2, g_1, g_2), \\
\tilde{N}_2(p_2, p_1, g_2, g_1) =& \gamma \tilde{D}_2(p_2, p_1, g_2, g_1),
\end{aligned} \tag{4}$$

with $\rho \triangleq \gamma\theta$ for ease of exposition. To ensure market coverage and profit concavity, we assume $\rho < \frac{1}{6}(3 - 2\phi), \Delta < \frac{1}{2}(3 - 6\rho - 2\phi)$.

Vertical separation

Under vertical separation, each platform maximizes its gross profit

$$\tilde{\Pi}_i(p_1, p_2, g_1, g_2) = p_i \tilde{D}_i(p_1, p_2, g_1, g_2) \qquad \forall i = 1, 2.$$

From the first-order conditions, we obtain the equilibrium prices

$$\begin{aligned} p_1^\star(g_1, g_2) =& \frac{1}{6}\Big(3 + 2\Delta + 2\phi(g_1 - g_2) - 6\rho\Big), \\ p_2^\star(g_2, g_1) =& \frac{1}{6}\Big(3 - 2\Delta - 2\phi(g_1 - g_2) - 6\rho\Big). \end{aligned} \tag{5}$$

Exclusivity with the low-quality platform. If the Superstar is exclusive on platform 2, then $g_1 = 0$ and $g_2 = 1$. Using (5) and (4), the gross equilibrium profits are

$$\begin{aligned} \Pi_1^\star(0, 1) =& \frac{(3 + 2\Delta - 6\rho - 2\phi)^2}{36(1 - 2\rho)}, \\ \Pi_2^\star(1, 0) =& \frac{(3 - 2\Delta - 6\rho + 2\phi)^2}{36(1 - 2\rho)}. \end{aligned}$$

Exclusivity with the high-quality platform. If the Superstar is exclusive on platform 1, then $g_1 = 1$ and $g_2 = 0$. Using (4) and (5), the gross equilibrium profits are

$$\Pi_1^\star(1, 0) = \frac{(3 + 2\Delta - 6\rho + 2\phi)^2}{36(1 - 2\rho)}, \qquad \Pi_2^\star(0, 1) = \frac{(3 - 2\Delta - 6\rho - 2\phi)^2}{36(1 - 2\rho)}.$$

Non-exclusivity. If the Superstar is non-exclusive, i.e., $g_1 = g_2 = 1$. Using (5) and (4), the equilibrium gross profits of the two platforms are as follows

$$\Pi_1^\star(1, 1) = \frac{(3 + 2\Delta - 6\rho)^2}{36(1 - 2\rho)}, \qquad \Pi_2^\star(1, 1) = \frac{(3 - 2\Delta - 6\rho)^2}{36(1 - 2\rho)}.$$

Exclusivity decision. In what follows, we first elicit the willingness to pay for exclusivity and non-exclusivity for each platform. Then, we present the contractual framework as in the baseline model.

The willingness-to-pay for exclusivity is denoted as $WTP_i \triangleq \Pi_i^\star(1,0) - \Pi_i^\star(0,1)$, where

$$\Pi_1^\star(1,0) - \Pi_1^\star(0,1) > \Pi_2^\star(1,0) - \Pi_2^\star(0,1).$$

The above inequality implies that, in an auction for exclusivity, $WTP_1 > WTP_2$. This implies that the high-quality platform can always outbid the low-quality platform by just offering $\Pi_2^\star(1,0) - \Pi_2^\star(0,1) + \varepsilon$, with $\varepsilon \to 0$. In turn, exclusivity (conditional on being chosen at equilibrium) only occurs with the high-quality platform. The optimal tariff under exclusivity is $T^\star(1,0) \triangleq \Pi_2^\star(1,0) - \Pi_2^\star(0,1)$ and, therefore,

$$T^\star(1,0) = \frac{2\phi(3 - 2\Delta - 6\rho)}{9(1 - 2\rho)},$$

which decreases in Δ.

Consider the case in which a tariff is non-discriminatory under non-exclusivity. The Superstar offers a non-exclusive contract that satisfies the participation constraint of the low-quality platform. Else, this platform would not sign the non-exclusive contract and the high-quality platform would be *a-fortiori* exclusive. The tariff that satisfies the participation constraint of the low-quality platform is $T^\star(1,1) = \Pi_2^\star(1,1) - \Pi_2^\star(0,1)$ and, therefore,

$$T^\star(1,1) = \frac{\phi(3 - 2\Delta - 6\rho - \phi)}{9(1 - 2\rho)},$$

As in the baseline model, this represents the reserve price of the auction set by the Superstar.

As exclusivity with the low-quality platform is never optimal, the choice of the Superstar is between exclusivity with the high-quality platform (via an auction) or non-exclusivity. Comparing the profit of the Superstar in the two cases, exclusivity occurs if $\Pi^{S,\star}(1,0) \equiv T^\star(1,0) + \gamma^S D_1^\star(1,0) > 2T^\star(1,1) + \gamma^S \equiv \Pi^{S,\star}(1,1)$, that is

$$\frac{3\gamma^S(2\Delta - 6\rho + 2\phi + 3) + 4\phi(3 - 2\Delta - 6\rho)}{18 - 36\rho} > \frac{9\gamma^S(1-2\rho) + 2\phi(3 - 2\Delta - 6\rho + \phi)}{9 - 18\rho}$$

$$\Leftrightarrow \gamma^S < \frac{4\phi^2}{9 - 6\Delta - 18\rho - 6\phi} \equiv \tilde{\gamma}^S$$

As in the baseline model, there exists a threshold value of γ^S above (resp. below) which non-exclusivity (resp. exclusivity) is chosen by the Superstar.

The total consumer surplus under exclusivity is

$$CS^\star(1,0) = v - \frac{(3 - 12\rho - 4\phi)}{8} + \frac{\Delta^2 + 9\Delta(1-2\rho)^2 + 2\Delta\phi + \phi^2}{18(1-2\rho)^2},$$

The total consumer surplus under non-exclusivity is

$$CS^\star(1,1) = v + \phi + \frac{3\rho}{2} - \frac{3}{8} + \frac{1}{18}\Delta\left(\frac{\Delta}{(1-2\rho)^2} + 9\right).$$

Denoting $\Delta CS \triangleq CS^\star(1,0) - CS^\star(1,1)$,

$$\Delta CS = \frac{\phi\left(2\Delta - 9(1-2\rho)^2 + \phi\right)}{18(1-2\rho)^2},$$

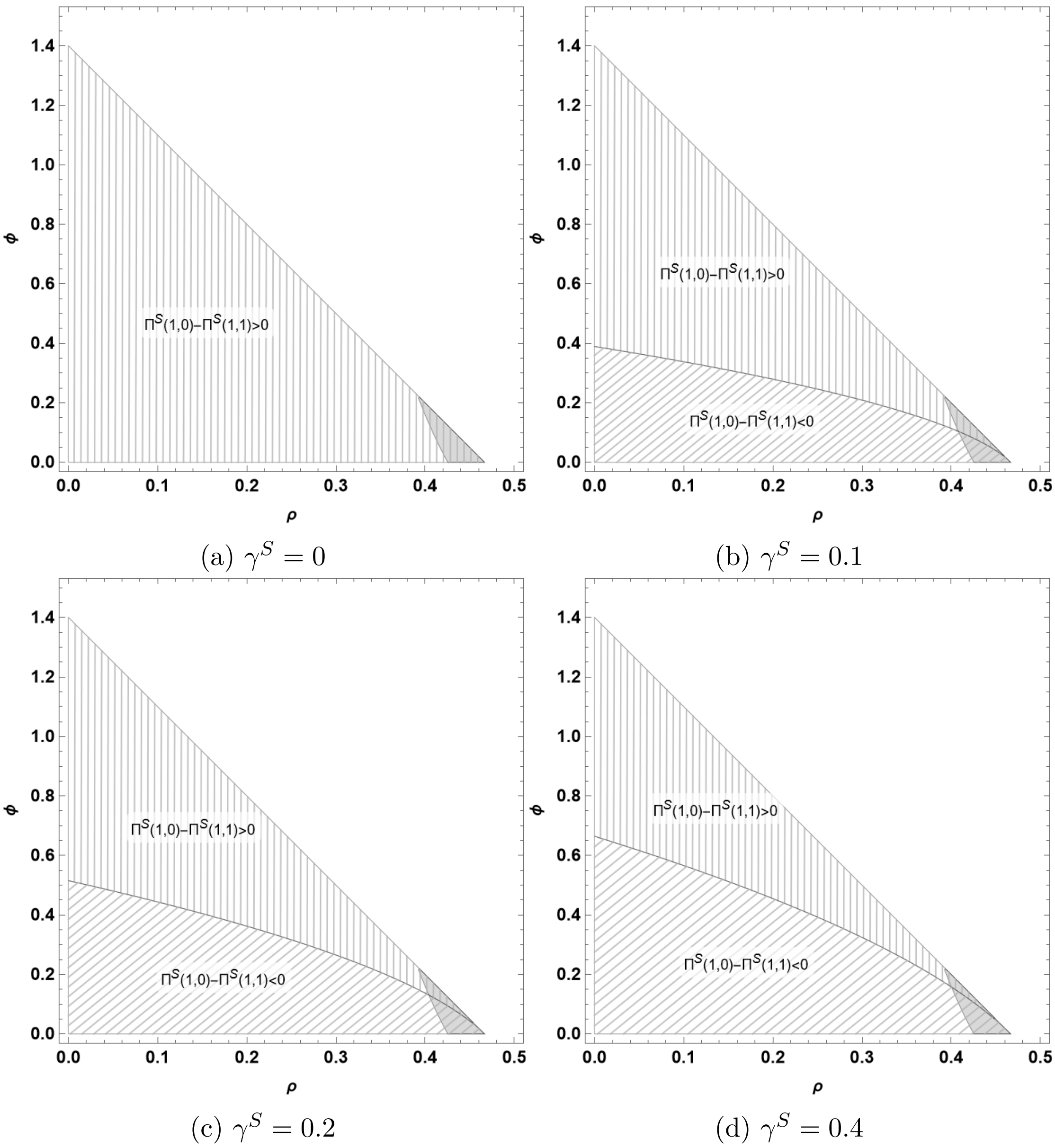
which is positive (resp. negative) if

$$\rho > (<) \frac{\left(3 - \sqrt{2\Delta + \phi}\right)}{6} \equiv \tilde{\rho}.$$

Note that the above expression does not depend on γ^S. Therefore, for $\gamma^S < \tilde{\gamma}^S$ and $\rho > \tilde{\rho}$, exclusivity is chosen by the Superstar ($\gamma \leq \tilde{\gamma}^S$) and this choice leads to a higher consumer surplus.

Figure (1) presents a graphical summary of the above results for different values of $\gamma^S = \{0, 0.1, 0.2, 0.4\}$. On the x-axis, there is the value of the cross-group network effects $\rho = \gamma\theta$, whereas on the y-axis there is the value generated by the Superstar ϕ. The white region identifies the area in which tipping occurs. Exclusivity increases consumer surplus relative to non-exclusivity, $CS^\star(1,0) - CS^\star(1,1) > 0$, in the the gray shaded region. The Superstar finds it optimal to choose non-exclusivity, $\Pi^S(1,0) - \Pi^S(1,1) < 0$, in the region shaded with diagonal lines. Finally, the Superstar finds it optimal to choose exclusivity, $\Pi^S(1,0) - \Pi^S(1,1) > 0$, in the region shaded with vertical lines.

In this example, we assume that $\Delta = 0.1$. We show that there exists an area in which exclusivity emerges and is beneficial to consumers. This area is identified by the overlap between the gray shaded area and the area shaded with vertical lines. Importantly, this area is small in size and is decreasing with γ^S.



(a) $\gamma^S = 0$ (b) $\gamma^S = 0.1$

(c) $\gamma^S = 0.2$ (d) $\gamma^S = 0.4$

Figure 1: Exclusivity and consumer welfare under vertical separation

Vertical Integration

Suppose now the high-quality platform is integrated with the Superstar.[10] Under exclusivity, the Superstar is merged with platform 1, demands are as in (4) when $g_1 = 1$ and $g_2 = 0$. The merged entity maximizes $\tilde{\Pi}^S(p_1, p_2, 1, 0) = \Big(p_1 + \gamma^S\Big)\tilde{D}_1(p_1, p_2, 1, 0)$ whereas the non-merged entity maximizes $\tilde{\Pi}_2(p_2, p_1, 0, 1) = p_2\tilde{D}_2(p_2, p_1, 0, 1)$.

Differentiating the profits of the merged entity and of platform 2 with respect to p_1 and p_2, respectively, and solving the first-order conditions simultaneously yields

$$\begin{aligned} p_1^\star(0) =& \frac{1}{6}(3 - 4\gamma^S + 2\Delta - 6\rho + 2\phi), \\ p_2^\star(0) =& \frac{1}{6}(3 - 2\gamma^S - 2\Delta - 6\rho - 2\phi), \end{aligned}$$

which decrease with γ^S. Using equilibrium prices and demands in (4), the net equilibrium profits of the two platforms are denoted by

$$\begin{aligned} \Pi^S(0) =& \frac{(3 + 2\gamma^S + 2\Delta - 6\rho + 2\phi)^2}{36(1 - 2\rho)} \\ \Pi_2^\star(0) =& \frac{(3 - 2\Delta - 2\gamma^S - 6\rho - 2\phi)^2}{36(1 - 2\rho)} \end{aligned}$$

Under non-exclusivity, the market outcome is equivalent to that of vertical separation. Therefore, $p_1^\star(1) = p_1^\star(1, 1)$, $p_2^\star(1) = p_2^\star(1, 1)$, $D_1^\star(1) = D_1^\star(1, 1)$, $D_2^\star(1) = D_2^\star(1, 1)$. The merged entity obtains

$$\Pi^S(1) = p_1^\star(1)D_1^\star(1) + T(1) + \gamma^S = \gamma^S + \frac{(3 + 2\Delta - 6\rho)^2}{36(1 - 2\rho)} + T(1),$$

whereas the non-merged entity (before paying the non-exclusive tariff $T(1)$) obtains

$$\Pi_2^\star(1) = p_2^\star(1)D_2^\star(1) = \frac{(3 - 2\Delta - 6\rho)^2}{36(1 - 2\rho)}.$$

[10] We rule out the case in which the small platform is integrated with the Superstar. This is a reasonable assumption as the Superstar provides a premium product and it is reasonable to assume that it is hosted by the platform of a higher rather than a lower quality.

Exclusivity decision. Under non-exclusivity, the owner of the merged sets the following tariff

$$T^{\star}(1) \triangleq \Pi_2^{\star}(1) - \Pi_2^{\star}(0) = \frac{(\gamma^S + \phi)(3 - \gamma^S - 2\Delta - 6\rho - \phi)}{9(1-2\rho)}.$$

Computing and comparing profits in two regimes, $\Pi^S(1) - \Pi^S(0) > 0$ if

$$\frac{\gamma^S(9 - 2\gamma^S - 4\Delta - 18\rho) - 4\phi(\gamma^S + \Delta) - 2\phi^2}{9(1-2\rho)} > 0$$

$$\Leftrightarrow \gamma^S \geq \frac{1}{4}\left(9 - 4\Delta - 18\rho - 4\phi - \sqrt{(4\Delta + 18\rho - 9)^2 + 72(2\rho - 1)\phi}\right) \equiv \tilde{\gamma}^{S,vi}.$$

Otherwise, for any $\gamma^S < \tilde{\gamma}^{S,vi}$, the merged entity will retain exclusivity, as in the baseline model.

The total consumer surplus under exclusivity is

$$CS^{\star}(0) = v + \frac{4(\gamma^S + \Delta + \phi)(9 + \gamma^S + \Delta + \phi - 36\rho(1-\rho)) - 27(1-2\rho)^2(1-4\rho)}{72(1-2\rho)^2}.$$

The total consumer surplus under non-exclusivity is

$$CS^{\star}(1) = \frac{1}{18}\Delta\left(\frac{\Delta}{(1-2\rho)^2} + 9\right) + \frac{3\rho}{2} + v + \phi - \frac{3}{8}.$$

Denoting $\Delta CS = CS^{\star}(0) - CS^{\star}(1)$, then

$$\Delta CS = \frac{(\gamma^S)^2 + \gamma^S(9 + 2\Delta - 36\rho(1-\rho) + 2\phi) + \phi\left(2\Delta + \phi - 9(1-2\rho)^2\right)}{18(1-2\rho)^2}$$

which is positive (resp. negative) if $\gamma^S > (<) \tilde{\gamma}^{CS,vi} \triangleq \frac{36\rho(1-\rho) - 9 - 2\Delta - 2\phi - \sqrt{(2\Delta + 9(1-2\rho)^2)^2 + 72\phi(1-2\rho)^2}}{2}$.

Numerical Example. In order to verify whether the insights from the baseline model extend to the case in which firms are asymmetric, we rely on a numerical example. We assume $\Delta = 0.1$, $\phi = 0.4$ and $\gamma^S = 0.2$. In this parameter constellation, $\tilde{\gamma}^{S,vi} > \tilde{\gamma}^{CS,vi}$, which implies that if $\tilde{\gamma}^{CS,vi} < \gamma^S < \tilde{\gamma}^{S,vi}$, the platform chooses exclusivity and consumers benefit from exclusivity. Specifically,

$$\Pi^S(0) - \Pi^S(1) = \frac{8}{75 - 150\rho} - \frac{1}{5} > 0 \Longleftrightarrow 0.23 \lessapprox \rho \lessapprox 0.26$$

and

$$\Delta CS = \frac{2}{75(1-2\rho)^2} - \frac{1}{10} > 0 \Longleftrightarrow 0.24 \lessapprox \rho \lessapprox 0.26$$

and $\Delta CS < 0 \Longleftrightarrow \rho \lessapprox 0.24$.

This analysis suggests that, as in the baseline model, there exists a non-empty interval in which exclusivity occurs at equilibrium and it benefits consumers relative to non-exclusivity (i.e., $0.24 \lessapprox \rho \lessapprox 0.26$). However, this interval is small in size. Likewise, there exists a non-empty interval in which exclusivity occurs at equilibrium but it hurts consumers relative to non-exclusivity (i.e., $0.23 \lessapprox \rho \lessapprox 0.24$). In Figure (2), the triangle identifying the overlap between the shaded gray area and the area with vertical lines represents cases in which exclusivity benefits consumers.

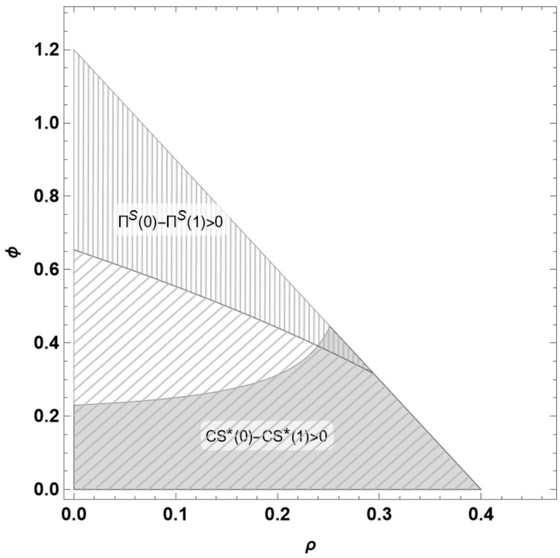


Figure 2: Example of exclusivity and consumer welfare under vertical integration ($\Delta = 0.1$ and $\gamma^S = 0.2$).

3.3 Alternative mode of competition and elastic demand participation

In this extension, we study the exclusivity decision of the Superstar and the merged entity in an uncovered market. We consider a variation of the baseline model in which consumers exhibit preferences *à la* Singh & Vives (1984) and we adapt these preferences to a two-sided market setting. For tractability, we assume that complementors' opportunity costs are distributed uniformly, i.e., $k \sim U[0,1]$. We consider a representative consumer, whose utility is

$$U \triangleq \sum_{i=\{1,2\}} \left[(1 + \theta N_i^e + \phi g_i) q_i - \frac{q_i^2}{2} \right] - \beta q_1 q_2 - \sum_{i=1,2} p_i q_i \tag{6}$$

where q_i and N_i^e is demand and the expected number of complementors at platform i, respectively, with $\beta \in (0,1)$ being the degree of product differentiation. For simplicity, we normalize $\beta = \frac{1}{2}$. Moreover, to ensure concavity in profits and rule out market tipping, we assume that cross-group externalities are not too strong:

$$\rho \triangleq \theta\gamma < \frac{7-\phi}{8} - \frac{1}{8}\sqrt{\phi^2 + 2\phi + 9}, \qquad \phi < \frac{5}{2} \tag{7}$$

The variable transformation ρ affords brevity in the exposition of our results.

The consumer demand for each platform i is represented by the quantity q_i for $i \in \{1,2\}$ that maximizes (6). Solving the system of $\partial U / \partial q_i = 0$ for $i \in \{1,2\}$, yields demands as a function of prices and the expected number of complementors

$$D_i(p_i, p_j, N_i^e, N_j^e, g_i, g_j) = \frac{2\Big(1 + \phi(2g_i - g_j) + \theta(2N_i^e - N_j^e) - (2p_i - p_j)\Big)}{3}, \tag{8}$$

with $j \neq i \in \{1,2\}$. The mass of complementors on platform $i \in \{1,2\}$ is $N_i(D_i^e) = \gamma D_i^e$. We assume fulfilled expectations so that $\tilde{D}_i = D_i^e$ and $\tilde{N}_i = N_i^e$. This yields consumer and complementor demands on platform i as functions of prices and exclusivity decisions only:

$$\begin{aligned}
\tilde{D}_i(p_i, p_j, g_i, g_j,) &\triangleq \frac{2}{3-2\gamma\theta}\left[1+\frac{\phi(2g_i(1-\gamma\theta)-g_j)+p_j-2(1-\gamma\theta)p_i}{1-2\gamma\theta}\right], \\
\tilde{N}_i(p_i, p_j, g_i, g_j) &\triangleq \frac{2\gamma}{3-2\gamma\theta}\left[1+\frac{\phi(2g_i(1-\gamma\theta)-g_j)+p_j-2(1-\gamma\theta)p_i}{1-2\gamma\theta}\right],
\end{aligned} \tag{9}$$

for $i \neq j \in \{1, 2\}$. The profit of the platforms is:

$$\Pi_i(p_i, p_j, g_i, g_j) - g_i T_i = p_i \tilde{D}_i(p_i, p_j, g_i, g_j) - g_i T_i. \tag{10}$$

where T_i represents the tariff paid to the Superstar, which depends on g_1 and g_2 as it differs in the two exclusivity regimes.

We consider two market structures. Under vertical separation, the Superstar total profit is:

$$\Pi^S(p_1, p_2, g_1, g_2) = \gamma^S\Big(g_1\tilde{D}_1(p_1, p_2, g_1, g_2) + g_2\tilde{D}_2(p_2, p_1, g_2, g_1)\Big) + g_1T_1 + g_2T_2.$$

Under vertical integration with platform 1, the profit of the merged entity is:

$$\Pi^S(p_1, p_2, 1, g_2) = p_1\tilde{D}_1(p_1, p_2, 1, g_2) + \gamma^S\Big(\tilde{D}_1(p_1, p_2, 1, g_2) + g_2\tilde{D}_2(p_2, p_1, g_2, 1)\Big) + g_2T_2. \tag{11}$$

Vertical separation

In the second stage of the game, platforms set prices. Differentiating equation (10) with respect to p_i and solving for the optimal prices yields

$$\begin{aligned}
p_1^\star(g_1, g_2) &= \frac{1-2\rho}{3-4\rho} + \frac{\phi(g_1(7-8\rho(2-\rho)) - 2g_2(1-\rho))}{(5-4\rho)(3-4\rho)}, \\
p_2^\star(g_2, g_1) &= \frac{1-2\rho}{3-4\rho} + \frac{\phi(g_2(7-8\rho(2-\rho)) - 2g_1(1-\rho))}{(5-4\rho)(3-4\rho)}.
\end{aligned}$$

Under non-exclusivity, prices and demands are symmetric and equal to:

$$\begin{aligned} p_1^\star(1,1) = p_2^\star(1,1) &= \frac{1-2\rho}{3-4\rho} + \frac{(2(1-4(2-\rho))\rho+5)\phi}{(5-4\rho)(3-4\rho)}, \\ D_1^\star(1,1) = D_2^\star(1,1) &= \frac{4(1-\rho)(1+\phi)}{(3-2\rho)(3-4\rho)}, \\ N_1^\star(1,1) = N_2^\star(1,1) &= \gamma D_1^\star(1,1). \end{aligned} \tag{12}$$

All active complementors multihome. Unlike the baseline model with full market coverage, the total consumer demand increases in the value generated by the Superstar. The profit of platform i under non-exclusivity is

$$\Pi_i^\star(1,1) - T_i(1,1) = p_i^\star(1,1) D_i^\star(1,1) - T_i(1,1) = \frac{4(1-\rho)(1-2\rho)(1+\phi)^2}{(3-4\rho)^2(3-2\rho)} - T_i(1,1).$$

Under exclusivity with platform 1, the optimal prices are:

$$\begin{aligned} p_1^\star(1,0) &= \frac{1-2\rho}{3-4\rho} + \frac{(8\rho^2-16\rho+7)\,\phi}{(5-4\rho)(3-4\rho)} > p^\star(1,1), \\ p_2^\star(0,1) &= \frac{1-2\rho}{3-4\rho} - \frac{2(1-\rho)\phi}{(5-4\rho)(3-4\rho)} < p^\star(1,1), \end{aligned}$$

The demands at the two platforms are:

$$\begin{aligned} D_1^\star(1,0) &= \frac{4(1-\rho)}{(3-2\rho)(3-4\rho)} \left(1 + \frac{(7-8\rho(2-\rho))\,\phi}{(1-2\rho)(5-4\rho)}\right) > D^\star(1,1), \\ D_2^\star(0,1) &= \frac{4(1-\rho)}{(3-2\rho)(3-4\rho)} \left(1 - \frac{2(1-\rho)\phi}{(1-2\rho)(5-4\rho)}\right) < D^\star(1,1). \end{aligned}$$

The masses of complementors are:

$$N_1^\star(1,0) = \gamma D_1^\star(1,0) > N_1^\star(1,1) > N_2^\star(0,1) = \gamma D_2^\star(0,1).$$

The profits of the two platforms are:

$$\Pi_1^\star(1,0) - T_1(1,0) = \frac{4(1-\rho)(5+7\phi-2\rho(7+8\phi-4\rho(1+\phi)))^2}{(3-4\rho)^2(5-4\rho)^2(3-4\rho(2-\rho))} - T_1(1,0)$$
$$\Pi_2^\star(0,1) = \frac{4(1-\rho)(5-2\phi-2\rho(7-\phi-4\rho(1+\phi)))^2}{(3-4\rho)^2(5-4\rho)^2(3-4\rho(2-\rho))}.$$

Comparing the total demand in the two exclusivity scenarios yields

$$\Big(D_1^\star(1,1) + D_2^\star(1,1)\Big) - \Big(D_1^\star(1,0) + D_2^\star(0,1)\Big) = \frac{4\phi(1-\rho)}{9-2\rho(9-4\rho)} > 0. \tag{13}$$

Differently from the baseline model with full market coverage, exclusivity entails a *market-shrinking effect*, which may impact the strategy of the Superstar.

Exclusivity decision. Following the auction-based contractual setting as in the baseline model, with $T^\star(1,0) \triangleq \Pi_1^\star(1,0) - \Pi_1^\star(0,1)$, the profit of the Superstar under exclusivity is

$$\begin{aligned}\Pi^S(1,0) = &\quad \gamma^S D_1^\star(1,0) + p_1^\star(1,0) D_1^\star(1,0) - p_2^\star(0,1) D_2^\star(0,1)\\ = &\quad \tfrac{4(1-\rho)}{(5-4\rho)(3-4\rho)}\left[\phi(2+\phi) + \gamma^S \tfrac{\left(8\rho^2-16\rho+7\right)(\phi+1)-2(1-\rho)}{(3-2\rho)(1-2\rho)}\right].\end{aligned} \tag{14}$$

The profit of the Superstar, under non-exclusivity, is

$$\begin{aligned}\Pi^S(1,1) = &\quad 2\left[\gamma^S D_1^\star(1,1) + p_1^\star(1,1) D_1^\star(1,1) - p_2^\star(0,1) D_2^\star(0,1)\right]\\ = &\quad \tfrac{8(1-\rho)}{(3-2\rho)(3-4\rho)}\left[\gamma^S(1+\phi) + \tfrac{\phi(7-8\rho(2-\rho))(10+3\phi-4\rho(7+3\phi-2\rho(2+\phi)))}{(5-4\rho)^2(1-2\rho)(3-4\rho)}\right].\end{aligned} \tag{15}$$

Comparing the profits of the Superstars in the two exclusivity regimes, we make the following observation.

Observation 1 *The Superstar finds it profitable to be exclusive if and only if*

$$\gamma^S < \tilde{\gamma}^S \triangleq \max\left\{0, \frac{\phi(\phi(3+4\rho(12-\rho(45-16\rho(3-\rho))))) - 2(5-2\rho(7-4\rho))^2}{(5-4\rho)(3-4\rho)(5+3\phi-2\rho(7+6\phi-4\rho(1+\phi)))}\right\}.$$

Note that for $\tilde{\gamma}^S > 0$ two further conditions are required, i.e. $\phi > \frac{2(5-2\rho(7-4\rho))^2}{3-4\rho(\rho(16(\rho-3)\rho+45)-12)}$

and $\rho \gtrapprox 0.23$. These two conditions should be evaluated together with the no-tipping conditions in (7). Observation 1 implies that if $\rho < 0.23$ exclusivity never arises at equilibrium. Figure 3, which we describe later, provides graphical evidence of the existence of a non-empty set in which exclusivity occurs at equilibrium.

Due to the presence of a representative consumer, consumer surplus is equal to the utility of the representative consumer. Under exclusivity, consumer surplus is

$$U^\star(1,0) = \frac{8(1+\phi)\left(3\left(5-19\rho+22\rho^2-8\rho^3\right)^2+(1-\rho)^2\phi^2(39-186\rho+324\rho^2-240\rho^3+64\rho^4)\right)}{\left(64\rho^4-256\rho^3+364\rho^2-216\rho+45\right)^2}.$$

Under non-exclusivity, consumer surplus is

$$U^\star(1,1) = \frac{24(1-\rho)^2(1+\phi)^2}{(2\rho(4\rho-9)+9)^2}.$$

Comparing consumer surplus in the two cases, $U^\star(1,0) > U^\star(1,1)$ if

$$\rho > \rho^{CS} \approx 0.437 \quad \text{and} \quad \phi > \phi^{CS} \triangleq \frac{3(5-2\rho(7-4\rho))^2}{2(3-4\rho(3-2\rho)(\rho(15-8\rho)-6))}.$$

Otherwise $U^\star(1,0) < U^\star(1,1)$.

As in the baseline model, there exists a non-empty interval in which exclusivity is chosen by the Superstar (i.e., $\gamma^S < \tilde{\gamma}^S$) and it benefits consumers (i.e., $\rho > \rho^{CS}$ and $\phi > \phi^{CS}$). We summarize the following.

Observation 2 *If $\rho > \rho^{CS}$, $\phi > \phi^{CS}$ and $\gamma^S < \tilde{\gamma}^S$, the Superstar chooses exclusivity and exclusivity benefits consumers relative to non-exclusivity.*

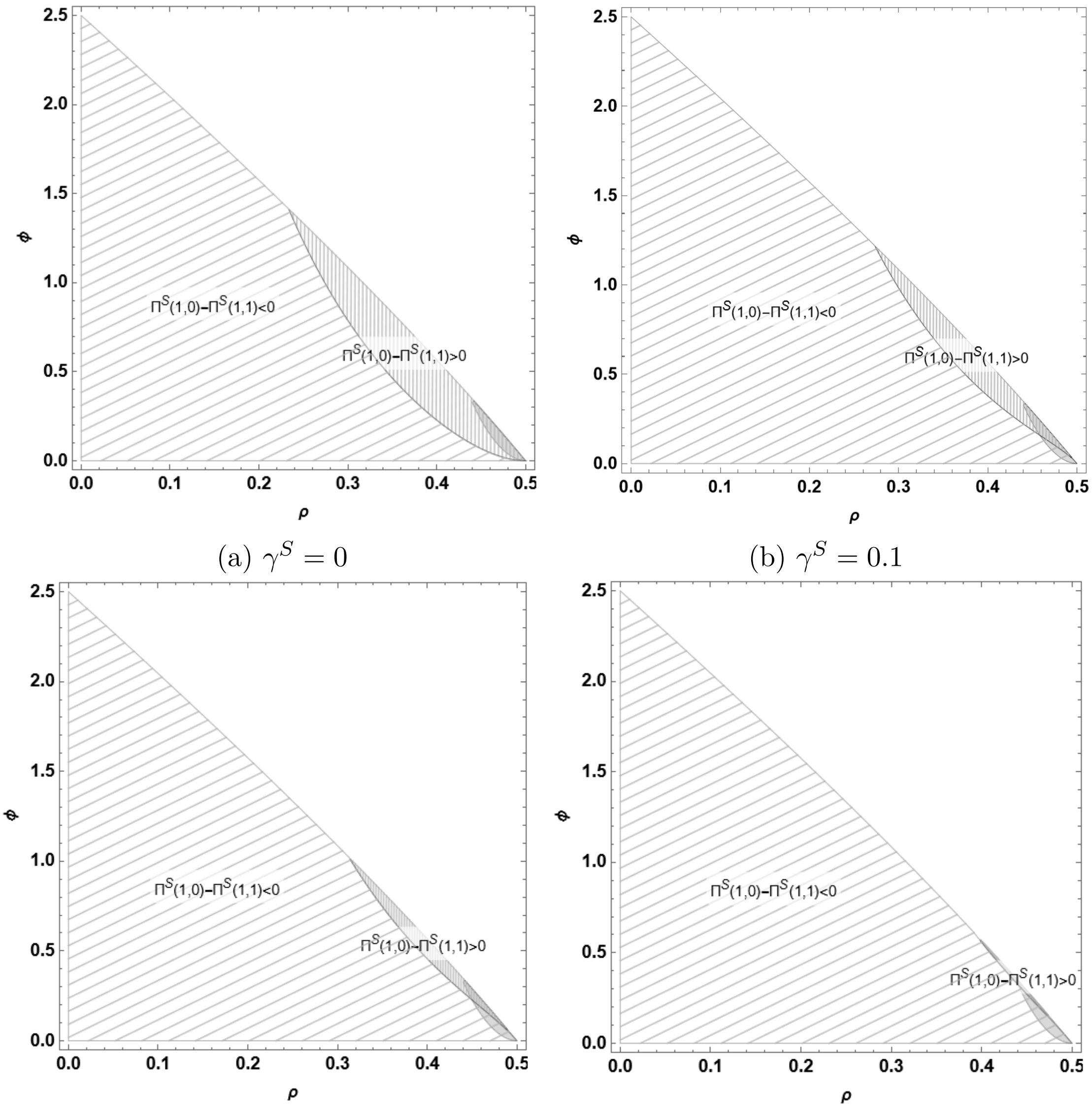
An implication of the above observation is that exclusivity is more likely to increase consumer surplus when the cross-group externalities are large enough and the value generated by the Superstar is not very large. Importantly, the parameter range in which exclusivity occurs at equilibrium and leads to a higher consumer surplus relative to non-exclusivity is small in size. This suggests that caution is required when deriving policy implications.

The following figures summarize our main results for different parameter values of $\gamma^S = \{0, 0.1, 0.2, 0.4\}$. In the white region, tipping occurs.[11]. Exclusivity benefits consumer, $U^\star(1,0) - U^\star(1,1) > 0$, in the gray shaded region. The Superstar finds it optimal to choose non-exclusivity, $\Pi^S(1,0) - \Pi^S(1,1) < 0$, in the region shaded with diagonal lines. Finally, the Superstar finds it optimal to choose exclusivity, $\Pi^S(1,0) - \Pi^S(1,1) > 0$, in the region shaded with vertical lines.

Moving from Figure (3a) to (3d), γ^S increases. The area with vertical lines shrinks and it almost disappears for $\gamma^S = 0.4$. In other words, exclusivity becomes less likely as γ^S increases. Moreover, the area in which exclusivity is beneficial to consumers is very small.

The graphical inspection offers evidence of the existence of a non-empty set in which exclusivity emerges at equilibrium and raises consumers' surplus. This case is identified by the overlap between the area with vertical straight lines and the shaded gray region. Importantly, this overlap is more pronounced for small enough γ^S and almost disappears for $\gamma^S = 0.4$. Moreover, in Figure (3c) and (3d), there exists an area in which non-exclusivity is chosen by the Superstar, but exclusivity benefits consumers (i.e., the overlap between the area with diagonal lines and the shaded gray region). In the remaining regions, instead, non-exclusivity occurs and is beneficial to consumers.

[11]Therefore, conditions in (7) are not satisfied



(a) $\gamma^S = 0$ (b) $\gamma^S = 0.1$

(c) $\gamma^S = 0.2$ (d) $\gamma^S = 0.4$

Figure 3: Exclusivity and consumer welfare under vertical separation

Vertical Integration

Consider now the case in which the Superstar is integrated with platform 1. The demands of the two platforms are determined the same way as under vertical separation (equation (9)) with $g_1 = 1$ by default. Differentiating the profit of the merged entity with respect to p_1 and the profit of platform 2 with respect to p_2 and solving simultaneously, we obtain the equilibrium prices

$$\begin{aligned}
p_2^\star(g_2) &= \frac{5-4\gamma^S(2-g_2)-2\rho(7-4\rho(1-\gamma^S)-2\gamma^S(4-g_2))+7\phi-2\phi(g_2(1-\rho)+4\rho(2-\rho))}{(5-4\rho)(3-4\rho)}, \\
p_2^\star(g_2) &= \frac{5-\gamma^S(2-g_2)-\phi(2-7g_2)-2\rho(7-\gamma^S-\phi+8g_2\phi-4\rho(1+g_2\phi))}{(5-4\rho)(3-4\rho)}.
\end{aligned}$$

Under exclusivity ($g_2 = 0$), equilibrium prices and demands are respectively

$$\begin{aligned}
p_1^\star(0) &= \frac{5-8\gamma^S+7\phi-2\rho(7-8\gamma^S+8\phi-4\rho(1+\phi-\gamma^S))}{(5-4\rho)(3-4\rho)}, \\
p_2^\star(0) &= \frac{5-2\gamma^S-2\phi-2\rho(7-4\rho-\gamma^S-\phi)}{(5-4\rho)(3-4\rho)}, \\
D_1^\star(0) &= \frac{4(1-\rho)\left[5+7(\gamma^S+\phi)+8\rho^2\ (\gamma^S+\phi+1)-2\rho(7+8(\gamma^S+\phi))\right]}{(3-2\rho)(1-2\rho)(5-4\rho)(3-4\rho)}, \\
D_2^\star(0) &= \frac{4(1-\rho)\left[5-2\gamma^S-2\phi-2\rho(7-4\rho-\gamma^S-\phi)\right]}{(3-2\rho)(1-2\rho)(5-4\rho)(3-4\rho)}.
\end{aligned}$$

Note that as γ^S increases, the price on both platforms decreases, i.e., $\frac{\partial p_1^\star(0)}{\partial \gamma^S} = -\frac{8(1-2\rho)}{(5-4\rho)(3-4\rho)} < 0$ and $\frac{\partial p_2^\star(0)}{\partial \gamma^S} = -\frac{2(1-2\rho)}{(5-4\rho)(3-4\rho)} < 0$. Further, comparing the negative impact of an increase in γ^S on $p_1^\star(0)$ relative to its impact on $p_2^\star(0)$, we observe $\frac{\partial p_1^\star(0)}{\partial \gamma^S} < \frac{\partial p_2^\star(0)}{\partial \gamma^S} < 0$. This is because an increase in γ^S impacts $p_1^\star(0)$ directly while the negative impact on $p_2^\star(0)$ is indirect and arises from the fact that prices are strategic complements.

At equilibrium, the profit of the merged entity and of the independent platform are

$$\begin{aligned}
\Pi^{S,\star}(0) =& \frac{4(1-\rho)(5+7(\gamma^S+\phi)+8\rho^2(1+\gamma^S+\phi)-2\rho(7-8(\gamma^S+\phi))^2}{(3-4\rho)^2(5-4\rho)^2(3-4\rho(2-\rho))}, \\
\Pi_2^\star(0) =& \frac{4(1-\rho)(5-2(\gamma^S+\phi)-2\rho(7-4\rho-\gamma^S-\phi))^2}{(3-4\rho)^2(5-4\rho)^2(3-4\rho(2-\rho))}.
\end{aligned}$$

Under non-exclusivity, the equilibrium prices and demands are

$$
\begin{aligned}
p_1^\star(1) &= \frac{(1-2\rho)(5-4\gamma^S-4\rho(1-\gamma^S+\phi)+5\phi)}{(5-4\rho)(3-4\rho)},\\
p_2^\star(1) &= \frac{(1-2\rho)((5-4\rho)(1+\phi)-\gamma^S)}{(5-4\rho)(3-4\rho)},\\
D_1^\star(1) &= \frac{2(7\gamma^S+10(1+\phi)+8\rho^2(1+\phi+\gamma^S)-2\rho(9+9\phi+8\gamma^S))}{(3-2\rho)(5-4\rho)(3-4\rho)},\\
D_2^\star(1) &= \frac{4(1-\rho)((5-4\rho)(1+\phi)-\gamma^S)}{(3-2\rho)(5-4\rho)(3-4\rho)}.
\end{aligned}
$$

Note that prices are falling as γ^S increases — $\frac{\partial p_1^\star(1)}{\partial\gamma^S} = -\frac{4(1-\rho)(1-2\rho)}{(5-4\rho)(3-4\rho)} < 0$ and $\frac{\partial p_2^\star(0)}{\partial\gamma^S} = -\frac{(1-2\rho)}{(3-4\rho)(5-4\rho)} < 0$. Further, comparing the negative impact of an increase in γ^S on $p_1^\star(1)$ relative to its impact on $p_2^\star(1)$ — i.e., $\frac{\partial p_1^\star(1)}{\partial\gamma^S} < \frac{\partial p_2^\star(1)}{\partial\gamma^S} < 0$.
The associated equilibrium profit of the merged entity is

$$
\Pi^{S,\star}(1) \triangleq p_1^\star(1)D_1^\star(1) + \gamma^S(D_1^\star(1)+D_2^\star(1)) + T_2(1).
$$

The equilibrium profit of the independent platform is

$$
\Pi_2^\star(1) = \frac{4(1-\rho)(1-2\rho)((5-4\rho)(1+\phi)-\gamma^S)^2}{(3-4\rho)^2(5-4\rho)^2(3-2\rho)} - T_2(1).
$$

Exclusivity decision. As in the baseline model, the tariff under non-exclusivity is $T_2^\star(1) \triangleq p_2^\star(1)D_2^\star(1) - p_2^\star(0)D_2^\star(0)$. Comparing profits of the merged entity under exclusivity and non-exclusivity, exclusivity is chosen if and only if $\Delta\Pi^S \triangleq \Pi^S(0) - \Pi^S(1) > 0$, where

$$
\begin{aligned}
\Delta\Pi^S &= \Big(p_1^\star(0)+\gamma^S\Big)D_1^\star(0) - \Big(p_1^\star(1)D_1^\star(1) + \gamma^S(D_1^\star(1)+D_2^\star(1))\Big) + T_2^\star(1))\\
&= \frac{2(3-4\rho)(\gamma^S)^2\mathcal{A} - 4(1-\rho)\phi\mathcal{B} - 2\gamma^S\mathcal{D}}{(3-4\rho)^2(5-4\rho)^2(3-4\rho(2-\rho))}.
\end{aligned}
$$

where $\mathcal{A} \triangleq 19-4\rho(14-\rho(13-4\rho))$, $\mathcal{B} \triangleq 2(5-2\rho(7-4\rho))^2 - \phi(3+4\rho(12-\rho(45-16\rho(3-\rho))))$, $\mathcal{D} \triangleq (5-4\rho)^2(1-2\rho)(7-2\rho(7-4\rho)) + \phi(63-4\rho(125-\rho(345-$

$4\rho(111-4\rho(17-4\rho)))))$. Moreover, the difference in consumer surplus is

$$\begin{aligned}\Delta U^{VI} &\triangleq U(0)-U(1)\\ &= \frac{2(3-4\rho)(\gamma^S)^2\mathcal{E}-8(1-\rho)^2\phi\mathcal{G}+4\gamma^S(1-\rho)\mathcal{H}}{(45-216\rho+364\rho^2-256\rho^3+64\rho^4)^2}\end{aligned}$$

with $\mathcal{E}\triangleq 39-2\rho(93-2\rho(81-4\rho(15-4\rho)))$, and $\mathcal{G}\triangleq 3(5+2\rho(7-4\rho))^2+2\phi(3-4\rho(3-2\rho))(6-\rho(15-8\rho))$ and $\mathcal{H}\triangleq 3(5+2\rho(7-4\rho))^2+\phi(81-2\rho(165-2\rho(93+2\rho(9-4\rho)(1-4\rho))))$.

Exclusivity and its consumer-welfare impact. In what follows, we prove by examples that there exists a parameter range in which exclusivity is chosen by the merged entity, $\Delta\Pi^{S,VI}>0$, and increases consumer surplus, $\Delta U^{VI}>0$. We provide examples for different levels of the cross-group externalities ρ.

Case (a) Cross-group externalities are low: $\rho=1/10$. In this case, exclusivity never occurs as

$$\Delta\Pi^S|_{\rho=1/10}=\frac{5\left(56485(\gamma^S)^2-2\gamma^S(19633\phi+75118)+9\phi(1933\phi-8464)\right)}{1251614}<0.$$

Nevertheless, this choice does not necessarily benefit consumers as

$$\Delta U^{VI}|_{\rho=1/10}=\frac{25\left(950885(\gamma^S)^2+54\gamma^S(26959\phi+21160)-81\phi(10763\phi+25392)\right)}{140180768},$$

which is positive only if

$$\gamma^S>\gamma^{S,L}\triangleq\frac{9\left(4\sqrt{14\phi(74890466\phi+153629535)+251856900}-80877\phi-63480\right)}{950885}.$$

Therefore, this result suggests that when cross-group externalities are low ($\rho\approx 1/10$), the merged entity chooses to be non-exclusive but consumers would be better off under exclusivity for large values of γ^S as in the latter case prices decrease faster with γ^S, thereby benefiting consumers.

Case (b): Cross-group externalities are intermediate: $\rho = 1/4$. In this case,

$$\Delta\Pi^S|_{\rho=1/4} = \frac{39\phi^2 - 2(24 + \gamma^S)\phi - 64(2 - \gamma^S)\gamma^S}{160},$$

which is positive if and only if

$$\gamma^S < \tilde{\gamma}^S \triangleq 1 + \frac{\phi - \sqrt{4096 + 5\phi(640 - 499\phi)}}{64}.$$

Therefore, exclusivity resp. non-exclusivity) arises for sufficiently low (resp. high) values of γ^S. Moreover, exclusivity benefits consumers if

$$\Delta U^{VI}|_{\rho=1/4} = \frac{296(\gamma^S)^2 + 18\gamma^S(16 + 29\phi) - 9\phi(48 + 11\phi)}{800}.$$

Solving for γ^S, we observe that consumer surplus under exclusivity is higher (resp. lower) than consumer surplus under non-exclusivity if

$$\gamma^S > (<)\gamma^{S,M} \triangleq \frac{\sqrt{5\phi(2165\phi + 4512) + 2304} - 48 - 87\phi}{296}.$$

Comparing $\tilde{\gamma}^S$ and $\gamma^{S,M}$, we note that $\tilde{\gamma}^S < \gamma^{S,M}$. Thus, for intermediate values of the cross-group externalities ($\rho \approx 1/4$)

- if $\gamma^S < \tilde{\gamma}^S$, the merged entity chooses exclusivity. Consumer surplus is higher under non-exclusivity.
- if $\tilde{\gamma}^S \leq \gamma^S < \gamma^{S,M}$, the merged entity chooses non-exclusivity. Consumer surplus is higher under non-exclusivity.
- if $\gamma^S \geq \gamma^{S,M}$, the merged entity chooses non-exclusivity. Consumer surplus is higher under exclusivity.

Case (c): Cross-group externalities are high: $\rho = 4/10$. In this case,

$$\Delta\Pi^S|_{\rho=4/10} = \frac{10\left(17045(\gamma^S)^2 + \gamma^S(14477\phi - 19363) + 6\phi(2531\phi - 578)\right)}{155771},$$

which is positive (resp. negative) if

$$\gamma^S < (>)\tilde{\tilde{\gamma}}^S \triangleq \frac{19363 - 14477\phi - \sqrt{374925769 - 11\phi(75072541\phi + 29471642)}}{34090}.$$

Exclusivity benefits consumers if

$$\Delta U^{VI}|_{\rho=4/10} = \frac{50\left(59465(\gamma^S)^2 + 18\gamma^S(6507\phi + 1445) - 36\phi(867 - 832\phi)\right)}{1713481}.$$

Solving for γ^S, consumer surplus under exclusivity is higher (resp. lower) than consumer surplus under non-exclusivity if

$$\gamma^S > (<)\gamma^{S,H} \triangleq \frac{3\left(\sqrt{18792225 + 11\phi(16651811\phi + 34133790)} - 4335 - 19521\phi\right)}{59465}.$$

Thus, for high values of the cross-group externalities ($\rho \approx 4/10$)

- if $\gamma^{S,H} < \gamma^S < \tilde{\tilde{\gamma}}^S$, the merged entity chooses exclusivity. Consumer surplus is higher under exclusivity.
- if $\tilde{\tilde{\gamma}}^S < \gamma^S < \gamma^{S,H}$, the merged entity chooses non-exclusivity. Consumer surplus is higher under exclusivity.
- in all other parameter constellations, the choice of the merged entity is misaligned with the interest of the consumers.

Consistently with the baseline model, there exists a non-empty interval in which exclusivity is chosen by the merged entity and may benefit consumers for sufficiently large cross-group externality. Interestingly, this result holds even if exclusivity leads to a market-shrinking effect relative to non-exclusivity but the parameter range in which it occurs is quite small in size.

The following figures summarize our main results for different values of $\gamma^S = \{0, 0.1, 0.2, 0.4\}$. Figure (4a) is the same as Figure (3a) because if $\gamma^S = 0$ vertical separation and vertical integration are equivalent. In Figure (4a) and Figure (4b), there exists an area in which exclusivity occurs and is also beneficial to consumers. This is the tiny area in which the gray shaded area ($U^\star(0) - U^\star(1) > 0$) intersects the area with the vertical lines ($\Pi^S(0) - \Pi^S(1) > 0$). In Figure (4c) and Figure (4d), exclusivity does not occur although it may be beneficial to consumers.

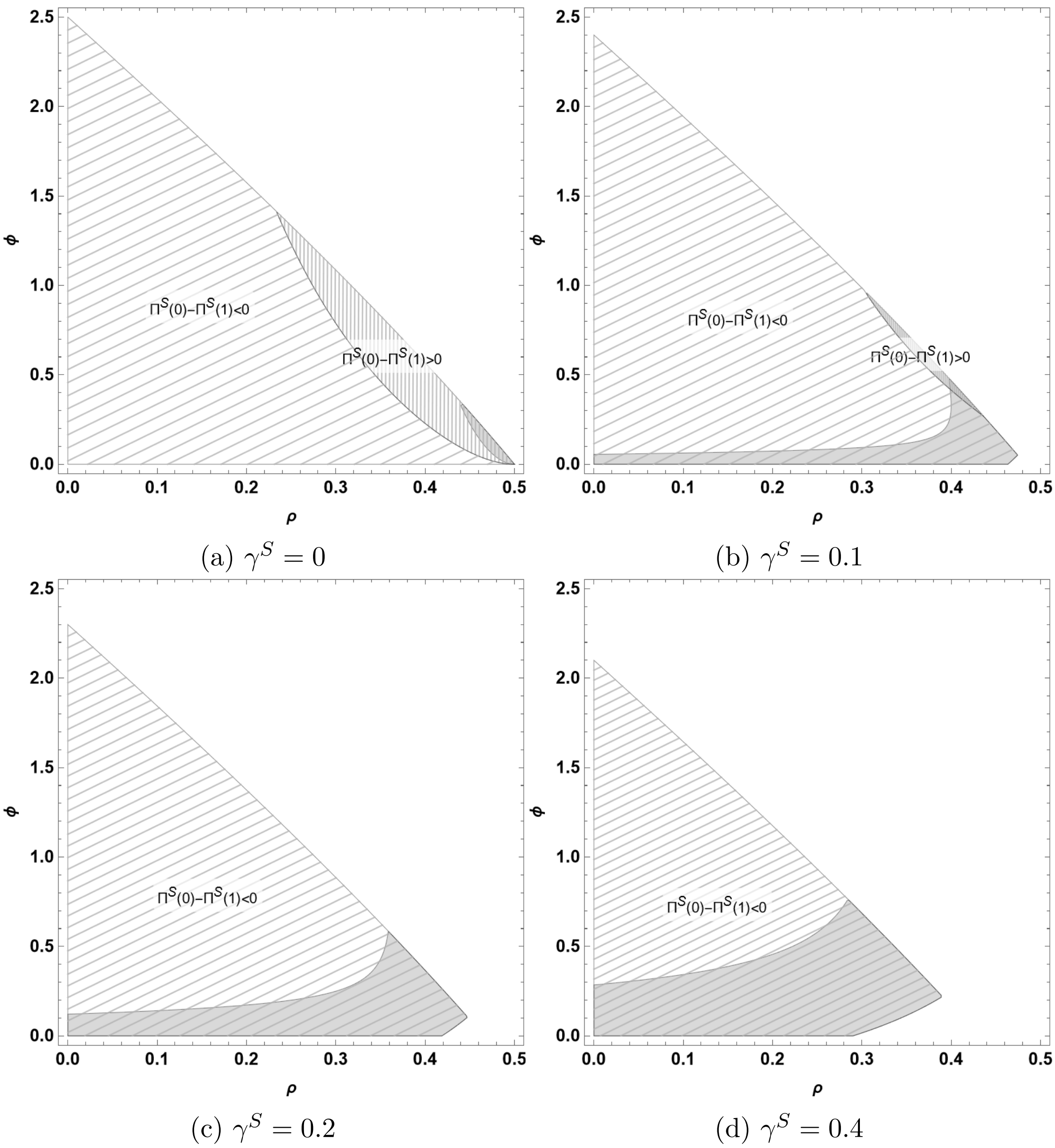


Figure 4: Exclusivity and consumer welfare under vertical integration

3.4 Coordination problem

The scenario proposed in our baseline model is well suited to explain the contractual decisions of a Superstar entering a market in which there are already inherited and symmetric market shares. In the latter case, exclusivity would stimulate switching decisions on the *favored* platform. Throughout the paper, we select an equilibrium compatible with sequential decisions as in Hagiu (2006). In such a setup, the typical "chicken-egg" problem is solved by letting sellers (i.e., complementors) move earlier than buyers, reducing the coordination problem to the sole decision of the "chicken". As discussed by Hagiu (2006), such a framework well suits most software and video game platforms wherein developers and game sellers join platforms before buyers, for example, for technological reasons.

To go deeper into the coordination issues that can arise under simultaneous moves on both sides, one shall note that if consumers believe that a sufficiently large number of other consumers and complementors will follow the Superstar, then the market can eventually tip. For instance, a device to solve this coordination problem is grouping homogeneous users as in Markovich & Yehezkel (2022). This would help an efficient platform to drive a less efficient focal rival out of the market. In our framework, the pivotal agent is the Superstar.

As coordination issues have important policy implications in the presence of market tipping, in what follows we discuss the impact of exclusivity on welfare in the limit case the *favored* platform (almost) conquers the entire market. Namely, we consider the limit of consumer welfare when the critical value $m^\star(1,0)$ is very close to the upper bound of the distribution. In this case, all consumers patronize the *favored* platform.

When this is the case, then $F(m^\star(1,0)) = 1$, and so $D_2^\star(0,1) = 0$, $\bar{N} \triangleq 1 \times N_1^\star(1,0) = \Lambda(\gamma \cdot 1)$ and $\bar{p} \triangleq 1 \times p_1^\star(1,0)$. As $N^\star(1,1) = \Lambda(\gamma/2)$, then ΔCS becomes equal to

$$\Delta CS_{m^\star \to \overline{m}} = \theta[\Lambda(\gamma \cdot 1) - \Lambda(\gamma/2)] - [p_1^\star(1,0) - p_1^\star(1,1)] - \int_0^{\overline{m}} mf(m)dm,$$

where $p_1^\star(1,1) = p_2^\star(1,1)$. Using prices in Lemma 1 and 2, we then have:

$$\Delta CS_{m^\star \to \overline{m}} = \theta\Big[\Lambda(\gamma) - \Lambda(\gamma/2)\Big] + \gamma\theta\Big[\lambda(\gamma) - \lambda(\gamma/2)\Big] - \left[\frac{1}{f(\overline{m})} - \frac{1}{2f(0)}\right] - \int_0^{\overline{m}} mf(m)dm.$$

One can easily see that the same trade-off as in the baseline model without market tipping applies. The first term is positive and captures the consumer benefit from the agglomeration of complementors on one platform. The second term is positive too and captures the price-reducing effect induced by the cross-group externalities. The third and fourth terms represent consumer loss due to market tipping. Specifically, the third term relates to the direct effect on prices (via consumer preferences), whereas the fourth one is the preference mismatch (which is now at its highest level).

In turn, market tipping towards the *favored* platform may lead to efficiency gains due to cross-group effects. On the negative side, however, consumers may bear the high costs of preference mismatch and the stronger direct effect on prices. When cross-group externalities get more substantial, these efficiency gains would outweigh the consumer welfare losses and, hence, exclusivity would be welfare-enhancing.

3.5 Multihoming Consumers

In most markets, consumer multihoming is quite common: platforms have overlapping market shares and they may lead to decreasing returns for the agents affiliated with multiple platforms (Ambrus et al. 2016, Athey et al. 2016, Anderson et al. 2018, Calvano & Polo 2020, D'Annunzio & Russo 2020). In this section, we provide an analysis of this case.

First, we characterize the utility, u^m, of a multihoming consumer as follows:

$$u^m \triangleq v + \phi \max\{g_1, g_2\} + \theta \max\{N_1^e, N_2^e\} - (p_1 + p_2). \tag{16}$$

Note that when consumers multihome, there is no longer a preference mismatch ($m = 0$). Indeed, consumers make their choice based on their preference $m/2$

and, when they affiliate with their preferred platform, $m/2$ is a benefit rather than a cost, and this offsets the cost of going to the rival platform. Moreover, we assume that the benefit of a consumer joining a platform, v, is only obtained once. The same happens when interacting with the same complementors or with the Superstar. This implies having access to $\max\{N_1^e, N_2^e\}$ complementors.[12]

A consumer decides between multihoming and singlehoming. Regardless of the Superstar's exclusivity, when a consumer affiliates her preferred platform, $m/2$ is a benefit rather than a cost, and its absolute value enters positively the utility function. Therefore, we compare u_1 to u^m for any $m < 0$ and u_2 to u^m for any $m > 0$. It follows that an agent with a relative preference to platform i is never indifferent between joining platform j and multihoming. This implies that $u_1 > u^m$ if

$$m < \tilde{m}_1(p_2, N_1^e, N_2^e, g_1, g_2) \triangleq 2\Big(\phi \min\{0, g_1 - g_2\} + \theta \min\{N_1^e - N_2^e, 0\} + p_2\Big), \quad (17)$$

and, similarly, $u_2 > u^m$ if

$$m > \tilde{m}_2(p_1, N_1^e, N_2^e, g_2, g_1) \triangleq 2\Big(\phi \max\{0, g_1 - g_2\} + \theta \max\{N_1^e - N_2^e, 0\} - p_1\Big). \quad (18)$$

Note that these critical values are not affected by the exclusivity decision of the Superstar. This implies that the demand for platform i is determined by the decision of the consumer indifferent between multihoming and singlehoming on the rival. The latter is not affected by the presence of the Superstar, which is guaranteed in both options. What changes, instead, is the demand of the rival platform, as multihoming always guarantees access to the Superstar, whereas singlehoming ensures it only under non-exclusivity.

To fix ideas, consider when vertical separation is present and focus on the decision of the Superstar. Under non-exclusivity ($g_1 = g_2$), consumer expectations regarding the number of complementors are symmetric, i.e., $N_1^e = N_2^e$. Then, the two

[12]This is a plausible assumption as consumers do not benefit differently from interacting with the same firms twice. However, when considering a less restrictive case, in which multihoming potentially gives rise to as much as 2ϕ and $2\min\{N_1^e, N_2^e\}$, this will only be a scale effect on the utilities and will not qualitatively change results and intuitions.

critical values $\tilde{m}_i$ with $i = \{1, 2\}$ can be rewritten as follows

$$\tilde{m}_1(p_2, 1, 1) = 2p_2 \text{ and } \tilde{m}_2(p_1, 1, 1) = -2p_1, \tag{19}$$

Under exclusivity on platform 1 (i.e., $g_1 = 1$ and $g_2 = 0$), the expected number of complementors is $N_1^e > N_2^e$. Thus,

$$\tilde{m}_1(p_2, 1, 0) = 2p_2 \text{ and } \tilde{m}_2(p_1, N_1^e, N_2^e, 0, 1) = 2\Big(\phi + \theta(N_1^e - N_2^e) - p_1\Big). \tag{20}$$

With fulfilled expectations, denote $\tilde{\tilde{m}}_2(p_1, 0, 1) \triangleq \tilde{m}_2(p_2, \tilde{N}_1(\cdot), \tilde{N}_2(\cdot), 0, 1)$. Platform 1's demand is $\tilde{D}_1(p_1, p_2, 1, 0) = F(\tilde{m}_1(p_2, 1, 0)) + F(\tilde{\tilde{m}}_2(p_1, 0, 1)) - F(\tilde{m}_1(p_2))$, where $F(\tilde{m}_1(p_2, 1, 0))$ represents the single homers, whereas the remaining $F(\tilde{\tilde{m}}_2(p_1, 0, 1)) - F(\tilde{m}_1(p_2, 1, 0))$ captures multihoming consumers. Consistently, $\tilde{N}_1(p_1, p_2, 1, 0) \triangleq \gamma\tilde{D}_1(p_1, p_2, 1, 0)$. In the same manner, platform 2's demand is$\tilde{D}_2(p_2, p_1, 0, 1) = 1 - F(\tilde{\tilde{m}}_2(p_1, 0, 1)) + F(\tilde{m}_1(p_2, 1, 0)) - F(\tilde{m}_1(p_2))$, where $1 - F(\tilde{\tilde{m}}_2(p_1, 0, 1))$ represents the single homers, whereas the remaining $F(\tilde{m}_1(p_2, 1, 0)) - F(\tilde{m}_1(p_2))$ captures multihoming consumers. Consistently, $\tilde{N}_2(p_2, p_1, 0, 1) \triangleq \gamma\tilde{D}_2(p_2, p_1, 0, 1)$..

From (19) and (20), for any p_1 and p_2, we note that $\tilde{D}_2(p_2, p_1, 0, 1) = \tilde{D}_2(p_2, p_1, 1, 1) \triangleq \tilde{D}_2(g_2, 1)$. As the gross profit of platform 2 is equal to $\tilde{D}_2(g_2, 1)p_2$ it also follows that the profit of platform 2 does not change in response to the decision of the Superstar to be exclusive on the platform 1. In turn, at equilibrium, $\Pi_2^\star(0, 1) = \Pi_2^\star(1, 1)$. This equality has an important implication for the Superstar, as non-exclusivity is profit-equivalent to exclusivity on platform 1. As a consequence, the Superstar cannot ask for any positive payment for non-exclusivity. In this case, the profit of the Superstar under non-exclusivity is only given by γ^S.

Following the contractual stage in the main model, if the Superstar finds it more profitable to be exclusive, she can reach this outcome by designing an auction with a reserve price. As discussed, under non-exclusivity, the Superstar gives her product free of charge. This implies a reserve price equal to 0.

To understand the highest bid for exclusivity, let us analyze how platform profits change with exclusivity. For any price p_1, we have:

$$p_1 F(\tilde{\tilde{m}}_2(p_1, 1, 0)) = p_1 F(\tilde{m}_2(p_1, 1, 1)) < p_1 F(\tilde{\tilde{m}}_2(p_1, 0, 1)),$$

because $F(\cdot)$ is increasing in its argument and $\tilde{\tilde{m}}_2(p_1, 1, 0) = \tilde{m}_2(p_1, 1, 1) < \tilde{\tilde{m}}_2(p_1, 0, 1)$. This also implies that, at equilibrium, we have $\Pi_1^\star(1, 0) > \Pi_1^\star(1, 1) = \Pi_1^\star(0, 1)$.

In turn, in the auction run by the Superstar, platforms will compete by bidding $T_i^\star(1, 0) = \Pi_i^\star(1, 0) - \Pi_i^\star(0, 1)$. Given symmetry, we assume that platform 1 is *favored*, and consequently Superstar's profits are given by:

$$\gamma^S D_1^\star(1, 0) + T_1^\star(1, 0) = \gamma^S D_1^\star(1, 0) + \Pi_1^\star(1, 0) - \Pi_1^\star(1, 1).$$

Comparing the profit of the Superstar, we conclude that there exists a threshold

$$\tilde{\gamma}^M \triangleq \frac{\Pi_1^\star(1, 0) - \Pi_1^\star(0, 1)}{1 - D_1^\star(1, 0)}$$

such that non-exclusivity emerges if, and only if, $\gamma^S \geq \tilde{\gamma}^S$. Exclusivity emerges otherwise.

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Market	Exclusives	Type	Vertical integration
Music on-demand	Drake, F. Ocean on Tidal; Rihanna, Beyoncé on Apple; Taylor Swift on Spotify	Full or "windowed release"	Spotify acquired Gimlet and Parcast
Gaming	Spider Man, Gran Turismo Sport, The Last of Us, Demon's Souls on PS; Super Mario Odyssey and Pokemon: Sword and Shield on Switch; Fornite on Epic Game Store	Console-specific, feature-specific, often limited in time	Historical feature of the industry (Lee 2013)
E-sport	Ninja, Shroud (top gamers) left Twitch for Mixer	Exclusive streaming of games	No(t yet)
Audio books	Garzanti, Loganesi, "Originals", Robert Caro, Jeffery Deaver, Michael Lewis on Amazon Audible; Bompiani on Storytel	Full	"Originals"
Apps	Bear, Timepage, Overcast on iOS; Steam Link, Tasker on Android	Full	Apple's Arcade and Shazam, Google's Suite
Shopping Malls	Anchor store	Often radius clauses	Departmental store
⋮	⋮	⋮	⋮

Table 1: Industry Background