MARCO BUSO

University of Padova

CESARE DOSI

University of Padova

MICHELE MORETTO

University of Padova

TAXATION OF PUBLIC FRANCHISES WITH PERSISTENT DEMAND SHOCKS

September 2023

Marco Fanno Working Papers – 306

Dipartimento di Scienze Economiche e Aziendali 'Marco Fanno'

Università degli Studi di Padova

Taxation of Public Franchises with Persistent Demand Shocks

Marco Buso* Cesare Dosi† Michele Moretto‡

August 30, 2023

Abstract

We study a contract between a public and a private entity, where the latter commits to pay the awarding body for an exclusive right to supply a public service by using a government-owned facility, when there is asymmetric information on demand parameters following a Brownian motion process. We show that optimal taxation requires an appropriate combination of fixed and time-adjusted payments from actual sales. We then analyze how the optimal combination of fixed and variable transfers is impacted by the private revenue potential, by the expected variability of consumer demand and by the importance assigned to tax receipts relative to other welfare concerns.

Keywords: Public-Private Partnerships. Public franchises. Monopoly. Taxation. Dynamic adverse selection. Persistent demand shocks.

JEL classification: D42, D81, D82, D86, H27, L51

---

*Department of Economics and Management, University of Padova and Interuniversity Centre for Public Economics (CRIEP), via del Santo 33 35123 Padova, Italy (marco.buso@unipd.it)

†Department of Economics and Management, University of Padova and Interuniversity Centre for Public Economics (CRIEP), via del Santo 33 35123 Padova, Italy (cesare.dosi@unipd.it)

‡Department of Economics and Management, University of Padova and Interuniversity Centre for Public Economics (CRIEP), via del Santo 33 35123 Padova, Italy (michele.moretto@unipd.it)

We thank for their useful comments and suggestions Edoardo Grillo and participants to the DSEA internal seminar (Padova, 2023), the 11th edition of the Oligo workshop (Padova, 2023) and the 50th Annual Conference of EARIE (Rome, 2023).

1 Introduction

Recent decades have witnessed an increasing contracting out of public services and Public-Private Partnership (PPP) has become a popular umbrella term for a variety of contracts where a government body enters into a long-term relationship with the private sector for performing an activity of general economic interest.

Many PPP models, e.g. DBFO, BOO, BOT[1], involve significant fixed capital investment and, indeed, greenfield projects have received most attention in the PPP literature.[2] However, there are also several contract types that assign the right (and obligation) to provide a product/service by exploiting an already-existing facility.

As in the case of greenfield initiatives, even among brownfield PPPs we can find either settlements where service provision is remunerated by the government ("management contracts") or arrangements where the operator is remunerated by end-user charges. Different terms are used in different legal contexts and sectors to describe "brownfield-user-pays agreements", notably franchise, lease, affermage, operation/service concession or, simply, concession (World Bank, 2017). Examples can be found in different sectors, e.g. water and sanitation, transport and logistics, urban public transport.

In this paper we restrict our attention on contracts granting a private entity an exclusive right to supply a product/service by operating an existing public facility, when there is asymmetric information about the stochastic variables affecting private profits. Specifically, in our model uncertainty comes from the demand side, as sales proceeds are assumed to depend both on the private sector's ability to seize market opportunities and on persistent demand shocks.[3]

The "public franchise" agreement we will focus on holds three main features. First, it provides for the exploitation of a government-owned asset (e.g.,

[1]Design-Build-Finance-Operate (DBFO), Build-Own-Operate (BOO) and Build-Operate-Transfer (BOT) are delivery methods usually for large-scale infrastructure projects.

[2]Issues addressed include, among others, the effects of bundling investment and operation (see, e.g., Hoppe and Schmitz, 2013; Iossa and Martimort, 2015; Martimort and Straub, 2016; Buso and Greco, 2023), the optimal timing of investment (Broer and Zwart, 2013; Soumare and Lai, 2016; Arve and Zwart, 2023) and the impacts of early-termination clauses upon the contract duration (Buso et al., 2021).

[3]For instance, household demand for water can vary according to weather conditions or the demand for local public transport can be influenced by fluctuations in the relative cost of private transport or by occasional restrictions on private mobility.

a municipal water distribution system, a rail corridor, vehicles and depots for urban public transport or a stretch of public sea-fronts). Second, the contract grants monopoly power, insofar as the franchisee enjoys an exclusive right to serve a specific market with a specific service. Third, the private party is required to pay a certain amount of money to the awarding authority. Indeed, payments to the public are a common element of all the aforementioned contract types. For instance, in the case of lease contracts the fee tends to be fixed irrespective of revenues, whereas in affermage it is typically a fixed rate per every unit sold (Asian Development Bank, 2008; World Bank, 2022).

Payments by the private sector are sometimes seen just as a rental price for public property use or, eventually, as a sort of compensation for the monopoly privilege granted to a private entity. However, taxation can be also regarded and used as a regulatory tool. For instance, long time ago Franklin Wilcox, who served as chief of the Bureau of Franchises in the New York Civil Service Commission, stated that when public utilities are run by companies enjoying "special privileges protected by judicial decisions and contractual rights, taxation may be resorted to both as a revenue measure and as a weapon for regaining public control over such utilities" (Wilcox, 1915, p.148).

In line with this view, in this paper we study how taxation can be used to regulate a state-sponsored monopolist under dynamic asymmetric information.

Monopoly regulation under adverse selection was first addressed by using static models where private information parameters (e.g., production costs) are assumed not to vary over time (Baron and Myerson, 1982; Laffont and Tirole, 1986; Riordan and Sappington, 1987). Further contributions introduced some dynamics by using two-period models (Baron and Besanko, 1984; Laffont and Tirole, 1993). Assuming perfect commitment, there are three possible environments for such a discrete-time setup (Laffont and Martimort, 2002).

First, if private information parameters change with time but are perfectly correlated between the two periods, it is optimal for the principal to commit to the repetition of the static contract. Second, if the realizations in each period are completely independent, the parties sign a long-term contract before the second-period information is disclosed, thus only the first-period private information is costly to the principal. Finally, if private information parameters are imperfectly correlated, the principal uses the first report to update his beliefs on the agent's second-period type. In this case, an intertemporal incentive-compatibility constraint needs to be added to the principal's maximization problem, such that the agent has no incentives to misreport his type in both periods.

However, when moving from the two-period to a continuous-time setting, things become more difficult. While the first and the second cases can still be solved as described above (see, e.g., Auriol and Picard, 2013; Tatoutchoup, 2015), when private information parameters are more realistically modelled as subject to imperfectly correlated shocks the solution is not anymore straightforward, because, since the space for deviations by the agent could be very rich, the standard incentive-compatible mechanism is in general hard to be implemented (Pavan et al., 2014; Bergemann and Valimaki, 2018).[4] Yet, this intertemporal adverse selection problem has been technically overcome by Bergemann and Strack (2015) who, by adopting a Brownian process as state variable, derive necessary conditions for a direct incentive-compatible mechanism and use their findings to obtain a closed-form solution for a dynamic contract between a revenue-maximizing seller and a privately informed buyer.

In this paper we exploit the mechanism design approach used in Bergemann and Strack (2015) which, to the best of our knowledge, has not been previously exploited for public-private arrangements. Indeed, an important difference between our and their setup is that while in Bergmann and Strack the principal is a revenue maximizer seller, in our environment the contracting authority has to manage two objectives, i.e., to maximize the economic surplus, while at the same time keeping tax receipts as high as possible. For instance, especially in countries where governments face financial constraints and the difficulty of further increasing fiscal receipts from other sources, levies against agents granted with special privileges (monopoly power, public asset usage) can provide additional resources for government spending or for reducing excess burden of taxation in other sectors.

Our main result is that optimal taxation requires combining an "entry fee", based on the private revenue potential and the expected volatility of sales proceeds, with a dynamically adjusted levy on the actual sales. Moreover, we study the effects of model parameters, namely the sales potential, the expected volatility of private revenues and the shadow value of public funds,

[4] An exception is Besanko (1985), who analyzes a continuing relationship between a regulator and a firm, where costs are private information and change over time following an AR(1) process. In this case, he shows that the distortions from the efficient allocation vanish as $t \to \infty$.

on the optimal mix of fixed and variable payments. As for the latter, we show how the optimal two-part tax schedule is impacted by the importance assigned by the contracting authority to tax receipts relative to other welfare concerns and how the combination of fixed and variable payments vary according to the franchisee's potential ability to seize market opportunities.

The paper proceeds as follows. Section 2 introduces the model. In Section 3 we derive the optimal taxation scheme, whose main features are illustrated in Section 4 through numerical simulations. Section 5 concludes. The proofs are presented in the appendices.

2 Set up

Consider a public authority (henceforth "the government", he) offering a private firm (she) a take-it-or-leave-it contract that gives her an exclusive right to sell a specific product or service ("good") by using a government-owned facility. By entering into the contract at $t = 0$, the firm commits to pay a sum of money ("tax") to the government. For sake of simplicity we assume that the contract term is sufficiently long to be approximated as infinitely long.[5]

The firm's activity generates in every period $t \geq 0$ a gross consumer surplus denoted by $S(Q_t, x_t)$, where Q_t is the output level and x_t is the demand shifter (Auriol and Picard, 2013), which captures exogeneous upward or downward movements in the demand curve, due, for instance, to changes in consumer preferences or in the number of consumers.

$S(Q_t, x_t)$ has the standard properties, namely $S_Q > 0$, $S_{QQ} < 0$, and $S_x > 0$, $S_{Qx} > 0$. The willingness to pay for an extra unit of the good and, thus, the per-unit-of-time ("current") surplus increases with x_t. Consumers cannot store and transfer goods to the next time periods and the whole production is sold at the market equilibrium price $P(Q_t, x_t) \equiv S_Q(Q_t, x_t)$ which defines the inverse demand function.

To illustrate the main features of the regulatory scheme and without loss of generality, we shape $S(Q_t, x_t)$ as a quadratic function with a linear demand shifter: $S(Q_t, x_t) = Q_t(x_t - \frac{Q_t}{2})$. Hence, $S_Q(Q_t, x_t) \equiv P(Q_t, x_t) = x_t - Q_t > 0 > S_{QQ}(Q_t, x_t) = -1$.[6]

[5] For instance, leases and affermage contracts generally last between 8 and 15 years (World Bank, 2022).

[6] A more general analysis of our results is provided in Appendix B.

Demand changes are assumed to be driven by a trendless geometric Brownian process:[7]

$$dx_t = \sigma x_t dZ_t \qquad x_{t=0} = x_0 \tag{1}$$

where $\sigma > 0$ is the constant instantaneous volatility and $Z_t \sim N(0, t)$ is a standard Wiener process having a normal distribution with zero mean and variance t. [8]

By solving (1), the demand shifter can be written as follows:

$$x_t \equiv \phi(t, x_0, Z_t) = x_0 \exp\left(-\frac{\sigma^2}{2}t + \sigma Z_t\right) \tag{2}$$

Eq. (2) highlights that x_t depends on the initial value x_0, on the uncertainty parameter σ and on the current shock Z_t. The process x_t is highly persistent because the time t shock Z_t has a non-vanishing effect on all x_{t+s} with $s > t$. Moreover, Eq. (2) has two interesting properties which will play an important role in later discussion.

First, the higher is x_0, the higher will, ceteris paribus, be the future demand of the good ($\frac{\partial\phi(t,x_0,Z_t)}{\partial x_0} > 0$), although this effect tends to fade over time ($\frac{\frac{\partial\phi(t,x_0,Z_t)}{\partial x_0}}{\phi(t,x_0,Z_t)} = \frac{1}{x_0}$). Second, the relative impact of x_0 versus $Z_{t,}$ (i.e., $\frac{\partial\phi(t,x_0,Z_t)}{\partial x_0} / \frac{\partial\phi(t,x_0,Z_t)}{\partial Z_t} = \frac{1}{\sigma x_0}$) is decreasing in x_0. In other words, the information potential of x_0 reduces as x_0 increases.[9]

Similarly, other things equal, the information value of x_0 decreases as the uncertainty parameter σ increases.

Whereas σ is public knowledge, the firm is better informed than the government about x_t ($t \geq 0$). The initial value x_0, reflecting the firm's innate ability to seize market opportunities ("the firm's type")[10], is distributed on $[x^l, x^h]$, according to the cumulative distribution function $G(x_0)$, with density

[7]The assumption of a trendless random walk allows us to focus on the pure effect of the uncertainty. However, by the Markov property of Eq. (1), our results would not be qualitatively altered by using a non-zero trend for x_t.

[8]The use of a Brownian process is quite common in the Principal-Agent literature (see, e.g., Demarzo and Sannikov, 2006; Sannikov, 2008; Broer and Zwart, 2013; Dosi and Moretto, 2013; Soumare and Lai, 2016; Di Corato et al, 2018; Bergemann and Strack, 2022; Arve and Zwart, 2023).

[9]Although the impact of x_0 on x_t reduces over time, it never vanishes. See Bergemann and Strack (2015) for a more in-depth discussion of these properties.

[10]In a standard (static) adverse selection problem, or in a dynamic model with perfectly correlated shocks, this would indeed be the only private information parameter of interest. See, e.g., Baron and Besanko (1984) and Auriol and Picard (2009; 2013).

$g(x_0) \geq 0$ and $g(x^l)$, $g(x^h) > 0$, which is common knowledge.[11] $G(x_0)$ is such that $\frac{1-G(x_0)}{g(x_0)x_0}$ is monotone and decreasing, with $x^l g(x^l) = k$.[12]

Assuming for simplicity that the tax is the only cost incurred, the firm's current utility function can be written as:[13]

$$u_t = \pi(Q_t, x_t) - T_t, \tag{3}$$

where $\pi(Q_t, x_t) = P(Q_t, x_t)Q_t$ and T_t are, respectively, gross profits and tax payments at time t.

Hence, the firm's expected intertemporal utility at $t = 0$ is given by:

$$U = E_0 \left[\int_0^\infty e^{-rt} u(Q_t, x_t) dt \right] \tag{4}$$

where r is the discount rate.

The government is benevolent and utilitarian. Specifically, we assume that, in choosing the optimal tax regime, the government simultaneously pursue two goals: on the one hand, to ensure a sufficiently high economic surplus, and on the other hand, to use the contract as a source of public income.

Thus, the government's objective is to maximize the following function:

$$\begin{aligned} W &= E_0 \left[\int_0^\infty e^{-rt} S[(Q_t, x_t) + \lambda T_t] dt \right] \qquad (5) \\ &= E_0 \left[\int_0^\infty e^{-rt} [S(Q_t, x_t) - \pi(Q_t, x_t) + (1+\lambda)T_t] dt \right] + U(x_0) \end{aligned}$$

where $\lambda > 0$ indicates that a unit of tax revenue from the firm yields a net welfare gain, by saving an excess burden of taxation in other markets.

Finally, we assume that the government and the firm share the same discount rate r.

[11] As in Arve and Zwart (2023), Skrzypacz and Toikka (2015) and Buso et al. (2021), this is equivalent to assuming that the firm's private information is represented by two stochastic processes, where the one representing the initial value is constant after time zero, but influences the transitions of the second one.

[12] Note that this condition is strictly weaker than the standard increasing hazard rate assumption (see, e.g., Guesnerie and Laffont 1984; Jullien 2000).

[13] The inclusion of operating costs, minimized for all production levels, would not qualitatively alter our results and conclusions.

3 The optimal contract

3.1 Incentive-compatibility conditions

The initial value x_0 is private information. Moreover, Eq. (2) implies that even if the government could get x_0 revealed, this would not provide sufficient information for inferring x_t after time zero. Thus, the government's problem consists of finding a mechanism capable of inducing the firm to truthfully report both her initial type and the values of x_t after time zero.

Having the stochastic process x_t the same properties as in Bergemann and Strack (2015), we can exploit their allocative mechanism, whose robustness relies on a class of deviations called as "consistent deviations".[14]

Borrowing this approach, the government's problem can be solved in two steps.

1. For any given initial value x_0, the government will find it optimal to commit himself to the repetition of a standard static regulatory contract where, at each $t > 0$, the firm reports x_t truthfully.

2. Since each future realization x_t depends both on the initial value x_0 and the contemporaneous shock Z_t, i.e., $x_t = \phi(x_0, Z_t)$, the government's problem reduces to induce the firm to report x_0 truthfully.[15]

By the separability of the problem, we get that the optimal regulatory scheme should include two components. First, an annuitized fixed fee $F(x_0)$ for the revelation of x_0.[16] Second, a time-varying transfer $TV(x_0, x_t)$ for the revelation of x_t $(t > 0)$.

As usual, it is convenient to work backward, starting from $t > 0$. Suppose the government has already obtained a truthful report of x_0. Thus, the firm's

[14]The concept of consistent deviations can be summarized as follows. If a firm, whose true initial type is x_0, misreports by reporting $\hat{x}_0$, then she will continue to misreport, by reporting $\hat{x}_t = \phi(\hat{x}_0, Z_t)$ instead of the true value $x_t = \phi(x_0, Z_t)$ in all $t > 0$. In other words, since x_t $(t > 0)$ depends on x_0, a firm misreporting with a consistent deviation continues to misreport her type x_t in all future periods. This means that, given the actual shock, Z_t, the type reported at time t, $\hat{x}_t$, would be the true value of x_t if the true initial value had been $\hat{x}_0$. Notice that this definition is well suitable with Eq. (2) which implies that each new realization of Z_t determines a new realization of x_t that depends only on time and x_0.

[15]Thereafter, we drop the direct dependence on time in $\phi(x_0, Z_t)$ for simplicity of the notation.

[16]Alternatively, the fixed component could be charged up front, in which case case it can be interpreted as a sort of "entry fee".

intertemporal utility at $t > 0$ becomes the sum over time of single standard problems.

Specifically, by (3) and (4) , we get:

$$U(x_0, \hat{x}_t, x_t) = E_0 \left[\int_0^\infty e^{-rt} [\underbrace{\pi(Q(x_0, \hat{x}_t), x_t) - TV(x_0, \hat{x}_t)} - F(x_0)] dt \right] \quad (6)$$

where $TV(x_0, \hat{x}_t)$ is such that the firm truthfully reports $\hat{x}_t = x_t$, for all $t > 0$.

Defining $\tilde{u}(x_0, \hat{x}_t, x_t) = \pi(Q(x_0, \hat{x}_t), x_t) - TV(x_0, \hat{x}_t)$, the necessary and sufficient conditions for incentive-compatibility are the following:

$$\frac{\partial \tilde{u}(Q(x_0, x_t), x_t))}{\partial x_t} = Q(x_0, x_t) \text{ for all } t > 0 \quad (7.1)$$

$$\frac{d\partial Q(x_0, x_t)}{\partial x_t} > 0 \quad \text{for all } t > 0 \quad (7.2)$$

$$Q(x_0, x_t) \geq 0 \quad \text{for all } t > 0 \quad (7.3)$$

Once $TV(x_0, x_t)$ has been determined, it remains to determine the fixed part $F(x_0)$, that is, the government's problem reduces to a new single standard adverse selection problem.

At time $t = 0$ the firm's utility becomes:

$$U(x_0, \hat{x}_0) = E_0 \left[\int_0^\infty e^{-rt} [\pi(Q(\hat{x}_0, x_t), \phi(x_0, Z_t))) - TV(\hat{x}_0, x_t) - F(\hat{x}_0)] dt \right] \quad (8)$$

where $\hat{x}_0$ is the report of the initial value and $F(\hat{x}_0)$ must be determined in such a way so as to induce the firm to report truthfully x_0.

Since $x_t = \phi(x_0, Z_t)$, the necessary and sufficient condition for incentive-compatibility is now:

$$\frac{\partial U(x_0)}{\partial x_0} = E_0 \left[\int_0^\infty e^{-rt} Q(x_0, \phi(x_0, Z_t)) \frac{\partial \phi(x_0, Z_t)}{\partial x_0} dt \right] \quad (9.1)$$

whereas the second order sufficient condition is:[17]

$$\frac{\partial Q(x_0, x_t)}{\partial x_0} \geq 0 \quad (9.2)$$

[17]The SOC for the two problems are presented in details in Appendix A.

3.2 The two-part tax scheme

Let's consider the first step. If x_0 has already been thruthfully revealed, the government can determine the optimal quantity to be produced at each time t and the corresponding variable transfer $TV(.)$ by solving the following problem:

$$\max_{Q(x_0,x_t),TV(x_0,x_t)} \int_{x^l}^{x^h} \left\{ E_0 \left[\int_0^\infty e^{-rt} [S(Q(x_0,x_t),x_t) + \lambda T(x_0,x_t)]dt \right] \right\} g(x_0)dx_0 \tag{10}$$

where $T(x_0,x_t) = F(x_0) + TV(x_0,x_t)$, subject to (7.1)-(7.2) and the following intertemporal participation constraint:

$$U(x_0) \geq 0 \tag{11}$$

Since $F(x_0)$ does not depend on the ex post values of x_t, we have a standard regulation problem under adverse selection (see, e.g., Baron and Myerson, 1982; Laffont and Tirole, 1993).

The general solutions for Q_t^* and TV_t^* for all $t > 0$ are (see Appendix B):

$$S_Q(Q^*(x_0,x_t),x_t) + \lambda \pi_Q(Q^*(x_0,x_t),x_t)) - \lambda \frac{(1-G(x_0))}{g(x_0)} \frac{\partial \phi(x_0,Z_t)}{\partial x_0} = 0 \tag{12.1}$$

$$TV^*(x_0,x_t) = \pi(Q^*(x_0,x_t),x_t)) - \int_0^{x_t} Q(x_0,z)dz \tag{12.2}$$

Given the optimal values of $TV^*(x_0,x_t)$ and $Q^*(x_0,x_t)$, by substituting (12.2) into (6) we get:

$$U(x_0) = E_0 \left[\int_0^\infty e^{-rt} \left[\int_0^{x_t} Q^*(x_0,z)dz - F(x_0) \right] dt \right] \tag{13}$$

Since $TV^*(x_0,x_t)$ is such that the firm, given her initial report x_0, will reveal x_t ($t > 0$) truthfully, the government's problem reduces to a static design problem where (9.1)-(9.2) are the first and second order conditions for incentive-compatibility.

By the Envelope theorem, Eq. (9.1) implies that:

$$U(x_0) = \int_{x^l}^{x_0} E_0 \left[\int_0^\infty e^{-rt} Q^*(y,x_t) \frac{\partial \phi(y,Z_t)}{\partial y} dt \right] dy \tag{14}$$

Using (13) and (14), we get the solution for $F^*(x_0)$:

$$F^*(x_0) = r\left\{E_0\left[\int_0^\infty e^{-rt}[\int_0^{x_t} Q^*(x_0,z)dz]dt\right] - \int_{x^l}^{x_0} E_0\left[\int_0^\infty e^{-rt}Q^*(y,x_t)\frac{\partial\phi(y,Z_t)}{\partial y}dt\right]dy\right\} \tag{15}$$

The following proposition characterizes the optimal contract under the assumptions presented in Section 2.

Proposition 1 *For any given x_0, the quantity supplied at each time t that solves problem (10) is given by:*

$$Q^*(x_0,x_t) = \frac{1+\lambda}{1+2\lambda}x_t - \frac{\lambda}{1+2\lambda}\frac{1-G(x_0)}{g(x_0)}\frac{x_t}{x_0} \quad \text{for all } t>0 \tag{16.1}$$

where $\frac{\partial\phi(x_0,Z_t)}{\partial x_0} = \frac{x_t}{x_0}$.

The time-dependent variable transfer (12.2) is:

$$TV^*(x_0,x_t) = (x_t - Q^*(x_0,x_t))Q^*(x_0,x_t) - Q^*(x_0,x_t)\frac{x_t}{2} \quad \text{for all } t>0 \tag{16.2}$$

and the fixed tax (15) is:

$$F^*(x_0) = rE_0\left\{\int_0^\infty e^{-rt}[Q^*(x_0,x_t)\frac{x_t}{2} - \int_{x^l}^{x_0} Q^*(y,x_t)\frac{x_t}{y}dy]dt\right\} \tag{16.3}$$

Proof. Proof: See Appendix B ■

On the LHS of (16.1), $\frac{1-G(x_0)}{g(x_0)}$ are the information rents, which depend on the initial value x_0, $\frac{1+\lambda}{1+2\lambda}x_t$ is the surplus-maximizing quantity (i.e., the profit-maximizing quantity $\frac{x_t}{2}$ plus the extra quantity $\frac{1}{1+2\lambda}\frac{x_t}{2}$ to maximize the surplus) and $\frac{\lambda}{1+2\lambda}\frac{1-G(x_0)}{g(x_0)}\frac{x_t}{x_0}$ is the allocative distortion due to regulation.

As usual there is no distortion for the highest-type firm: $Q^*(x^h,x_t) = \frac{1+\lambda}{1+2\lambda}x_t$, $t>0$.[18]

By simple algebra, Eq.(16.1) can be rewritten as:

$$Q^*(x_0,x_t) = Q^*(x_0)\frac{x_t}{x_0}, \tag{17}$$

where $Q^*(x_0) = \frac{1+\lambda}{1+2\lambda}x_0 - \frac{\lambda}{1+2\lambda}\frac{1-G(x_0)}{g(x_0)}$ is the optimal quantity at time zero.

[18] Note that if the government could observe x_t, he would tax all firm's revenues and choose an allocation Q_t ($t>0$) that maximizes the welfare value $(x_t - \frac{Q_t}{2})Q_t + \lambda(x_t - Q_t)Q_t$, which properly defines $\frac{1+\lambda}{1+2\lambda}x_t$.

Eq.(17) highlights that the optimal quantity at each time $t > 0$ is given by the time zero quantity $Q^*(x_0)$ multiplied by $\frac{x_t}{x_0} = \frac{\partial\phi(x_0,Z_t)}{\partial x_0} > 0$ which, in the language of Pavan et al. (2014), is the "impulse response" of x_0 to x_t. Hence, quantities at time points close to $t = 0$ are more strongly and positively correlated than quantities at time points distant from $t = 0$. Notice that both $\frac{\partial Q^{**}(x_0,x_t)}{x_t}$ and $\frac{\partial Q^*(x_0,x_t)}{\partial x_0} = \frac{\partial Q^*(x_0,x_t)}{x_0} + \frac{\partial Q^*(x_0,x_t)}{x_t}\frac{\partial\phi(x_0,Z_t)}{\partial x_0}$ are positive.

To avoid corner solutions we make the following assumption, stating that the consumers' willingness-to-pay for the first unit of the good is sufficiently high so as to allow even the lowest-type firm to earn a non-negative revenue.[19]

Assumption 1. $k \geq \frac{\lambda}{1+\lambda}$

Under Assumption 1, it is easy to show that the optimal quantity decreases as the parameter λ increases ($\frac{\partial Q^*(x_0,x_t)}{\partial\lambda} < 0$). The simple intuition is that the higher is the shadow value of public funds, the more the government will tend to prioritize budgetary issues at the expense of the output level, in so doing sacrificing part of the consumer surplus.[20]

Let's look more in detail at total current tax payments:

$$\begin{aligned} T^*(x_0,x_t) &= F^*(x_0) + TV^*(x_0,x_t) \\ &= rE_0\left[\int_0^\infty e^{-rt}[Q^*(x_0,x_t)\frac{x_t}{2} - \int_{x^l}^{x_0} Q^*(y,x_t)\frac{x_t}{y}dy]dt\right] \\ &+ (x_t - Q^*(x_0,x_t))Q^*(x_0,x_t) - Q^*(x_0,x_t)\frac{x_t}{2} \end{aligned} \tag{18}$$

The third line of Eq. (18) indicates that, like in any standard adverse selection problem, the variable component $TV^*(.)$ is given by the firm's revenues, $\pi(Q^*(x_0,x_t),x_t)) = (x_t - Q^*(x_0,x_t))Q^*(x_0,x_t)$, minus the information rents paid to the firm to reveal x_t, i.e., $Q^*(x_0,x_t)\frac{x_t}{2}$.

Note that the same rents also appear, but with positive sign, in the fixed component (first term in the second line). The intuition is that, since x_0 provides some elements for predicting future demand levels ($\frac{\partial\phi(x_0,Z_t)}{\partial x_0} = \frac{x_t}{x_0}$), its

[19]Since $\frac{x_0 g(x_0)}{1-G(x_0)}$ is increasing in x_0, the quantity of the lowest type is $Q^*(x^l) = \frac{1+\lambda}{1+2\lambda}\left[k - \frac{\lambda}{1+\lambda}\right]\frac{x^l}{k}$, which is always non-negative if $k \geq \frac{\lambda}{1+\lambda}$.

[20]Notice that, as $\lambda \to \infty$, the optimal quantity reduces to $Q^*(x_0,x_t) \to \frac{x_t}{2} - \frac{1}{2}\frac{1-G(x_0)}{g(x_0)}\frac{x_t}{x_0}$, where $\frac{x_t}{2}$ is the profit-maximizing quantity, and the allocative distortion reduces to the information rents.

knowledge allows to tax the firm of an amount equal to the rents that the government expects to pay after time zero ($E[\int_0^\infty e^{-rt} Q^*(x_0, x_t)\frac{x_t}{2} dt]$). However, since extracting information on x_0 is costly, the government, in determining the fixed rate, will have to substract from the first term the amount related to the informative content of x_0 (second term in the second line). This ultimately leads to an always positive value of the fixed fee (see Appendix B).

Using (17), $F^*(x_0)$ can be simplified as follows:

$$F^*(x_0) = \frac{r}{r-\sigma^2}\left[\frac{Q^*(x_0)x_0}{2} - \int_{x^l}^{x_0} Q^*(y)dy\right] > 0 \tag{19}$$

which is increasing in x_0 (see Appendix B). The reason is that, since high values of x_0 are relatively less useful to make predictions about future demand, also the rents to be paid to induce the firm to disclose her type (second term in Eq.(19)), grow less then the first term when x_0 is high.

Notice that, unlike the fixed tax, the positivity of the variable component (16.2) is not always guaranteed, i.e., the variable tax can turn into a subsidy. For any given x_0, this could occur when the shadow value of public funds is relatively small, namely, when:

$$\lambda < \frac{1}{2}\frac{g(x_0)x_0}{1-G(x_0)} \tag{20}$$

or, for any given λ, when x_0 is sufficiently high.

Now consider the discounted value at time zero of the intertemporal public revenues. From (18), it is easy to prove that:

$$\begin{aligned} TT^*(x_0) &= E_0\left[\int_0^\infty e^{-rt}T^*(x_0, x_t)\right] dt \\ &= \frac{(x_0 - Q^*(x_0))Q^*(x_0)}{r-\sigma^2} - \frac{\int_{x^l}^{x_0} Q^*(y)dy}{r-\sigma^2} \end{aligned} \tag{21}$$

Unlike standard adverse selection settings, where tax levies monotonically increase with the firm's type, in our model the intertemporal public revenues (21) exhibit an "anomalous" trend. Indeed, according to (20), it is easy to show that, for any given value of λ, there exists a cutoff type $\bar{x}_0(\lambda) \in [x^l, x^h]$ below which $TT^*(x_0)$ is increasing in x_0 and decreasing otherwise.[21]. In addition, if λ is particularly low, namely $\lambda < \frac{1}{2}k$ (where $k = x^l g(x^l)$), $TT^*(x_0)$ monotically

[21] The cutoff $\bar{x}_0(\lambda)$ is given by simply inverting Eq.(20).

decreases in x_0 for all the range $[x^l, x^h]$.

Finally, by substituting (21) into (4), we obtain the firm's intertemporal utility:

$$U^*(x_0) = \int_{x^l}^{x_0} \frac{Q^*(y)}{r-\sigma^2} dy \tag{22}$$

with $U^*(x^l) = 0$, and from (5)-(21) the government's expected payoff:

$$\begin{aligned} W^*(x_0) &= \frac{S(Q_0^*, x_0)}{r-\sigma^2} + \lambda E_0 \left[\int_0^\infty e^{-rt} T^*(x_0, x_t) \right] dt \\ &= \frac{S(Q_0^*, x_0) + \lambda \pi(Q_0^*, x_0)}{r-\sigma^2} - \lambda U^*(x_0) \end{aligned} \tag{23}$$

3.3 Perfectly correlated shocks

The two-part tax presented above is derived in a continuous-time setting where sales proceeds are affected by demand conditions which are private information to the firm.

Models with a similar adverse selection problem have been considered by a variety of authors, by restricting to environments where the realizations of the private information state variable are perfectly correlated over time. For instance, this is the framework used by Auriol and Picard (2013), who argue that the optimal regulatory process consists of the repetition of a static contract with time-independent transfers.

Notice that the same result can be replicated in our model by simply assuming that there is no uncertainty. Indeed, by defining $T_{\sigma=0}(x_0)$ as the instantaneous tax payments under $\sigma = 0$, from (21) it is easy to get the Auriol and Picard's time-independent transfer:

$$T^*_{\sigma=0}(x_0) = rTT_{\sigma=0}(x_0) = (x_0 - Q^*(x_0)Q^*(x_0) - \int_{x^l}^{x_0} Q^*(y) dy \tag{24}$$

Eq. (24) implies that the firm's intertemporal utility is given by:

$$U_{\sigma=0}(x_0) = \frac{\int_{x^l}^{x_0} Q^*(y) dy}{r} \tag{25}$$

whereas the government's payoff is:

$$W_{\sigma=0}(x_0) = \frac{S(Q_0^*, x_0) + \lambda \pi(Q_0^*, x_0)}{r} - \lambda U_{\sigma=0}(x_0) \tag{26}$$

However, if $\sigma > 0$, the use of fixed levies, without subsequent adjustments, would make both parties worse off:

$$U^*(x_0) > U_{\sigma=0}(x_0)$$
$$W^*(x_0) > W_{\sigma=0}(x_0)$$

4 A numerical example

For illustration purposes we assume a uniform distribution $G(x_0) = \frac{x_0 - x^l}{x^h - x^l}$, which implies that $\frac{1-G(x_0)}{g(x_0)} = x^h - x_0$. Moreover, we shall assume (unless otherwise indicated) that $x^l = 1$, $x^h = 3$, $\lambda = 0.5$, $r = 0.05$ and $\sigma = 0.20$.

4.1 Tax payments

Figure 1 shows that the fixed fee $F^*(x_0)$ monotonically increases with the firm's type x_0. The reason is that $F^*(x_0)$ positively depends on the information rents that the government expects to pay over the contract period. Since the higher is x_0 the lower is its informational value, the higher is x_0 the greater is $F^*(x_0)$.

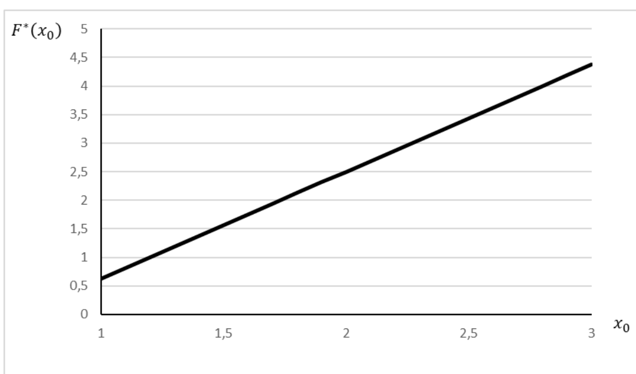

Fig. 1: Change of $F^*(x_0)$ with x_0

Turning to the variable component, it is convenient to rewrite the current transfer $TV^*(x_0, x_t)$ as an excise tax:

$$TV^*(x_0, x_t) = \beta(x_0, x_t) Q^*(x_0, x_t), \tag{27}$$

where $\beta(x_0, x_t) = \frac{x_t}{2} - Q^*(x_0, x_t) = \frac{x_t}{1+2\lambda}\left[\lambda \frac{1-G(x_0)}{g(x_0)x_0} - \frac{1}{2}\right] x_t$ is the tax rate.

Figure 2a (where black $x_t = 1$, red $x_t = 3$, blue $x_t = 5$; this legend will remain the same in all following figures) shows that the tax base Q^* monotonically increases with the firm's type and, for any given x_0, with the current realization x_t .

Regarding the excise tax rate, Figure 2b shows how β varies with x_0 and, for any given x_0, with x_t. Three comments are in order. First, for any given realization x_t, the higher is the initial value x_0, the lower will be the tax rate. Second, beyond a threshold value of x_0 (in our example, $\bar{x}_0(\lambda = 0.5) = 1.5$), β turns out to be negative, that is, the tax becomes a subsidy. Third, beyond the threshold, the higher is x_t the greater will be the subsidy rate.

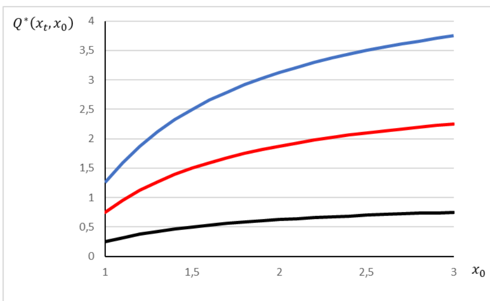

Fig. 2a: Change of Q^* with x_0 and x_t

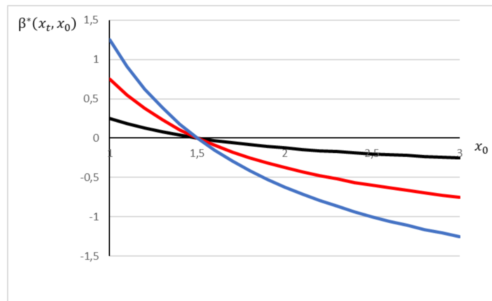

Fig. 2b: Change of β with x_0 and x_t

Taken together, Figure 1 and Figure 2b indicate that the two-part tax schedule can be regarded as a "risk sharing device", insofar as firms with a greater revenue potential assume a risk by signing a contract that involves high fixed payments, in exchange of the government's commitment to subsequently adjusting the variable tax rate. Indeed, when x_0 is high, any upward demand shifts require the government to reduce β (up to the point it can become negative), so that the firm will have no incentives to misreport x_t and to reduce the output at the detriment of the consumer surplus.

Conversely, when x_0 is low, the firm will be charged with a low entry fee, in exchange of the commitment to pay an increasing share of her revenues should demand prove to be higher than expected.

Notice that, as shown by Figure 3, the current net total amount of government revenue T^* can turn out to be negative. For instance, this could occur at values of x_t significantly higher than the initial value x_0 (e.g., $x_0 = 2.3$ and $x_t = 5$), in which case, in some periods, the subsidies received by the firm would be greater than the fixed fee.

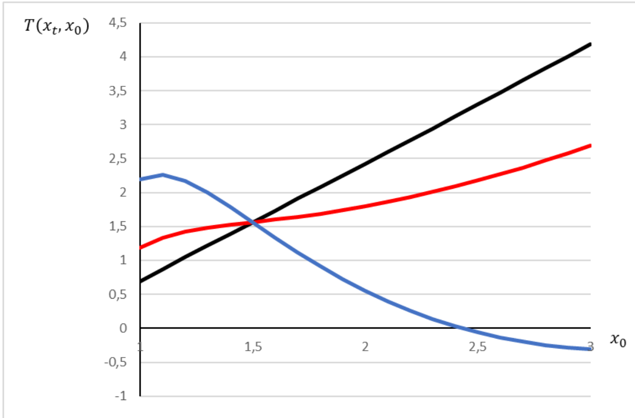

Fig. 3: Change of T^* with x_0 and x_t

4.2 The effect of λ

The tax schedule is affected, among other variables, by the parameter λ that captures the weight assigned by the government to tax receipts. For instance, an increase in λ causes the government to prioritize budgetary issues at the expense of the output level. That said, it is worth analysing the effect of λ on the composition of total tax receipts, namely on the optimal mix of intertemporal fixed and variable transfers.

From (18) and (21), the present value of total expected variable transfers is given by:

$$TTV^*(\lambda, x_0) = \frac{1}{r - \sigma^2}\left[(x_0 - Q^*(\lambda, x_0))Q^*(\lambda, x_0) - \frac{Q^*(\lambda, x_0)x_0}{2}\right] \tag{28}$$

whereas, from (19), the present value of total fixed payments is given by:[22]

$$TF^*(\lambda, x_0) = \frac{1}{r - \sigma^2}\left[\frac{Q^*(\lambda, x_0)x_0}{2} - \int_{x^l}^{x_0} Q^*(\lambda, y)dy\right] \tag{29}$$

Figures 4a and 4b describe the effect of $\lambda \in (0, 1]$ for a "high-type" ($x_0 = 2.2$) firm (red line) and a "low-type" firm ($x_0 = 1.2$) (black line) on (28) and on (29) respectively.[23]

[22] In the formulas we add λ to highlight how this parameter impacts on the optimal tax payments.

[23] In the example $g(x^l)x^l = \frac{1}{2}$, thus, by Assumption 1, the feasible interval for λ is $\in (0, 1]$.

The effect of an increase of λ on the variable component is similar for both types (see Figure 4a), in the sense that an increase of the shadow value of public funds leads to an increase of the intertemporal variable payments for a low-type firm or, equivalently, to a reduction of the total subsidies received by a high-type firm. The simple intuition is that a government assigning more importance to budgetary aspects will find it convenient to reduce the information rents paid over the contract period, even though this implies sacrificing part of consumer surplus.

On the contrary, the effect of an increase λ on the fixed payments is not univocal (see Figure 4b). Whereas total fixed charges always increase with λ for the high-type firms, they decrease for the low-type firms.[24]

The reason is that the fixed tax must be calibrated against the rents that the government expects to save in the future by inducing the firm to reveal her initial type x_0. Since high values of x_0 have a relatively low information value for predicting future demand leveles, a government assigning more importance to budgetary issues will find it more "productive" to gain information on x_0 when it is low (i.e., to reduce the fixed tax) instead of adjusting (lowering) later the variable tax rate to gain information on the ex-post demand levels.

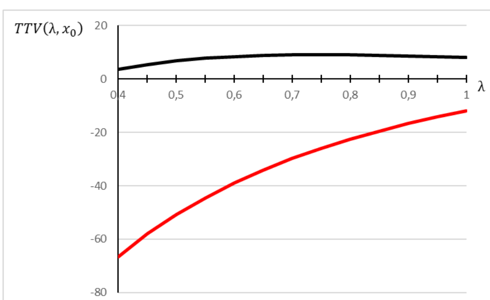

Fig. 4a: Change of TTV^* with λ and x_0

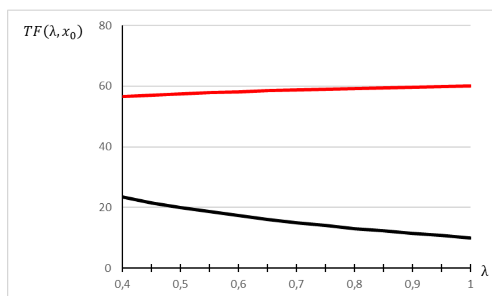

Fig. 4b: Change of TF^* with λ and x_0

[24]This result is analytically proven in the Appendix C, where it is shown that, when x_0 is low, the derivative of TF with respect to λ can be negative if the rents that the government expects to pay in the future (first term in square brackets of Eq. (29) decreases more with λ than the rents paid to induce the firm to truthfully reveal x_0 (second term in square brackets of Eq. (29)).

4.3 Implementing the contract

Instead of using a truthful direct mechanism the government can use an indirect mechanism which does not require exchange of information between the parties (Laffont and Tirole, 1993). Specifically, in our framework an indirect mechanism would consist of letting the firm to decide the production level and determining the tax payments on the basis of the observable output.

Since under our assumptions all functions are invertible, i.e., $x_0 = Q^{-1}(Q_0)$ and $x_t = \frac{Q^{-1}(Q_0)Q_t}{2}$, by substituting in Eq. (18) and rearranging we get the optimal tax schedule as a function of the time zero and the ex-post production levels, which replicates the same choice of output as with the direct revelation mechanism:

$$\begin{aligned} T^*(Q_0, Q_t) &= F^*(Q^{-1}(Q_0)) + TV^*(Q^{-1}(Q_0), \frac{Q^{-1}(Q_0)Q_t}{2}), \quad \text{for} \quad t \geq 0. \\ &= F^*(Q^{-1}(Q_0)) + \gamma(Q_0)Q_t^2 \end{aligned} \tag{30}$$

with $\gamma(Q_0) = \frac{Q^{-1}(Q_0)}{2Q_0} - 1$.

Notice that Eq. (30) can be reformulated as an equivalent adaptive regulatory mechanism capable of adjusting the levies that the firm will bear during the contract period in response to (and to be suitable for) new market conditions.

After some algebraic steps and by approximating the tax schedule in discrete time, we get:

$$T_t^* = T_{t-1}^* + \gamma(Q_0)\left(Q_t^2 - Q_{t-1}^2\right) \quad \text{for } t \geq 0 \tag{31}$$

where $T_0^* = F^*(Q_0) + \left(\frac{Q^{-1}(Q_0)}{2} - Q_0\right)Q_0$, and $\gamma(Q_0)$ now representing an adjustment coefficient that applies to the difference between time t and time $t-1$ supplied quantity.

The firm chooses at time zero the adjustment coefficient $\gamma(Q_0)$, and thus the payment T_0^*, and then the adaptive regulatory policy can start.

Using the same parameters and the uniform distribution as in previous sections, if $Q_0 = \frac{9}{8}$ (i.e., the firm's type is $x_0 = 1.5$), Eq. (31) becomes:

$$T_t^* = T_{t-1}^* - \frac{1}{3}\left(Q_t^2 - Q_{t-1}^2\right), \quad \text{with } \gamma = -\frac{1}{3} \text{ and } T_0^* = 1.56$$

Instead, if $Q_0 = 3$ (i.e., $x_0 = 3$), the adaptive mechanism becomes:

$$T_t^* = T_{t-1}^* - \frac{1}{2}\left(Q_t^2 - Q_{t-1}^2\right), \qquad \text{with } \gamma = -\frac{1}{2} \text{ and } T_0^* = 4.38$$

Eq. (31) allows to further highlight the impact of the policy parameter λ on the optimal tax schedule. As already pointed out, an increase of λ brings about a reduction of the role that time-adjusted transfers play within the two-part tax schedule. Indeed, Eq. (31) implies that it would be optimal to simply use a flat tax only when $\gamma(Q_0) = 0$, that is when $\lambda = \frac{x_0}{3-x_0}$.

5 Final remarks

There are several examples worldwide, across different sectors, of public contracts where a private entity agrees to pay the awarding body for an exclusive right to supply a product/service by exploiting a government-owned asset. These settlements generally involve either lump-sum payments or the transfer of a fixed share of sales proceeds or, more rarely, a combination of fixed and variable payments.

In this paper we have studied optimal taxation when the awarding authority intends to use taxation both as an efficiency-enhancement tool and as a source of public income. Our main result is that if contracting authorities face agents holding private information on project's returns evolving in such a way that the information obtained at the award stage is not fully indicative of future earnings, optimal contracting requires taxing state-sponsored monopolists with an appropriate combination of fixed and time-adjusted payments from actual sales.

Not surprisingly, we found that agents with a greater expected revenue potential should be charged with relatively high entry fees. However, in order to ensure incentive-compatibility, the government must simultaneously commit to subsequent downward adjustments of tax rates, notably when sales proceeds perform beyond expectations, up to the point where variable payments might turn into a subsidy. Conversely, agents with lower profit prospects should be charged with lower fixed fees, in exchange for committing to pay an increasing share of their revenues if demand happens to be higher than expected.

We have also analyzed how the optimal mix of fixed and variable payments is impacted by the expected volatility of consumer demand and by the government's trade-off between efficiency gains and public income.

As for the uncertainty, we have shown that the lower the expected demand variability, the higher the optimal proportion of fixed payments within the two-part tax schedule. This implies that the quite common practice of charging franchisees with fixed fees can find justification only when there is very little uncertainty on consumer demand or, equivalently, when demand is expected to be hit by perfectly correlated shocks. If this condition is not met, the lack of time-adjusted variable transfers would result in persistent and increasing allocative distortions.

About the trade-off between efficiency gains and public income, the effect of an increase of the shadow value of public funds is not univocal as it varies according to the firm's type. While for agents with high revenue expectations the relative weight of fixed payments tends to increase with the importance assigned to tax receipts relative to other welfare concerns, the opposite occurs when contracting authorities face agents with lower profit prospects.

Appendix A

Neglecting to indicate the dependence on x_0, we can write:

$$\tilde{u}(x_t, \hat{x}_t) = (x_t - Q(\hat{x}_t))Q(\hat{x}_t) - TV(\hat{x}_t) \tag{A.1}$$

where $\hat{x}_t$ is the report by the firm's type x_t

The FOC with respect to $\hat{x}_t$ is:

$$\frac{\partial \tilde{u}(x_t, \hat{x}_t)}{\partial \hat{x}_t} = (x_t - Q(\hat{x}_t))\frac{dQ(\hat{x}_t)}{d\hat{x}_t} - Q(\hat{x}_t)\frac{dQ(\hat{x}_t)}{d\hat{x}_t} - \frac{dTV(\hat{x}_t)}{d\hat{x}_t} = 0 \tag{A.2}$$

A truthful report is optimal if at $\hat{x}_t = x_t$:

$$\left.\frac{\partial \tilde{u}(x_t, \hat{x}_t)}{\partial \hat{x}_t}\right|_{\hat{x}_t = x_t} = 0$$

Further, the local SOC is:

$$\frac{\partial^2 \tilde{u}(x_t, \hat{x}_t)}{\partial \hat{x}_t^2} = (x_t - Q(\hat{x}_t))\frac{d^2 Q(\hat{x}_t)}{d\hat{x}_t^2} - 2(\frac{dQ(\hat{x}_t)}{d\hat{x}_t})^2 - Q(\hat{x}_t)\frac{d^2 Q(\hat{x}_t)}{d\hat{x}_t^2} - \frac{d^2 TV(\hat{x}_t)}{d\hat{x}_t^2} \,|_{\hat{x}_t = x_t} \leq 0 \tag{A.3}$$

Since, when $\hat{x}_t = x_t$, Eq. (A.2) is an identity, the derivative is zero, i.e.:

$$\frac{dQ(\hat{x}_t)}{d\hat{x}_t}+(x_t-Q(\hat{x}_t))\frac{d^2Q(\hat{x}_t)}{d\hat{x}_t^2}-2(\frac{dQ(\hat{x}_t)}{d\hat{x}_t})^2-Q(\hat{x}_t)\frac{d^2Q(\hat{x}_t)}{d\hat{x}_t^2}-\frac{d^2TV(\hat{x}_t)}{d\hat{x}_t^2}\ |_{\hat{x}_t=x_t}=0 \tag{A.4}$$

By replacing (A.4) in (A.3), we get:

$$\left.\frac{\partial^2\tilde{u}(x_t,\hat{x}_t)}{\partial\hat{x}_t^2}\right|_{\hat{x}_t=x_t}=\frac{\partial Q(x_t)}{\partial x_t}\geq 0 \tag{A.5}$$

As shown by Laffont and Martimort (2001, pp. 134-136), local incentive constraints also imply global incentive constraints. Thus, from Eqs. (A.2)-(A.5), a truthful revelation mechanism can be characterized by the following conditions:

$$\frac{\partial\tilde{u}(Q(x_0,x_t),x_t))}{\partial x_t}=Q(x_0,x_t) \text{ for all } t>0$$

$$\frac{\partial Q(x_t)}{\partial x_t}\geq 0 \quad \text{for all } t>0$$

Once $x_t>0$ is known, the firm's intertemporal utility becomes:

$$U(x_0,\hat{x}_0)=E_0\left[\int_0^\infty e^{-rt}[\pi(Q(\hat{x}_0,\phi(\hat{x}_0,W_t)),\phi(x_0,W_t))-TV(\hat{x}_0,\phi(\hat{x}_0,W_t))-F(\hat{x}_0)]dt\right] \tag{A.6}$$

where $\hat{x}_0$ is the report by the firm's type x_0 and the time-varying x_t depends on $\hat{x}_0$ through the function $x_t=\phi(\hat{x}_0,W_t)$.

By using Eq. (A.2), the FOC with respect to $\hat{x}_0$ is:

$$\frac{\partial U(x_0,\hat{x}_0)}{\partial\hat{x}_0}=E_0\left[\int_0^\infty e^{-rt}[\frac{\partial\pi_t}{\partial Q_t}\frac{dQ_t}{d\hat{x}_0}-\frac{dTV_t}{d\hat{x}_0}-\frac{dF}{d\hat{x}_0}]dt\right]=0 \tag{A.7}$$

while the local SOC is:

$$\frac{\partial^2U(x_0,\hat{x}_0)}{\partial\hat{x}_0^2}=E_0\left[\int_0^\infty e^{-rt}[\frac{\partial^2\pi_t}{\partial Q_t^2}\frac{dQ_t}{d\hat{x}_0}+\frac{\partial\pi_t}{\partial Q_t}\frac{d^2Q_t}{d\hat{x}_0^2}-\frac{d^2TV_t}{d\hat{x}_0^2}-\frac{d^2F}{d\hat{x}_0^2}]dt\right]\leq 0 \tag{A.8}$$

A truthful report is optimal if at $\hat{x}_0=x_0$:

$$\left.\frac{\partial U(x_0,\hat{x}_0)}{\partial\hat{x}_0}\right|_{\hat{x}_0=x_0}=0$$

and:
$$\left.\frac{\partial^2 U(x_0,\hat{x}_0)}{\partial \hat{x}_0^2}\right|_{\hat{x}_0=x_0} \leq 0$$

Totally differentiating Eq. (A.7), we get:

$$E_0\left[\int_0^{\infty} e^{-rt}[\frac{\partial^2 \pi_t}{\partial Q_t^2}\frac{dQ_t}{d\hat{x}_0}+\frac{\partial \pi_t}{\partial Q_t}\frac{d^2 Q_t}{d\hat{x}_0^2}+\frac{\partial x_t}{\partial \hat{x}_0}\frac{dQ_t}{\partial \hat{x}_0}-\frac{d^2 TV_t}{d\hat{x}_0^2}-\frac{d^2 F}{d\hat{x}_0^2}]dt\right]|_{\hat{x}_0=x_0}=0 \tag{A.9}$$

Finally, by replacing Eq. (A.9) in Eq. (A.8), we obtain:

$$\frac{\partial^2 U(x_0,\hat{x}_0)}{\partial \hat{x}_0^2}=-E_0\left[\int_0^{\infty} e^{-rt}\left(\frac{\partial x_t}{\partial \hat{x}_0}\frac{dQ_t}{\partial \hat{x}_0}\right)dt\right]|_{\hat{x}_0=x_0}\leq 0 \tag{A.10}$$

Thus, from (A.7)-(A.10), a truthful revelation mechanism can be characterized by:

$$\frac{\partial U(x_0)}{\partial x_0}=E_0\left[\int_0^{\infty} e^{-rt}Q(x_0,\phi(x_0,Z_t))\frac{\partial \phi(x_0,Z_t)}{\partial x_0}dt\right]$$

together with the sufficient condition:

$$\frac{\partial Q_t}{\partial x_0}\geq 0$$

Appendix B

Proof of Proposition 1

The standard approach to solve Eq. (10) is to ignore, for the moment, the second order conditions, Eqs. (7.2)-(9.2), and to solve the relaxed problem.

By the Envelope Theorem (see Milgrom and Segal, 2002, Theorem 1 and Theorem 2), Eq. (9.1) implies that:

$$U(x_0)=\int_{x^l}^{x_0} E_0\left[\int_0^{\infty} e^{-rt}Q(y,x_t)\frac{\partial \phi(y,Z_t)}{\partial y}dt\right]dy \tag{B.1}$$

where the lowest demand gets zero utility, i.e. $U(x^l)=0$. Integrating B.1 by part we get:

$$\int_{x^l}^{x^h} U(x_0)g(x_0)dx$$
$$= \int_{x^l}^{x^h} E_0\left[\int_0^\infty e^{-rt}Q(x_0,x_t)\frac{\partial\phi(x_0,Z_t)}{\partial x_0}dt\right]\frac{(1-G(x_0))}{g(x_0)}g(x_0)dx_0 \qquad \text{(B.2)}$$

From Eq. (4) we get:

$$\int_{x^l}^{x^h}\left\{E_0\left[\int_0^\infty e^{-rt}T(x_0,x_t)dt\right]\right\}g(x_0)dx_0 \qquad \text{(B.3)}$$
$$= -\int_{x^l}^{x^h} U(x_0)g(x_0)dx + \int_{x^l}^{x^h}\left\{E_0\left[\int_0^\infty e^{-rt}(\pi(Q(x_0,x_t),x_t))dt\right]\right\}g(x_0)dx_0$$
$$= -\int_{x^l}^{x^h}\left\{E_0\int_0^\infty e^{-rt}[Q(x_0,x_t)\frac{(1-G(x_0))}{g(x_0)}\frac{\partial\phi(x_0,Z_t)}{\partial x_0} + \pi(Q(x_0,x_t),x_t))]dt\right\}g(x_0)dx_0$$

Substituting (B.3) in the objective function (10), we obtain:

$$\max_{Q(x_0,x_t)}\int_{x^l}^{x^h}\left\{E_0\left[\begin{array}{c}\int_0^\infty e^{-rt}[S(Q(x_0,x_t),x_t)+\lambda\pi(Q(x_0,x_t),x_t)) \\ -\lambda Q(x_0,x_t)\frac{(1-G(x_0))}{g(x_0)}\frac{\partial\phi(x_0,Z_t)}{\partial x_0}]dt\end{array}\right]\right\}g(x_0)dx_0 \qquad \text{(B.4)}$$

Differentiating Eq. (B.4) with respect to Q_t we obtain the first order condition for the optimal output:

$$S_Q(Q^{**}(x_0,x_t),x_t)+\lambda\pi_Q(Q^{**}(x_0,x_t),x_t))-\lambda\frac{(1-G(x_0))}{g(x_0)}\frac{\partial\phi(x_0,Z_t)}{\partial x_0}=0 \qquad \text{(B.5)}$$

Since $\frac{\partial\phi(x_0,Z_t)}{\partial x_0}=\frac{x_t}{x_0}$ and given the assumption on $\frac{(1-G(x_0))}{g(x_0)}$, both the second order conditions, Eqs. (7.2) and (9.2), are satisfied, i.e.:

$$\frac{dQ^*(x_0,x_t)}{dx_t}>0 \quad \text{and} \quad \frac{dQ^*(x_0,x_t)}{dx_0}>0 \qquad \text{(B.6)}$$

Let's now derive the time-variant payment contract $TV^*(x_0,x_t)$. For each time $t>0$, integrating Eq. (7.1), we obtain:

$$\tilde{u}(Q^*(x_0,x_t),x_t)) = \int_0^{x_t} Q(x_0,z)dz \qquad \text{(B.7)}$$

where $\tilde{u}(Q^*(x_0,0),0)) = 0$.

By substituting Eq. (B.7) into Eq. (A.1) we get:

$$TV^*(x_0,x_t) = \pi(Q^*(x_0,x_t),x_t)) - \int_0^{x_t} Q^*(x_0,z)dz, \tag{B.8}$$

and by substituting (B.8) into (6) we get:

$$U(x_0,x_t) = E_0\left[\int_0^\infty e^{-rt}[\int_0^{x_t} Q^*(x_0,z)dz - F(x_0)]dt\right] \tag{B.9}$$

We now turn to the second problem (9.1)-(9.2). Since by construction of $TV^*(x_0,x_t)$, independently of his initial report x_0, the firm finds it optimal to report x_t truthfully, the firm's value can be rewritten as:

$$U(x_0,\hat{x}_0) = E_0\left[\int_0^\infty e^{-rt}((\pi(Q^*(\hat{x}_0,x_t),\phi(x_0,Z_t)) - TV^*(\hat{x}_0,x_t)) - F(\hat{x}_0))dt\right] \tag{B.10}$$

where x_0 is the true initial shock and $\hat{x}_0$ is the one reported.

In addition, since it is optimal to report x_t truthfully for all t, we get that $\frac{\partial}{\partial x_t}\left(\pi(Q^*(\hat{x}_0,x_t),x_t) - TV^*(\hat{x}_0,x_t)\right) = Q^*(x_0,x_t)$. Thus, since $x_t = \phi(x_0,Z_t)$, the derivative of Eq. (B.10) with respect to the initial shock x_0 reduces to Eq. (9.1), whereas the integral of Eq. (9.1) with respect to x_0 is simply Eq. (B.1):

$$U(x_0) = \int_{x^l}^{x_0} E_0\left[\int_0^\infty e^{-rt}Q^*(y,x_t)\frac{\partial\phi(y,Z_t)}{\partial y}dt\right]dy$$

Finally, equalizing (B.1) and (B.9) we get:

$$\begin{aligned} U(x_0) &= \int_{x^l}^{x_0} E_0\left[\int_0^\infty e^{-rt}Q^*(y,x_t)\frac{\partial\phi(y,Z_t)}{\partial y}dt\right]dy \\ &= E_0\left[\int_0^\infty e^{-rt}[\int_0^{x_t} Q^*(x_0,z)dz - F(x_0)]dt\right] \\ &= U(x_0,x_t) \end{aligned}$$

and solving for $F^*(x_0)$ we get:

$$F^*(x_0) = r\left\{E_0\left[\int_0^\infty e^{-rt}\left[\int_0^{x_t} Q^*(x_0,z)dz\right]dt\right] - \int_{x^l}^{x_0} E_0\left[\int_0^\infty e^{-rt}Q^*(y,x_t)\frac{\partial\phi(y,W_t)}{\partial y}dt\right]dy\right\} \tag{B.11}$$

Proof of formula (19)

Recalling that $Q^*(y,x_t) = Q^*(y)\frac{x_t}{y}$ and $\frac{\partial\phi(y,W_t)}{\partial y} = \frac{x_t}{y}$, we get:

$$E_0\left[Q^*(y,x_t)\frac{x_t}{y}\right] = Q^*(y)\frac{1}{y^2}E_0\left[x_t^2\right] = Q^*(y)e^{\sigma^2 t}$$

where $E_0(x_t^2) = y^2 e^{\sigma^2 t}$.

Thus, the second term on the R.H.S. of Eq, (B.11) reduces to:

$$\begin{aligned}
&\int_{x^l}^{x_0} E_0\left[\int_0^\infty e^{-rt}Q^*(y,x_t)\frac{\partial\phi(y,Z_t)}{\partial y}dt\right]dy \\
&= \int_{x^l}^{x_0}\left[\int_0^\infty e^{-rt}E_0[Q^*(y,x_t)\frac{x_t}{y}]dt\right]dy \qquad \text{(B.12)} \\
&= \frac{1}{r-\sigma^2}\int_{x^l}^{x_0} Q^*(y)dy
\end{aligned}$$

Let's now consider the first term of Eq. (B.11). Applying the same approach as before, we can write:

$$\begin{aligned}
E_0\int_0^{x_t} Q^*(x_0,z)dz &= \frac{Q^*(x_0)}{x_0}E_0\int_0^{x_t} zdz \qquad \text{(B.13)} \\
&= Q^*(x_0)\frac{x_0 e^{\sigma^2 t}}{2}
\end{aligned}$$

the first term on the R.H.S. of Eq. (B.11) reduces to:

$$\begin{aligned}
&E_0\left[\int_0^\infty e^{-rt}[\int_0^{x_t} Q^*(x_0,z)dz]dt\right] \qquad \text{(B.14)} \\
&= \frac{x_0}{2(r-\sigma^2)}Q^*(x_0)
\end{aligned}$$

Putting together (B.12) and (B.14) we get:

$$F^*(x_0) = \frac{r}{(r-\sigma^2)}\left[\frac{Q^*(x_0)x_0}{2} - \int_{x^l}^{x_0} Q^*(y)dy\right] \qquad \text{(B.15)}$$

Since $\frac{dQ^*(x_0)}{dx_0} > 0$ and $\frac{1-G(x_0)}{g(x_0)}$ is decreasing in x_0, the fixed part $F^*(x_0)$ is positive and increasing in x_0.

Appendix C

By some algebra we can write $TF^*(x_0)$ as:

$$TF^*(x_0) = \frac{1}{(r-\sigma^2)}\left[\frac{Q^*(x_0)x^l}{2} + \frac{1}{2}\left(\int_{x^l}^{x_0}(Q^*(x_0) - 2Q^*(y))dy\right)\right] \qquad \text{(C.1)}$$

where $Q^*(x_0) = \frac{x_0}{2} + \frac{1}{1+2\lambda}\left[x_0 - \lambda\frac{1-G(x_0)}{g(x_0)}\right]$.

Thus, substituting $Q^*(x_0)$ in (C.1), the sign of $\frac{\partial F^*}{\partial\lambda}$ is driven by the sign of the following term:

$$sign\frac{\partial F^*}{\partial\lambda} = sign\left\{\left[x_0^2 - (x^l)^2 - \frac{1}{2}\frac{1-G(x_0)}{g(x_0)}x_0\right] + \int_{x^l}^{x_0}\frac{1-G(y)}{g(y)}dy\right\}$$

The second term on the LHS is always positive, whereas the first term is negative when x_0 is low.

References

[1] Arve, M., and G. Zwart. 2023. "Optimal Procurement and Investment in New Technologies Under Uncertainty," 147 *Journal of Economic Dynamics and Control*, 1-26.

[2] Asian Development Bank. 2008. *Public-Private Partnership (PPP) Handbook.* Manila https://www.adb.org/sites/default/files/institutional-document/31484/public-private-partnership.pdf Last access: July 16, 2023

[3] Auriol, E., and P.M. Picard. 2009. "Government Outsourcing: Public Contracting with Private Monopoly," 119(540) *Economic Journal*, 1464-1493.

[4] Auriol, E., and P.M. Picard. 2013. "A Theory of BOT Concession Contracts," 89 *Journal of Economic Behavior and Organization*, 187-209.

[5] Baron, D., and D. Besanko. 1984. "Regulation and Information in a Continuing Relationship," 1(3) *Information Economics and Policy*, 447-470.

[6] Baron, D., and R. Myerson. 1982. "Regulating a Monopolist with Unknown Costs," 50(4) *Econometrica*, 911-930.

[7] Bergemann, D. and P. Strack. 2015. "Dynamic Revenue Maximization: A Continuous Time Approach," 159(PB) *Journal of Economic Theory*, 819-853.

[8] Bergemann, D. and P. Strack. 2022. "Progressive participation," 17 *Theoretical Economics*, 1007-1039.

[9] Bergemann, D., and G. Valimaki. 2018. "Dynamic Mechanism Design: An Introduction," Cowles Foundation Discussion Paper No. 2102R.

[10] Besanko, D. 1985. "Multi-Period Contracts Between Principal and Agent with Adverse Selection," 17(1-2) *Economics Letters*, 33-37.

[11] Broer, P. and G. Zwart. 2013,."Optimal Regulation of Lumpy Investments," 44 *J. Regul. Econ.* 177-196.

[12] Buso, M., C., Dosi, and M. Moretto. 2021. "Do Exit Options Increase the Value for Money of Public-Private Partnerships?," 30 *Journal of Economics & Management Strategy*, 721-742.

[13] Buso, M., and L. Greco. 2023. "The Optimality of Public-Private Partnerships under Financial and Fiscal Constraints," *Journal of Economics & Management Strategy*, 1-26.

[14] Demarzo, Peter M., and Y. Sannikov. 2006. "Optimal Security Design and Dynamic Capital Structure in a Continuous-Time Agency Model." 61 *The Journal of Finance*, 2681-2724.

[15] Di Corato, L, Dosi, C. and M. Moretto. 2018. "Multidimensional auctions for long-term procurement contracts with early-exit options: The case of conservation contracts," 267(1) *European Journal of Operational Research*, 368-380.

[16] Dosi, C. and M. Moretto. 2015. "Procurement with Unenforceable Contract Time and the Law of Liquidated Damages," 31(11) *Journal of Law, Economics, & Organization*, 160-186.

[17] Guesnerie, R. and J.J. Laffont. 1984. "A Complete Solution to a Class of Principal-Agent Problems with an Application to the Control of a Self-Managed Firm," 25 *Journal of Public Economics*, 329-369.

[18] Hoppe, E.I. and P.W. Schmitz, 2013, "Public-Private Partnerships versus Traditional Procurement: Innovation Incentives and Information Gathering," 44 *The RAND Journal of Economics*, 56-74.

[19] Iossa, E. and D. Martimort. 2015. "The Simple Micro-Economics of Public Private Partnerships," 17 *Journal of Public Economic Theory*, 4-48.

[20] Jullien, B. 2000. "Participation Constraints in Adverse Selection Models,"93 *Journal of Economic Theory*, 1-47.

[21] Laffont, J.J, and D. Martimort. 2002. *The Theory of Incentives The Principal-Agent Model.* N.J: Princeton University Press, Princeton.

[22] Laffont, J.J., and J. Tirole. 1986. "Using Cost Observation to Regulate Firms," 94(3) *Journal of Political Economy*, 614-641.

[23] Laffont, J.J, and J. Tirole. 1993. *A Theory of Incentives in Procurement and Regulation.*, vol. 1, 1 ed., The MIT Press Books, Cambridge.

[24] Martimort, D. and S. Straub. 2016. “How to Design Infrastructure Contracts in a Warming World: a Critical Appraisal of Public Private Partnerships,” 57 *International Economic Review,* 61-88.

[25] Milgrom, P. and I. Segal. 2002. “Envelope Theorems for Arbitrary Choice Sets,” 70 *Econometrica*, 583-601.

[26] Pavan, A., I. Segal, and J. Toikka. 2014. “Dynamic Mechanism Design: A Myersonian Approach,” 82 *Econometrica*, 601-653.

[27] Riordan, M. and D. Sappington, David. 1987. “Awarding Monopoly Franchises,” 77(3) *American Economic Review*, 375-87.

[28] Sannikov, Y. 2008. “A Continuous-Time Version of the Principal-Agent Problem,” 75(3) *Rev. Econ. Stud.*, 957-984.

[29] Skrzypacz, A., and J. Toikka. 2015. “Mechanisms for Repeated Trade,” 7 (4) *American Economic Journal: Microeconomics*, 252-93.

[30] Soumare, I. and V.S. Lai. 2016. “An Analysis of Government Loan Guarantees and Direct Investment through Public-Private Partnerships,” 59 *Economic Modelling*, 508-519.

[31] Tatoutchoup, F.D. 2015. “Optimal Forestry Contracts under Asymmetry of Information,” 117(1) *Scand. J. of Economics*, 84-107.

[32] Wilcox, D. F. 1915. “Taxation of Public Utilities,” 59(1) *The ANNALS of the American Academy of Political and Social Science*, 140-148.

[33] World Bank. 2017. *PPP Reference Guide - PPP Basics.* Public-Private Partnership Legal Resource Center: Washington DC. https://ppp.worldbank.org/public-private-partnership/library/ppp-reference-guide-ppp-basics Last access: July 16, 2023

[34] World Bank. 2022. *Leases and Affermage Contracts.*. Public-Private Partnership Legal Resource Center: Washington DC. https://ppp.worldbank.org/public private-partnership/agreements/leases-and-affermage-contracts Last access: July 16, 2023