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Optimal Climate and Monetary-Fiscal Policy in a Climate-DSGE Framework*

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Abstract

This paper points to the welfare enhancing effect of policies to regulate emissions in the face of climate shock. Fiscal and monetary policies alone would achieve suboptimal outcomes. We consider the Ramsey-optimal long-run and dynamic policy interactions between climate and fiscal-monetary policies in a climate-monetary DSGE model under sticky prices. In the model, the planner – on top of a fiscal and a monetary instrument – controls also a carbon tax (or, equivalently, an emission abatement technology) to manage emissions, and therefore temperatures and climate damages. In this setup, the presence of carbon taxation sharply reduces the fall in key macroeconomic variables such as output, consumption, and welfare, to a shock to emissions compared with the case without carbon taxation in place. We also show that it is essential to consider climate-specific shocks to appreciate the importance of carbon policies; the Ramsey optimal solution to typical TFP or government spending shocks is not very different whether or not the planner has access to carbon taxation. In the face of climate shocks, the optimal monetary is very similar to the one under no climate policy, while the optimal fiscal policy set distortionary labor income taxation at a lower rate, as the planner now raises revenue also through carbon taxes. Finally, in an extension of the model, we show that the optimal environmental implications can significantly change as we explicitly include climate fiscal outlays in the government budget constraint. As climate change becomes more costly for the government, the optimal abatement increases and the magnitude of carbon emissions and thus output damages decreases.

JEL Classification: E52, E61, E62, H21, H23, Q5

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1 Introduction

Climate change has become an important challenge for policy. While fiscal policy is considered to be the most impactful domain of government action to combat the negative consequences caused by greenhouse gases emissions that result in global warming (Hansen [2021]), monetary policy is also increasingly believed to be able to support fiscal policy in designing policies to confront climate change (Smith Jr [2022]). Keeping also in mind that fiscal and monetary policies are tightly linked to one another and are not independent in having effects on the macroeconomy (Leeper [2021]). Indeed, central bank officials around the world have turned their attention to climate change. For instance, in a recent speech at "The Economics of Climate Change" the then Federal Reserve Vice Chair Lael Brainard mentioned that the Fed is trying to understand the implications of climate change for monetary policy and how environmental issues may have consequences for interest rates:

> As policies are implemented to mitigate climate change, they will affect prices, productivity, employment and output in ways that could have implications for monetary policy ... the large amount of uncertainty regarding climate-related events and policies could hold back investment and economic activity (Brainard [2019], p. 4).[1]

At the same time, the current main objective of most central banks around the world is price stability. The current leading theories of monetary non-neutrality that include nominal rigidities in the form of sluggish price adjustment, but abstract from environmental economic externalities, feature price stability as the central policy outcome even under a mild degree of price stickiness. This is considered to be the case even if we explicitly take the fiscal dynamic budget constraint into account and assume that fiscal policy is active in a Leeper [1991] sense (Schmitt-Grohé and Uribe [2004, 2010]).

Motivated by these facts, the focus of this paper is to develop a dynamic, stochastic "Integrated Assessment Model" (IAM), which is a setting that integrates a leading theory of monetary non-neutrality and the climate into a unified framework, to study optimal carbon pricing and fiscal-monetary policies and their interaction at the same time. More precisely, the article takes the seminal paper of Schmitt-Grohé and Uribe [2004], which is a DSGE

[1]Furthermore, in another speech she recently mentioned:

> It is increasingly clear that climate change could have important implications for the Federal Reserve ... Given the implications of climate change for both individual financial institutions and the financial sector as a whole....(Brainard [2021], p. 2).

See Hansen [2021] for an analysis on how climate change poses an important policy challenge for governments around the world and what the role of central banks should be. See also Campiglie et al. [2018] on the role of central banks in addressing climate-related issues to financial risks.

framework with a demand for fiat money, sticky prices and distortionary fiscal policy, and augments it with: i) a carbon circulation system that specifies how emissions over time translate into a path of CO2 concentration; ii) a climate system that specifies the link between the atmospheric CO2 concentration and the global average atmospheric temperature near the surface; and iii) a damage function that models the economic losses from climate change as a function of the global mean temperature.

Our aim is to investigate the long-run and dynamic implications of the interactions between optimal climate policy (optimal carbon taxation) and optimal fiscal-monetary policy. To this end, along with the standard technology and government spending shocks, we introduce a climate-specific shocks (to the stock of emissions in the atmosphere) into our model and argue that to appreciate the essentiality of designing carbon tax policies and using appropriate fiscal-monetary policies to fight global warming it is important to look at the implications of such shocks. We show that the typical aggregate macroeconomic shocks such as TFP and government spending shocks, which are widely assumed in the macroeconomic literature, do not provide much insight into the macroeconomic consequences of climate change and the need for a tax on carbon emissions. As a matter of fact, the optimal response to TFP and government spending shocks does not require use of a carbon tax.[2]

Our paper can be seen as an extension of Barrage [2020b] to monetary policy. She considers the DGE model of Golosov et al. [2014] and augments it with distortionary fiscal policy and solves for the Ramsey optimal policy. In her work there are distortionary taxes and the Pigouvian climate tax, but no monetary policy. We share a key friction of New Keynesian (NK) models, which is the inability to subsidize labor to undo firms' monopoly power, and the fact that the carbon tax is an additional instrument to correct for this distortion (in the spirit of Correia et al. [2008]).

An important finding of the existing literature on the optimal taxation of carbon in static or dynamic settings with distortionary taxes is that when the economic damages of climate change are pure production losses, the optimal carbon taxation fully internalizes output damages under quite general assumptions (Barrage [2020b] and references therein). We analytically show that, under flexible prices and perfectly competitive product markets, this is the case only if the abatement cost function is linear in output. Otherwise profits are not necessarily zero, and the optimal carbon tax is not Pigouvian and does not fully

[2]We abstract in this study from sectoral heterogeneity in terms of emission intensity, and also from unconventional monetary policies. Each of these factors on its own is relevant and can lead to different results from those presented below. We leave their implications for optimal climate and monetary and fiscal policy to future studies, and prefer as a first step to extend the literature on homogeneous and representative agents to incorporate climate considerations. This focus will help us to understand later how heterogeneity in terms of, for instance, endogenous movements in relative prices of green versus brown goods, or unconventional monetary policy, might lead to different implications.

internalize output damages. Whether the carbon tax over or under internalizes the damages depends on whether the abatement cost function is strictly increasing or decreasing in GDP. We then link this environmental policy result to the well-known Friedman rule in monetary economics and show that if the optimal carbon tax ceases to be Pigouvian, deviations from the Friedman rule also happen to be optimal.[3]

We calibrate our model to some common parameters values, and assume that we are in a laissez-faire world (business-as-usual scenario) where with continuously increasing carbon emissions the mean atmospheric temperature rises to 4°C above its pre-industrial level in the steady state (the steady state can be interpreted as "by end of the century"). We then allow for the possibility of mitigating (i.e., reducing) emissions and endogenize climate and fiscal-monetary policies and solve for the optimal long-run Ramsey policies. The Ramsey-optimal policy calls for reducing emissions by about 36% - compared to the baseline, where emission concentrations reach a level consistent with a 4°C mean atmospheric temperature increase - thus lowering the increase in temperature to 2.12°C in the steady state, very close to IPCC target of 2°C by the end of century. Of course, the results are quite sensitive to the assumed parameters values, in particular to the parameter governing the size of output damages from higher temperatures. Higher values for this parameter sharply increase the optimal abatement rate and significantly lowers the increase in the steady-state level of temperature. When the Ramsey planner does not have access to a carbon tax, the temperature barely changes and remains at 4°C in the Ramsey steady-state with no climate policy.

We find that, in line with Barrage [2020b], there is a negative relationship between the optimal rate of labor tax and the carbon tax rate in the steady state as, moving from the Ramsey equilibrium with no climate policy to the one with carbon taxation, an increase in the carbon tax rate leads to a lower rate for the labor income tax. The optimal inflation rate barely changes and remains near zero for a wide range of parameter values with the exception of the intratemporal elasticity of substitution across the different varieties of intermediate goods, as we will discuss shortly below.

Government budget can be exposed to climate change in several ways, including through existing program costs such as health care and also the fact that some of publicly funded adaptation programs have to be provided by the government. Indeed, Barrage [2020a, 2021] documents that climate change is becoming a fiscal burden for many governments around the world. We follow Barrage [2021] and in section 4 present an extension of the baseline model where we assume that government spending may depend on the climate, or more precisely on the average temperature. We solve for the Ramsey-optimal results under this assumption

[3]The Friedman rule calls for minimizing the opportunity cost of holding cash balances by setting the nominal interest rate to zero.

and show that the optimal abatement rate sharply rises and the optimal long-run average temperature significantly falls the higher becomes the direct costs of climate change for the government budget.

We consider a climate-specific shock on CO2 emissions and study the Ramsey-optimal dynamic implications under the scenarios that the Ramsey planner has and has not access to a tax on carbon emissions. The emissions shock generates changes in temperatures via the relation that governs the long-run change in the earth's mean temperature as a function of the atmospheric CO2 concentration. In response to an unexpected rise in the amount of carbon concentration in the atmosphere (or similarly, to an unexpected shock the climate sensitivity, that is the average temperature sensitivity to the amount of carbon concentration), the declines in output, consumption, and welfare with a carbon taxation policy are much more modest than without a climate policy. The Ramsey planner induces sharp rises in optimal carbon tax rate and significantly lowers emissions, leading to modest rises in temperature over time, compared with the case of no carbon tax policy. This result points to the importance to have an additional instrument (on top of a fiscal and a monetary one) in order to max welfare in response to climate shocks. Moreover, the planner also finds it optimal to use an appropriate combination of contractionary fiscal and monetary policies to limit the damages of the climate shocks. This is particularly evident when we plot the responses to a climate-specific shock for a range of different levels of output damages. The higher are the climate change production damages, the sharper are the rises in labor income tax and the nominal interest rate in response to the shock.[4]

In response to unexpected shocks to TFP and government spending, key macroeconomic variables in the climate-macroeconomy model with a climate component exhibit substantially larger responses and variability, compared with their counterparts in the standard NK model where there is no environmental externality. For instance, the response of inflation is about three times as large in the environmental NK model than in the standard NK version. However, the responses of almost all key common macroeconomic variables to both shocks in the climate-macroeconomy model with or without a carbon tax policy are essentially the same. This is quite in contrast to what emerges from the responses to a climate-specific shock such as an unexpected rise in the CO2 concentration, as we just discussed above. Our conclusion from these dynamic exercises is that to understand the importance of a climate

[4] Lemoine and Traeger [2014] and Cai and Lontzek [2019] in related works study the role of climate uncertainty and tipping points for the social cost of carbon and thus, efficient carbon tax. They find that climate uncertainty and tipping points substantially affect the optimal carbon tax. This paper adds to this literature by studying the implications of such shocks for optimal climate policy in a macro environment with distortionary fiscal policy and money nonneutrality. Our study helps to distinguish between quite different implications of climate and macro aggregate shocks for optimal climate and carbon policy and also their implications for optimal distortionary fiscal and monetary policy and their interactions with carbon taxes.

policy in combating climate change we need to look at the right impulse responses, at least in the context of such DSGE models.

The paper consider a number of robustness exercises (Sections 3 and 4). First with respect to the intensity of the damage function, i.e. the impact of global temperatures on output. Second, with respect to the relationship between CO2 concentrations in the atmosphere and global temperature. As expected, as CO2 concentrations have a larger effect on temperatures and, in turn, temperatures have a more significant effect on output, the optimal response of carbon taxation is higher and the welfare gains associated with having a carbon tax is also higher.

This paper contributes also to a quite large literature in environmental economics that has discussed the importance of emissions trading plans or carbon taxation for the design of pollution mitigation policies (see, Bovenberg and Goulder [2002] for a review). And to that strand of literature that develops computable general equilibrium models and tax system to study the interactions between environmental policy and fiscal policy.[5] Our paper is also related to Heutel [2012] on the optimal carbon taxation in the context of an RBC model with TFP shocks and climate externality. Annicchiarico and Di Dio [2015] updates the model of Heutel [2012] with nominal price rigidities and capital, and studies the dynamic implications of the model for different environmental policy regimes. Both of these papers focus on the business cycle implications of their climate-model economies in response to typical TFP or/and government spending shocks and abstract from distortionary fiscal policies geared toward mitigating emissions.

2 The model

We consider an economy that consists of a representative household with an infinite planning horizon where higher temperatures affect welfare directly. A collection of monopolistically competitive firms that produce differentiated goods and face nominal rigidities, with climate damages appearing in the production function (temperature negatively affects total factor productivity (TFP)). The government finances exogenous expenditures with distortionary labor taxes, carbon taxes, issuing fiat money, and one-period nominal riskless debt. As

[5] Regarding both strands, see, e.g., W.Jorgenson et al. [2013], H. [1995], Bovenberg and Goulder [1996], Bovenberg and de Mooij [1994, 1997], de Mooij and Bovenberg [1998], Cremer et al. [2001, 2010], Babiker et al. [2003], de Bruin et al. [2009], Arbex and Batu [2020], Burke et al. [2015], Carbone and Smith [2008a], Jared C Carbone and Burtraw [2013], Carbone and Smith [2008b], Fullerton and Kim [2008], Anthoff and Tol. [2014], Goulder et al. [1996, 2019], Goulder and Hafstead [2017], Hope [2011], Ligthart and van der Ploeg [1994], Ballard and Fullerton [1992], DeVries et al. [2017], Oueslati [2014], Parry et al. [1999b,a], Burtraw et al. [1998], Rezai et al. [2012], Rezai and van der Ploeg [2016], Williams [2002], West and Williams [2007], van der Ploeg and Rezai [2021]

in the related literature, we assume a linear carbon circulation system where the average temperature is approximated by a logarithmic function of the carbon concentration in the atmosphere.

2.1 Households

The economy is populated by a continuum of identical households. Each household's preferences depend on consumption, c_t, hours worked, h_t, and the mean global surface temperature rise over pre-industrial levels, T_t. Preferences are:

$$E_0 \sum_{t=0}^{\infty} \beta^t U\left(c_t, h_t, T_t\right) = E_0 \sum_{t=0}^{\infty} \beta^t \left[u\left(c_t, h_t\right) + d\left(T_t\right)\right] \tag{1}$$

where E_t is the conditional expectation operator evaluated at period t, and $0 < \beta < 1$ is the discount factor. The single-period utility function $U(.)$ is assumed to be strictly increasing in consumption, strictly decreasing in hours worked and temperature, and strictly concave. We assume that:

$$u\left(c_t, h_t\right) = \frac{c_t^{1-\sigma} - 1}{1-\sigma} - \psi \frac{h_t^{1+\frac{1}{\theta}}}{1+\frac{1}{\theta}}, \tag{2}$$

$$d\left(T_t\right) = \frac{\left(1 + \alpha_0 T_t^2\right)^{-(1-\sigma)}}{1-\sigma}, \tag{3}$$

where (2) is taken from MaCurdy [1981] and Boppart and Krusell [2020] and (3) from Barrage [2020b].

Based on data from many countries, Boppart and Krusell [2020] document that going back 100 years or more across many countries hours, worked per worker have been falling at a steady rate of roughly half a percentage point per year. They show that the MaCurdy utility function (2) corresponds to their general formulation of preferences over consumption and leisure and propose a calibration of 1.34 for σ (the inverse of the constant intertemporal elasticity of substitution) and 2.84 for θ (the Frisch elasticity of labor supply) based on their long-run data. We adopt these two values as our baseline calibration.

We assume household consumption is a CES aggregator made up of a basket of goods defined by $c_t = \left[\int_0^{\infty} c_{it}^{\frac{\epsilon-1}{\epsilon}} di\right]^{\frac{\epsilon}{\epsilon-1}}$, where c_{it} represents the quantity of good i consumed by the household in period t. The parameter $\epsilon > 1$ stands for the intratemporal elasticity of substitution across different varieties of consumption goods. As it is standard in the literature, we assume each good i is produced using a linear technology that takes labor of type i, h_{it}, as the sole input in period t. The household supplies all types of labor. Denoting P_{it} as the nominal price of the final goods produced in industry i, the optimal allocation for consumption across individual goods results in the following demand for good i:

$$c_{it} = \left(\frac{P_{it}}{P_t}\right)^{-\epsilon} c_t, \tag{4}$$

where P_t is the aggregate price index in period t and is defined as $P_t = \left[\int_0^\infty P_{it}^{1-\epsilon} di\right]^{\frac{1}{1-\epsilon}}$.

As in Schmitt-Grohé and Uribe [2004, 2010], we introduce a money demand friction into the model by assuming that nominal money holdings, denoted by M_t, facilitate consumption purchases. More specifically, consumption purchases are subject to a proportional transaction cost $\Theta(v_t)$, where,

$$v_t = \frac{P_t c_t}{M_t} \tag{5}$$

is consumption-based money velocity. The transaction cost function satisfies the same assumptions as in Schmitt-Grohé and Uribe [2004, 2010]. It is assumed to be decreasing *[increasing?]* in v_t and satisfying the following four assumptions: (a) $\Theta(v_t) \geq 0$ and is continuous and twice differentiable; (b) there exists a level of velocity $\tilde{v}_t > 0$ – known as the satiation level of money – at which $\Theta(\tilde{v}_t) = \Theta'(\tilde{v}_t) = 0$; (c) $(v_t - \tilde{v}_t)\Theta'(\tilde{v}_t) > 0$ for $v_t \neq \tilde{v}_t$; and (d) $2\Theta'(v_t) + v_t\Theta''(v_t) > 0$ for all $v_t \geq \tilde{v}_t$. Assumption (b) is made to ensure that, in order to get a zero nominal interest, the demand for money need not to be infinite. When this assumption holds, the nominal net interest rate will be zero, and the transaction cost and the distortion it introduces disappear. Assumption (c) is made to guarantee that money velocity in equilibrium is always greater than or equal to the satiation level $\tilde{v}_t$. Assumption (d) is to make sure that the money demand function is decreasing in the nominal interest rate.

Each period, the household allocates its income between consumption, nominal money holdings, and one-period nominal riskless bonds, denoted by B_t, carrying a gross nominal interest rate of R_t when held from period t to period $t+1$. The household derives its income from net-of-tax labor income, money, government bond repayments, and share of profits, $\mathit{\Pi}_t$, received from the ownership of the imperfectly competitive firms producing intermediate goods.

The household's objective is to choose $\{c_t, v_t, h_t, M_t, B_t\}_{t=0}^{\infty}$ to maximize expected utility (1), subject to (5) and the period-t flow budget constraint:

$$P_t c_t [1 + \Theta(v_t)] + M_t + B_t = P_t (1 - \tau_{ht}) w_t h_t + M_{t-1} + R_{t-1} B_{t-1} + P_t \mathit{\Pi}_t, \tag{6}$$

where τ_{ht} is a wage income tax rate, and w_t is the period-t real wage. To make our results on optimal fiscal-monetary policy as comparable as possible to Schmitt-Grohé and Uribe [2004], we follow them and focus only on the case that the Ramsey planner does not have access to profit taxation. Notice that households are also assumed to be subject to a borrowing limit that prevents them from engaging in Ponzi schemes.

The first-order conditions associated with the above maximization problem are as follows:

$$-\frac{u_h(c_t,h_t)}{u_c(c_t,h_t)} = \frac{(1-\tau_{ht})\,w_t}{1+\Theta(v_t)+v_t\Theta'(v_t)}, \tag{7}$$

$$v_t^2\Theta'(v_t) = \frac{R_t-1}{R_t}, \tag{8}$$

$$\frac{u_c(c_t,h_t)}{1+\Theta(v_t)+v_t\Theta'(v_t)} = \beta R_t E_t \frac{1}{\pi_{t+1}} \frac{u_c(c_{t+1},h_{t+1})}{1+\Theta(v_{t+1})+v_{t+1}\Theta'(v_{t+1})}, \tag{9}$$

where $\pi_{t+1} \equiv \frac{P_{t+1}}{P_t}$ denotes the gross rate of price inflation in period $t+1$. Optimality condition (7) shows that the labor income tax τ_{ht}, and an interest rate greater than zero (which is associated with a level of money velocity above the satiation level $\tilde{v}_t$) introduce wedges between the marginal rate of substitution of consumption for leisure and the real wage rate. The first-order condition (8) defines a demand for fiat money or can be interpreted as a liquidity preference function. Optimality condition (9) is a Fisher equation. It represents a standard Euler equation and states that the nominal interest rate must be equal to the sum of the real rate of interest and the expected inflation rate.

2.2 Firms

There is a continuum of monopolistically competitive firms that produce differentiated goods. Production of good i is given by:

$$y_{it} = (1 - D(T_t))\, z_t h_{it} \tag{10}$$

where via $D(T_t)$ climate damage is formulated as a fraction of output lost. This way of formulating damages was pioneered by Nordhaus [1991] and is widely used in the literature. z_t is an exogenous stochastic aggregate technology shock, which is common across firms, like

$D(T_t)$. Firm i faces the following demand schedule:

$$y_{it} = \left(\frac{P_{it}}{P_t}\right)^{-\epsilon} y_t. \tag{11}$$

Notice that due to the presence of imperfectly price-elastic demand, each firm i has some market power, which results in inefficiency in product and factor markets. There are two other distortions. One stemming from quadratic adjustment costs in changing prices, as in Rotemberg [1982], and the other from emissions abatement costs. Specifically, the price adjustment costs faced by firm i take the form $\frac{\phi}{2}\left(\frac{P_{it}}{P_{it-1}} - 1\right)^2$, where ϕ measures the degree of nominal price rigidity. Output of firm i is assumed to entail carbon emissions, but we assume firms can mitigate (i.e., reduce) them by a factor μ_{it} at an additional cost of $x(\mu_{it} y_{it})$, as in Barrage [2018, 2020b,a]. $x(.)$ is an increasing function in the amount of abatement or clean output$\mu_{it} y_{it}$. Notice that in Nordhaus [2008] $x(.)$ is linear in output, but this is not always the case in the literature, as it is not in Barrage's works.

Nominal profits of firm i at date t are given by:

$$\mathit{\Pi}_{it} = P_{it} y_{it} - P_t w_t h_{it} - P_t \frac{\phi}{2}\left(\frac{P_{it}}{P_{it-1}} - 1\right)^2 - P_t \tau_{ct}(1 - \mu_{it})\Gamma(y_{it}) - P_t x(\mu_{it} y_{it}), \tag{12}$$

where τ_{ct} denotes the excise tax on carbon emissions. $\Gamma(.)$ is a function that determines how emissions are related to output, holding μ constant. If $\Gamma(.)$ is convex, emissions are increasing more rapidly than output, and if it is concave they are increasing less rapidly.

The objective of the firm is to choose contingent plans for P_{it}, h_{it}, and μ_{it} to maximize the present discounted value of profits, given by:

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{\lambda_t}{\lambda_0} \mathit{\Pi}_{it}, \tag{13}$$

subject to (10) and (11). Here $\lambda_t = \frac{u_c(c_t, h_t)}{1 + \Theta(v_t) + v_t \Theta'(v_t)}$. $\beta^t \frac{\lambda_t}{\lambda_0}$ is the period 0 value to the household of period t goods, which the firm uses to discount profit flows. Technically, λ_t is the Lagrangian multiplier associated with the household budget constraint (6).

As is common in the literature, we restrict attention to symmetric equilibria, where all firms charge the same price, produce the same amount of goods, and hire an equal amount of labor. Therefore, we can drop the index i. The first-order conditions for the firm's profit maximization result in the following conditions:

$$-\pi_t(\pi_t - 1) + E_t \beta \frac{\lambda_{t+1}}{\lambda_t} \pi_{t+1}(\pi_{t+1} - 1) + \epsilon y_t / \phi \left(\frac{(1 - \epsilon)}{\epsilon} + mc_t\right) = 0, \tag{14}$$

$$\tau_{ct} = \frac{x^{'}(\mu_t y_t)\, y_t}{\Gamma(y_t)}, \tag{15}$$

$$w_t = (1 - D(T_t))\, z_t \left[mc_t - \tau_{ct}(1-\mu_t)\, \Gamma^{'}(y_t) - \mu_t x^{'}(\mu_t y_t) \right], \tag{16}$$

where mc_t is the Lagrange multiplier associated with constraint (11), which is simply the real marginal cost. Condition (14) is a New Keynesian Phillips curve which shows an equilibrium relationship between current inflation, π_t, the current deviation of marginal cost, mc_t, from marginal revenue, $\frac{\epsilon-1}{\epsilon}$, and the expected future inflation. The first-order conditions (15) and (16) state that the firm engages in emission reductions until the carbon price τ_t^c equals the adjusted marginal abatement cost and hires labor until the net marginal revenue product of labor (marginal revenue product adjusted for the marginal cost of carbon tax per unit of output, for the marginal abatement costs and for damages) equates its price w_t. Condition (16) makes it clear that both carbon taxes and the climate externality distort the equality between the real wage w_t and the marginal revenue product of labor $z_t mc_t$ that would prevail in the absence of any pollution externalities.

The New Keynesian Phillips curve (14) shows that climate externalities affect the inflation rate through affecting the marginal product of labor. To see this more clearly, consider the long-run version of (14):

$$\pi(\pi - 1) = \frac{(1 - D(T))(1 - (1-\epsilon)\, mc)\, h}{(1-\beta)\, \phi},$$

where we used the fact that $z = 1$ in the steady state. Compared to the standard formulation, this expression shows that in the steady state inflation is inversely dependent also on the damage function $D(.)$ and on the long run level of temperature T. Of course, the values of endogenous variables are determined simultaneously as a solution to a system of equations in the model.

2.3 Carbon Circulation

We assume a linear carbon cycle model with a constant decay rate of $(1-\zeta)$. Let the variable s_t denotes the stock of carbon in the atmosphere in period t. Hence, the law of motion for s_t is given by:

$$s_t = \zeta s_{t-1} + (1-\mu_t)\, \mathit{\Gamma}(y_t) + \kappa_t, \tag{17}$$

where κ_t is an exogenous stochastic process following a univariate autoregressive process of the form:

$$\ln(\kappa_t/\bar{\kappa}) = \rho^{\kappa} \ln(\kappa_{t-1}/\bar{\kappa}) + \varepsilon_t^{\kappa}, \tag{18}$$

where $\bar{\kappa}$ is a constant, $\rho^{\kappa} \in (-1, 1)$, and ε_t^{κ} is an i.i.d. innovation with mean zero and standard deviation $\sigma_{\varepsilon^{\kappa}}$, e.g. land-based emissions (as in Nordhaus, 2018). We assume κ_t to be 30 percent of emissions in the non-stochastic steady state of a competitive equilibrium (this is based on what Hassler et al. [2016] discuss in section 3.2).

We can interpret κ_t as a combination of different exogenous shifters that might lead to an unexpected rise in the amount of carbon concentration in the atmosphere. As Hassler et al. [2016] discuss in detail, a tipping point mainly refers to a phenomenon either in the carbon cycle or in the climate system where in each there can be a strong nonlinearity. Thus, we can interpret a strong positive shock to κ_t as such a tipping point in the carbon circulation model. Moreover, the stochastic process κ_t can be also interpreted as representing the uncertainties that the scientific community still faces regarding issues such as the oceans' and forests' ability to decay pollution; how the carbon cycle responds to the possible future higher temperatures; and the possibility of a switch in current carbon sinks from being a sink to a carbon source due to extreme heat, wildfires or melting of glaciers (see, e.g., Pörtner et al. [2019], DeVries et al. [2017], Pangala et al. [2017], Nordhaus [2018]). Considering an emission shock is also particularly useful in this regard for understanding the implications of climate change and its risks for aggregate macroeconomic variables, where the evolution of key climate variables is uncertain. It also helps to better understand an assessment of optimal environmental policies for fighting climate change in the case of unexpected climate shocks.

As is standard in the related literature (see, e.g., DICE and Hassler et al. [2016]), we assume the following relation between the long-run change in the average temperature above its pre-industrial level and the carbon concentration in the atmosphere:

$$T_t = \frac{\nu}{\log 2} \log \left(\frac{s_t}{\bar{s}} \right), \tag{19}$$

where $\bar{s}$ stands for the pre-industrial stock of carbon in the atmosphere. The parameter ν is the equilibrium climate sensitivity parameter and reflects the long-run increase in the earth's average temperature (in degrees of Celsius) relative to pre-industrial times from doubling the amount of atmospheric carbon concentrations relative to pre-industrial levels.

2.4 Government and Aggregate Resource Constraint

As in the case of private consumption, aggregate government consumption takes the form[6]

$$g_t = \left[\int_0^{\infty} g_{it}^{\frac{\epsilon-1}{\epsilon}} di\right]^{\frac{\epsilon}{\epsilon-1}}.$$

The optimal level of g_{it} is then given by

$$g_{it} = \left(\frac{P_{it}}{P_t}\right)^{-\epsilon} g_t.$$

Note that government expenditures g_t are exogenous, stochastic, and unproductive. The government is assumed to finance its public consumption by imposing labor income taxes at rate τ_{ht}, carbon taxes at rate τ_{ct}, and printing money M_t and issuing one-period, riskless nominal bonds B_t. Then, the government's sequential budget constraint can be written as:

$$\tau_{ht} P_t w_t h_t + \tau_{ct} P_t (1-\mu_t) y_t + M_t + B_t = M_{t-1} + R_{t-1} B_{t-1} + P_t g_t. \tag{20}$$

The climate and fiscal/monetary regime includes announcing state-contingent plans for the abatement μ_t, the labor tax rate τ_{ht}, and the nominal interest rate R_t.

The aggregate resource constraint in the present model is given by the following two expressions:

$$c_t + g_t + x(\mu_t y_t) + \frac{\phi}{2}(\pi_t - 1)^2 = y_t, \tag{21}$$

$$y_t = (1 - D(T_t)) z_t h_t. \tag{22}$$

Before turning to model calibration and analyzing the Ramsey-optimal long-term and short-term results, we define the competitive equilibrium and the Ramsey problem in the next subsection.

2.5 Competitive equilibrium and Ramsey problem

A competitive equilibrium is a set of sequences $\{c_t, h_t, v_t, y_t, s_t, mc_t, w_t, T_t, M_t, B_t, P_t\}_{t=0}^{\infty}$ satisfying equations (5), (7)-(9), (14)-(17), (19)-(22) and the fact that, in equilibrium, the nominal interest rate must be non-negative,

$$R_t \geq 1 \tag{23}$$

[6]We could have equivalently assumed an 'aggregator' firm which converts the individual goods to a final output which will be purchased by households and the government. In the model we have considered here, equivalently, the government and households perform this aggregation themselves.

given policies $\{\tau_{ht}, \mu_t, R_t\}$, exogenous stochastic processes $\{z_t, \kappa_t, g_t\}$, and the initial condition $M_{-1}+R_{-1}B_{-1}$. Notice that the optimal carbon tax rate is a function of the optimal abatement rate μ_t, they are not independent endogenous variables. When we determine μ_t optimally, τ_{ct} is automatically determined.

We characterize optimal environmental and fiscal/monetary policy under the assumption that the government has the ability to commit to policy announcements. This policy optimality concept is known as Ramsey optimality. The optimal climate and monetary-fiscal policy in the context of the present framework is the process $\{\mu_t, R_t, \tau_{ht}\}$ associated with the competitive equilibrium that maximizes the expected lifetime utility of the representative household (1). In other words, it is the best competitive equilibrium that yields the highest level of utility to the household.

2.6 Calibration

We calibrate the model at an annual frequency for a global economy. Table (2.6) summarizes the values of the deep structural parameters of the model implied by our calibration strategy. It is well understood in the climate literature that the discount factor greatly matters for what optimal carbon tax to recommend (Hassler et al. [2016]). We follow Nordhaus as our baseline calibration and set β to 0.985. We will also report our Ramsey result for a much higher value for β in the spirit of Stern [2006].

We adopt the environmental production damage $D(T_t)$ and the calibration of its parameter from DICE (Nordhaus [2008]) as follows:

$$D(T_t) = 1 - \frac{1}{1+\bar{\vartheta}T_t^2}, \tag{24}$$

where $\bar{\vartheta} = 0.0028388$.

We assume that $\Gamma(y_t) = y_t$ as in much of the literature. We assume the following form for the abatement cost function:

$$x(\mu_t y_t) = \gamma_1 (\mu_t y_t)^{\gamma_2}. \tag{25}$$

In DICE (2008) the abatement cost function is linear in output, that is, it is $\gamma_1 (\mu_t)^{\gamma_2} y_t$ where $\gamma_2 = 2.8$ and γ_1 is estimated to be 0.029 for the US (see also Cline [2011], chapter 4, for a discussion of calibration of the abatement cost function based on DICE). However, as we mentioned before, we follow Barrage [2018, 2020b,a] and assume that the abatement cost function is nonlinear in the amount of clean output and takes the form (25). In calibrating γ_1 and γ_2 we follow DICE (2008) and set γ_2 to 2.8 but we pin down γ_1 in such a way that the abatement cost function (25) gives the same magnitude of abatement cost as the form $\gamma_1 (\mu_t)^{\gamma_2} y_t$, where GDP is assumed at the same level as that implied by the competitive equilibrium. This calibration strategy gives $\gamma_1 = 0.41$.

As our baseline calibration, we set the pollution decay parameter ζ to 0.9946, meaning that the excess CO2 has a mean lifetime of about 300 years (see, e.g., Archer [2005] and Hassler et al. [2016]). The long-run climate sensitivity parameter ν in (19) is set to 3, which is the current best estimate according to Masson-Delmotte et al., 2021.

To facilitate comparison to the results reported by Schmitt-Grohé and Uribe [2004], we adopt, where possible, their calibration of the model for our macroeconomic part of the model. Although we assume our economy is a global economy model, nevertheless we think the parameter values of Schmitt-Grohé and Uribe [2004], which are calibrated based on the US economy, can be considered as an average for the world economies.

The functional form of the transaction cost technology is assumed to be:

$$\Theta (v_t) = A v_t + B / v_t - 2\sqrt{AB}, \tag{26}$$

where $A = 0.0111$ and $B = 0.07524$.

Given that our New Keynesian non-linear Phillips curve has the same form as in Schmitt-Grohé and Uribe [2004], following them we set initially the elasticity of substitution between intermediate goods ϵ to 6 and the degree of price stickiness ϕ to 17.5/4. The debt-to-output ratio is assumed to be 44 percent.

Total factor productivity z_t and government spending g_t are assumed to follow independent AR(1) processes in their logarithms:

$$\ln (z_t) = \rho^z \ln (z_{t-1}) + \varepsilon_t^z, \tag{27}$$

$$\ln (g_t / \bar{g}) = \rho^g \ln (g_{t-1} / \bar{g}) + \varepsilon_t^g, \tag{28}$$

where $\varepsilon_t^z \sim N (0, \sigma_{\varepsilon^z}^2)$ and $\varepsilon_t^g \sim N (0, \sigma_{\varepsilon^g}^2)$. We assume $(\rho^z, \sigma_{\varepsilon^z}) = (0.82, 0.02)$ and $(\rho^g, \sigma_{\varepsilon^g}) = (0.9, 0.03)$. In (28), $\bar{g}$ is the long-run value of government spending which we assume to be 20 percent of the steady-state of GDP in a competitive equilibrium (Schmitt-Grohé and Uribe [2004]).

We adopt the calibration of Boppart and Krusell [2020] for the inverse of the intertemporal elasticity of substitution σ and the Frisch elasticity of labor supply θ and set them to $\sigma = 1.34$ and $\theta = 2.84$. Together, they imply that hours worked decline about 0.5% per year. As for the climate change disutility, $d(T_t)$, we borrow the value of α_0 from Barrage [2020b, 2021] and assume $\alpha_0 = 0.0023808$, which means an aggregate global consumption loss-equivalent of disutility from climate change at 2.5^o C equals 0.8 percent of output.

Next, we need to calibrate the utility function parameter ψ and the pre-industrial amount of carbon in the atmosphere $\bar{s}$. In the literature on climate change the value of $\bar{s}$ is assumed to be 600 GtC (Hassler et al. [2016]). In our framework where the working time unit is normalized to one, hours worked are between $(0, 1)$ and thus also output. Together, with the fact that we consider one-box linear carbon circulation system we need to modify the assumed magnitude of $\bar{s}$ accordingly. More precisely, to calibrate ψ and $\bar{s}$, we assume that up to period 0, the economy is in the non-stochastic steady state of the business-as-usual (BAU) scenario of the laissez-faire equilibrium with constant paths for output, pollution, consumption, hours worked, nominal interest rates, inflation, tax rates, etc. As in Hassler et al. [2016], business as usual is the laissez-faire allocation where $\mu = 0$ and emissions, based on Scenario RCP8.5 from IPCC's 5th Assessment Report, lead to a global temperature increases of about 4°C at the end of the century. Thus, we can interpret the steady-state of our model as the end of this century. We assume that in the steady state the inflation rate is about 1.5 percent per year, which is in line with the average growth rate of the US GDP deflator over the period 1990:Q1 to 2019:Q4. Consistent with Aguiar and Hurst [2016] and the macro literature, we assume that in the flexible-price steady-state households allocate 24 percent of their time to work. Taken together, we obtain $\bar{s} = 21.9$ (thus 600 GtC is equivalent to 21.9 in arbitrary units in our framework) and $\psi = 9.76$. Notice that in the business-as-usual scenario of non-Ramsey steady state the amount of carbon, given our baseline calibration, is roughly equal to $s = 55.3$.[7]

Table 1: Benchmark Calibration

Parameter	Value	Description	Parameter	Value	Description
β	0.985	discount factor	ν	3	climate sensitivity parameter
θ	2.84	elasticity of labor effort	A	0.0111	Parameter of transaction cost function
ψ	9.76	utility scale parameter	B	0.07524	Parameter of transaction cost function
α_0	.00023808	utility damage function	ϵ	6	Gross value-added markup
σ	1.34	consumption utility (damage) exponent	ϕ	17.5/4	Degree of price stickiness
$\bar{\vartheta}$	.0028388	production damage function	ρ^g	0.9	serial correlation of $\ln g_t$
γ_1	0.41	abatement function-multiplier	σ_{ε^g}	0.03	standard deviation of innovation to $\ln g_t$
γ_2	2.8	abatement function-exponent	ρ^z	0.82	serial correlation of $\ln z_t$
ζ	0.9946	decay parameter	σ_{ε^z}	0.02	standard deviation of innovation to $\ln z_t$
$\frac{b}{y}$	0.44	debt-to-GDP ratio	$\frac{g}{y}$	0.20	government spending to GDP ratio

[7] See the appendix for a detailed description.

3 Ramsey Results

In this section we describe Ramsey-optimal policy results under several scenarios. We first analytically show that when the abatement cost function is non-linear in output the optimal carbon tax to internalize production damages is not a Pigouvian tax. We then turn to the quantitative results and solve for the steady state and then the dynamics implied by the Ramsey equilibrium of our model, following the general procedure used by Schmitt-Grohé and Uribe [2004, 2005, 2007, 2010, 2012]. The procedure yields an exact numerical solution for the steady state of the Ramsey problem. We compute business-cycle properties of the optimal policy by solving second-order logarithmic approximations to the Ramsey planner's policy functions around a non-stochastic Ramsey steady state.

The government is assumed to be benevolent in the sense that its objective is to maximize the welfare of the representative household. At $t = 0$, the government is supposed to have been operating for an infinite number of periods. In choosing optimal environmental and fiscal-monetary policy, the government is assumed to honor commitments made in the past. This form of policy commitment is referred to as 'optimal from the timeless perspective' (Woodford [2003]).

For the quantitative part, we first report and discuss the Ramsey-optimal policy results, steady-state as well as dynamics, for our baseline model with sticky prices and distortionary taxes, including for a range of values for the key structural parameters. To better understand the interactions between optimal climate and fiscal-monetary policies, we also report the long-run and dynamic implications of Ramsey policy under different scenarios: under perfect competition and flexible prices and with and without distortionary fiscal policy; under sticky prices and with and without distortionary taxation.

3.1 Optimal climate policy under flexible prices and perfect competition

Before turning to present our quantitative results, we analytically prove that the optimal carbon tax deviates from the Pigouvian rate if profits are not fully taxed in the case of a non-linear emission abatement cost function in output. This is the case even if prices are fully flexible and we have perfect competition. In this case the Friedman rule also ceases to be optimal. That is, it is no longer optimal to set the net nominal interest rate to zero, so that $R_t > 1$.

Proposition 1 *If the abatement cost function is non-linear in output, the optimal carbon tax rate does not fully internalize output damages and is not Pigouvian. The Friedman rule also* *ceases to be optimal in this case. The optimal tax on carbon is equal to (assuming $\Gamma(y_t) = y_t$ and climate change only affects production possibilities and not also consumers' utility)*

$$\tau_c = x'(\mu y) = \frac{\left(\left(1 - x'(\mu y)\mu\right) - \frac{\bar{\lambda}}{\lambda^3}\Delta^3\right) T'(s) D'(T(s)) h + \frac{-T'(s) d'(T(s))}{\lambda^3}}{\left(1 + \frac{\bar{\lambda}}{\lambda^3}\Delta^3\right)\left((1 - \beta\zeta) + (1 - \mu) T'(s) D'(T(s)) h\right)} \tag{29}$$

where $\Delta^3 = \frac{u_c(c,h)\mu^2 y x''(\mu y)}{1+\Theta(v)+v\Theta'(v)}$ and $\bar{\lambda}$ and λ^3 are Lagrangian multipliers and strictly positive.

Proof. See The Appendix C. ■

If the abatement cost function is linear in output, as is the case in DICE, as we prove in detail in the appendix, $\Delta^3 = 0$. In this case the Friedman rule is optimal and the optimal carbon tax when climate change does not impact utility is equal to the social cost of carbon (SCC) (see Appendix B for the formal derivation of SCC).[8] However, if the abatement costs are non-linear in μy, as is the case in Barrage [2018, 2020b, 2021], profits will be different from zero, making the carbon tax to be different from its Pigouvian rate (see the Appendix C for the proofs). To see this intuitively, consider that firms will abate based on condition (15). They will be willing to abate up to the point in which the marginal cost of abatement is equal to the carbon tax; if the former is nonlinear there will some extra profits/losses. Whether the optimal carbon tax would over or under internalize production damages depends on whether we assume it is strictly increasing or decreasing in output.[9]

This analytical finding means that the main theoretical result in Barrage [2020b] that the optimal carbon tax can fully internalize production damages of climate change regardless of the presence of distortionary taxes is true only if profits are fully taxed in the energy sector, otherwise that result does not follow.[10]

Under the assumption of a non-linear emission abatement cost function, also the Friedman rule also ceases to be optimal, despite the fact that prices are fully flexible and we have perfect competition (see the Appendix C), as the Ramsey planner uses a positive nominal interest rate to tax profits indirectly. Note that the Friedman rule would be optimal if the government did not have access to a carbon tax policy. This non-optimality of the Friedman rule in a monetary model with climate change and carbon taxation policy adds to the cases discussed in Schmitt-Grohé and Uribe [2010] where the Friedman rule might cease to be optimal.

[8]Note that, as is discussed in detail in Kiarsi and Masoudi [2021], when climate change affect utility directly the optimal carbon tax under distortionary taxes may or may not be Pigouvian.

[9]It is important to note that this deviation of carbon tax from a Pigouvian rate is different from the well-known result in the related literature that when the Hotelling profits are different from zero–presence of a non-renewable energy resource–the optimal carbon tax is higher than Pigouvian if profits are fully taxed (see Barrage [2020b] and references therein).

[10]This is not discussed in Barrage [2020b] where it's taken for granted that profits in the energy sector are zero.

We already know from Schmitt-Grohé and Uribe [2010] that in our assumed monetary model with no environmental externality and imperfect competition but flexible prices the Friedman rule is not optimal. In Appendix D we formally show that in such circumstance the optimal carbon tax will also under internalize production damages.

3.2 Quantitative results: baseline model

We first present the Ramsey long run results for our baseline model where prices are sticky and the government has to finance its spending via imposing distortionary labor income taxes. We assume that carbon tax revenues are insufficient to cover the government expenditures. We then present Ramsey dynamics and discuss the impulse responses of the model to three unexpected shocks.

3.2.1 The Ramsey steady state

Table (2) displays the steady-state values of key macroeconomic variables under the Ramsey policy and for a range of values for key climate structural parameters. The figures reported in the table correspond to the exact numerical solution to the steady state of the Ramsey problem.

The first row of table (2) reports the results in the deterministic steady state of a competitive laissez-faire (no-policy) equilibrium under the benchmark calibration explained in subsection 2.6. The second row documents the Ramsey results for the standard New Keynesian model where there is no environmental externality. The results here are in line with what Schmitt-Grohé and Uribe [2004] report. The third row displays the optimal results for the case that there is no climate policy. That is, the government does not have access to a carbon tax policy. The fourth row reports the Ramsey steady-state results for the baseline model where the Ramsey planner can impose a carbon tax on firms' emissions. The other rows display the Ramsey results of the baseline framework when we reset one of the structural parameters to a new calibration, while the rest are kept the same at their baseline values. In what follows we use the benchmark Ramsey results (line 4) as the reference point.

Table 2: Ramsey steady state climate and monetary/fiscal policies and their sensitivity

Scenario	wf	τ_h	τ_c	R	π	μ	s	T	$D(T)$	h	y	c
CE (BAU)-base. calibr.	-423.23	24.37	0	3.05	1.5	0	55.3	4	.0434	.24	.2296	.1809
RE–NK model only	-218.64	25.4	-	1.4	-.12	-	-	-	-	.2348	.2348	.1868
RE–no carbon policy	-423.12	26.2	0	1.51	-.019	0	54.6	3.96	.0426	.237	.2269	.1789
RE–base. calibr.	-417.6	25.04	1.3	1.51	-.015	35.8	35.8	2.12	.0127	.2348	.2318	.1834
Medium damage $2\bar{\vartheta}$	-417.7	25.03	1.84	1.51	-.013	43.6	31.4	1.56	.0136	.2344	.2312	.1825
High damage $4\bar{\vartheta}$	-417.44	25.02	2.4	1.51	-.011	50.6	27.5	.98	.0108	.2337	.2311	.1821
Low abatement cost $\gamma_1 = 0.41/3$	-415.65	25.2	.82	1.51	-.014	50.7	27.7	1.02	.0029	.2344	.2337	.1853
Low discount $\beta = 0.998$	-3119.7	23.9	2.4	.33	.13	50	28.2	1.1	.0034	.2353	.2345	.1854
Low decay rate $\zeta = 0.9976$	-419.7	26.5	1.1	1.52	-.004	32.4	85	2.35	.0154	.2352	.2316	.1806
High utility damage $2\alpha_0$	-417.7	25	1.57	1.51	-.015	39.7	33.7	1.86	.01	.2345	.2322	.1836
Low Frisch elasticity $\theta = 1.5$	-371.3	19.8	2.17	1.45	-.07	38.4	42.7	2.9	.0231	.2943	.2875	.2386

Note: τ_h, τ_c, π, R, and μ are expressed in percent while the other variables are in level. wf stands for the level of welfare.

Comparison of the Ramsey results under no climate policy (third line) with the laissez-faire economy (first line) indicates that the optimal temperature in this scenario barely changes from its laissez-faire level which is calibrated to be 4 degrees above its pre-industrial level under no mitigation policy. As a result climate damages remain quite high in the Ramsey economy in this case. Regarding monetary policy, observe that in the Ramsey economy the optimal long-run rate of inflation falls from the 1.5 percent prevailing in the competitive economy to near zero and on average slightly negative. That is, the Ramsey planner finds it optimal to stabilize the price level, in line with the existing New Keynesian literature (Schmitt-Grohé and Uribe [2004, 2005]). It is well known that in this class of models the Ramsey-optimal steady-state rate of inflation is on average negative and lies between the one called for by the Friedman rule and the one corresponding to full price stabilization, zero net inflation rate. In other words, the optimal rate of inflation is determined by the tradeoff between minimizing the opportunity cost of holding money (that means setting the inflation rate equal to minus the real interest rate) and minimizing price adjustment costs. It is shown that in calibrated monetary economies, the optimal deflation rate is, however, small (see Schmitt-Grohé and Uribe [2010] for a thorough discussion). We show below that this result is very sensitive to the calibration of gross value-added markup. Indeed, we find out that under high market powers the Ramsey planner finds it optimal to impose a quite high and positive inflation in the steady state and calls for much higher inflation volatility than the existing results in the context of similar models.

Moving to the Ramsey economy with climate policy, line 4, where the key results of this table are reported, we see that under the benchmark parameterization, the Ramsey planner chooses to conduct environmental policy in such a way as to abate around 36 percent of the emissions. This in turn implies an optimal carbon taxation at about 1.3 percent of carbon emissions. The drop in emissions leads to a significantly lower amount of carbon concentration in the atmosphere than what we get in the absence of a tax on carbon, and therefore, a much lower temperature level. More precisely, the optimal environmental policy features an increase in mean atmospheric temperature of 2.12°C, significantly lower than 4°C in the competitive economy under BAU scenario, or the Ramsey economy with no carbon taxation. This level is also very much in line with 2°C targeted by the IPCC by the end of century. It also features a significant reduction by more than 70% in output damages (a decline from 0.0434 to 0.0127) as a result of a decline in the amount of carbon, compared with the cases that there is no carbon taxation. The Ramsey policy also induces higher consumption level, higher output, and higher welfare, and lower hours of work than in the competitive economy or the Ramsey economy with no climate policy. The optimal monetary policy remains the same as that under no climate policy. However, the optimal fiscal policy is to set the average value of distortionary labor income tax at a lower rate. The Ramsey planner imposes a relatively lower rate of labor tax because it now also raises revenue through carbon taxes in financing its exogenous stream of purchases in the long run.

The next two rows (fifth and sixth lines) demonstrate that the optimal rate of abatement, the optimal carbon tax rate, and thus, the mean temperature rise above its pre-industrial level are quite sensitive to the value of the damage function parameter. The lines document the Ramsey results for two higher values for the output damage function parameter $\bar{\vartheta}$, all else equal. Both lines show that since a higher $\bar{\vartheta}$ induces higher output damages, the planner finds it optimal to sharply increase the optimal abatement rate and impose a higher carbon tax rate. More specifically, when $\bar{\vartheta}$ increases from the baseline value $\bar{\vartheta} = 0.0028388$ to $2\bar{\vartheta}$ and $4\bar{\vartheta}$, the optimal abatement rate rises from 35.8% to 43.6% and 50.6%, respectively and the optimal carbon tax rate rises from 1.3% to 1.84% and 2.4%, respectively. That is, the optimal carbon tax increases by more than 40 percent and 84 percent, respectively, in these cases. The increase in output damages decrease output and consumption via two channels: a higher $\bar{\vartheta}$ induces higher output loss, all else equal. Second, since a higher $\bar{\vartheta}$ leads to a higher optimal abatement rate, it induces lower output and consumption level.

Next, consider the case that the coefficient of the abatement cost function γ_1 is a third of its baseline calibration (the seventh row). In this case, because of smaller abatement costs, the Ramsey policy calls for much higher abatement which results in a drop in the amount of carbon concentration and thus less increase in the deviation of mean temperature from the preindustrial average temperature. This in turn will result in lower losses of output and therefore higher level of GDP.

Line 8 of the table displays the results for the case of a discount factor near 1, in the spirit of what is suggested by Stern [2006]. This leads to the optimality of higher abatement rate, by more than 14%, and almost doubling of carbon tax rate, which in turn features a lower level of the stock of carbon and of the average temperature. People find it optimal to work more, which along with a significant fall in output damage, results in a much higher level of GDP and thus, consumption. Notice that the steady state of optimal nominal interest rate in this case falls by about 1 percentage point and the optimal inflation rate becomes positive, about 0.13% per year. The optimal labor tax also falls by around 1.5 percent, due to higher carbon taxes and a higher baseline labor income.

Next, we discuss the sensitivity of the Ramsey results to the decay parameter of carbon emission in the atmosphere, ζ, in (17).[11] At a lower decay rate, $\zeta = 0.9976$, the optimal policy implies a relatively lower optimal abatement rate and carbon tax rate since now the decay of carbon is significantly slower and therefore reducing emissions costlier (see17) . As a result of slower decay, the optimal average temperature also rises to 2.34 and output damages go up. Observe that the optimal steady-state rate of labor tax also rises by about 1.5%. This is because the labor income tax base, $w_t h_t$, falls significantly (w falls since output damages sharply rise), as a result of a decline in the real wage. Hence, the Ramsey planner finds it optimal to increase the labor tax rate to finance its exogenous amount of spending.

A higher value (double) of the utility damage parameter α_0 (line 10) features qualitatively similar Ramsey results as those in the case of a larger value for the output damage parameter (lines 5 and 6) regarding the optimal values of abatement rate, carbon tax rate, the stock of carbon, and the average temperature. That is, the optimal abatement rises from 35.8% to 39.7% and the carbon tax rate increases by more than 20%. As a result, the amount of carbon and the average temperature decline relative to the baseline Ramsey results. But the magnitude of other variables does not change much in this case.

The last line of the table documents the optimal policy results when the Frisch elasticity of labor supply is changed from the benchmark 2.84 – a value based on the work of Boppart and Krusell [2020] – to a lower value of 1.5, keeping all other parameters unchanged. The main implication of a lower Frisch elasticity is the sharp rise in hours worked which leads to a significant increase in output and therefore the amount of carbon and average tempera-

[11] Since with a higher ζ the steady-state value of carbon emission significantly rises, we adjust the initial value of $\bar{s}$ accordingly in our baseline calibration and then solve for the Ramsey optimal policies.

ture. The optimal rate of abatement also significantly increases because production climate damages go up. The increase in optimal abatement results in an increase in optimal carbon taxation. Since the labor tax base significantly rises, due to a big increase in h, the optimal labor income tax sharply falls by more than 5 percentage points, from 25.4% to 19.8%.

3.2.2 Ramsey dynamics

In this subsection, we turn to study the dynamics induced by the Ramsey climate and fiscal-monetary policy. Table (3) reports a number of unconditional moments (the standard deviation, serial correlation, and correlation with output) of key macroeconomic variables of interest under the Ramsey policy and for the benchmark calibration. The second moments reported in this scenario are driven by productivity and government spending shocks presented by expressions (27) and (28). The top panel corresponds to the Ramsey dynamics for the case of New Keynesian model with no climate change. The results reported in this panel are simply a replication of the main results of Schmitt-Grohé and Uribe [2004]. The middle panel reports the results for the baseline New Keynesian model. The bottom panel corresponds to the case in which there is no carbon taxation. In calculating these moments, we first generate simulated time series of length 200 years for the selected variables and compute the moments. We repeat this procedure 1000 times and then compute the average of these figures. All economies are assumed to start out in period 0 with the same level of real total government liabilities $\frac{M_{-1}+R_{-1}B_{-1}}{P_0}$. Thus, the moments reported in the table are unconditional with respect to the three exogenous shocks, but not with respect to the initial level of government liabilities.

Table 3: Models dynamics

	y	μ	T	n	c	τ^n	τ^c	$R-1$	$\pi-1$
	A. Ramsey economy with no environmental externality								
std. dev.	.64	-	-	.43	.6	.71	-	.53	.07
autocorrelation	.85	-	-	.8	.84	.9	-	.9	.47
correlation with y	1	-	-	-.4	.87	.15	-	-.89	.48
	B. Baseline Ramsey-climate economy								
std. dev.	.56	.35	.89	.4	.51	.69	.03	.46	.07
autocorrelation	.84	.88	.99	.8	.84	.9	.83	.9	.1
correlation with y	1	-.42	.06	-.3	.84	.14	.8	-.86	.42
	C. Ramsey-climate economy with no carbon taxation								
std. dev.	.56	-	.9	.41	.5	.69	-	.46	.06
autocorrelation	.84	-	.99	.82	.83	.9	-	.91	.1
correlation with y	1	-	.06	-.3	.83	.14	-	-.85	.4

Note: The standard deviations, serial correlations, and correlations with output of the variables correspond to percent deviations from their steady-state values.

The main result on Ramsey-optimal dynamics of climate policy is that carbon taxes and abatement, in particular the former, are remarkably stable over the business cycle. The optimal volatility of temperature, on the other hand, is relatively high, about 0.9%, and extremely persistent. Given that the optimal mean atmospheric temperature, reported in table (2), under the baseline calibration is 2.12°C, a two-standard deviation interval around the mean temperature is thus about, 2.1–2.14.[12] Comparing the unconditional moments reported in the middle panel with the bottom one makes clear that the optimal dynamics of temperature remain unchanged whether or not there is a carbon tax policy.

Regarding optimal fiscal-monetary policy results, and in line with the related literature (Schmitt-Grohé and Uribe [2004] and Siu [2004]) the optimal inflation volatility under sticky prices is virtually zero. More precisely, the table shows that the optimal unconditional moments of inflation, the nominal interest rate, and the labor tax rate remain virtually the same across different models and scenarios. Since changes in the inflation rate due to the assumption that firms face nominal rigidities come at a cost, the optimal policy calls for near-zero inflation volatility. Instead, the Ramsey planner finds it optimal to vary distortionary income taxes over the business cycle in financing the government outlays. However, as it is well known, the optimal volatilities of inflation and labor tax are quite sensitive to the calibration of market power. As the market power sharply rises, so do the volatilities of inflation and labor tax rate.

In sum, the presence of environmental externalities and the absence or presence of a carbon tax policy per se do not have significant implications for the optimality of price stability under the baseline calibration. It is also important to note that to appreciate the macroeconomic consequences of climate factors and carbon taxation policy we need to study the conditional dynamic implications of shocks, and in particular climate-specific shocks. This is the task that we will take up in the next subsection.

3.3 Impulse responses

In this subsection we present theoretical impulse responses to the two aggregate macroeconomic shocks driving business cycles in our climate-model economy, and to a climate-specific shock, which is a shock to the stock of emissions, as presented in (18). We fix the size of the shock so it can generate about 10 percent increase in the stock of carbon relative to its

[12] Although we have not reported here, an interesting result emerging from unconditional Ramsey dynamics is that the presence or the absence of the central source of monetary non-neutrality in the current leading theories of monetary non-neutrality, that is price stickiness, has no significant implication for the optimal volatility of key environmental variables. That is, the cyclical properties of climate variables under sticky prices are very similar to those arising in (im)perfectly competitive environments with flexible prices. The results can be provided upon request.

steady-state value. In the case of an emission shock (figure (1)), we present both the Ramsey responses for the baseline climate-economy model and the case that there is no climate policy, that is when the government does not have access to a carbon taxation policy. In the cases of a positive government spending shock (figure (2)), and a positive productivity shock (figure (3)), we also add the responses of the standard New Keynesian model where there are no environmental externalities. The solid red line displays the response of the baseline climate-New Keynesian model. The dashed blue line displays the responses in the case of no climate policy, and the dash-dotted green line displays the responses of the baseline NK model with no environmental factors. As should be the case, the latter responses are the same as those reported in Schmitt-Grohé and Uribe [2004]. In all three cases the size of the innovation is one standard deviation and it is one-time shock where the persistence of the shock is set to zero.

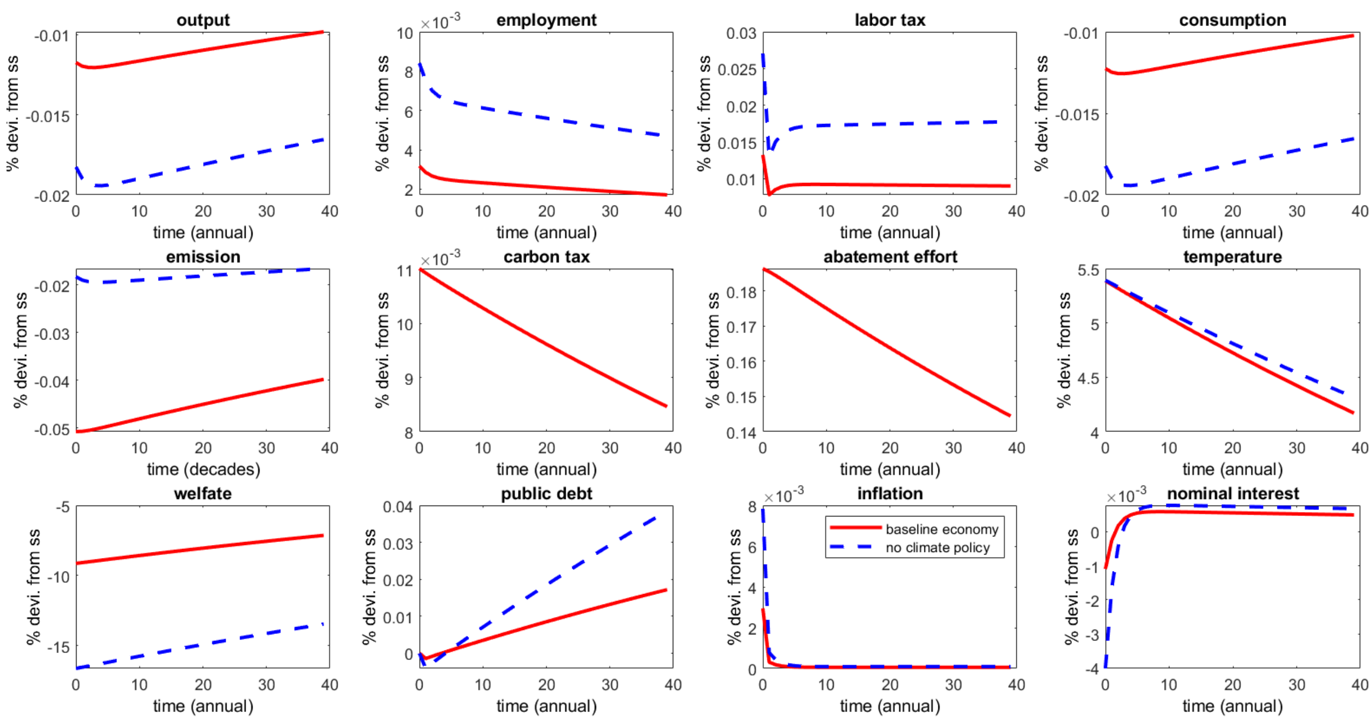

Figure 1: Impulse responses to a one-time and one standard deviation positive shock to the stock of emissions under two scenarios: when the Ramsey planner has access to carbon taxation policy (red line), and when it does not (blue dashed line). The size of the initial innovation to the emission stock shock is one standard deviation.

We may think of the shock in the carbon cycle as a tipping point like shock, although we understand that under tipping point scenario the unexpected change in the amount of carbon still can be quite larger than what we assume in our baseline calibration. Nevertheless, the exercise provides qualitatively good insights regarding what we should expect in such scenario. Examples leading to tipping points in this regard are the case of forests turning from sinks to sources due to extreme heat stress and fires, or carbon release due to melting of permafrost. Figure (1) shows that under such a scenario, the important result is that the presence of carbon taxation policy sharply reduces the fall in key macroeconomic variables such as output, consumption, and welfare, compared with the case that there is no carbon taxation policy. More specifically, when the Ramsey planner has access to the climate policy, in response to the unexpected increase in the amount of CO2 circulation it sharply rises the abatement effort and therefore lowers the emissions significantly. As a result of large abatement and carbon tax rates, the increase in carbon concentration in the atmosphere will be lower than the case of no carbon taxation policy and thus will bring about a lower increase in the average temperatures.

As a result of the availability of mitigation policy, the increase in employment is lower than the case with no carbon policy. In fact, employment initially falls on impact in this case and then starts rising. Given the significant fall in output and consumption under no carbon policy employment increases, and in order to persuade people to work less to limit the damages of climate change through lowering emissions, the Ramsey planner sharply increases the labor tax on impact, where the increase is much higher than the case with a carbon tax policy. However, at the same time the planner finds it optimal to lower the nominal interest rate on impact. That is, the optimal policy features contractionary fiscal policy and expansionary monetary policy in response to such a shock. The combination results in a sharper rise in public debt over time in the economy with no carbon tax policy. All in all, the impulse responses show that the presence of the carbon tax policy significantly limits the negative consequences of an adverse climate shock. In our assessment, this speaks in favor of having a proper carbon taxation policy in place.

Next, we present the implications of the government spending shock and TFP shock, figures (2) and (3), respectively, for the key macroeconomic variables of the three economies. Two key results emerge from these two figures: first, the presence or absence of a tax on carbon does not imply any significant difference in the behavior of key non-climate macroeconomic variables. All variables, with no exception, behave the same in response to both shocks. Second, compared with the standard NK model with no environmental externality (green lines), the presence of climate externalities also do not induce significant response or variation in any common aggregate variable.

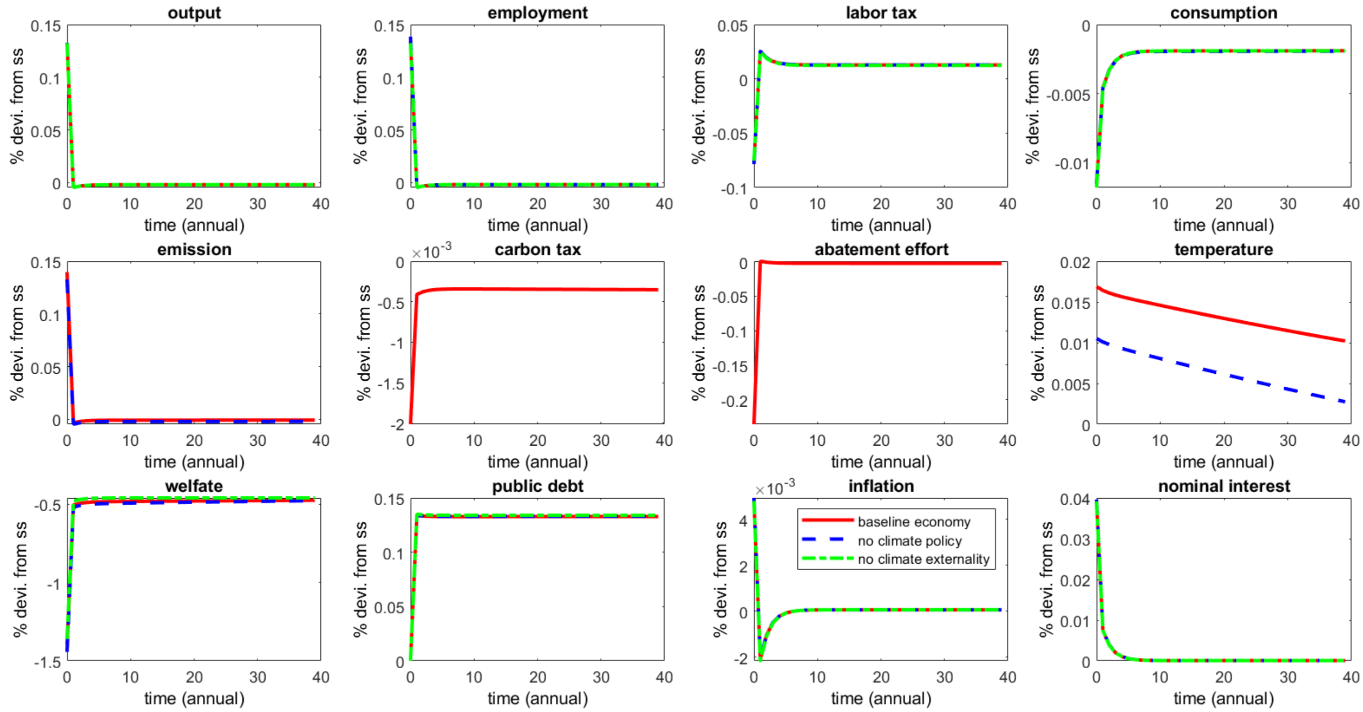

Figure 2: Impulse responses to a positive shock to government spending in three economies: the baseline model where the Ramsey planner has access to carbon taxation policy (red line), and when it does not (blue dashed line), and the standard New-Keynesian economy with no climate externality (green dash-dotted line). The size of the initial innovation to the emissions stock shock is one standard deviation.

More specifically, figure (2) shows that the increase in government purchases, with the exception of welfare that falls less in the baseline NK model, does not induce any significance difference in the responses of aggregate GDP, employment, labor tax rate, public debt, and inflation rate in the economies with climate change externalities compared with the standard NK framework. All these common variables respond in the same direction and almost with the same size. Regarding the responses of climate variables, it is important to note that the sharp rise in GDP makes abatement costlier, therefore, the Ramsey planner finds it optimal to lower the abatement rate and the carbon tax rate. These responses lead to higher emissions in the economy with climate policy, which in turn leads to higher temperature.

Note that the figure also replicates the well-known results of Schmitt-Grohé and Uribe [2004] regarding the responses of inflation, debt, and labor tax rate to a positive fiscal shock under sticky prices. More precisely, in line with what Schmitt-Grohé and Uribe [2004] established, the presence of price stickiness induces the Ramsey planner not to use inflation surprises in response to an unexpected increase in government spending, since it is costly. Instead, the planner finds it optimal to rely on taxes and issuing debt and induce near random walk behavior in them in response to the shocks. All in all, these results show that the Ramsey-optimal policy under sticky prices implies that monetary policy should be used to stabilize inflation, while an increase in income tax rates together with an increase in public debt ensure fiscal sustainability.

Figure (3) plots the outcomes for the macroeconomic variables of interest following a rise in the total factor productivity by one standard deviation, equal to 2%: GDP and consumption rise on impact and labor effort falls on impact. Due to the fall in labor the Ramsey government in financing its exogenous spending finds it optimal to increase taxes and inflation immediately. These increases allow the government to lower its public debt. Note that, as in figure (2), the implied changes in common macroeconomic variables in the climate-economy models are the same as those implied by the standard NK model. This basically means that, under a reasonable calibration of structural parameters, the presence of environmental externalities and an optimal climate policy do not make per se any significant changes in macroeconomic implications of typical aggregate shocks.

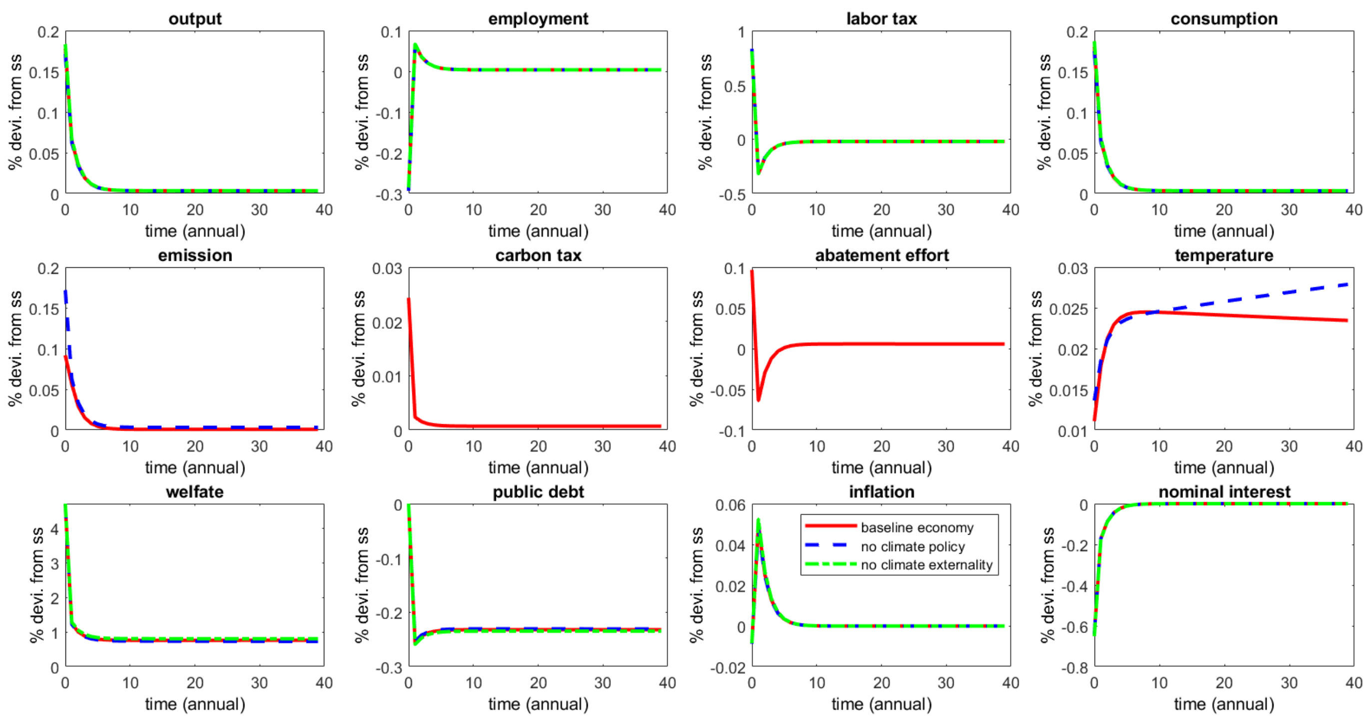

Figure 3: Impulse responses to a positive shock to TFP in three economies: the baseline model where the Ramsey planner has access to carbon taxation policy (red line), and when it does not (blue dashed line), and the standard New-Keynesian economy with no climate externality (green dash-dotted line). The size of the initial innovation to the emission stock shock is one standard deviation.

Regarding climate variables, observe that in the climate-economy model with a carbon tax, the abatement effort rises on impact due to the large rise in output, but then, as the output increase begins to fade away, it also falls sharply and returns to zero after several periods. Since the optimal carbon tax is also a function of output, it sharply increases on impact, and in line with output and abatement returns to zero. Taken together, in the economy with carbon taxation, the increase in emissions is lower than in the economy without a carbon tax, although this smaller increase, due to the already very large amount of carbon in the atmosphere, is not significant enough to induce a noticeable difference in the response of temperature.

One of the central lessons from figures (2) and (3) is that, in order to better appreciate the importance of a carbon tax policy, it is more useful to look at the right shocks. That is, shocks that are climate-specific, and not standard aggregate macroeconomic shocks, which do not highlight the importance of having an environmental policy in place.

3.4 Robustness on impulse responses

As we have seen in the previous section, the response of the main macro variables to government spending and TFP shocks is not different in our baseline economy with or without optimal carbon taxation policy. In this section, we will therefore focus in the responses to emissions shocks when we change two important parameters regulating the impact of climate on the economic activity, i.e. the climate damage function and the relation between carbon concentration and temperatures.

3.4.1 Impulse responses – different values for $\bar{\vartheta}$

The damage function plays a central role in integrated-assessment models, where it is used by economists as a simplified way to capture the economic consequences of climate change. Damage function uncertainty is considered as an example of "physical risk". Along with transition risks, physical risks are the main types of risks which are repeatedly referenced in the climate literature (see, Hansen [2021] for a discussion of these types of climate risks). Moreover, new studies such as Burke et al. [2015] and Hänsel and Sterner [2020] suggest that the climate damages are much larger than what is assumed in DICE. Therefore, in this section we study the Ramsey impulse responses of the sticky-price climate macroeconomy for three different values of the damage function parameter $\bar{\vartheta}$. We plot the impulses for the baseline value of $\bar{\vartheta} = 0.0028388$, a value three times, and a value five times higher than the benchmark calibration.

Figure (4) reports the optimal policy responses to a negative pollution shock. It shows that the Ramsey responses of key aggregate variables to an increase in the stock of pollution exhibit significantly notable differences in the size of responses of selected variables for different values of $\bar{\vartheta}$. Higher damage costs call for a larger response in optimal abatement and carbon taxes and lead to substantially lower emissions over time. A larger response of abatement is associated with a sharper fall in employment, output, and consumption. To induce people to work less, the Ramsey planner increases the labor tax more the higher is $\bar{\vartheta}$. As a matter of fact, for the baseline value of $\bar{\vartheta}$, the labor tax returns very close to its pre-shock level after about two periods, and employment rises to a level above zero. However, in the case that output damage is very high, i.e., the value of $\bar{\vartheta}$ is very large, employment is permanently negatively affected, and large damages lead to the optimality of working and producing less. Unlike fiscal policy, monetary policy is accommodative with climate change. It is accommodative in the sense that, as the figure shows, the higher is $\bar{\vartheta}$ the larger will be the fall in the nominal interest rate on impact, although after several periods the economy with larger output damages will have a higher interest rate. Taken all these responses together, the optimal temperature in response to an emission shock rises much less the higher are climate damage costs.

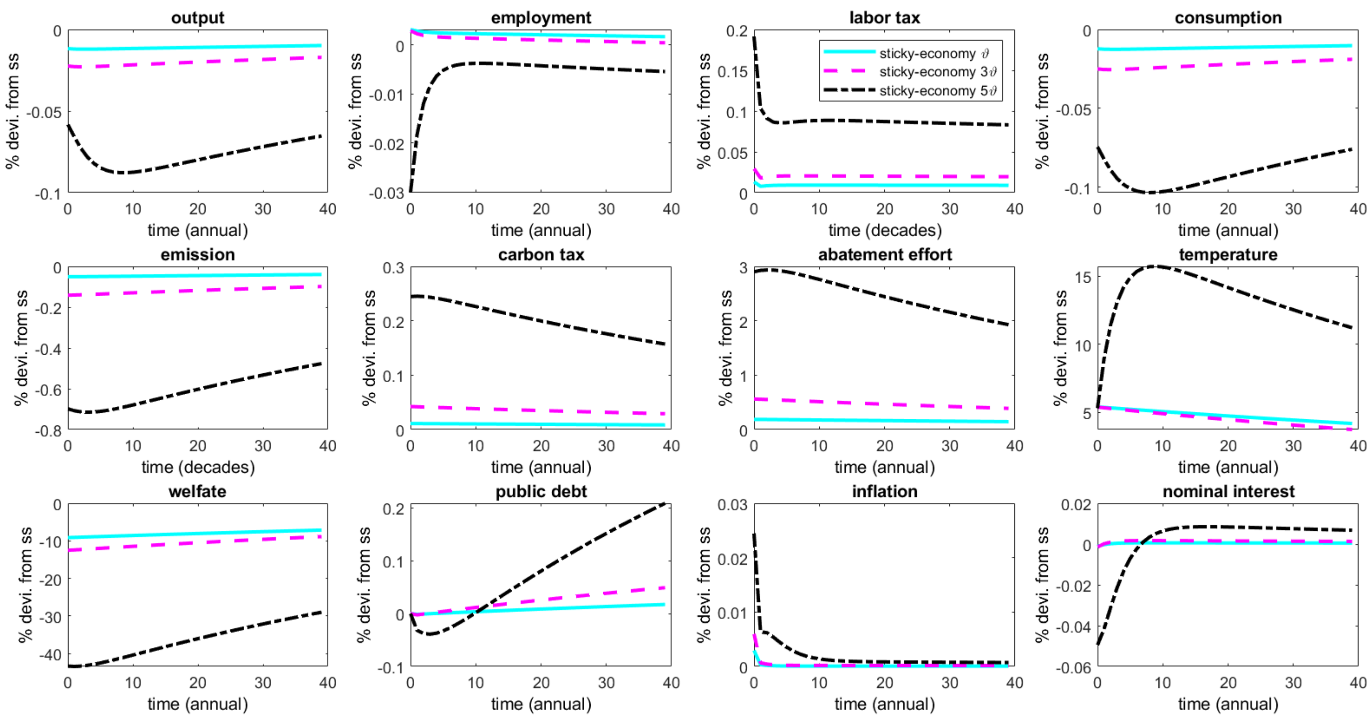

Figure 4: Impulse responses to a positive shock to the stock of pollution in the benchmark sticky-price economy under three different values for the damage function parameter $\bar{\vartheta}$. The size of the initial innovation is one standard deviation.

3.4.2 A tipping point shock in the climate system

In this subsection we introduce a disturbance Λ_t into the mapping between $\frac{s_t}{s}$ and the global mean temperature (19) in order to capture, to some extent, the economic consequences of what in the related climate literature is referred to as a tipping point shock. That is, suppose now that

$$T_t = \frac{\nu \Lambda_t}{\log 2} \log\left(\frac{s_t}{\bar{s}}\right).$$

As discussed in Hassler et al. [2016], one way to express a tipping point is to assume that ν sharply rises beyond some critical CO2 concentration level. An unexpected large increase in the random variable Λ_t is a simple way to capture such a scenario. We assume the tipping point shock Λ_t follows an i.i.d. process, with an unconditional mean of unity:

$$\ln(\Lambda_t) = \rho^{\Lambda} \ln(\Lambda_{t-1}) + \varepsilon_t^{\Lambda}. \tag{30}$$

As another example of what we discussed before, our aim of introducing this shock is show that to see better the importance of having a carbon tax policy in place in mitigating the contraction in the economy in the case of unexpected adverse climate events, we need to consider such shocks and not simply looking at some typical aggregate macroeconomic shocks like TFP shock.

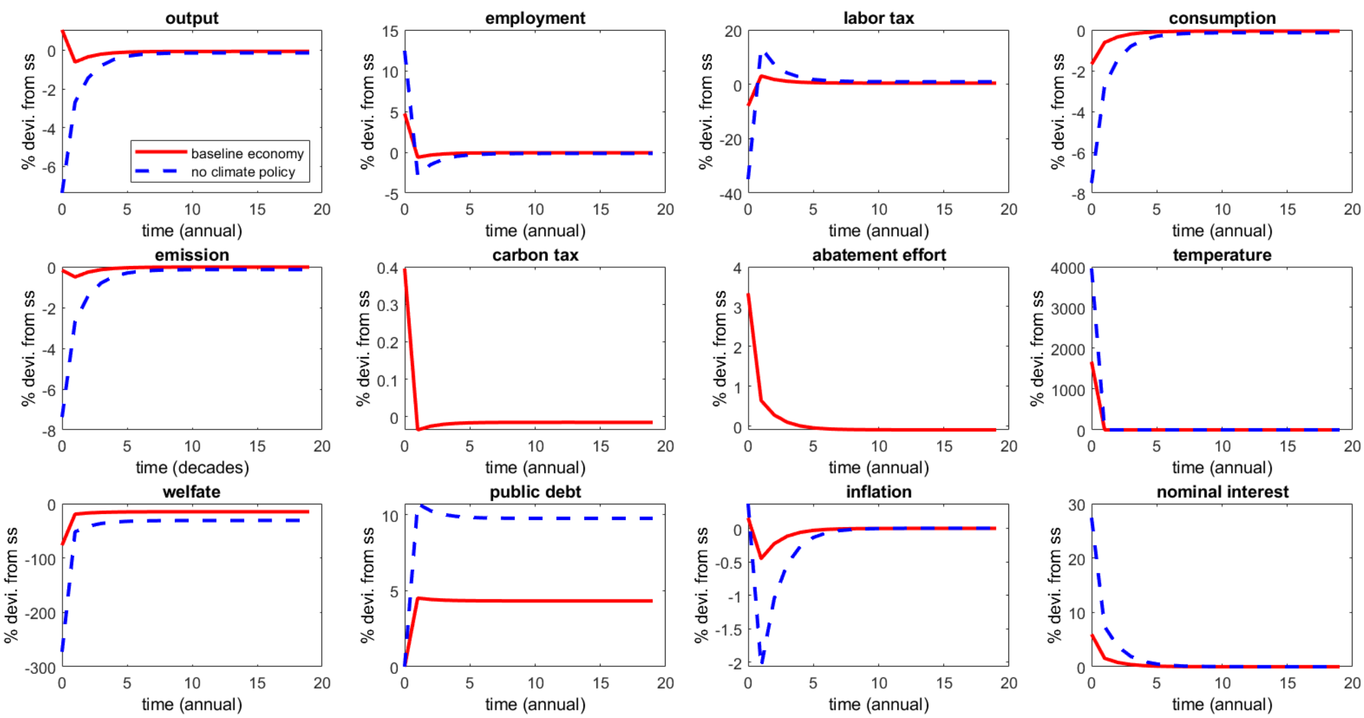

Figure 5: Impulse responses to a positive shock in the temperature mapping under two scenarios: when the Ramsey planner has access to carbon taxation policy (red line), and when it does not (blue dashed line). The size of the initial innovation to the pollution stock shock is one standard deviation.

Figure (5) shows the responses of the baseline climate-economy with and without a carbon tax to a positive one standard deviation (0.10%) shock to Λ_t. For both cases the shock reduces output, consumption, and welfare on impact, nevertheless the drops in the presence of a carbon tax policy are significantly lower because the shock induces much smaller output damages. The reason behind much smaller drops in these variables under a carbon tax policy is that the carbon tax rises significantly inducing firms to move a significant fraction of their technology toward zero-emissions technologies (abatement effort). Regarding employment, note that while the Ramsey planner in both cases lowers the labor tax to boost labor effort to compensate to some extent for the output drop, the decrease in the tax in the case of the economy with carbon taxation is much lower. At the same time the Ramsey planner increases the nominal interest rate in both economies, but again the increase in the economy with a carbon tax is smaller. Note that, in response to this shock fiscal and monetary policy are moving in the opposite directions. While fiscal policy works in the direction of boosting employment and therefore GDP through lower labor tax rates, monetary policy works in the direction of cooling the economy through higher nominal interest rates and therefore lower consumption levels. All in all, with a carbon tax at hand, the Ramsey planner is able to use a combination of higher abatement and carbon tax rates along with appropriate fiscal and monetary policy responses to mitigate the negative consequences of such shocks in order to maximize the welfare.

4 Government Spending as a Function of Climate Change

As discussed in detail in Barrage [2020a, 2021], climate change has become a fiscal risk for governments around the world, through affecting, for instance, their existing program costs such as healthcare and disaster assistance or hurricane-related public spending, and crop insurance subsidies, or through the need for publicly funded adaptation such as coastal protective infrastructure. In this section, we follow Barrage [2021] and assume that climate change directly affects government expenditures and evaluate the optimal long-run implications of such costs. More specifically, assume that total government spending is now equal to $\bar{g}_t = g_t\left(1 + \alpha_g T_t^2\right)$, where g_t is the same as before.

Table (4) presents the Ramsey optimal policy choices for the relevant macroeconomic variables in the climate sticky-price model for different values of α_g. All other structural parameters take the same value as in the benchmark calibration. For comparison, the first row of the table reports the baseline results. The main result emerging from the table is that the optimal environmental implications can significantly change as we explicitly include climate fiscal costs in the government budget constraint. As climate change becomes more costly for the government, the optimal abatement and optimal carbon taxes increase and the magnitude of carbon emission and thus output damage decrease. They, in turn, result in lower global average temperature. Notice that higher climate costs also induce higher labor tax rates to finance government expenses, which increase with an increase in α_g.

Table 4: Ramsey steady-state climate and monetary/fiscal policies when climate affects government expenditures

Scenario	wf	τ_h	τ_c	R	π	μ	s	T	$D(T)$	h	y	c
Benchmark ($\alpha_g = 0$)	-417.6	25.04	1.3	1.51	-.015	35.8	35.8	2.12	.0127	.2348	.2318	.1834
($\alpha_g = .005$)	-417.7	25.4	1.56	1.51	-.012	39.5	33.8	1.88	.01	.2345	.2322	.1828
($\alpha_g = .006$)	-417.74	25.45	1.6	1.51	-.012	40.2	33.4	1.83	.0095	.2345	.2322	.1827
($\alpha_g = .008$)	-417.75	25.5	1.69	1.51	-.01	41.3	32.8	1.75	.0086	.2343	.2323	.1825
($\alpha_g = .01$)	-417.77	25.6	1.78	1.51	-.01	42.4	32.22	1.67	.008	.2342	.2324	.1824

Note: τ_h, τ_c, π, R, and μ are expressed in percent while the other variables are in level.

5 Conclusion

We study the long-run and dynamic interactions between optimal carbon taxation and monetary-fiscal policy in a macroeconomic model of climate change. The main findings from our work can be summarized as follows. First, if the abatement cost function is not linear in output the optimal carbon tax cannot fully internalize production damages of climate change unless the government is able to fully tax firms' profits. If the carbon tax is not Pigouvian the Friedman rule is also suboptimal. Second, the long-run results on optimal climate, fiscal and monetary policies indicate that the Ramsey planner finds it optimal to sharply rise the optimal carbon tax as output damages from climate change increase. The optimal carbon tax can substantially lower the increase in the mean temperature by the end of century to around 2°C and improve significantly welfare compared to a situation in which the planner doe not have carbon taxation as an instrument. Third, our dynamic exercises indicate that it matters a great deal to macroeconomic implications of climate change in response to an adverse unexpected climate shock whether there is a carbon taxation policy in place or not. This does not happen to be the case if we look at impulse responses to typical macroeconomic shocks such as a TFP or government consumption shock. Fourth, the optimal responses of macroeconomics variables in a standard NK framework and a climate-NK framework show that the variables in the latter respond more to shocks regardless of whether there is a carbon policy in place or not. The optimal inflation rate, for instance, increases three times more in the latter model in response to a government spending shock. Fifth, the Ramsey planner uses also a combination of fiscal and monetary policy to mitigate the macroeconomic consequences of climate shocks. In response to a sudden release of carbon in the atmosphere, the planner increases the labor tax rate on impact in order to reduce employment and lowers the nominal interest rate to support consumption. The responses are larger the greater are climate damage costs.

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A Calibration of ψ and $\bar{s}$ in the non-stochastic competitive equilibrium

This appendix derives the steady state of the competitive equilibrium as well as the long-run restrictions used for calibration of ψ and $\bar{s}$.

Given that the gross inflation rate is assumed to be equal to $\pi = 1.015$, from the steady-state of Euler equation (9) the gross nominal interest rate is pinned down by

$$R = \frac{\pi}{\beta} \tag{31}$$

The consumption-based money velocity v is determined using (8) and (26) as

$$v = \sqrt{\frac{B}{A} + \frac{1}{A}\frac{R-1}{R}} \tag{32}$$

Given $h = 0.24$ and $T = 4$ in the steady-state of the business-as-usual (BAU) of the competitive economy, output and carbon magnitude, from (22) and (17), respectively, are equal to (notice that in the long-run $z = 1$ and we also calibrate $\kappa = 0.3\,(1-\mu)\,y$, where in the BAU scenario $\mu = 0$)

$$y = \frac{h}{1 + \bar{\vartheta}T^2} \tag{33}$$

$$s = \frac{4y}{\zeta} \tag{34}$$

Now, we can derive the magnitude of preindustrial carbon using relation (19)

$$\bar{s} = \frac{s}{e^{\frac{T\log 2}{\nu}}} \tag{35}$$

From the steady-state of (14), the marginal cost mc is pinned down to

$$mc = \frac{\epsilon - 1}{\epsilon} + \frac{\phi\,(1-\beta)\,\pi\,(\pi-1)}{\epsilon y}$$

From the aggregate resource constraint we get

$$c = \frac{y - g - \frac{\phi}{2}\,(\pi - 1)^2}{1 + \Theta\,(v)}$$

where $g \equiv g = .2y$. Next, solving for the labor tax from the government budget constraint (20) and (5) yield:

$$\tau_h = \frac{\epsilon - 1}{\epsilon} \left\{ (1/\beta - 1) \frac{b}{y} + \frac{c}{vh} (1/\pi - 1) + \frac{g}{y} \right\}$$

where $\frac{b}{y} = 0.44$ from our baseline calibration. Finally, use (7) to tie down ψ.

B The Social Planner

The social planner solves the following problem in our model:

$$\begin{aligned} L &= E_0 \sum_{t=0}^{\infty} \beta^t \{ u(c_t, h_t) + d(T(s_t)) \\ &\quad + \lambda_t^{1^*} [-y_t + (1 - D(T(s_t))) z_t h_t] \\ &\quad + \lambda_t^{2^*} [-s_t + \zeta s_{t-1} + (1 - \mu_t) \Gamma(y_t) + \kappa_t] \\ &\quad + \lambda_t^{3^*} [-(1 + \Theta(v_t)) c_t - g_t - x(\mu_t y_t) + y_t] \}. \end{aligned}$$

where $\lambda_t^{1^*}$, $\lambda_t^{2^*}$and $\lambda_t^{3^*}$ are the Lagrange multipliers.

First, consider the social planner's choice of money velocity, v_t. Money velocity enters only in the aggregate resource constraint. Given our assumptions regarding the transaction cost function $\Theta(v_t)$, it has a global minimum at $\tilde{v}_t$. Thus, the social planner will set $v_t = \tilde{v}_t$, which makes the transaction costs vanish, that is $\Theta(\tilde{v}_t) = 0$. Given this, the first-order conditions with respect to c_t, n_t, y_t, s_t and μ_t in that order are the following:

$$c_t: \quad u_c(c_t, h_t) - \lambda_t^{3^*} = 0, \tag{36}$$

$$h_t: \quad u_h(c_t, h_t) + \lambda_t^{1^*} (1 - D(T(s_t))) z_t = 0, \tag{37}$$

$$y_t: \quad -\lambda_t^{1^*} + \lambda_t^{2^*} (1 - \mu_t) \Gamma'(y_t) + \lambda_t^{3^*} \left(1 - \mu_t x'(\mu_t y_t) \right) = 0, \tag{38}$$

$$s_t: \quad T'(s_t) d'(T(s_t)) - \lambda_t^{1^*} T'(s_t) D'(T(s_t)) z_t h_t - \lambda_t^{2^*} + E_t \beta \zeta \lambda_{t+1}^{2^*} = 0, \tag{39}$$

$$\mu_t: \quad -\lambda_t^{2^*} \Gamma(y_t) - \lambda_t^{3^*} y_t x'(\mu_t y_t) = 0. \tag{40}$$

From (36) we obtain $\lambda_t^{3^*} = u_c(c_t, h_t)$, and from (37) we obtain $\lambda_t^{1^*} = \frac{-u_h(c_t,h_t)}{(1-D(T(s_t)))z_t}$. (36) and (40) together give us $\lambda_t^{2^*} = -\frac{u_c(c_t,h_t)x'(\mu_t y_t) y_t}{\Gamma(y_t)}$. Plugging these results into (38) and (40) result, respectively, in:

$$\frac{-u_h(c_t, h_t)}{u_c(c_t, h_t)} = \left(1 - \mu_t x'(\mu_t y_t) - \frac{(1 - \mu_t) \Gamma'(y_t) x'(\mu_t y_t) y_t}{\Gamma(y_t)} \right) (1 - D(T(s_t))) z_t,$$

$$\frac{T^{'}(s_t)d^{'}(T(s_t))}{u_c(c_t,h_t)} + \frac{u_h(c_t,h_t)T^{'}(s_t)D^{'}(T(s_t))z_th_t}{u_c(c_t,h_t)(1-D(T(s_t)))z_t} + \frac{x^{'}(\mu_t y_t)y_t}{\Gamma(y_t)}$$
$$-E_t\beta\zeta\frac{u_c(c_{t+1},h_{t+1})x^{'}(\mu_{t+1}y_{t+1})y_{t+1}}{u_c(c_t,h_t)\Gamma(y_{t+1})} = 0$$

Combining the above two equations yields:

$$\frac{E_t\zeta\beta\frac{u_c(c_{t+1},h_{t+1})}{u_c(c_t,h_t)}\frac{x^{'}(\mu_{t+1}y_{t+1})y_{t+1}}{\Gamma(y_{t+1})}}{\frac{x^{'}(\mu_t y_t)y_t}{\Gamma(y_t)}} - 1 = \frac{T^{'}(s_t)d^{'}(T(s_t))}{u_c(c_t,h_t)\frac{x^{'}(\mu_t y_t)y_t}{\Gamma(y_t)}}$$
$$+ (1-\mu_t)\Gamma^{'}(y_t)T^{'}(s_t)D^{'}(T(s_t))z_th_t - \frac{T^{'}(s_t)D^{'}(T(s_t))z_th_t\left(1-x^{'}(\mu_t y_t)\mu_t\right)}{\frac{x^{'}(\mu_t y_t)y_t}{\Gamma(y_t)}} \tag{41}$$

The steady-state version of (41) is as follows:

$$\frac{x^{'}(\mu y)y}{\Gamma(y)} = \frac{\left(1-x^{'}(\mu y)\mu\right)T^{'}(s)D^{'}(T(s))h - \frac{T^{'}(s)d^{'}(T(s))}{u_c(.)}}{1-\beta\zeta+(1-\mu)\Gamma^{'}(y)T^{'}(s)D^{'}(T(s))h}, \tag{42}$$

where we used the assumption that in the steady state $z = 1$.

C Ramsey problem under flexible prices and perfect competition

Suppose that λ_t^1, λ_t^2, λ_t^3, and $\bar{\lambda}$ (that is a constant) are the Lagrange multipliers, the Lagrangian of the Ramsey problem under flexible prices and perfect competition consists of choosing $\{c_t, h_t, v_t, \mu_t, y_t, s_t\}_{t=0}^{\infty}$ to maximize:[13]

$$\begin{aligned} L = \; & E_0\sum_{t=0}^{\infty}\beta^t\{u(c_t,h_t) + d(T(s_t)) \\ & +\lambda_t^1[-y_t + (1-D(T(s_t)))z_th_t] \\ & +\lambda_t^2[-s_t + \zeta s_{t-1} + (1-\mu_t)\Gamma(y_t) + \kappa_t] \\ & +\lambda_t^3[-(1+\Theta(v_t))c_t - g_t - x(\mu_t y_t) + y_t]\} \\ & +E_0\sum_{t=0}^{\infty}\beta^t\bar{\lambda}\left[u_c(c_t,h_t)c_t + u_h(c_t,h_t)h_t - u_c(c_t,h_t)\Delta_t^1\right] \\ & -\bar{\lambda}\left[\frac{u_c(c_0,h_0)}{1+\Theta(v_0)+v_0\Theta'(v_0)}\frac{R_{-1}B_{-1}+M_{-1}}{P_0}\right] \end{aligned} \tag{43}$$

[13] Since the procedure of the derivation of the implementability constraint is the same as the ones presented presented in detail in Schmitt-Grohé and Uribe [2004, 2010] and Kiarsi and Masoudi [2021], to save space we do not present it here.

where $\Delta_t^1 = \frac{(1-\mu_t)x^{'}(\mu_t y_t)\left(\frac{\Gamma^{'}(y_t)y_t}{\Gamma(y_t)}-1\right)+\mu_t y_t x^{'}(\mu_t y_t)-x(\mu_t y_t)}{1+\Theta(v_t)+v_t\Theta^{'}(v_t)}$, and $v_t^2\Theta^{'}(v_t) < 1$ must hold, and also given $\frac{R_{-1}B_{-1}+M_{-1}}{P_0}$.

Define the following terms

$$\Delta_t^2 = \frac{u_c(c_t,h_t)\left\{\left((1-\mu_t)y_t x^{''}(\mu_t y_t)-x^{'}(\mu_t y_t)\right)\left(\frac{\Gamma^{'}(y_t)y_t}{\Gamma(y_t)}-1\right)+\mu_t y_t^2 x^{''}(\mu_t y_t)\right\}}{1+\Theta(v_t)+v_t\Theta^{'}(v_t)}$$

$$\Delta_t^3 = \frac{u_c(c_t,h_t)\left\{(1-\mu_t)\mu_t x^{''}(\mu_t y_t)\left(\frac{\Gamma^{'}(y_t)y_t}{\Gamma(y_t)}-1\right)+(1-\mu_t)x^{'}(\mu_t y_t)\left(\frac{\left(\Gamma^{''}(y_t)y_t+\Gamma^{'}(y_t)\right)\Gamma(y_t)-\left(\Gamma^{'}(y_t)\right)^2 y_t}{\Gamma(y_t)^2}\right)+\mu_t^2 y_t x^{''}(\mu_t y_t)\right\}}{1+\Theta(v_t)+v_t\Theta^{'}(v_t)}$$

$$\Delta_t^4 = -\frac{2\Theta^{'}(v_t)+v_t\Theta^{''}(v_t)}{1+\Theta(v_t)+v_t\Theta^{'}(v_t)}u_c\left(c_t,h_t\right)\Delta_t^1$$

$$x_t^1 = u_{cc}\left(c_t,h_t\right)c_t + u_c\left(c_t,h_t\right) + u_{hc}\left(c_t,h_t\right)h_t - u_{cc}\left(c_t,h_t\right)\Delta_t^1$$

$$x_t^2 = u_{ch}\left(c_t,h_t\right)c_t + u_h\left(c_t,h_t\right) + u_{hh}\left(c_t,h_t\right)h_t - u_{ch}\left(c_t,h_t\right)\Delta_t^1$$

Given the above definitions, the Ramsey first-order conditions with respect to c_t, h_t, y_t, s_t, μ_t and v_t in that order are the following:

$$c_t: \quad u_c\left(c_t,h_t\right) - \left(1+\Theta\left(v_t\right)\right)\lambda_t^3 + \bar{\lambda}x^1 = 0, \tag{44}$$

$$h_t: \quad u_h\left(c_t,h_t\right) + \lambda_t^1\left(1-D\left(T\left(s_t\right)\right)\right)z_t + \bar{\lambda}x^2 = 0, \tag{45}$$

$$y_t: \quad -\lambda_t^1 + \lambda_t^2\left(1-\mu_t\right)\Gamma^{'}\left(y_t\right) + \lambda_t^3\left(1-x^{'}\left(\mu_t y_t\right)\mu_t\right) - \bar{\lambda}\Delta_t^3 = 0, \tag{46}$$

$$s_t: \quad T^{'}\left(s_t\right)d^{'}\left(T\left(s_t\right)\right) - \lambda_t^1 T^{'}\left(s_t\right)D^{'}\left(T\left(s_t\right)\right)z_t h_t - \lambda_t^2 + E_t\beta\zeta\lambda_{t+1}^2 = 0, \tag{47}$$

$$\mu_t: \quad -\lambda_t^2\Gamma\left(y_t\right) - \lambda_t^3 x^{'}\left(\mu_t y_t\right)y_t - \bar{\lambda}\Delta_t^2 = 0, \tag{48}$$

$$v_t: \quad -\lambda_t^3\Theta^{'}\left(v_t\right)c_t + \bar{\lambda}\Delta_t^4 \leq 0\left(=0 \text{ if } v_t > \tilde{v}_t\right), \tag{49}$$

where we need to check the solution also satisfies $v_t^2\Theta^{'}\left(v_t\right) < 1$.

For simplicity we discuss our analytical results for the case that $\Gamma\left(y_t\right) = y_t$. When $\Gamma\left(y_t\right)$ is nonlinear it only reinforces our main point here. First, suppose that the cost of mitigation is nonlinear in μ_t but linear in output. That is, abatement costs as in DICE are equal to $x\left(\mu_t\right)y_t$. In this case, profits will be zero and $\Delta_t^i = 0$ for $i = 1,2,3,4$. Thus, the first-order condition with respect to money velocity reduces to

$$\frac{\partial L}{\partial v_t} = -\lambda_t^3\Theta^{'}\left(v_t\right)c_t,$$

where we know that the Lagrange multiplier on the feasibility constraint λ_t^3 is always greater than zero. Thus, the only solution to this optimality condition is $v_t = \tilde{v}_t$. Recall from assumption 1 that $\Theta\left(\tilde{v}_t\right) = \Theta^{'}\left(\tilde{v}_t\right) = 0$. Given this, from the first-order condition (8) we have that $R_t = 1$ over time.

Next, from conditions (48), (46) we have:

$$\begin{aligned} \lambda_t^2 &= -\,x^{'}\left(\mu_t y_t\right)\lambda_t^3, \\ \lambda_t^1 &= \left(1 - x^{'}\left(\mu_t y_t\right)\right)\lambda_t^3. \end{aligned} \tag{50}$$

Furthermore, from the steady-state of (47) we get

$$T^{'}\left(s\right)d^{'}\left(T\left(s\right)\right) - \lambda^1 T^{'}\left(s\right)D^{'}\left(T\left(s\right)\right)h - \lambda^2(1-\beta\zeta) = 0. \tag{51}$$

Substituting (50) in (51) and solving for $x^{'}\left(\mu y\right)$ gives us

$$x^{'}\left(\mu y\right) = \frac{\left(1 - x^{'}\left(\mu y\right)\mu\right)T^{'}\left(s\right)D^{'}\left(T\left(s\right)\right)h - \frac{T^{'}(s)d^{'}(T(s))}{\lambda^3}}{1 - \beta\zeta + \left(1-\mu\right)T^{'}\left(s\right)D^{'}\left(T\left(s\right)\right)h} \tag{52}$$

which is the same as expression (42) with the only exception that here in (52) instead of having $u_c\left(.\right)$ in the denominator of the last term on the numerator we have λ^3. As is discussed in detail in Kiarsi and Masoudi [2021], the optimal carbon tax may over, under, or exactly internalizes the utility damages depending on the assumed preferences. Similar results hold here as well, thus we do not discuss them again. Notice that a comparison of (52) with (42) makes clear that when climate change affects productivity only and is not affecting preferences then the optimal carbon tax is Pigouvian. That is, it internalizes the full environmental damage costs of carbon emissions.

Next, suppose that abatement costs are nonlinear in y_t and are as what we assumed in our baseline model, that is strictly increasing in $\mu_t y_t$. Under this scenario the first-order condition with respect to v_t is:

$$\frac{\partial L}{\partial v_t} = -\lambda_t^3 \Theta^{'}\left(v_t\right)c_t - \bar{\lambda}\Delta_t^4 \leq 0\,(=0 \text{ if } v_t > \tilde{v}_t), \tag{53}$$

The Friedman rule is optimal if $v_t = \tilde{v}_t$. To see this cannot be a solution, evaluate the above expression at $v_t = \tilde{v}_t$. From our assumptions regarding the transaction cost function, $\Theta^{'}\left(\tilde{v}_t\right) = 0$, this makes the first term on the right-hand side of (53) to vanish. Note that $\bar{\lambda}$ must always be positive, because otherwise it would be possible to increase welfare through an increase in initial government debt, which cannot be the case in this model. Since $2\Theta^{'}\left(v_t\right) + v_t\Theta^{''}\left(v_t\right)$ by assumption is always positive it follows that the optimality condition (53) is satisfied only if $\mu_t y_t x^{'}\left(\mu_t y_t\right) - x\left(\mu_t y_t\right) \leq 0$. However, given that the abatement costs are assumed to be nonlinear and strictly increasing in $\mu_t y_t$ this cannot be a solution, because $\mu_t y_t x^{'}\left(\mu_t y_t\right) - x\left(\mu_t y_t\right) > 0$. Therefore, the Friedman rule cannot be optimal and $R_t > 1$.

Now the long-run versions of (48), (46) change to

$$\lambda^2 = -\left(\lambda^3 x^{'}(\mu y) + \frac{\bar{\lambda}\Delta^2}{y}\right),$$
$$\lambda^1 = \lambda^3\left(1 - x^{'}(\mu y)\right) - \bar{\lambda}\left(\frac{\Delta^2}{y}(1-\mu) + \Delta^3\right). \tag{54}$$

Using the last two expressions along with the steady-state versions of (44), (45), and (47) we arrive at:

$$x^{'}(\mu y) = \frac{\left(\left(1 - x^{'}(\mu y)\mu\right) - \frac{\bar{\lambda}}{\lambda^3}\Delta^3\right)T^{'}(s)D^{'}(T(s))h + \frac{-T^{'}(s)d^{'}(T(s))}{\lambda^3}}{\left(1 + \frac{\bar{\lambda}}{\lambda^3}\Delta^3\right)\left((1-\beta\zeta) + (1-\mu)T^{'}(s)D^{'}(T(s))h\right)}$$

where we have from above that $\Delta^3 = \frac{u_c(c,h)\mu^2 y x^{''}(\mu y)}{1+\Theta(v)+v\Theta^{'}(v)}$.

D Ramsey Problem under Imperfect Competition

In this section, we analytically discuss the Ramsey-optimal solution under imperfect competition. We restrict our attention to the case that $\Gamma(y_t) = y_t$ and abatement costs are linear in output as in DICE, that is they are equal to $x(\mu_t)y_t$. Obviously, for the case that they are nonlinear in clean output $(\mu_t y_t)$ as in our baseline model the results presented below will simply be reinforced.

Suppose that λ_t^1, λ_t^2, λ_t^3, and $\bar{\lambda}$ (that is a constant) are the Lagrange multipliers, the Lagrangian of the Ramsey problem under flexible prices and imperfect competition consists of choosing $\{c_t, h_t, v_t, \mu_t, y_t, s_t\}_{t=0}^{\infty}$ to maximize:[14]

$$\begin{aligned}
L \;=\; & E_0 \sum_{t=0}^{\infty} \beta^t \{u(c_t, h_t) + d(T(s_t)) \\
& +\lambda_t^1\left[-y_t + (1 - D(T(s_t)))z_t h_t\right] \\
& +\lambda_t^2\left[-s_t + \zeta s_{t-1} + (1-\mu_t)y_t + \kappa_t\right] \\
& +\lambda_t^3\left[-(1+\Theta(v_t))c_t - g_t - x(\mu_t)y_t + y_t\right]\} \\
& +E_0 \sum_{t=0}^{\infty} \beta^t \bar{\lambda}\left[u_c(c_t,h_t)c_t + u_h(c_t,h_t)h_t - \frac{u_c(c_t,h_t)}{1+\Theta(v_t)+v_t\Theta'(v_t)}\frac{(1-D(T(s_t)))z_t h_t}{\epsilon}\right] \\
& -\bar{\lambda}\left[\frac{u_c(c_0,h_0)}{1+\Theta(v_0)+v_0\Theta'(v_0)}\frac{R_{-1}B_{-1}+M_{-1}}{P_0}\right]
\end{aligned} \tag{55}$$

[14] Observe that due to the presence of market power a third term appears in the PVIC that contains ϵ. For details regarding how to derive this condition, see Schmitt-Grohé and Uribe [2004, 2010].

where $v_t^2\Theta^{'}(v_t) < 1$ must hold and given $\frac{R_{-1}B_{-1}+M_{-1}}{P_0}$.

The Ramsey first-order conditions with respect to $c_t, h_t, y_t, s_t, \mu_t$ and v_t in that order are the following:

$$c_t: \quad u_c(c_t, h_t) - (1+\Theta(v_t))\lambda_t^3 + \bar{\lambda}x^1 = 0, \tag{56}$$

$$h_t: \quad u_h(c_t, h_t) + \lambda_t^1(1 - D(T(s_t)))z_t + \bar{\lambda}x^2 = 0, \tag{57}$$

$$y_t: \quad -\lambda_t^1 + \lambda_t^2(1-\mu_t) + \lambda_t^3(1 - x(\mu_t)) = 0, \tag{58}$$

$$s_t: \quad T^{'}(s_t)d^{'}(T(s_t)) - \lambda_t^1 T^{'}(s_t)D^{'}(T(s_t))z_t h_t - \lambda_t^2 + E_t\beta\zeta\lambda_{t+1}^2 + \bar{\lambda}\frac{u_c(c_t,h_t)D^{'}(T(s_t))z_t h_t}{\epsilon(1+\Theta(v_t)+v_t\Theta^{'}(v_t))} = 0, \tag{59}$$

$$\mu_t: \quad -\lambda_t^2 y_t - \lambda_t^3 y_t x^{'}(\mu_t) = 0, \tag{60}$$

$$v_t: \quad -\lambda_t^3\Theta^{'}(v_t)c_t + \bar{\lambda}\frac{u_{cc}(c_t,h_t)\left(2\Theta^{'}(v_t) + v_t\Theta^{''}(v_t)\right)(1-D(T(s_t)))z_t h_t}{\epsilon(1+\Theta(v_t)+v_t\Theta^{'}(v_t))} \leq 0\,(=0 \text{ if } v_t > \tilde{v}_t), \tag{61}$$

where $x_t^1 = \left\{u_{cc}(c_t,h_t)c_t + u_c(c_t,h_t) + u_{hc}(c_t,h_t)h_t - \frac{u_{cc}(c_t,h_t)(1-D(T(s_t)))z_t h_t}{\left(1+\Theta(v_t)+v_t\Theta^{'}(v_t)\right)\epsilon}\right\}$ and

$$x_t^2 = u_{ch}(c_t,h_t)c_t + u_h(c_t,h_t) + u_{hh}(c_t,h_t)h_t$$
$$-\frac{u_c(c_t,h_t)}{1+\Theta(v_t)+v_t\Theta^{'}(v_t)}\frac{(1-D(T(s_t)))z_t}{\epsilon} - \frac{u_{ch}(c_t,h_t)}{1+\Theta(v_t)+v_t\Theta^{'}(v_t)}\frac{(1-D(T(s_t)))z_t h_t}{\epsilon}.$$

Notice that we need to check the solution also satisfies $v_t^2\Theta^{'}(v_t) < 1$.

When there is imperfect competition in product markets, we already know from Schmitt-Grohé and Uribe [2004, 2010] that the Friedman rule ceases to be optimal. To see the reason, notice that

$$\frac{\partial L}{\partial v_t} = -\lambda_t^3\Theta^{'}(v_t)c_t + \bar{\lambda}\frac{u_{cc}(c_t,h_t)\left(2\Theta^{'}(v_t)+v_t\Theta^{''}(v_t)\right)}{1+\Theta(v_t)+v_t\Theta^{'}(v_t)}\frac{(1-D(T(s_t)))z_t h_t}{\epsilon} \leq 0\,(=0 \text{ if } v_t > \tilde{v}_t), \tag{62}$$

The Friedman rule is optimal if $v_t = \tilde{v}_t$. To see this cannot be a solution under imperfect competition, evaluate the above expression at $v_t = \tilde{v}_t$. From our assumptions regarding the transaction cost function, this makes the first term on the right-hand side of (62) to vanish. Note that $\bar{\lambda}$ must always be positive, because otherwise it would be possible to increase welfare through an increase in initial government debt, which cannot be the case in this model. Since $\epsilon > 0$, and $(1 - D(T(s_t))) > 0$, it follows that the optimality condition (62) is satisfied only if $\tilde{v}_t\Theta^{''}(\tilde{v}_t) \leq 0$. However, based on assumption 1 this cannot be the case. Therefore, the Friedman rule cannot be optimal and $R_t > 1$.

Given $\epsilon > 0$, condition (52) modifies to

$$x^{'}(\mu) = \frac{(1-x(\mu))\,T^{'}(s)\,D^{'}(T(s))\,h + \frac{-T^{'}(s)d^{'}(T(s)) - \bar{\lambda}\frac{u_c(c_t,h_t)D^{'}(T(s_t))z_t h_t}{\epsilon\left(1+\Theta(v_t)+v_t\Theta^{'}(v_t)\right)}}{\lambda^3}}{1-\beta\zeta+(1-\mu)\,T^{'}(s)\,D^{'}(T(s))\,h} \tag{63}$$

Now, suppose climate change impacts affect production possibilities only. Then condition (63) reduces to

$$x^{'}(\mu) = \frac{(1-x(\mu))\,T^{'}(s)\,D^{'}(T(s))\,h - \bar{\lambda}\frac{u_c(c_t,h_t)D^{'}(T(s_t))z_t h_t}{\lambda^3\epsilon\left(1+\Theta(v_t)+v_t\Theta^{'}(v_t)\right)}}{1-\beta\zeta+(1-\mu)\,T^{'}(s)\,D^{'}(T(s))\,h} \tag{64}$$

Since $\bar{\lambda}, \lambda^3, \epsilon$ are greater than zero as well as $\left(1+\Theta(v_t)+v_t\Theta^{'}(v_t)\right)$, $D^{'}(T(s_t))$, $u_c(c_t,h_t)$, and $z_t h_t$, in sharp contrast to the case of perfect competition, a comparison of (64) with the first-best solution (42) which in the case of production damages only takes the form of

$$\chi^{'}(\mu) = \frac{(1-x(\mu))\,T^{'}(s)\,D^{'}(T(s))\,h}{1-\beta\zeta+(1-\mu)\,T^{'}(s)\,D^{'}(T(s))\,h},$$

makes clear that the optimal carbon tax under internalizes production damages relative to the Pigouvian rate.