Interfaces with Other Disciplines

Multidimensional auctions for long-term procurement contracts with early-exit options: The case of conservation contracts

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Conservation contracts, aimed at encouraging preservation and maintenance of natural areas, generally involve long-term obligations. Yet, contractors can find it profitable to breach the agreement when the opportunity cost of keeping their land idle for environmental purposes increases, and contracts do not provide for adequate early termination penalties. In this paper, we study how exit options can affect bidding behavior and the buyer’s and the seller’s expected payoffs in multidimensional procurement auctions. First, we show that bidders’ payoff is lower when competing for contracts with unenforceable contract terms. Second, we show that neglecting the risk of opportunistic behavior by sellers can lead to contract awards that do not maximize the buyer’s potential payoff. Third, we make suggestions about how to mitigate potential misallocations by pointing out the role of eligibility rules and competition among bidders.

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1. Introduction

Procurement contracts can be broadly divided into two categories. On the one hand, there are contracts governing a single transaction, such as those related to the construction of public infrastructure facilities without operational duties. On the other hand, there are contracts where sellers commit themselves to supply a flow of goods or services over an extended period of time.

Conservation contracts (or “Payments for ecosystem services”, PES) fit the latter category, as they typically require long-term commitments to preserve natural habitats or to set-aside croplands, in order to provide environmental services (ES), such as maintenance of biodiversity, carbon sequestration, soil erosion control or visual amenities. Meanwhile, landowners will be entitled to receive a flow of payments for the foregone market earnings and the direct additional costs related to conservation management activities.

Traditionally, governments have offered flat-rate payments for compliance with a predetermined combination of management goods, which reduces the chances for private sector involvement in funding PES (Tacconi, 2012; Vatn, 2010). In this paper, we will restrict ourselves to government-financed PES.

PES schemes generally involve “a contract between the conservation agent and the landowner [where] the term “landowner” denotes any entity that is in the position (de jure or de facto) to supply environmental services through its influence on the ecosystem” (Ferraro, 2008, p. 810). For example, in the US, the Conservation Reserve Program (CRP) provides payments for “agricultural producers”, where the term “producers” has to be intended as including “an owner, operator, or tenant of the land for at least 12 months prior to the close of the CRP sign-up period, and show control of the land for the duration of the contract” (Stubbins, 2012, pp. 2–3).

Some PES schemes simply require “no action”, i.e. preserving natural/semi-natural resources (e.g. forestlands, wetlands) in their current state (or removing cropland from production). Other programs, however, also require on-site activities aimed at restoring or enhancing environmental/ecosystem services (e.g., native plant restoration, placement of buffer strips, etc.). Examples of “active” conservation initiatives include the CRP in the US, and several agri-environmental schemes in the EU. In this paper, we will consider the latter situation.
prescriptions. However, following a general trend in the public procurement sector, interest in bidding mechanisms has gradually grown, in order to increase the cost-effectiveness, transparency and political acceptance of environmental payments (Latacz-Lohmann & Schilizzi, 2005). Competitive bidding, when used, often comes in the form of multi-dimensional auctions where bidders are asked to submit proposals on both price and conservation activities, and offers are ranked according to pre-specified scoring rules. Examples of competitive tendering can be found in the US, where bidding mechanisms have been pioneered under the Conservation Reserve Program (CRP), in Australia (Bush Tender and Auction for Landscape Recovery), Germany (Grassland Conservation Pilot Tender) and Scotland (Challenge Funds) (Zandersen, Braten, & Lindhjem, 2009).

Starting with the seminal paper by Che (1993), there is by now a large body of theoretical literature on scoring procurement auctions and several studies have specifically focused on the design and evaluation of conservation tenders. An issue, however, which has not received much attention in either the general literature on multi-dimensional auctions or conservation literature is the impact of premature termination of supply on bidding behavior and on auction performance. This is largely explained by the fact that the standard auction theory has mainly focused on single transactions, in so doing directing attention to contract breaches stemming from failure to comply with quality and/or quantity specifications. Conservation auction models, in turn, though recognizing the long-term nature of PES schemes, are generally built on the implicit assumption that time commitments, set forth in the agreement, will be honored by contract winners (see, e.g., Claassen, Cattaneo, & Johansson, 2008; Espinosa-Arredondo, 2008; Kirwan, Lubowski, & Roberts, 2005; Vukina, Zheng, Marra, & Levy, 2008; Wu & Lin, 2010).

Yet, premature termination of procurement contracts and public-private partnerships is not uncommon. For instance, breaches of contract can be tied, on the one hand, to unanticipated reductions in private profits, and on the other to the lack of adequate incentives against opportunistic behavior by suppliers, notably the absence of reputational mechanisms (Kelman, 1990; Spagnolo, 2012), the weakness of penalties for contract infringements, or the weak enforcement of contractual claims (Dosi & Moretto, 2015).

In the case of conservation agreements, early termination can be traced to changes in the private opportunity cost of keeping land idle for environmental purposes (e.g., sharp rises in crop prices, increases in land price to urban expansion). In turn, governments can face political pressure, or other outside influences, to soft early termination fees. For example, in the USA, agricultural associations have frequently lobbied for reducing payments for early release of CRP acres and in 2011 some Members of the Congress asked President Obama to release CRP land without penalty for the purpose of grain production (Stubbs, 2012). Moreover, legal remedies for breach of contracts can be threatened by institutional failures leading to costly litigation or inefficient settlement processes (Guash, Laffont, & Straub, 2006). For instance, institutional failures weakening the effectiveness of contractual claims have been pointed out by several studies analyzing PES programs aimed at reducing degradation of tropical forests in developing countries (see, e.g., Cordero Salas, 2013; Cordero Salas & Roe, 2012; Palmer, 2011).

This study contributes to the literature on procurement and conservation auctions by investigating the effects of “exit options” on bidding behavior in multi-dimensional tenders for long-term supply contracts. Given the focus of the paper, we will restrict the analysis to the effects of the lack of sufficiently strong and credible exit “penalties”, by leaving aside topics addressed by other authors in the conservation literature such as (i) the imperfect monitoring of conservation activities (or final environmental outputs) and (ii) bidding behavior in budget-constrained conservation tenders (Latacz-Lohmann & van der Hamsvoort, 1997).

Up to our knowledge, ours is the first paper examining in a dynamic framework the impact of the moral hazard associated with the exercise of the option to breach a supply contract. The main findings can be summarized as follows. First, we show that bidders’ payoff is lower when competing for contracts which do not provide for enforceable time commitments. Secondly, that neglecting the risk of ex post opportunistic behavior by sellers can lead to contract awards that do not maximize the buyer’s potential payoff. Finally, we make suggestions about how to mitigate potential misallocations, by pointing out the role of eligibility rules and competition.

The remainder is organized as follows. The next section provides a brief overview of the related literatures. In Section 3, we set up the model. Section 4 illustrates the benchmark case in which the contractual duration is enforceable. In Section 5, we derive the equilibrium of the auction game when bidders do not face sufficiently strong incentives against early-exit and in Section 6, we discuss the impacts of ignoring the risk of a premature termination of contracts and possible remedies. We conclude in Section 7. The Appendix contains the proofs omitted from the text.

2. Related literature

This article is related to several lines of literature which have been developed in a largely independent fashion.

The first is the literature on multidimensional auctions in which bidders compete on both price and quality dimensions (see, for instance, Asker & Cantillon, 2008; 2010; Bushnell & Oren, 1994; Che, 1993; Lorentziadis, 2010; Wang, 2013), where the term “quality” conveys different meanings depending on the nature of the exchange being made. The guarantee of supply over the stipulated contract period clearly represents an important element of the quality mix in a long-term procurement setting. However, as already noted, the possibility that the seller could prematurely walk away has not been deeply addressed in the auction literature, which has mostly focused on single (“one-shot”) transactions rather than on long-term supply contracts. In his seminal paper, the term “penalty” is used here in a broad sense, to encompass both informal and formal remedies against contract infringements and, as far as the latter are concerned, both contractual provisions aimed at enforcing compliance with contractual obligations (penalties stricto sensu) or at protecting the promisee from the expected costs of breach (“liquidated damages”) in the legal jargon. On the distinction between penalties and liquidated damages in Contract Law see, e.g., DiMatteo (2001). On the analysis of imperfect monitoring and non-compliance of agri-environmental contracts in a static setting see, among others, Giannakas and Kaplan (2005), Hart and Latacz-Lohmann (2005) and Kawasaki, Fujie, Koito, Inoue, and Sasaki (2012).
Che (1993) showed that, under specific conditions on the cross partial derivatives of the cost function, an auction in which price enters linearly into the scoring rule implements the optimal scheme by distorting quality downward with respect to the efficient level. A contribution of our paper is that it proves that Che’s result still holds even when bidders, facing on-going changes in outside conditions, incorporate into their bidding strategies potential terminations of supply, as they do not face binding enforcement constraints.

The second literature is the one on real options analysis. Proprietary options, that is exclusive rights (but not obligations) to invest capital in productive assets, can result from copyrights, patents, or from a company’s managerial resources which competitors cannot replicate (Dixit & Pindyck, 1994; Kester, 1984; Trigeorgis, 1996). Real options can also be incorporated in public contracts, such as concessions for public services, oil and gas leases or radio spectrum licenses (see, e.g., Scandizzo & Ventura, 2010). This occurs when allottees have contractual discretion as to when, if at all, to develop the lease or to supply the market by using the assigned spectrum (Dosi & Moretto, 2013). Moreover, even when contracts do not explicitly provide such discretion, real options may emerge as a result of the contracting authority’s inability to enforce compliance with contractual obligations. For instance, in a procurement context, this can occur when contractors do not face sufficiently strong penalties against delays in project implementation or against premature termination of supply. Several works, using real option theory, have analyzed the value of managerial flexibility from the promisee’s perspective (see, e.g., D’Alpaos, Moretto, Valleri, & Vergalli, 2013; Ford, Landier, & Voyer, 2002; Garvin & Cheah, 2004; Ho & Liu, 2002; Lo, Lin, & Yan, 2007; Löfler, Pfeffer, & Schneider, 2012; You & Tam, 2006). These works, however, have somehow overlooked the feedback effects of flexibility on bidding behavior, as well as the impacts in terms of contract allocation which can be substantial (Kogan & Morgan, 2010). One of the few exceptions is the paper by Dosi and Moretto (2015) that, viewing the imperfect enforcement of contract terms as a source of real options, studies the effects of the inability to enforce compliance with delivery schedules on competitive bids for public works. The authors, however, limit their analysis to homogeneous projects, where, unlike what we do here, bidding is restricted to the price dimension.

A third line of literature is the one merging real options and game theory to analyze investment decisions involving strategic considerations. In particular, our paper relates to the literature focusing on opportunistic behavior and moral hazard potentially arising in non-cooperative exit option games (see, e.g., Alvarez, 1999; Bayer, 2007; Mutro, 2004; Scandizzo & Ventura, 2016).

A fourth strand of literature, broadly related to the present work, is that on auctions with contingent payments.8 Within this literature, DeMarzo, Kremer, and Skrzypacz (2005) compared the parties’ expected payoffs when bids are independent on future events with those achievable when bids are securities whose value derives from the value of an underlying asset. They showed that the former yields higher bidders’ payoffs than security-bid auctions when the underlying asset is a call-like option. Even though our framework is different, that is, consistently with current practices, we assume here that the conservation contract does not provide for subsequent price adjustments, our results are similar to those of DeMarzo et al. (2005) since we find that when the contract expiration date is not enforceable, the impact on the bidders’ payoff is similar to the effect of auctioning a contract with contingent payments, as it might occur if, for example, subsidies for setting aside cropland were indexed to changes in agricultural prices.9

Finally, it is worth mentioning that the continuous-time setting that we use here is rather common in the literature studying the principal-agent relationship under moral hazard and/or adverse selection, when some asset (a natural resource, an investment project, etc.) owned by a risk-neutral principal is contracted out to a risk-neutral agent. De Marzo and Sannikov (2006), for instance, consider a continuous-time financial contracting model where the agent may divert cash for personal gain, while Sung (2005) and Sannikov (2007) present a continuous-time model of dynamic agency with both moral hazard and adverse selection.

3. Model set up

Consider $n \geq 2$ risk-neutral agents, each one holding one unit of a natural asset $L$ that is currently kept "idle", meaning that it does not provide the owners with any (relevant) private benefit.

For the sake of simplicity and without loss of generality we assume that all parcels are suitable for producing one unit of a homogeneous marketable product ($good$). However, this requires developing the asset (e.g., draining a wetland for agricultural use), by affording a sunk investment cost $K(\theta_i)$, where $\theta_i \in [0^{\ell_i}, \theta_i^0]$ denotes the agent’s innate managerial skills, with $K_0 < 0$.10

Agents have private information about their own type $\theta_i$. Agent $i$ only knows that $\theta_i$, $j \neq i$ is drawn from a common prior cumulative distribution $F(\theta_i)$, with continuously differentiable density $f(\theta)$ defined on a positive support $[0^{\ell_i}, \theta_i^0] \subseteq R_+$. Normalizing at zero operational costs, the investment’s returns depend only on the exogenous market price of good 1, $p(t)$. We assume that $p(t)$ is publicly observable and evolving according to the following, commonly known, process:12

$$dp(t) = \sigma p(t) dx(t) + \mu dt, \quad \text{with } p(0) = p_0$$

where $\sigma$ is the instantaneous volatility and $dx(t)$ is the increment of a standard Wiener process.13 Furthermore, we assume that agents’ information about $\theta_i$ and $p(t)$ is independently distributed among agents.14

Consider now a government agency (henceforth, “the buyer”) that wishes to procure by competitive bidding an environmental service ($good$ 2) over a period of time. Provision of good 2 (e.g., soil erosion control) requires keeping $L$ in its current natural state

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8 See, e.g., Hendricks, Pinkse, and Porter (2003), Board (2007), Haile, Hendricks, and Porter (2010), and for an excellent survey (Skrzypacz, 2013).

9 In the literature on conservation contracts this possibility has been explored by Engel, Palme, Teschke, and Urech (2005) who, however, assume that the degree of indexation is chosen a priori by the contracting agency. In our frame, state-contingent payments could be introduced by adding to a constant payment $a > 0$ an indexed payment component $\lambda \cdot I$ with $0 < \lambda < 1$ where $I$ is an observable index $1$ and $\lambda$ a scale coefficient. One may then require bids on either $a$ and/or $\lambda$. Note that, although an indexed payment may partially counter an early exit, the quality of our results remains unchanged.

10 $K(\theta_i)$ can also be thought as the present value of a flow of periodic fixed costs $k(\theta_i) = CR(\theta_i) \bar{K}$ to which the agent commits once investment is undertaken.

11 Here and in the rest of the paper a subscript is used to denote a partial derivative.

12 In Eq. (1) we abstract from the drift in order to focus on the impact that uncertainty has on outcome of the bidding process. Note, however, that by the Markov property of Eq. (1), our results would not be qualitatively altered by using a non-zero trend for $p(t)$. It can be also easily shown that Eq. (1) is consistent with the case of a firm maximizing instantaneous operating profits under a Cobb–Douglas production technology (see Dixit and Pindyck, 1994, pp. 185-199).

13 In a Wiener process the relationship between the increment $dx(t)$ and $dt$ is given by $dx(t) = \epsilon(t) \sqrt{dt}$ where $\epsilon(t)$ is a normally distributed random variable with mean and standard deviation equal to 0 and 1, respectively. It follows that $E_x[dx(t)dt] = 0$ and $E_x[dx(t)^2] = dt$ where $E_x$ is the expectation taken at time zero over the process $x(0, t, 0)$ (see Dixit and Pindyck, 1994, pp. 63-65).

14 The assumption that bidders have symmetric rational expectations about $p(t)$ and asymmetric private information on $\theta$, allow us setting the auction format into the independent private value framework. Note that this assumption rules out full surplus extraction à la (Crémér & Mclean, 1988).
as well as undertaking on site activities aimed at enhancing the overall quality of environmental services.

The per-unit-of-time (say, annual) direct cost of service provision is assumed to depend on the agents’ efficiency, i.e. type \( \theta_i \), and is defined as \( c(g, \theta_i) \) where \( g \) denotes the annual environmental outcome resulting from (i) land conservation and (ii) on-site environmental quality-enhancing activities.\(^{15}\) This cost function satisfies:\(^{16}\)

**Assumption 1.** \( c(0, \theta_i) = 0 \), \( c_0 > 0 \), \( c_g > 0 \), \( c_0 \leq 0 \) and \( c_g < 0 \). Costs are, irrespective of the agent’s type \( \theta_i \), null when no environmental services are provided. Costs are increasing and (strictly) convex in the service level and costs and marginal cost of services are not increasing and decreasing in types, respectively.

Since conservation contracts are usually long-lasting,\(^{17}\) we assume that the expiration date set forth by the buyer is sufficiently far in the future that it can be approximated by infinity.\(^{18}\)

The contract is awarded by a first-score-sealed-bid auction. Specifically, agents are solicited at time \( t = 0 \) to bid on the environmental service \( (g > 0) \) and on the annual payment \( (b > 0) \). Offers are then evaluated according to a scoring rule announced prior to bid opening, and the winner is the bidder with the highest score.\(^{19}\)

In line with the literature on conservation auctions and current practices (see, e.g., Kirwan et al., 2005; Yukina et al., 2008; Wu & Lin, 2010), we assume the following scoring rule:

\[
\tilde{S}(b, g) = \int_0^\infty s(b, g)e^{-rt}dt = \frac{v(g) - b}{r}. \tag{2}
\]

\(^{15}\) The environmental outcome \( g \) can be seen as an index number resulting from the sum of points associated with the specific practices that the bidder commits to undertake should the contract be awarded. This is in line with the common procedure followed, for instance, when awarding contracts within the CRP in U.S. In this program, in fact, bids are ranked on the basis of a score that comprises also the consideration of the Environmental Benefit Index resulting from the sum of points given when assessing the specific set of conservation practices offered by the landowner (see e.g. Kirwan et al., 2005).

\(^{16}\) The cost function \( c(g, \theta_i) \) can be defined as:

\[
c(g, \theta_i) = \min_{x \geq 0} w \cdot x \quad \text{s.t.} \quad \psi(x, \theta_i) \geq g
\]

where \( w \) is the vector of input prices and \( \psi(x, \theta_i) \) is a strictly concave production function indicating the efficient level of environmental output for any given input vector \( x \), with \( \psi \geq 0 \).

\(^{17}\) For example, under the CRP, the contract duration is between 10 and 15 years. Longer durations characterize other conservation initiatives such as the Forest Carbon Sequestration program (Vietnam, 20 years), the Social Forestry Programme (Indonesia, 25 years), the International Forest Conservation program (Guyana, 30 years) and the Forest Conservation Fund program (Australia, 12 years, 24 years, 48 years, perpetuity). On contract durations, see Lennox and Armsworth (2011) and OECD (2010).

\(^{18}\) This assumption allows us avoiding to resort to numerical methods for solving the dynamic programming problem faced by the bidder. Then, at no cost in terms of generality of our results, we are able to provide a closed-form solution which immediately allows for the use of comparative statics.

\(^{19}\) Note that the simplifying assumption that each bidder holds only one eligible asset can be relaxed. The model can, in fact, be easily extended to a multi-item procurement auction where each bidder holds multiple assets, as long as each agent is allowed to bid only for one contract. In addition, most of our general results hold also in the case of auctions with a discriminatory format, as long as the number of bidders is sufficiently high (see Krishna, 2010, Chapter 12, for a discussion on this point).

\(^{20}\) The assumption that on-site (environmental quality – enhancing) activities require more sophisticated skills than, say, conventional farming allows the introduction of a potentially relevant and theoretically interesting trade-off. As shown later in the paper, while the most efficient (least-cost) bidders have more chances to win the auction, they are also more prone to walk away, because of their higher opportunity cost of land conservation.

\(^{21}\) This principle requires that any payment leads to additional benefits relative to the status quo. For example, “payments for habitat conservation are only additional if in their absence the habitat would be lost” (OECD, 2010, p. 51). On the additivity principle, see also Ferraro (2008).
contract. Implicitly, we assume that the buyer does not face any constraints to stipulate and to costlessly enforce arbitrarily large exit penalties.

4.1. Preferences

Prior to bidding each agent contemplates the opportunity of developing \( L \) for producing good \( i \). Denoting by \( \hat{p}_i \) the market price triggering investment in such venture for the \( i \)th agent, the value of the development project is given by:

\[
\hat{\Pi}(\theta_i) = E_0 \left[ e^{-rT} \int_{t_i}^{\infty} p(z)e^{-r(z-T_i)}dz - K(\theta_i) \right]
\]

\[
= E_0 [e^{-rT_i} \left( \frac{\hat{p}_i}{r} - K(\theta_i) \right)],
\]

where the time of investment is a random variable defined as \( T_i = \inf\{t \geq 0 \mid p(t) = \hat{p}_i\} \), and \( E_0 \) is the expectation taken at time zero over the process \( \{p(t), t \geq 0\} \). Notice that Eq. (3) can be interpreted both as the winning bidder’s opportunity cost of keeping \( L \) in its natural state and the value of the asset for all losing bidders.

The optimal trigger \( \hat{p}_i \) is the solution of the following problem:

\[
\hat{\nu}(\theta_i) = \max_{\hat{p}_i} E_0 [e^{-rT_i} \left( \frac{\hat{p}_i}{r} - K(\theta_i) \right)] = \max_{\hat{p}_i} \left( \frac{p_0}{\hat{p}_i} \right) \left( \frac{\hat{p}_i}{r} - K(\theta_i) \right),
\]

where \( E_0 [e^{-rT_i}] = \left( \frac{p_0}{\hat{p}_i} \right)^{\hat{p}_i} \) is the expected discount factor, and \( \hat{p}_0 > 1 \) is the positive root of the characteristic function \( \phi(\hat{p}_0) = (\sigma^2/2)(\hat{p}_0 - 1) - \hat{r} = 0 \).

Solving problem (4), we get:

\[
\hat{p}_i = \hat{\nu}(\theta_i) = [\beta/(\beta - 1)]rK(\theta_i),
\]

(4.1)

\[
\hat{\nu}(\theta_i) = [\beta/(\beta - 1)]rK(\theta_i)^{1 - \beta},
\]

(4.2)

Thus, as standard in the real-option literature, we find that the optimal trigger is given by the user cost of capital, \( rK(\theta_i) \), adjusted by the option multiple \( \beta/(\beta - 1) \) which reflects the uncertainty about the profits generated by developing \( L \) for productive use. Notice that \( d\hat{p}(\theta_i)/d\theta_i > 0 \) and \( d\hat{\nu}(\theta_i)/d\theta_i > 0 \). That is, the more efficient is the agent, the lower is the market price making it convenient to implement the development project and the higher is the opportunity cost of keeping \( L \) idle.

According to the additivity principle, public payments are allowed only for supporting private decisions that would have not been made anyway. In our frame, additivity passes through the actual threat of having \( L \) developed for productive use. Thus, in order to consider the case where all \( n \) bidders are eligible for conservation funding at the time of bidding, market revenues should be such that none of the agents participating in the auction would continue to keep \( L \) idle without conservation subsidies.

Now consider the winning bidder. The ex post value of the contract is given by:

\[
\Pi(b_i, g_i; \theta_i) = \frac{b_i - c(g_i, \theta_i)}{r}
\]

(6)

By subtracting Eq. (5) from Eq. (6), we get:

\[
\Pi(b_i, g_i; \theta_i) - \hat{\Pi}(\theta_i) = \frac{b_i - c(g_i, \theta_i)}{r} - (K(\theta_i) - K(\theta_i)), \text{ for } \theta_i \in [\theta^l, \theta^u].
\]

(6.1)

In words, Eq. (6.1) simply says that the seller’s pay-off (accounting for the reservation value) is given by the difference between the present value of rental payments (net of the direct cost of supplying \( g_i \)) and the opportunity cost of keeping \( L \) in its current natural state.

4.2. Equilibrium strategy

Given the information available at time \( t = 0 \), each agent will choose the optimal bidding strategy by maximizing the following function:

\[
\Pi(\bar{S}_i) = \Pi(b_i, g_i; \theta_i) \cdot \text{Pr(\text{of win} \mid \bar{S}_i)} + \hat{\Pi}(\theta_i) \cdot (1 - \text{Pr(\text{of win} \mid \bar{S}_i)}).
\]

(7)

where \( \text{Pr(\text{of win} \mid \bar{S}_i)} \) is the probability of winning the auction, conditional on the reported score \( \bar{S}_i(b_i, g_i) \) given in (2), and \( \hat{\Pi}(\theta_i) \) is the reservation value given in (3.1). In other words, with probability \( \text{Pr(\text{of win} \mid \bar{S}_i)} \), agent \( i \) will be entitled to receive a flow of net payments worth \( (b_i - c(g_i, \theta_i))/r \), while with probability \( (1 - \text{Pr(\text{of win} \mid \bar{S}_i)}) \) she will simply get \( \hat{\Pi}(\theta_i) \geq 0 \).

Agents participate in the auction only if the following individual rationality constraint holds:

\[
\Pi(\bar{S}_i) = \hat{\Pi}(\theta_i) \geq 0.
\]

(8)

Notice that the assumption that sellers will always comply with time frames established in the contract implies that the probability of winning is equivalent to \( \text{Pr(\text{of win} \mid \bar{S}_i)} \), where \( S_i \) measures the annual net benefit that the buyer receives. Thus, using Eq. (6.1), we can rearrange Eq. (7) and define agent \( i \)’s objective function as follows:

\[
\hat{W}(b_i, g_i; \theta_i) = \left[ \frac{b_i - c(g_i, \theta_i)}{r} - (K(\theta_i) - K(\theta_i)) \right] \text{Pr(\text{of win} \mid \bar{S}_i)}.
\]

(9)

The following proposition summarizes the solution of the bidding game.

**Proposition 1.** Under perfect enforcement, for any finite number of bidders there will always exist an equilibrium in symmetric and strictly increasing strategies \( S(\theta_i) = \nu(g_i(\theta_i)) - b(\theta_i) \) characterized by:

(i) the annual environmental output:

\[
g(\theta_i) = \arg \max \{\nu(g_i) - c(g_i, \theta_i)\}
\]

with \( d\nu(\theta_i)/d\theta_i = c_{\nu \theta}(\theta_i) > 0 \), for all \( \theta_i \in [\theta^l, \theta^u] \).

(ii) the bidding function:

\[
\bar{b}(\theta_i) = c(g(\theta_i), \theta_i) + r(K(\theta_i) - K(\theta_i)) - \int_{\theta_i}^{\theta^l} \Delta(\theta) F(\nu(\theta_i)) d\theta.
\]

where \( F(\nu(\theta_i)) = F(\nu)^{\nu - 1} \). 

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22 The expected present value \( E_0 [e^{-rT_i}] \) can be determined by using dynamic programming (see, e.g., Dixit and Pindyck, 1994, pp. 315–316).
(iii) the expected profits:
\[
\hat{W}(\theta_i) = -\int_0^\theta \frac{\Delta_0(x)}{r} F^{(\alpha)}(x) dx,
\]
where \( \Delta_0(x) = c_0(g(x), x) - rK_0(x) < 0 \).

**Proof.** See Appendix A.1 □

By Proposition 1, we can illustrate the agents’ bidding strategy by determining \( s_i \) and the output level \( g_i \). Moreover, since we are assuming that agents will comply with service time requirements, the auction efficiency is ensured by showing that the score \( s(\theta_i) = v(g(\theta_i)) - \bar{b}(\theta_i) \) is increasing in the bidder’s efficiency type, that is, \( ds(\theta_i)/d\theta_i > 0 \) (see Appendix A.1).

Notice that, with respect to a standard price-only independent private value auction, the bid price (11) internalizes the direct cost of performing the promised environmental service, \( c(g(\theta_i), \theta_i) \), as well as the opportunity cost of keeping \( L \) in its current natural state, \( r(K(\theta^f) - K(\theta_i)) \). Moreover, agents must be compensated by information rents, \( -\int_0^\theta \Delta_0(x) F^{(\alpha)}(x) dx \). As standard in the auction literature, no rents will be paid to the least cost-efficient agent, i.e. \( \bar{b}(\theta^f) = c(g(\theta^f), \theta^f) \).

5. Non-enforceable contract duration

Now suppose that sellers do not face sufficiently strong and credible exit penalties. As already argued, this can be due to several reasons, e.g., the weakness of contractual penalties or to the weak enforcement of contractual claims. Further, let us suppose that the buyer turns a blind eye on the risk of premature termination of supply and, as above, ranks bids on the basis of the annual payoff \( s_i \). In the following, we will show how opportunistic behavior gives rise to moral hazard and, thus, distortions in the actual contract duration.

5.1. Preferences

As above, prior to bidding, each agent contemplates the opportunity of developing \( L \), which is worth \( \hat{\Pi}(\theta_i) \) as defined by Eq. (3.1). Now consider the winner. Since sellers can walk away (at the only cost of losing from the breaching date onwards the rental payments), agent i’s optimization problem consists of choosing the optimal exit time, that is, when stopping the provision of conservation services and developing \( L \) for productive use.

The termination date is a random variable depending only on the information at time \( t \) concerning the process \( \{p(t)\} \). Thus, agent i’s optimization problem consists of choosing the exit time \( T_i \) maximizing the following present value:

\[
\Pi(b_i, g_i; \theta_i) = E_0 \left[ \int_0^{T_i} (b_i - c(g_i, \theta_i)) e^{-r z} dz + e^{-rT_i} \left( \int_0^\infty p(z) e^{-r(z-T_i)} dz - K(\theta_i) \right) \right] = (b_i - c(g_i, \theta_i)) \frac{1 - E_0[e^{-rT_i}]}{r} + (p^*_i - rK(\theta_i)) \frac{E_0[e^{-rT_i}]}{r},
\]

where \( T_i = \inf(t \geq 0 | p(t) = p^*_i) \) and \( p^*_i \) is the optimal threshold triggering the asset development. The first term in Eq. (14) is the expected net present value of conservation payments, while the second term is the option value of switching to the production of good 1.

By rearranging Eq. (14), the seller’s optimal trigger is given by the solution of the following problem:

\[
V(b, g, \theta_i) = \max_{T_i} E_0[e^{-rT_i}] \left[ (b_i - c(g_i, \theta_i)) + rK(\theta_i) \right] = \max_{T_i} \left( \frac{p_0}{p^*_i} \right)^\beta \left[ (b_i - c(g_i, \theta_i)) + rK(\theta_i) \right].
\]

where \( E_0[e^{-rT_i}] = (p_0/p^*_i)^\beta < 1 \). In (15) the term in squared brackets represents the cost of switching to the production of good 1, which, besides the direct cost, \( rK(\theta_i) \), must also account for the forgone rental payments (net of conservation maintenance costs), \( b_i - c(g_i, \theta_i) \).

Solving problem (15), we get:

\[
p^*_i = p^*(b_i, g_i, \theta_i) = \frac{\beta}{\beta - 1} \left[ (b_i - c(g_i, \theta_i)) + rK(\theta_i) \right].
\]

\[
V(b, g, \theta_i) = \left[ \Gamma(p_0)/(\beta - 1) \right] \left( \frac{b_i - c(g_i, \theta_i)}{r} + K(\theta_i) \right)^{1-\beta}.
\]

Subtracting \( \hat{\Pi}(\theta_i) \) from Eq. (14) yields the value attached to having the contract awarded:

\[
\Pi(b_i, g_i; \theta_i) - \hat{\Pi}(\theta_i) = \left[ (b_i - c(g_i, \theta_i)) - r(K(\theta^f) - K(\theta_i)) \right] \times \frac{1 - E_0[e^{-rT_i}]}{r} + (p^*_i - rK(\theta^f)) \left( \frac{E_0[e^{-rT_i}]}{r} \right),
\]

for \( \theta_i \in [\theta^f, \theta^s] \).

Comparison between Eq. (6.1) and Eq. (16) points out the value of the opportunistic behaviour stemming from the possibility of the possibility to prematurely terminate the contract. It also shows that the exit option alters the seller’s expected payoff, namely, by lowering the opportunity cost of entering into the conservation program.

5.2. Equilibrium strategy

Bidders will make their bids by maximizing the following function under the individual rationality constraint \( \Pi(s_i) \geq \hat{\Pi}(\theta_i) \):

\[
\Pi(s_i) = \Pi(b_i, g_i; \theta_i) \cdot \Pr(\text{of win}|s_i) + \hat{\Pi}(\theta_i) \cdot (1 - \Pr(\text{of win}|s_i)).
\]

Using Eq. (16) and Eq. (17), bidder i’s objective can be defined as follows:

\[
W(b, g, \theta_i) = \left\{ (b_i - c(g_i, \theta_i)) - r(K(\theta^f) - K(\theta_i)) \right\} \frac{1 - E_0[e^{-rT_i}]}{r} + (p^*_i - rK(\theta^f)) \left( \frac{E_0[e^{-rT_i}]}{r} \right) \Pr(\text{of win}|s_i).
\]

The following proposition summarizes the solution of the bidding game.

**Proposition 2.** When bidders do not face exit penalties and bids are ranked only on the reported \( g_i \) and \( b_i \), for any finite nthere will always exist an equilibrium in symmetric, strictly increasing strategies \( s(\theta_i) = v(g(\theta_i)) - b(\theta_i) \), characterized by:

(i) the environmental output \( g(\theta_i) \) (defined by Eq. (10)),
(ii) the bidding function:
Proof. See Appendix A.2 □

First note that the annual environmental output is identical to that offered when agents bid knowing that they cannot prematurely terminate the contract. This is an important result, because it allows us to extend Che (1993)’s Lemma 1 to the case where bids incorporate exit strategies (see Appendix A.2). It follows that, once secured its monotonicity, i.e., $ds(\theta_t)/dt > 0$, the use of the annual score still allows the buyer selecting the most efficient (least-cost) provider.

The second remark concerns the bid price. Comparison of Eq. (19) with Eq. (11) shows that when bidders do not face binding commitments they will bid more aggressively, by reducing the price by the amount $(p^*(\theta_t) - rK(\theta_t)) \frac{E_0[e^{-rT(\theta_t)}]}{\beta - (1 - E_0[e^{-rT(\theta_t)}])}$ which reflects the potential gains associated with early-exit.\(^{25}\)

Putting together Results (i) and (ii) in Proposition 2, it is easy to show that:

$$s(\theta_t) = v(g(\theta_t)) - b(\theta_t) > v(g(\theta_t)) - \bar{b}(\theta_t) = \bar{s}(\theta_t).$$

As information rents are null for agent $\theta_t$, we get:

$$b(\theta_t) = c(g(\theta_t), \theta_t) - rK(\theta_t) \frac{E_0[e^{-rT(\theta_t)}]}{\beta - (1 - E_0[e^{-rT(\theta_t)}])} < \bar{b}(\theta_t),$$

from which it is easy to show that:

$$\beta(\beta - 1)(b(\theta_t) - c(g(\theta_t), \theta_t) + rK(\theta_t)) = p^*(\theta_t) > p_0,$$

i.e., the managerial flexibility spurred by the buyer’s inability to enforce compliance with time schedules tends to intensify the competition among bidders.

The following proposition summarizes the effects of managerial flexibility on bidders’ payoff.

Proposition 3. Both the price received by sellers for conservation services and the overall expected payoff are lower when sellers compete for contracts which do not provide for enforceable time commitments, i.e. $b(\theta_t) < \bar{b}(\theta_t)$ and $W(\theta_t) < \bar{W}(\theta_t)$.

Proof. See Appendix A.3. □

The first result is consistent with other findings, such as those of Spulber (1990), who pointed out that, in the absence of contract enforcement, the most efficient bidders will be forced to bid low in order to preserve their chances of winning. This, in turn, raises the probability of breach of contracts. However, unlike Spulber, we find that the possibility of adjusting the service period allows preserving the efficiency of the bidding process; that is, the auction does not fail to allocate the contract to the most efficient bidder, i.e. the bidder with the lowest cost for conservation activities.

The second result states that bidders’ expected payoff is lower when sellers can prematurely terminate the contract, because the benefits obtainable from exercising exit options are outweighed by the price competition effect. This result is in line with the literature on auctions with contingent payments, in particular with DeMarzo et al. (2005) where, comparing bidders’ payoffs when bids are independent on future events (“cash auctions”) with those achievable when bids are securities whose value derive from the value of an underlying asset, it is found that security-bid auctions yield lower payoffs when the underlying asset is a call-like option. In our frame, bidders bid knowing that, upon winning, they will give up the opportunity to develop $L$ for productive use in exchange of a flow of fixed rental payments. By taking the buyer’s perspective, when the contract duration is enforceable, this is essentially equivalent to purchasing the option to develop through a cash auction. Note in fact that the winner will receive in exchange the present value of the flow of fixed rental payments. On the other hand, without enforcement, the price received for selling the option to develop also includes the option to prematurely terminate the contract, i.e., a call-like option. The inclusion of this element in the bid price makes, de facto, state-contingent the value attached to the contract. Thus, similarly to what DeMarzo et al. (2005) show in a static frame, we find that, also in a dynamic setting, bidders are better off (in terms of extended net present value) when competing for contracts with (effective) lock-in provisions.

Finally, it is worth remarking that the bid reduction, under non-enforcement, will depend on the uncertainty about the profits resulting from breaching the contract. For instance, it is easy to show that $\lim_{\sigma \to \infty} E_0[e^{-rT(\theta_t)}] = 0$ for all $\theta_t$. In other words, the higher is the uncertainty about the profits resulting from developing the asset, the lower is the difference between the bid price with and without contract enforcement, since the today’s value of exploiting the asset for productive use tends to vanish as uncertainty about market returns increases.

6. The buyer’s payoff and the risk of opportunistic behavior

6.1. The buyer’s payoff

When the contract duration is not enforceable, the buyer’s total expected payoff is given by:

$$S_i = E_0 \left[ \int_0^{T_i} s_i e^{-rt} dt \right] = s_i \frac{1 - E_0[e^{-rT}]}{r}, \quad (21)$$

where $T_i$ is the seller’s optimal time for breaching the contract.

Proposition 3 implies that allocating the contract on the basis of the highest annual score $s_i$ may turn out not be the best decision for the buyer because the winner (least-cost) agent can be more prone than others to early-exit.

This becomes clear if we take a closer look at the derivative of Eq. (15.1) with respect to bidders’ types:

$$\frac{dp^*(\theta_t)}{d\theta_t} = - \frac{\beta}{\beta - 1} \left( \Delta_{\theta}(\theta_t) + ds(\theta_t)/d\theta_t \right). \quad (22)$$

Since $\Delta_{\theta}(\theta_t) < 0$ and $ds(\theta_t)/d\theta_t > 0$, the sign of $dp^*(\theta_t)/d\theta_t$ is ambiguous, that is, higher values of $\theta$ can either translate into an increase or a reduction of the optimal trigger for breaching the contract. In the latter case, the combination of high annual benefits and a short service period can turn out not being the one giving the buyer the highest total payoff.

It is worth remarking that, in our framework, Assumption 6 (“additionality”) implies that losing bidders will put $L$ into productive use by undertaking an irreversible investment. Therefore, in case of early exit by the winner, the possibility of procuring the environmental service contractualized in the first place by running a new auction with the same agents is ruled out.

\(^{25}\) Notice that both the information rents and the gains associated with contract breach are annualized by the term $(1 - E_0[e^{-rT(\theta_t)}])/r$.\)
6.2. Accounting for the risk of opportunistic behavior

The risk of not selecting the agent providing the highest potential payoff could be avoided if the buyer exploited the information gathered through the bidding process and used the total expected payoff, $S(\theta_i)$, instead of $s(\theta_i)r$, to allocate the contract.

This would come at no cost in terms of auction efficiency, provided we strengthen Assumption 1 by adding the following assumption on information rents.

**Assumption 7.** $-\Delta_0(\theta_i)(F(\theta_i)/f(\theta_i))$ is increasing in $\theta_i$ and is bounded above by $(n-1)/r$ for each $\theta_i \in [\theta^l, \theta^u]$.

Notice that, if $\Delta (\theta_i) = [c(g, \theta_i) - rK(\theta_i)]$ is concave in $\theta_i$, Assumption 1 would suffice for having $-\Delta_0(\theta_i)(F(\theta_i)/f(\theta_i))$ increasing in $\theta_i$. This, for instance, corresponds to the well-known sufficient conditions (i.e., Spence–Mirrlees single-crossing and monotonicity conditions) for implementing the optimal contract in a static framework (see, e.g., Fudenberg & Tirole, 1991; Guesnerie & Laffont, 1984).

However, in our dynamic frame, the standard conditions do not ensure the monotonicity of $T(\theta_i)$ and, thus, the monotonicity of $S(\theta_i)$. In fact, the information rents for the most efficient agents could be so high that they can competitively bid on the annual score, $s(\theta_i)$, and win the auction, even though there might be other bidders able to ensure a longer service provision and, as a whole, higher total benefits to the buyer.

Therefore, the risk of opportunistic behavior calls for a restriction not just on $\Delta_0(\theta_i)(F(\theta_i)/f(\theta_i))$, but on both $T(\theta_i)$ and $\Delta_0\theta_i(F(\theta_i)/f(\theta_i))$. In fact, by rearranging Eq. (22) as follows (see Appendix A.4):

$$\frac{d\theta_i}{d\theta} = \frac{\beta}{\beta - 1} \Delta_0(\theta_i) \left[ (n-1) \frac{f(\theta_i)}{F(\theta_i)} \frac{W(\theta_i)}{dW(\theta_i)/d\theta_i} - 1 \right]$$

we can notice that a sufficient condition for $d\theta_i/d\theta > 0$ is that:

$$\Delta_0(\theta_i) \frac{1 - E_0(\theta_i)}{r} F^{(n)}(\theta_i) + f^{(n)}(\theta_i) > 0 \text{ for all } \theta_i \in [\theta^l, \theta^u]$$

(22.1)

This condition is satisfied by Assumption 7, which represents the minimum level of adjustment required for meeting the monotonicity condition in the presence of opportunistic sellers competing a potential early-exit.\(^{26}\)

The following proposition captures this result.

**Proposition 4.** Under Assumption 7, for any finite $n$ there will always exist an equilibrium in symmetric strictly increasing strategies $s(\theta_i)$ with a non-decreasing optimal expected service period in $\theta_i$, that is, $dS(\theta_i)/d\theta_i > 0$.

**Proof.** See Appendix A.4. □

It is worth remarking that the regularity condition imposed by Assumption 7 is a sufficient (not necessary) condition for the equilibrium existence. For instance, should the condition not be met in a subset of the space $[\theta^l, \theta^u]$, the solution in Proposition 2 could still constitute an equilibrium, since there can be a value $\theta \in [\theta^l, \theta^u]$ beyond which, despite $d\theta_i/d\theta_i > 0$, $dS(\theta_i)/d\theta_i < 0$ for every $\theta_i \in [\theta^l, \theta^u]$ (see Appendix A.4). In this case, even though not ensuring the longest service provision, the most efficient (least-cost) agent would provide the buyer with the highest total payoff, by compensating the shortening of service period with higher annual benefits.

6.3. Competition and eligibility rules

Proposition 4 implies that a strong competition for winning the contract reduces the risk of not selecting the agent providing the highest potential payoff to the buyer. Otherwise, when competition is relatively low, the buyer could still get the highest payoff by restricting eligibility for conservation payments.

To illustrate these statements, consider the following simplifying example. Suppose that the agent-types are uniformly distributed between $\theta^l = 0$ and $\theta^u = 1$, i.e. $F(\theta) = \theta$ and $f(\theta) = 1$. Moreover, let us assume that the sunk cost for developing $L$ is $K(\theta) = 1 - \theta$, the cost of conservation activities is $c(g, \theta) = (g^2/2) - g(\theta - \theta^l)$ and the annual social benefit is $v(g) = g$.

By Eq. (10), we get:

$$g(\theta) = 1 + \theta \geq 1$$

As shown in Appendix A.4, for the case of a trendless geometric Brownian motion, Assumption 7 is implied by the following condition:

$$\Delta_0(\theta)(F(\theta)/f(\theta)) > r(n-1)$$

(24)

where $\Delta_0(\theta) = r - g(\theta) > 0$ is decreasing in $\theta$.

Substituting Eq. (23) into condition (24) and rearranging, we obtain:

$$Q(\theta; n) = (1 + \theta - r\theta) - r(n-1) < 0$$

(24.1)

Notice that, for any $n$, $Q(\theta; n)$ is increasing in $\theta$ in the interval $[0, 1]$, with $Q(0) = -r(n-1)$ and $Q(1) = 2 - nr$.

Denoting by $\theta^e$ the solution of the equation $Q(\theta^e; n) = 0$, this is given by:

$$\theta^e = \sqrt{\frac{1 - r}{2}} n - 1 - \frac{1 - r}{2}$$

where:

$$0 < \theta^e < 1 \quad \text{for } n < 2/r$$

$$1 \geq \theta^e \quad \text{for } n \geq 2/r$$

$\theta^e$ represents the highest eligible type that the buyer should admit to the auction. Our example shows, on the one hand, that when the number of bidders is relatively high (i.e., $n \geq 2/r$), no restrictions on the eligibility rules are needed in order to ensure that $dS(\theta_i)/d\theta_i > 0$ over the entire range $[0, 1]$. On the other hand, when competition is relatively low ($n < 2/r$), the buyer can increase the total payoff by restricting the range of eligible agents, namely, by excluding from award bidders belonging to the subset $[\theta^e, 1]$ that, is, those having relative high opportunity costs of compliance with conservation requirements which are more likely to breach the contract.

7. Final remarks

Contracts providing payments for preserving and enhancing natural assets generally require long-term commitments. Agents, however, can find it profitable to breach the contract when the opportunity cost of conservation requirements increase and contracts do not provide for sufficiently strong and credible exit penalties.

The question addressed in the paper is how this can affect bidding behavior and the parties’ individual payoffs in procurement auctions where contractors are selected on the basis of both the

\(^{26}\) Sung (2005) presents a similar condition in a continuous-time agency model, with both adverse selection and moral hazard, where the optimal contract is linear in the effort. Note, however, that condition (22.2) is more general, since it accounts for the non-linearity of the optimal contract (i.e., the bid function) in the level of “effort” (i.e., the contract duration $T$).
promised quality or level of environmental services and the required payments. While the paper is written with a focus on procurement contracting, the contents can be applicable to other procurement bidding processes, whereby buyers face the risk of interruption of supply due to outside opportunities that are valuable for the sellers.

The novelty of our model, with respect to the previous literature on scoring auctions, mostly focused on single transactions, is that agents can adjust their bidding strategies by exploiting the managerial flexibility spurred by the lack of exit penalties. This implies that, unlike when service time commitments are enforceable, bidding strategies will be affected by endogenous timing considerations.

A first result of the paper is that procurement contracts embedding (implicit) early-exit options do not translate into higher expected payoffs for the sellers, because, in a competitive environment, the potential benefits, stemming the opportunity to terminate the contract, are outweighed by the stronger bid competition spurred by the lack of enforcement of time commitments. Thus, besides increasing the risk of failure of conservation programs, the weak enforcement of contractual obligations may not even be in the contractors’ best interest.

A second result is that failure to account for the risk of opportunistic behavior could lead to the choice of contractors who will not ensure the highest payoff to the buyer. In the case of conservation contracts, this possibility relies on the correlation between the direct cost of undertaking conservation activities and the opportunity cost of not exploiting land for alternative uses. For instance, if costs are negatively correlated, that is, if agents that are able to more efficiently exploit the asset for productive uses are also able to undertake conservation activities at lower costs, the most efficient bidders, while providing the buyer with the maximum annual payoff, can be more prone than others to early-exit. Therefore, the combination of higher annual benefits and shorter service periods can turn out not being the one giving the buyer the highest total value.

The paper makes suggestions which may contribute to avoid such potential bias in contract allocation. A policy implication of our paper is that when, for whatever reasons, contracting agencies cannot rely on sufficiently strong and credible incentives against premature termination of contracts, they should “internalize” the risk of early-exit. In practice, this means exploiting the information gathered through the auction process in order to assess, and include in bid evaluation, the bidders’ actual prospective compliance. As shown in the paper, this would not affect the auction’s allocative efficiency, so long as the number of bidders is sufficiently high to downsize the most efficient agents’ information rents.

Otherwise, when competition is relatively low, it might be profitable for the buyer to restrict the range of eligible agents, by excluding from award those with relatively high opportunity costs of compliance with conservation requirements and, therefore, more prone to early-exit.

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Appendix A

A.1 Proposition 1

In spite of being quite standard in the auction literature, we include the following proof for the reader’s convenience. Consider a common prior cumulative distribution $F(\theta)$ with continuously differentiable density $f(\theta)$ defined on a positive support $\Theta = [\theta^l, \theta^n] \subset R_+$, where the lowest value $\theta^l$ is such that $\theta^l = \inf\{\theta : f(\theta) > 0\}$ and the highest value is $\theta^n = \sup\{\theta : f(\theta) > 0\}$. Now consider the agent’s bidding behavior. It is immediate to note that the maximization of the objective in (9) is equivalent to maximize the instantaneous expected net payoff, i.e. $\left[ \frac{b_i - c(g(\theta_i))}{r} - (K(\theta^l) - K(\theta_i)) \right] \Pr(\text{win}|s(\theta_i))$, and that $\left[ \frac{b_i - c(g(\theta_i))}{r} - (K(\theta^l) - K(\theta_i)) \right]$ must be non-negative to guarantee a positive expected payoff (otherwise, winning the auction would never be profitable). Assume that all other bidders use a strictly monotone increasing bid function $s(\theta_j)$, i.e., $s(\theta_j) : [\theta^l, \theta^n] \rightarrow [s(\theta_j)^{(1)}, s(\theta_j)^{(n)}] \forall j \neq i$. Since, by assumption, $s(\theta_j)$ is monotone in $[\theta^l, \theta^n]$, the probability of winning by bidding $s(\theta_j)$ is $\Pr(s(\theta_j) > s(\theta_i)) \big| \forall j \neq i = \Pr(\theta_j < s^{-1}(s(\theta_i))) \big| \forall j \neq i = F(s^{-1}(s(\theta_i)))^{n-1} = F(\theta_i)^{n-1} = F^{(n)}(\theta_i)$.

Bayes–Nash equilibrium – At the Bayes–Nash equilibrium of the direct revelation (auction) game, truth-telling by any bidder (supplier) is a best response to truth-telling by all other bidders (suppliers). Hence, consider any bidder $i = 1, \ldots, n$ and suppose that all other bidders correctly reveal their efficiency type $\theta_j$ and bid accordingly on the basis of the strictly monotone increasing bid function $s(\theta_j) : [\theta^l, \theta^n] \rightarrow [s(\theta_j)^{(1)}, s(\theta_j)^{(n)}] \forall j \neq i$. Now let us consider the problem faced by bidder $i$. For $s(\theta_i)$ to be an equilibrium strategy, it must be shown that bidder $i$ cannot do better than bidding $s(\theta_i)$ or, in other words, that bidder $i$ with efficiency type $\theta_i$ gains nothing by pretending, bidding $s(\tilde{\theta}_i)$ for some $\tilde{\theta}_i \neq \theta_i$, that his efficiency type is $\tilde{\theta}_i$.

If all other bidders report truthfully, bidder $i$ with efficiency type $\theta_i$ chooses his report $\hat{\theta}_i$ in order to maximize the following payoff:

$$\hat{\theta}_i = \left( \frac{b(\hat{\theta}_i) - c(g(\hat{\theta}_i))}{r} - (K(\theta^l) - K(\theta_i)) \right) F^{(n)}(\hat{\theta}_i)$$

where, by Che (1993, Lemma 1, p. 672), $g_i = g(\hat{\theta}_i)$ is determined by Eq. (10). This in turn implies that:

$$\hat{\theta}_i = \left( \frac{b(\hat{\theta}_i) - c(g(\hat{\theta}_i), \theta_i)}{r} - (K(\theta^l) - K(\theta_i)) \right) F^{(n)}(\hat{\theta}_i),$$

(A.1.1)

where $F^{(n)}(\hat{\theta}_i) = F(\hat{\theta}_i)^{n-1} = F(s^{-1}(s(\theta_i)))^{n-1}$.

Using the Envelope Theorem at $\hat{\theta}_i = \theta_i$ we get:

$$\frac{\partial W(\theta_i, \theta_i)}{\partial \theta_i} \big|_{\theta_i = \theta_i} = \left( \frac{db(\theta_i)}{d\theta_i} - c_g(\theta_i, \theta_i) \frac{dg(\theta_i)}{d\theta_i} \right) F^{(n)}(\theta_i)$$

$$+ \left( (b(\theta_i) - c(g(\theta_i), \theta_i)) - r(K(\theta^l) - K(\theta_i)) \right) F^{(n)}(\theta_i) = 0$$

(A.1.2a)

or

$$\left( \frac{db(\theta_i)}{d\theta_i} - c_g(\theta_i, \theta_i) \frac{dg(\theta_i)}{d\theta_i} \right) F^{(n)}(\theta_i)$$

$$= -\left( (b(\theta_i) - c(g(\theta_i), \theta_i)) - r(K(\theta^l) - K(\theta_i)) \right) F^{(n)}(\theta_i).$$

(A.1.2b)

Integrating Eq. (A.1.2b) on both sides yields:

$$\int_{\theta_i}^{\theta_i} \left( \frac{db(x)}{d\theta_i} - c_g(g(x), x) \frac{dg(x)}{d\theta_i} \right) F^{(n)}(x) dx$$

$$= -\int_{\theta_i}^{\theta_i} \left( (b(x) - c(g(x), x)) - r(K(\theta^l) - K(x)) \right) F^{(n)}(x) dx$$
Using integration by parts, one can easily show that:
\[
\tilde{B}(\theta_i) = c(g(\theta_i), \theta_i) + r(K(\theta_i') - K(\theta_i)) - \int_{\theta_i}^{\theta_i'} \Delta_0(x) \frac{F^{(n)}(x)}{F^{(n)}(\theta_i')} dx.
\]
(A.13)

By differentiating Eq. (A.13) with respect to \( \theta_i \), we obtain:
\[
d\tilde{B}(\theta_i) = \frac{\partial}{\partial \theta_i} \tilde{B}(\theta_i) = c_s(g(\theta_i), \theta_i) \frac{dg(\theta_i)}{d\theta_i} + (n - 1) f(\theta_i) \int_{\theta_i}^{\theta_i'} \Delta_0(x) \frac{F^{(n)}(x)}{F^{(n)}(\theta_i')} dx.
\]
(A.14)

Eq. (A.14) implies that \( \tilde{B}(\theta_i) \) is increasing in the cost of producing higher quality, \( g(\theta_i) \), and decreasing in the information rent, i.e. \(- \int_{\theta_i}^{\theta_i'} \Delta_0(x) \frac{F^{(n)}(x)}{F^{(n)}(\theta_i')} dx \), to be paid.

By differentiating Eq. (2) with respect to \( \theta_i \) and using Eqs. (10) and (A.14), we can immediately prove the assumed monotonicity of the optimal strategy \( \tilde{s}(\theta_i') \):
\[
d \tilde{s}(\theta_i) = - (n - 1) \int_{\theta_i}^{\theta_i'} \Delta_0(x) \frac{F^{(n)}(x)}{F^{(n)}(\theta_i')} dx > 0.
\]
(A.15)

Let us now show that \( \tilde{s}(\theta_i) \) is a direct revelation mechanism. As one can immediately see, \( \tilde{B}(\theta_i) \) is identified on the basis of the necessary condition to be set so that \( \tilde{s}(\theta_i) \) is a Bayes–Nash equilibrium, i.e. Eq. (A.12a). Now, it remains to be verified that using payment \( \tilde{B}(\theta_i) \), the equilibrium strategy \( \tilde{s}(\theta_i) \) indeed arises as such. In order to show it, suppose that \( \tilde{B}(\theta_i) \) is the equilibrium payment function and assume that all bidders correctly reveal their efficiency type \( \theta_i \) and bid accordingly on the basis of the strictly monotone increasing bid function \( \tilde{s}(\theta_i') : [\theta_i, \theta_i'] \rightarrow [\tilde{s}(\theta_i), \tilde{s}(\theta_i')] \forall i \neq i \). Plugging Eq. (A.13) into Eq. (A.11) yields:
\[
\tilde{W}(\theta_i, \tilde{\theta}_i) = \left[ \tilde{B}(\theta_i) - c(g(\tilde{\theta}_i), \theta_i) - (K(\theta_i) - K(\theta_i')) \right] F^{(n)}(\tilde{\theta}_i).
\]

Taking the derivative with respect to \( \tilde{\theta}_i \), we obtain:
\[
\frac{\partial \tilde{W}(\theta_i, \tilde{\theta}_i)}{\partial \tilde{\theta}_i} = \left( \frac{d\tilde{B}(\tilde{\theta}_i)}{d\tilde{\theta}_i} - c_s(g(\tilde{\theta}_i), \theta_i) \frac{dg(\tilde{\theta}_i)}{d\tilde{\theta}_i} \right) \frac{F^{(n)}(\tilde{\theta}_i)}{r} + \left[ (\tilde{B}(\tilde{\theta}_i) - c(g(\tilde{\theta}_i), \theta_i)) - r(K(\theta_i) - K(\theta_i')) \right] \frac{F^{(n)}(\tilde{\theta}_i)}{r}.
\]
(A.16)

Note that by evaluating Eq. (A.12b) at \( \tilde{\theta}_i \) and rearranging we have:
\[
\frac{d\tilde{B}(\tilde{\theta}_i)}{d\tilde{\theta}_i} = c_s(g(\tilde{\theta}_i), \tilde{\theta}_i) \frac{dg(\tilde{\theta}_i)}{d\tilde{\theta}_i} F^{(n)}(\tilde{\theta}_i) + c(g(\tilde{\theta}_i), \tilde{\theta}_i) F^{(n)}(\tilde{\theta}_i).
\]
(A.12c)

Substituting Eq. (A.12c) into Eq. (A.16) yields:
\[
\frac{\partial \tilde{W}(\theta_i, \tilde{\theta}_i)}{\partial \theta_i} = - (c_s(g(\tilde{\theta}_i), \tilde{\theta}_i) - c(g(\tilde{\theta}_i), \tilde{\theta}_i)) \frac{dg(\tilde{\theta}_i)}{d\tilde{\theta}_i} \frac{F^{(n)}(\tilde{\theta}_i)}{r} - (c(g(\tilde{\theta}_i), \theta_i) - c(g(\tilde{\theta}_i), \tilde{\theta}_i)) \frac{F^{(n)}(\tilde{\theta}_i)}{r} \frac{\partial}{\partial \theta_i} \tilde{B}(\theta_i) = - c_{\theta_0}(g(\tilde{\theta}_i), \theta_i)(\tilde{\theta}_i - \tilde{\theta}_i) \frac{dg(\tilde{\theta}_i)}{d\tilde{\theta}_i} \frac{F^{(n)}(\tilde{\theta}_i)}{r}.
\]
(A.17)

Note that, as by assumption, \( c_{\theta_0} \leq 0 \), \( c_{\theta_0} > 0 \) and, by Eq. (10), \( dg(\tilde{\theta}_i)/d\tilde{\theta}_i > 0 \), one may immediately see that \( \tilde{W}(\theta_i, \tilde{\theta}_i) \) is globally maximized at \( \tilde{\theta}_i = \theta_i \). This in turn implies that \( \tilde{s}(\theta_i) \) is indeed a revealing Bayes–Nash equilibrium.

Eq. (A.1.3) can be used in order to define the agent's expected payoff. That is:
\[
\tilde{W}(\theta_i) = - \int_{\theta_i}^{\theta_i} \Delta_0(x) \frac{F^{(n)}(x)}{r} dx + \tilde{W}(\theta_i'),
\]
(A.18)

where, by Eq. (8), \( \tilde{W}(\theta_i') = 0 \).

Finally, by using Eq. (A.1.5), we can easily prove the auction efficiency. That is,
\[
d\tilde{s}(\theta_i) = \frac{1}{r} \frac{d\tilde{s}(\theta_i)}{d\theta_i} > 0.
\]
(A.19)

This concludes the proof.

A.2. Proposition 2

Separability – Let us first show that the following lemma holds:

**Lemma 1.** With a first-score auction, the equilibrium output \( g(\theta_i) \) is chosen such that
\[
\theta_i = \arg \max \left\{ [v(g_i) - c(g_i, \theta_i)] \right\}, \text{ for all } \theta_i \in [\theta_i^l, \theta_i^u].
\]

(A.2.1)

This lemma can be easily shown by adapting to our case the proof provided by Che (1993, Lemma 1). Suppose that any equilibrium bid, \( b_i, g_i \), with \( g_i \neq g(\theta_i) \), is dominated by an alternative bid, \( b_i', g_i' \) where \( b_i' = b_i + \nu(g_i') - \nu(g_i) \), and \( g_i' = g(\theta_i) \). It follows that:
\[
b_i - c(g(\theta_i), 0) = b_i' - c(g(\theta_i), 0) + \left( \nu(g_i') - \nu(g_i) - c(g(\theta_i), 0) \right)
\]
(A.2.2)

This in turn implies \( p^* (b_i, g_i) < p^* (b_i', g_i') \) and \( E_0 e^{-r \Delta_1} > E_0 e^{-r \Delta_1} \).

Given Eq. (A.2.2), it can be easily shown that:
\[
S_i = \frac{(v(g_i) - b_i)}{r} \left[ 1 - E_0 e^{-r \Delta_1} \right] = \frac{(v(g_i) - b_i')}{r} \left[ 1 - E_0 e^{-r \Delta_1} \right] = S_i',
\]
(A.2.3)

or, equivalently, that \( \Pr(\text{of win} | S_i') > \Pr(\text{of win} | S_i) \). Note also that \( \Pi(b_i, g_i) = \tilde{\Pi}(\theta_i) \) (see Eq. (16)) is increasing in \( h(b_i, g_i) = b_i - c(g(\theta_i), 0) \) as:
\[
\frac{\partial}{\partial h} \left[ \Pi(S_i) - \Pi(0, 0; \theta_i) \right] = \frac{1 - \left( \frac{b_i}{c} \right)^\beta \left( \frac{\gamma}{\beta} \right)^\beta}{r} > 0.
\]

This in turn implies that:
\[
W(h_i, g_i; \theta_i) = \left\{ \left( b_i - c(g(\theta_i), 0) - [r(K(\theta_i') - K(\theta_i))] \right) - E_0 e^{-r \Delta_1} \right\} \frac{1 - E_0 e^{-r \Delta_1}}{r} 
\]
(A.2.4)

Finally, given Lemma 1, the equilibrium quality, \( g(\theta_i) \), can be determined using a standard set of optimality conditions. Note that the problem is equivalent to (10). Hence, the same set of first- and second-order conditions hold also in this case (see condition (10.1)).
Bayes-Nash equilibrium – Let us now consider agent i’s bidding behavior. Assume that all other bidders use a strictly monotone increasing bid function \( s(\theta_i) \), i.e., \( s(\theta_i) : [\theta^l, \theta^u] \rightarrow [s(\theta^l), s(\theta^u)] \mid \forall j \neq i \). Since, by assumption, \( s(\theta_i) \) is monotone in \([\theta^l, \theta^u] \), the probability of winning by bidding \( s(\theta_i) \) is \( \Pr(s(\theta_i) > s(\theta_j) \mid \forall j \neq i) = \Pr(\theta_j < s^{-1}(s(\theta_i)) \mid \forall j \neq i) = F(s^{-1}(s(\theta_i)))^{-1} = F(\theta_i)^{-1} = \frac{\theta_i}{F(\theta_i)} \).

It follows that agent i chooses his report \( \tilde{\theta}_i \) by solving the following problem:

\[
W(\theta_i, \tilde{\theta}_i) = \max_{\theta} \left\{ (b(\tilde{\theta}_i) - c(g(\tilde{\theta}_i), \theta_i)) - r(K(\theta^l) - K(\theta_i)) \right\}
\]

\[
= \frac{1}{r} \left[ -E_0 e^{-\theta^l(\tilde{\theta}_i)} + (p^*(\tilde{\theta}_i, \theta_i) - rK(\theta^l)) E_0 e^{-\theta^l(\tilde{\theta}_i)} \right] \Pr(s(\tilde{\theta}_i))
\]

\[
= \frac{1}{r} \left[ -E_0 e^{-\theta^l(\tilde{\theta}_i)} + (p^*(\tilde{\theta}_i, \theta_i) - rK(\theta^l)) E_0 e^{-\theta^l(\tilde{\theta}_i)} \right] \Pr(s(\tilde{\theta}_i))
\]

\[
= \frac{1}{r} \left[ -E_0 e^{-\theta^l(\tilde{\theta}_i)} + (p^*(\tilde{\theta}_i, \theta_i) - rK(\theta^l)) E_0 e^{-\theta^l(\tilde{\theta}_i)} \right] \Pr(s(\tilde{\theta}_i))
\]

\[
\leq \max_{j \neq i} s(\theta_j)
\]

\[
= \frac{1}{r} \left[ -E_0 e^{-\theta^l(\tilde{\theta}_i)} + (p^*(\tilde{\theta}_i, \theta_i) - rK(\theta^l)) E_0 e^{-\theta^l(\tilde{\theta}_i)} \right] \Pr(s(\tilde{\theta}_i))
\]

\[
\leq \frac{1}{r} \left[ -E_0 e^{-\theta^l(\tilde{\theta}_i)} + (p^*(\tilde{\theta}_i, \theta_i) - rK(\theta^l)) E_0 e^{-\theta^l(\tilde{\theta}_i)} \right] \Pr(s(\tilde{\theta}_i))
\]

where, by Lemma 1, \( g_i = g(\tilde{\theta}_i) \) is still determined by solving Eq. (10).

In order to derive the equilibrium strategies we could solve the ordinary differential equation that follows from the maximization problem in Eq. (A.2.5). However, differently from the problem in Eq. (A.1.1), this ordinary differential equation, due to the presence of the optimal trigger \( \tilde{\theta}(\theta_i, \theta_i) \) and the stopping time \( T_i(\tilde{\theta}(\theta_i, \theta_i)) \), does not have a closed-form solution. We then determine the integral equation that describe the utility \( W(\tilde{\theta}_i) \) by using the generalized Envelope Theorem provided by Milgrom and Segal (2002, Theorem 2).

Suppose that each agent acts \( y \rightarrow Y \) to maximize \( \Psi(y, \omega) \), where \( \omega \in [0, 1] \) and \( Y \) is arbitrary. Denote the set of maximizers by \( F(\omega) := \arg \max_{y} \Psi(y, \omega) \) and let \( \Psi(\omega) = \sup_{\psi \in Y} \Psi(y, \omega) \).

Lemma 2 (Milgrom and Segal, 2002, Theorem 2). Suppose \( a(y, \omega) \) is differentiable and absolutely continuous in \( \omega \) \((\forall \psi) \); \( b(y, \omega) \) is uniformly bounded \((\forall \psi) \) and \( c(\omega) \) is nonempty. Then, for any \( y^{*} = y^{*}(\omega) \in F(\omega) \),

\[
\Psi(\omega) = \int_{0}^{\omega} \psi^*(\omega^*, x) \frac{\partial \omega}{\partial x} dx + \Psi(0).
\]

Now, let set \( \omega = \theta_i \) and \( y = (\tilde{\theta}_i, T_i) \) (or equivalently \( (\tilde{\theta}_i, p^*_i) \)), and apply Milgrom and Segal’s theorem to the problem in Eq. (A.2.5).

Note that: (i) \( W(\theta_i, \tilde{\theta}_i) \) is always differentiable and continuous in \( \tilde{\theta}_i \); (ii) the derivative of W is bounded because of Assumption 1; (iii) Eq. (15.1) says that the stopping time attains its maximum for given \( \tilde{\theta}_i \) and \( \theta_i \). Since, by the revelation principle, \( \tilde{\theta}_i = \theta_i \), it follows

\[
W(\theta_i) = -\int_{\theta_i}^{\theta} \Delta_0(\theta) \frac{1}{1 - E_0[e^{-rT(\theta)}]} F(\theta) dx + W(\theta^l).
\]

where, by Eq. (8), \( W(\theta^l) = 0 \).

Now, we can use Eq. (A.2.7) in order to define the equilibrium payment. The equilibrium payment, \( b(\theta_i) \), solves the following implicit equation:

\[
[(b(\theta_i) - c(g(\theta_i), \theta_i)) - r(K(\theta^l) - K(\theta_i))] \frac{1}{r} - E_0 e^{-rT(\theta)}
\]

\[
+ (p^*(\tilde{\theta}_i, \theta_i) - rK(\theta^l)) E_0 e^{-rT(\theta)} \frac{1}{r} \frac{F(\theta)}{F(\theta_i)} dx.
\]

which can be rearranged as in Eq. (19). Although it is not possible to express \( b(\theta_i) \) in a closed form, we may easily show some of its properties. In fact, by totally differentiating Eq. (A.2.8) with respect to \( \theta_i \), and rearranging, we obtain:

\[
\frac{db(\theta_i)}{d\theta_i} = c_\theta(\theta_i, \tilde{\theta}_i, \theta_i) \frac{d\theta_i}{d\theta_i} + (n-1) \frac{f(\theta_i)}{F(\theta_i)}
\]

\[
\int_{\theta_i}^{\theta} \Delta_0(\theta) \frac{1}{1 - E_0[e^{-rT(\theta)}]} F(\theta) dx.
\]

We notice that the payment, \( b(\theta_i) \), is increasing in the cost of producing higher quality and decreasing in the information rent to be paid. Finally, by using Eq. (A.2.9), it is easy to show that the bid function \( s(\theta_i) \) is strictly monotone and increasing in \( \theta_i \):

\[
ds(\theta_i) = -(n-1) \frac{f(\theta_i)}{F(\theta_i)} \int_{\theta_i}^{\theta} \Delta_0(\theta) \frac{1}{1 - E_0[e^{-rT(\theta)}]} F(\theta) dx > 0.
\]

It remains to show that Eq. (A.2.7) provides the unique local maximum for the problem in Eq. (A.2.5). As usual, the uniqueness follows from the fact that the function in Eq. (A.2.7) is increasing in \( \theta_i \) and has Eq. (8) as boundary condition. For showing that it is a maximum, it suffices to prove that \( s_i \) is incentive compatible:

\[
W(\theta_i, \tilde{\theta}_i) \geq W(\theta_i, \tilde{\theta}_i).
\]

for all \( (\tilde{\theta}_i, \theta_i) \in [\theta^l, \theta^u] \times [\theta^l, \theta^u] \). We prove this by following the approach in adopted by Milgrom (2004, Chapter 4).

Lemma 3. The equilibrium strategy \( s(\theta_i) \) is incentive compatible if and only if Eq. (A.2.7) holds and if \( s(\theta_i) \) is strictly increasing.

Necessity. Since \( s(\theta_i) \) is strictly increasing then by the Monotonic Selection Theorem (Milgrom, 2004, Theorem 4.1), the function \( W(\theta_i, \tilde{\theta}_i) \) satisfies the strict single-crossing difference condition.

Sufficiency. Suppose that \( W(\theta_i, \tilde{\theta}_i) \) satisfies the strict-single crossing difference condition. If \( s(\theta_i) \) is strictly increasing and Eq. (A.2.7) holds then by the Sufficient Theorem (Milgrom, 2004, Theorem 4.2), \( s(\theta_i) \) is a local maximum for the problem in Eq. (A.2.5).

This concludes the proof.

A.3 Proposition 3

By rearranging Eq. (19) we obtain:

\[
b(\theta_i) = \frac{c(\theta_i, \tilde{\theta}_i, \theta_i) + r(K(\theta^l) - K(\theta_i))}{\int_{\theta_i}^{\theta} \Delta_0(\theta) \frac{1}{1 - E_0[e^{-rT(\theta)}]} F(\theta) dx + \frac{E_0 e^{-rT(\theta)}}{1 - E_0[e^{-rT(\theta)}]} (p^*(\tilde{\theta}_i) - rK(\theta^l))}
\]

\[
= \frac{1}{E_0[e^{-rT(\theta)}]} \frac{1}{1 - E_0[e^{-rT(\theta)}]} (p^*(\tilde{\theta}_i) - rK(\theta^l))
\]
Now, note that:
\[
p^*(\theta_i) - rK(\theta^i) = p^0(\theta_i) - rK(\theta^i) - \frac{\beta}{\beta - 1} M(\theta_i), \tag{A.3.2}
\]
where
\[
p^0(\theta_i) = \frac{\theta_i}{\theta_i - \bar{B}(\theta_i) - c(g(\theta_i), \theta_i) + rK(\theta_i)} = \frac{\theta_i}{\theta_i - \bar{B}(\theta_i) - c(g(\theta_i), \theta_i) + rK(\theta_i)} - \int_0^{\theta_i} \Delta_0(x) f^0(\theta_i) dx.
\]
Substituting Eq. (A.3.2) into \( M(\theta_i) \) yields:
\[
M(\theta_i) = -\frac{\beta}{\beta - 1} \int_0^{\theta_i} \Delta_0(x) E_0[e^{-rT_1(x)}] dx - E_0[e^{-rT_1(\theta_i)}] p^0(\theta_i) > 0. \tag{A.3.4}
\]
This in turn implies that:
\[
b(\theta_i) < \bar{B}(\theta_i), \tag{A.3.5}
p^*(\theta_i) < \bar{p}(\theta_i). \tag{A.3.6}
\]
It follows that:
\[
\frac{\partial}{\partial \theta_i} (\theta_i = \int_0^{\theta_i} \Delta_0(x) E_0[e^{-rT_1(x)}] dx - E_0[e^{-rT_1(\theta_i)}] p^0(\theta_i) \). \tag{A.4.1}
\]
As can be easily seen, substituting for \( ds(\theta_i) \) in Eq. (A.4.2) is positive or, equivalently, if:
\[
W(\theta_i) < F^0(\theta_i) \tag{A.4.3}
\]
Rearranging condition (A.4.3), we obtain:
\[
-\int_0^{\theta_i} \Delta_0(x) \frac{1 - E_0[e^{-rT_1(x)}]}{F^0(x)} F^0(x) dx < 0 \tag{A.4.4}
\]
which, by Assumption 7, is always satisfied. Further, note that, by Jensen’s inequality, \( E_0[e^{-rT_1(x)}] \geq e^{-rE_0[T_1(x)]} \). This implies that:
\[
\frac{n - 1}{1 - e^{-rT_1(x)}} > \frac{n - 1}{1 - e^{-rT_1(\theta_i)}} \tag{A.4.5}
\]
This means that Assumption 6 is implied by the following condition:
\[
-\Delta_0(x) F(x)/f(x) < r(n - 1) \tag{A.4.6}
\]
which, for the case of a trendless geometric Brownian motion, reduces to:
\[
-\Delta_0(x) F(x)/f(x) < r(n - 1) \tag{A.4.7}
\]
Let us now check the properties of the scoring rule \( S(\theta_i) \). We notice that:
\[
\frac{dS(\theta_i)}{d\theta_i} = \frac{dS(\theta_i)}{d\theta_i} (1 - E_0[e^{-rT_1(\theta_i)}]) \frac{dL(\theta_i)}{d\theta_i} = \frac{dS(\theta_i)}{d\theta_i} (1 - E_0[e^{-rT_1(\theta_i)}]) \frac{dL(\theta_i)}{d\theta_i} > 0. \tag{A.4.8}
\]
This implies that \( S(\theta_i) \) is increasing in \( \theta_i \) when the following condition holds:
\[
(n - 1) f(\theta_i) > \beta S(\theta_i) \frac{dp^*(\theta_i)}{d\theta_i} \frac{dp^*(\theta_i)}{d\theta_i} \tag{A.4.9}
\]
This condition is always satisfied for \( dp^*(\theta_i)/d\theta_i > 0 \).

References


